经典例题：

1.已知矩形脉冲信号$f(t)=E\cdot G_{\tau}(t)=E\cdot\left[u\left(t+\frac{\tau}{2}\right)-u\left(t-\frac{\tau}{2}\right)\right]$，$E\cdot G_{\tau}(t)\leftrightarrow E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)$。

问题(1)：求有限长非周期函数$f_k(t)=\sum_{n=-k}^{+k}\left[E\cdot G_{\tau}(t+n\cdot L)\right]=\sum_{n=-k}^{+k}\left(E\cdot\left\{u\left[(t+n\cdot L)+\frac{\tau}{2}\right]-u\left[(t+n\cdot L)-\frac{\tau}{2}\right]\right\}\right)$，其中$L>0$；

问题(2)：求无限长周期函数$f_{\infty}(t)=f_{\infty}(t)=\sum_{n=-\infty}^{+\infty}\left[E\cdot G_{\tau}(t+n\cdot L)\right]=\sum_{n=-\infty}^{+\infty}\left(E\cdot\left\{u\left[(t+n\cdot L)+\frac{\tau}{2}\right]-u\left[(t+n\cdot L)-\frac{\tau}{2}\right]\right\}\right)$，其中$L>0$。

解：由图1即可知，$f_k(t)$、$f_{\infty}(t)$都可以由典型非周期函数$f(t)$进行时移(即平移$n\cdot L$长度)得到，

根据傅里叶变换的线性性质和时移性质(时加频正)，得到：

$$f_k(t)=\sum_{n=-k}^{+k}\left[E\cdot G_{\tau}(t+n\cdot L)\right]=\sum_{n=-k}^{-1}\left[E\cdot G_{\tau}(t+n\cdot L)\right]+E\cdot G_{\tau}(t)+\sum_{n=1}^{+k}\left[E\cdot G_{\tau}(t+n\cdot L)\right]$$

则F $\left[f_k(t)\right]=F_k(\omega)=\sum_{n=-k}^{+k}\left[E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)\cdot e^{j\omega(n\cdot L)}\right]=\sum_{n=-k}^{-1}\left[E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)\cdot e^{j\omega(n\cdot L)}\right]+E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)+\sum_{n=1}^{+k}\left[E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)\cdot e^{j\omega(n\cdot L)}\right]$

$$=\sum_{n=1}^{+k}\left[E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)\cdot e^{-j\omega(n\cdot L)}\right]+E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)+\sum_{n=1}^{+k}\left[E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)\cdot e^{j\omega(n\cdot L)}\right]$$

$$=\sum_{n=1}^{+k}\left\{E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)\cdot\left[e^{j\omega(n\cdot L)}+e^{-j\omega(n\cdot L)}\right]\right\}+E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)\cdot 1$$

$$=\overbrace{\left[E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)\right]}^{\text{决定}F_k(\omega)\text{的包络线}}\times\overbrace{\left\{1+\sum_{n=1}^{+k}\left[2\cos(n\cdot L)\cdot\omega\right]\right\}}^{\text{这是余弦函数族，其周期为}\frac{2\pi}{1\cdot L}}$$

同理，F $\left[f_{\infty}(t)\right]=F_{\infty}(\omega)=\overbrace{\left[E\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)\right]}^{\text{决定}F_k(\omega)\text{的包络线}}\times\overbrace{\left\{1+\sum_{n=1}^{+\infty}\left[2\cos(n\cdot L)\cdot\omega\right]\right\}}^{\text{这是余弦函数族，其周期为}\frac{2\pi}{1\cdot L}}$

2.已知三角脉冲信号$s(t)=E\cdot\left\{\left(\frac{2}{\tau}\cdot t+1\right)\cdot\left[u\left(t+\frac{\tau}{2}\right)-u(t)\right]+\left(-\frac{2}{\tau}\cdot t+1\right)\cdot\left[u(t)-u\left(t-\frac{\tau}{2}\right)\right]\right\}$。

问题(1)：求该三角脉冲信号的傅里叶变换$S(\omega)$；

问题(2)：求有限个该三角脉冲信号$s_k(t)=\sum_{n=-k}^{+k}s(t+n\cdot L)$的傅里叶变换$S_k(\omega)$；

问题(3)：求无限个该三角脉冲信号$s_{\infty}(t)=\sum_{n=-\infty}^{+\infty}s(t+n\cdot L)$的傅里叶变换$S_{\infty}(\omega)$。

解：根据例题1，考虑将三角脉冲信号$S(\omega)$用矩形脉冲信号求解。

$\because\frac{d[s(t)]}{dt}=E\cdot\left[\left(\frac{2}{\tau}\cdot\left[u\left(t+\frac{\tau}{2}\right)-u(t)\right]\right)+\left(-\frac{2}{\tau}\cdot\left[u(t)-u\left(t-\frac{\tau}{2}\right)\right]\right)\right]$，此处求导一定要省去门函数(因为它是分段用的)

$=E\cdot\left[\left(\frac{2}{\tau}\cdot\left[u\left(t+\frac{\tau}{2}\right)-u(t)\right]\right)+\left(-\frac{2}{\tau}\cdot\left[u(t)-u\left(t-\frac{\tau}{2}\right)\right]\right)\right]$，具体原因是：这里不是普通意义上乘积函数求导数

$$=\frac{2E}{\tau}\cdot\left[u\left(t+\frac{\tau}{2}\right)-u(t)\right]-\frac{2E}{\tau}\cdot\left[u(t)-u\left(t-\frac{\tau}{2}\right)\right];$$

$$\frac{d^2[s(t)]}{dt^2}=\frac{2E}{\tau}\cdot\left[\delta\left(t+\frac{\tau}{2}\right)-\delta(t)\right]-\frac{2E}{\tau}\cdot\left[\delta(t)-\delta\left(t-\frac{\tau}{2}\right)\right]=\frac{2E}{\tau}\cdot\left[\delta\left(t+\frac{\tau}{2}\right)-2\cdot\delta(t)+\delta\left(t-\frac{\tau}{2}\right)\right]$$

$\therefore$ 根据$\begin{cases}\text{时域}n\text{次积分}\Leftrightarrow\text{频域}\div(j\omega)^n+\pi\cdot F(0)\cdot\delta(\omega)\rightarrow\text{已知导函数信息，求原函数信息}\\\text{时域}n\text{次微分}\Leftrightarrow\text{频域}\times(j\omega)^n\rightarrow\text{适用于已知原函数信息，求导函数信息}\end{cases}$

则有两种方法求三角波形的傅里叶变换$S(\omega)$:

①使用其一阶导数，即门函数来求，由于 $\dfrac{d[s(t)]}{dt}=\dfrac{2E}{\tau}\cdot\left[u\left(t+\dfrac{\tau}{2}\right)-u(t)\right]-\dfrac{2E}{\tau}\cdot\left[u(t)-u\left(t-\dfrac{\tau}{2}\right)\right]$

即 $\dfrac{d[s(t)]}{dt}=\dfrac{2E}{\tau}\cdot\left[G_{\frac{\tau}{2}}\left(t+\dfrac{\tau}{4}\right)-G_{\frac{\tau}{2}}\left(t-\dfrac{\tau}{4}\right)\right]\Leftrightarrow\dfrac{2E}{\tau}\cdot\dfrac{\tau}{2}\left[Sa\left(\dfrac{\tau}{4}\cdot\omega\right)\cdot e^{j\omega\cdot\frac{\tau}{4}}-Sa\left(\dfrac{\tau}{4}\cdot\omega\right)\cdot e^{-j\omega\cdot\frac{\tau}{4}}\right]$

$\Leftrightarrow E\cdot Sa\left(\dfrac{\tau}{4}\cdot\omega\right)\cdot\left(e^{j\omega\cdot\frac{\tau}{4}}-\cdot e^{-j\omega\cdot\frac{\tau}{4}}\right)=E\cdot Sa\left(\dfrac{\tau}{4}\cdot\omega\right)\cdot 2j\sin\left(\dfrac{\tau}{4}\cdot\omega\right)=j2E\cdot Sa\left(\dfrac{\tau}{4}\cdot\omega\right)\cdot\sin\left(\dfrac{\tau}{4}\cdot\omega\right)$

$\because\dfrac{d[s(t)]}{dt}\Leftrightarrow j2E\cdot Sa\left(\dfrac{\tau}{4}\cdot\omega\right)\cdot\sin\left(\dfrac{\tau}{4}\cdot\omega\right),\int_{-\infty}^{t}f(\lambda)d\lambda\Leftrightarrow\dfrac{F(\omega)}{j\omega}+\pi\cdot F(0)\cdot\delta(\omega)$

$\therefore S(\omega)=\int_{-\infty}^{t}\dfrac{d[s(\lambda)]}{d\lambda}d\lambda=\dfrac{j2E\cdot Sa\left(\frac{\tau}{4}\cdot\omega\right)\cdot\sin\left(\frac{\tau}{4}\cdot\omega\right)}{(j\omega)}+\pi\cdot(0)\cdot\delta(\omega)=\dfrac{E\tau}{2}\cdot Sa^2\left(\dfrac{\tau}{4}\cdot\omega\right)$

②$\dfrac{d^2[s(t)]}{dt^2}=\dfrac{2E}{\tau}\cdot\left[\delta\left(t+\dfrac{\tau}{2}\right)-2\delta(t)+\delta\left(t-\dfrac{\tau}{2}\right)\right]\Leftrightarrow\dfrac{2E}{\tau}\cdot\left(1\cdot e^{j\omega\cdot\frac{\tau}{2}}-2+1\cdot e^{-j\omega\cdot\frac{\tau}{2}}\right)$

$\Leftrightarrow\dfrac{2E}{\tau}\cdot\left[2\cos\left(\dfrac{\tau}{2}\cdot\omega\right)-2\right]=\dfrac{4E}{\tau}\cdot\left[\cos\left(\dfrac{\tau}{2}\cdot\omega\right)-1\right]=-\dfrac{8E}{\tau}\cdot\sin^2\left(\dfrac{\tau}{4}\cdot\omega\right)$

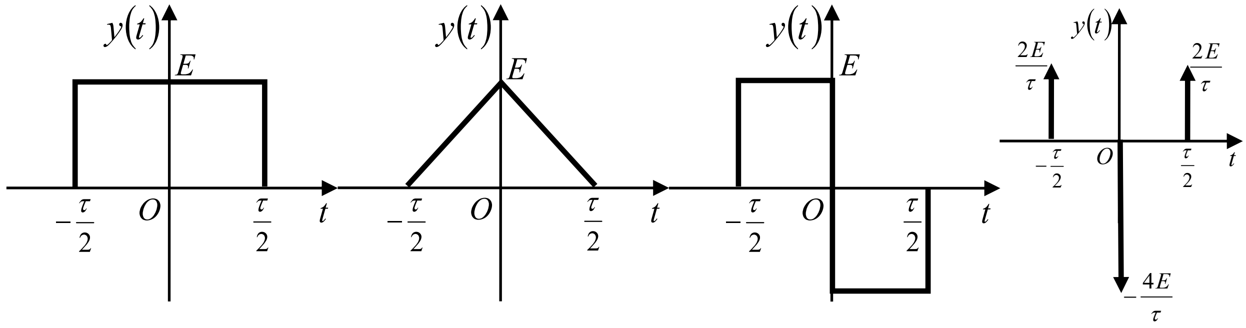
$\dfrac{d[s(t)]}{dt}\Leftrightarrow\dfrac{-\frac{8E}{\tau}\cdot\sin^2\left(\frac{\tau}{4}\cdot\omega\right)}{j\omega}+\pi\cdot(0)\cdot\delta(\omega)=j2E\cdot Sa\left(\dfrac{\tau}{4}\cdot\omega\right)\cdot\sin\left(\dfrac{\tau}{4}\cdot\omega\right);$

$s(t)\Leftrightarrow\dfrac{j2E\cdot Sa\left(\frac{\tau}{4}\cdot\omega\right)\cdot\sin\left(\frac{\tau}{4}\cdot\omega\right)}{j\omega}+\pi\cdot(0)\cdot\delta(\omega)=\dfrac{E\tau}{2}\cdot Sa^2\left(\dfrac{\tau}{4}\cdot\omega\right)$

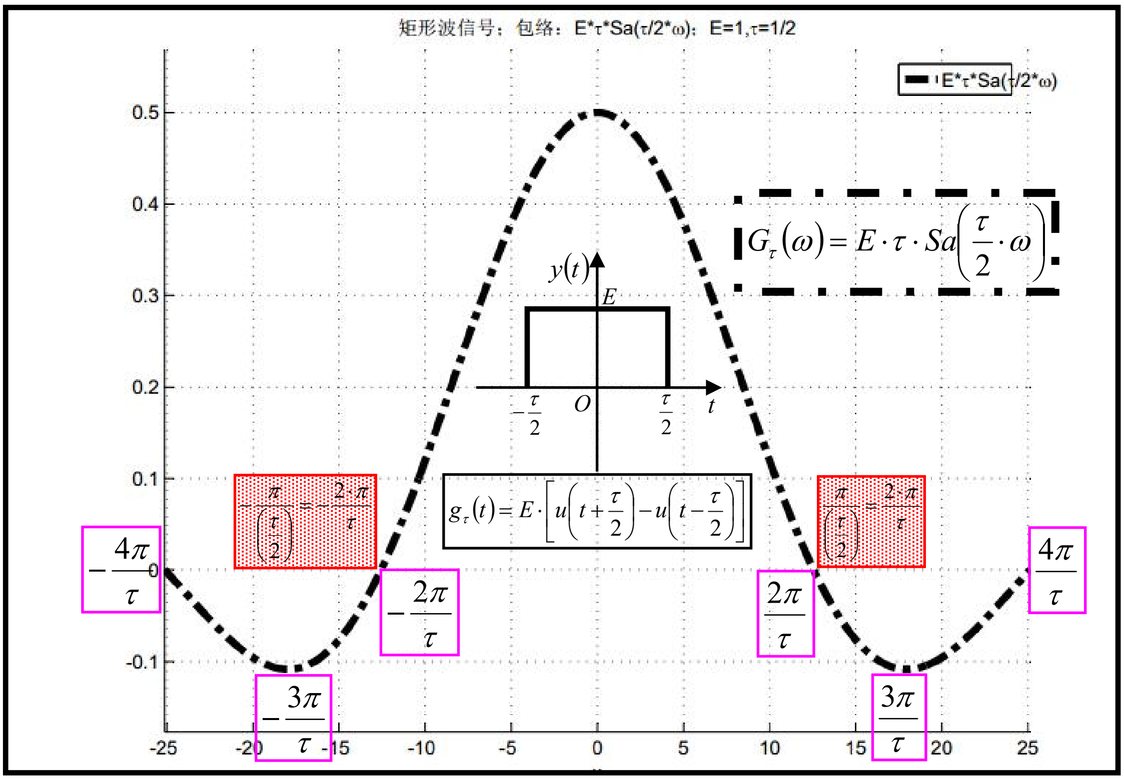
即(1)典型三角波$s(t)=E\left[\left(1-\dfrac{2}{\tau}\cdot|t|\right)\cdot\left[u\left(t+\dfrac{\tau}{2}\right)-u\left(t-\dfrac{\tau}{2}\right)\right]\right]$，$\mathcal{F}\ [s(t)]=\dfrac{E\tau}{2}\cdot Sa^2\left(\dfrac{\tau}{4}\cdot\omega\right)$

(2)有限长三角波$s_k(t)=\sum_{n=-k}^{+k}[s(t+n\cdot L)]\Leftrightarrow\mathcal{F}\ [s_k(t)]=\overbrace{\left[\dfrac{E\tau}{2}\cdot Sa^2\left(\dfrac{\tau}{4}\cdot\omega\right)\right]}^{\text{决定}S_k(\omega)\text{的包络线}}\cdot\overbrace{\left\{1+\sum_{n=-k}^{+k}2\cos[(n\cdot L)\cdot\omega]\right\}}^{\text{余弦函数族}}$

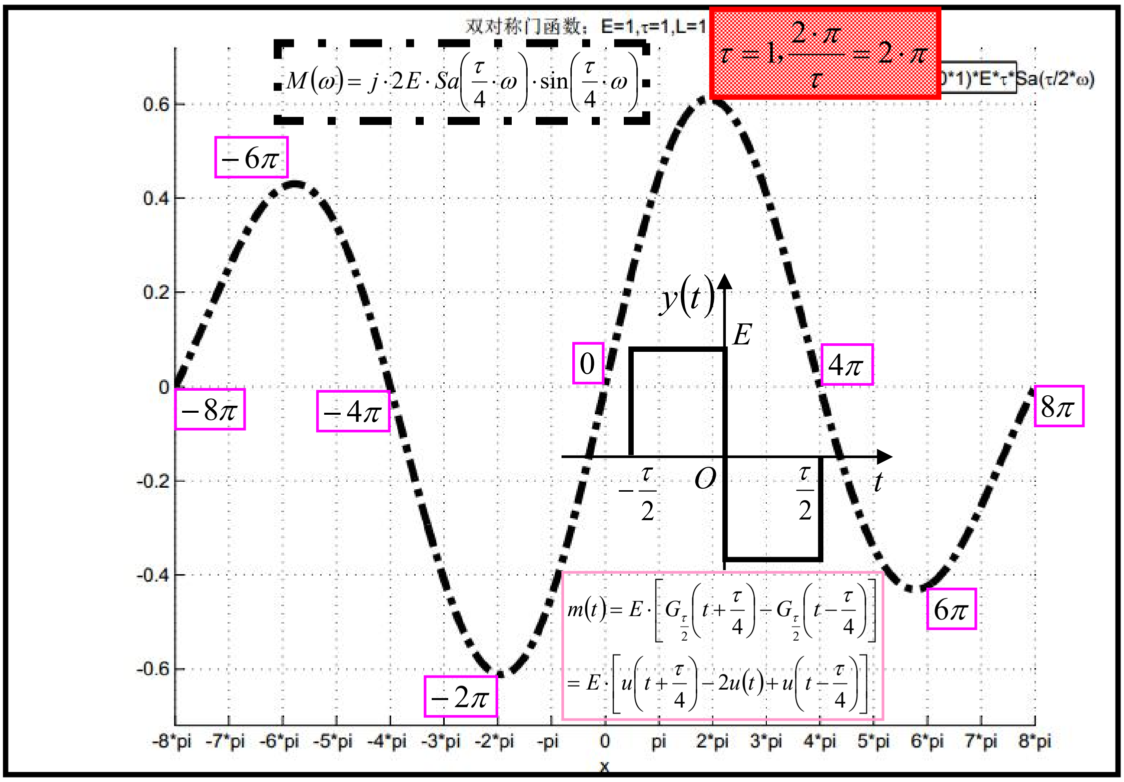
(3)无限长三角波$s_\infty(t)=\sum_{n=-\infty}^{+\infty}[s(t+n\cdot L)]\Leftrightarrow\mathcal{F}\ [s_\infty(t)]=\overbrace{\left[\dfrac{E\tau}{2}\cdot Sa^2\left(\dfrac{\tau}{4}\cdot\omega\right)\right]}^{\text{决定}S_\infty(\omega)\text{的包络线}}\cdot\overbrace{\left\{1+\sum_{n=-\infty}^{+\infty}2\cos[(n\cdot L)\cdot\omega]\right\}}^{\text{余弦函数族}}$

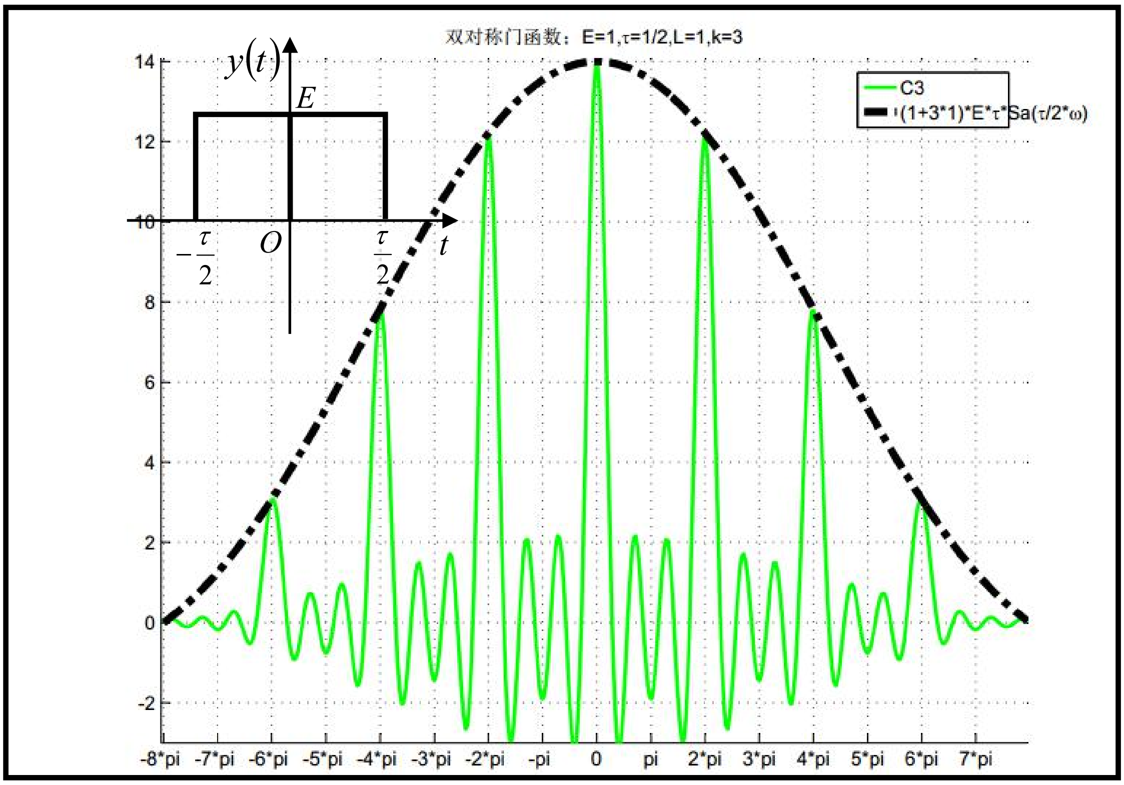


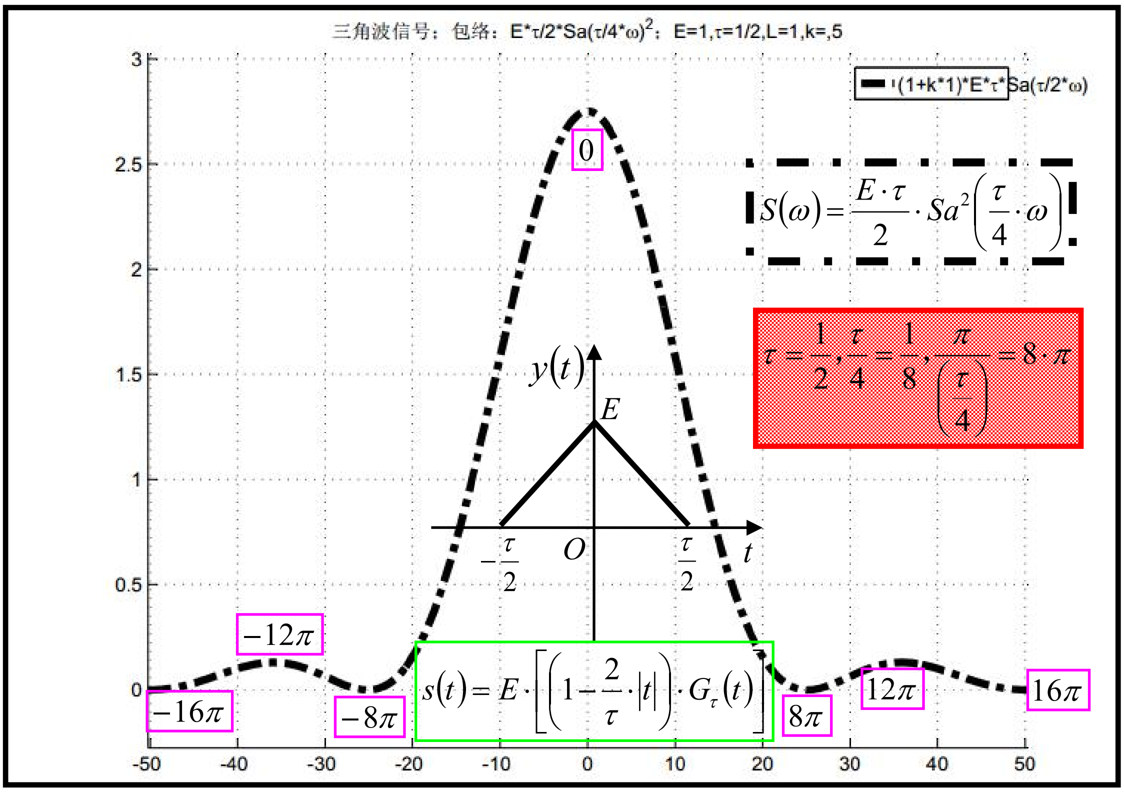
y(t)
E
−τ/2
O
τ/2
t
2E/τ
−4E/τ

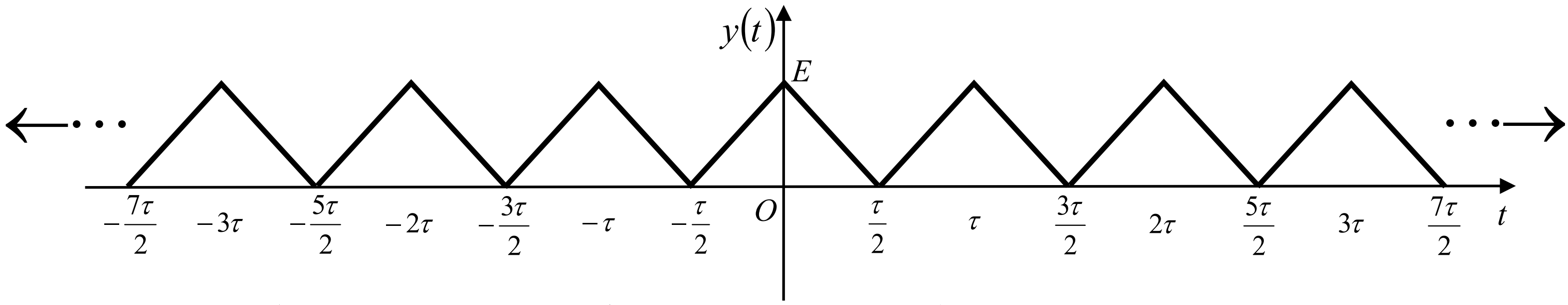


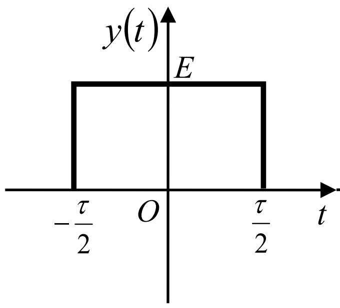
矩形波信号：包络：E*τ*Sa(τ/2*ω)：E=1,τ=1/2
E*τ*Sa(τ/2*ω)
$G_\tau(\omega)=E\cdot\tau\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)$
$g_\tau(t)=E\cdot\left[u\left(t+\frac{\tau}{2}\right)-u\left(t-\frac{\tau}{2}\right)\right]$
$-\frac{\pi}{\left(\frac{\tau}{2}\right)}=-\frac{2\cdot\pi}{\tau}$
$\frac{\pi}{\left(\frac{\tau}{2}\right)}=\frac{2\cdot\pi}{\tau}$
$-\frac{4\pi}{\tau}$
$-\frac{3\pi}{\tau}$
$-\frac{2\pi}{\tau}$
$\frac{2\pi}{\tau}$
$\frac{3\pi}{\tau}$
$\frac{4\pi}{\tau}$

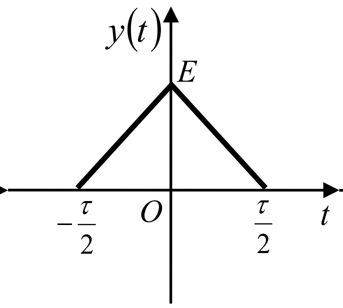

双对称门函数：E=1,τ=1,L=1
$M(\omega)=j\cdot 2E\cdot Sa\left(\frac{\tau}{4}\cdot\omega\right)\cdot\sin\left(\frac{\tau}{4}\cdot\omega\right)$
$\tau=1,\frac{2\cdot\pi}{\tau}=2\cdot\pi$
-6π
-8π
-4π
-2π
0
4π
8π
6π
$y(t)$
E
$-\frac{\tau}{2}$
O
$\frac{\tau}{2}$
t
$m(t)=E\cdot\left[G_{\frac{\tau}{2}}\left(t+\frac{\tau}{4}\right)-G_{\frac{\tau}{2}}\left(t-\frac{\tau}{4}\right)\right]$
$=E\cdot\left[u\left(t+\frac{\tau}{4}\right)-2u(t)+u\left(t-\frac{\tau}{4}\right)\right]$

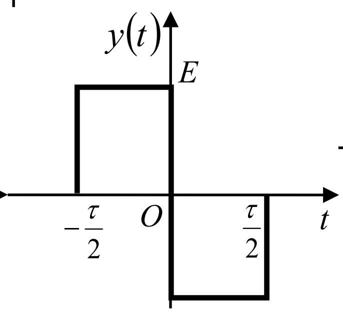

双对称门函数：E=1,τ=1/2,L=1,k=3
C3
(1+3*1)*E*τ*Sa(τ/2*ω)
$y(t)$
E
$-\frac{\tau}{2}$
O
$\frac{\tau}{2}$
t

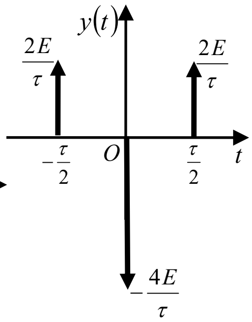

三角波信号：包络：E*τ/2*Sa(τ/4*ω)²；E=1,τ=1/2,L=1,k=,5
'(1+k*1)*E*τ*Sa(τ/2*ω)
$S(\omega)=\frac{E\cdot\tau}{2}\cdot Sa^2\left(\frac{\tau}{4}\cdot\omega\right)$
$\tau=\frac{1}{2},\frac{\tau}{4}=\frac{1}{8},\frac{\pi}{\left(\frac{\tau}{4}\right)}=8\cdot\pi$
$s(t)=E\cdot\left[\left(1-\frac{2}{\tau}\cdot|t|\right)\cdot G_\tau(t)\right]$
0
-12π
-16π
-8π
8π
12π
16π
$y(t)$
E
$-\frac{\tau}{2}$
O
$\frac{\tau}{2}$
t

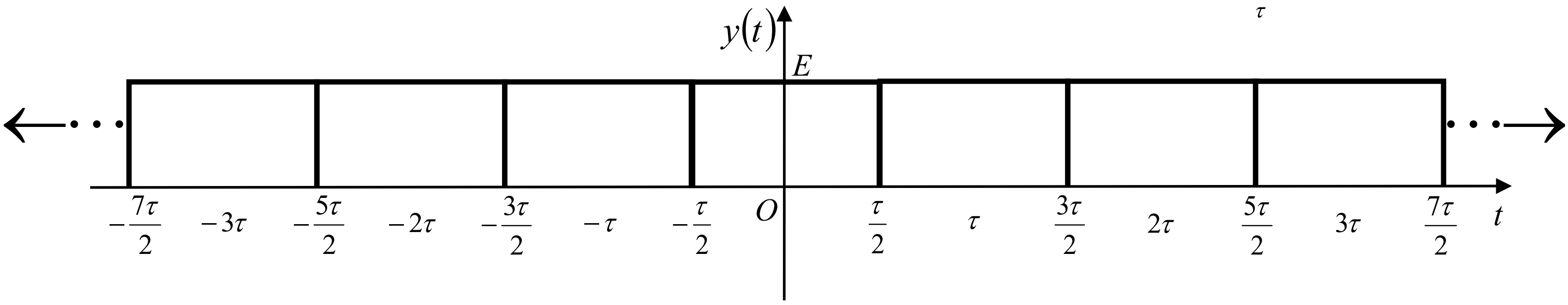

$y(t)$
E
$-\frac{7\tau}{2}$ -3τ $-\frac{5\tau}{2}$ -2τ $-\frac{3\tau}{2}$ $-\tau$ $-\frac{\tau}{2}$ O $\frac{\tau}{2}$ τ $\frac{3\tau}{2}$ 2τ $\frac{5\tau}{2}$ 3τ $\frac{7\tau}{2}$ t

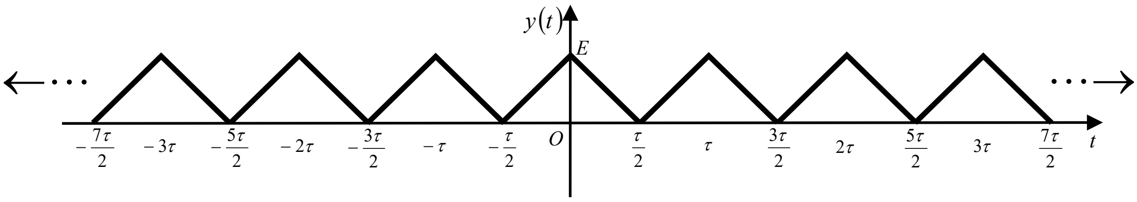

$y(t)$
E
$-\frac{\tau}{2}$ O $\frac{\tau}{2}$ t


$y(t)$
E
$-\frac{\tau}{2}$ O $\frac{\tau}{2}$ t

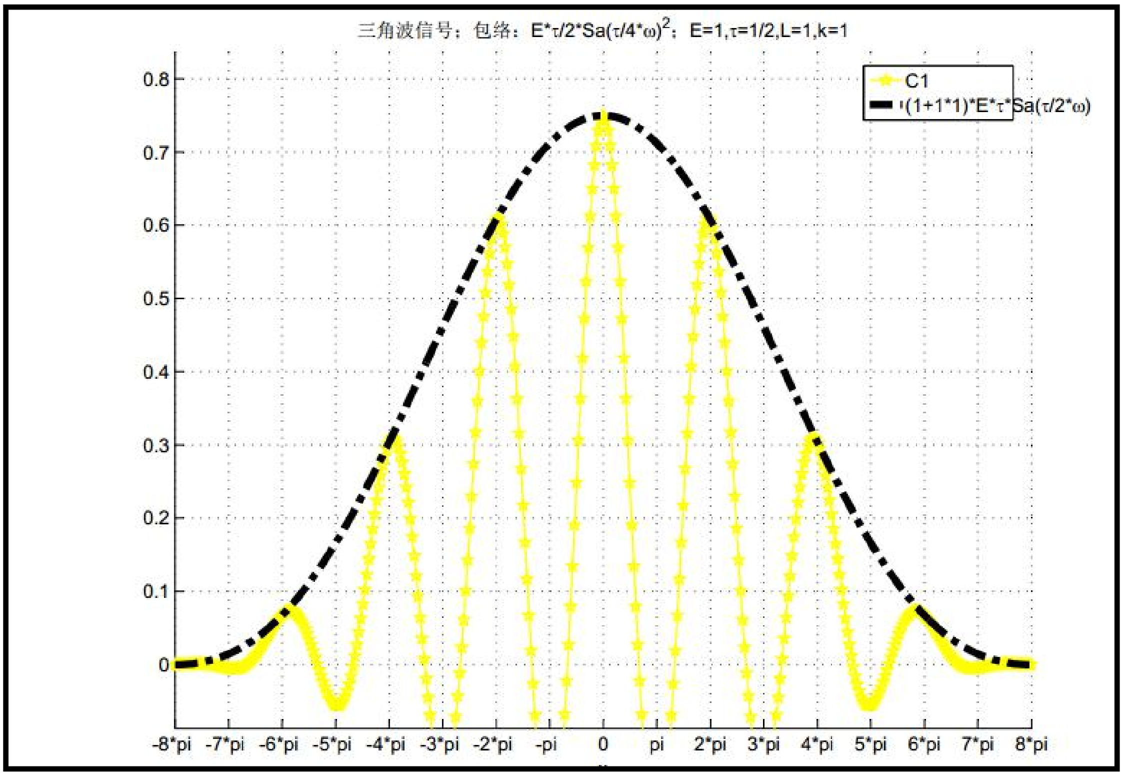

$y(t)$
E
$-\frac{\tau}{2}$ O $\frac{\tau}{2}$ t


$y(t)$
$\frac{2E}{\tau}$
$\frac{2E}{\tau}$
$-\frac{\tau}{2}$ O $\frac{\tau}{2}$ t
$-\frac{4E}{\tau}$

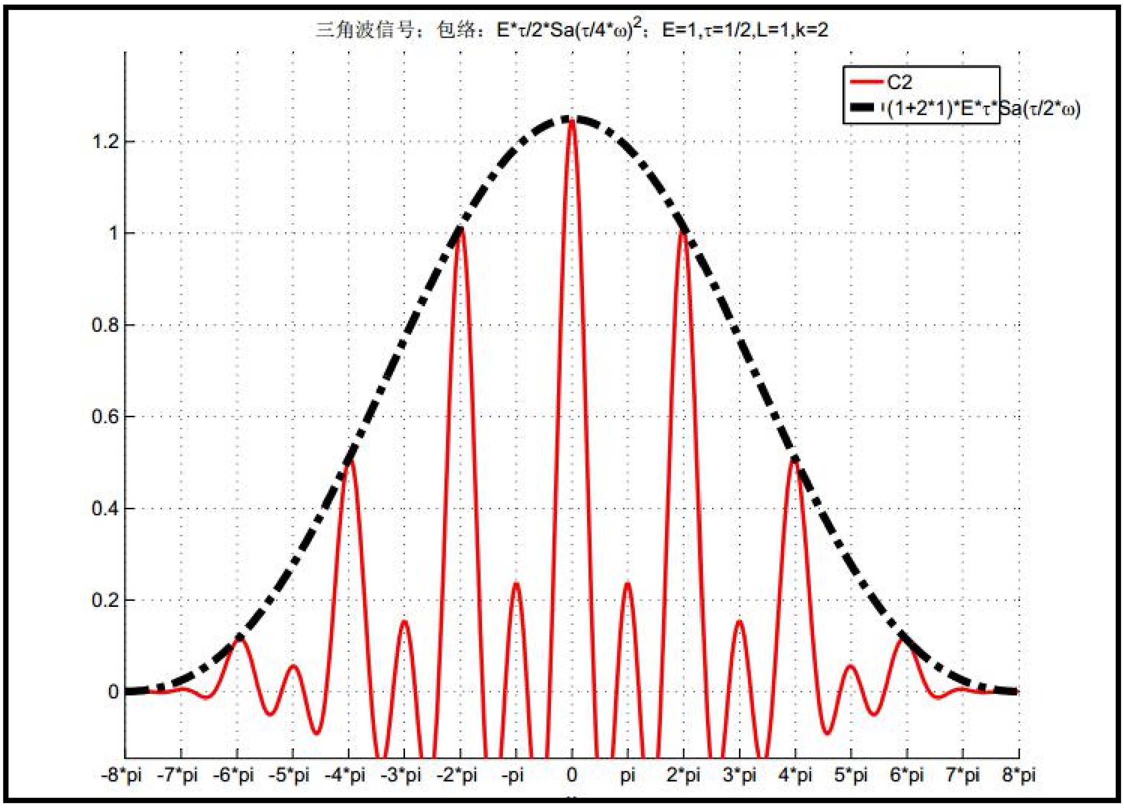

$y(t)$
E
$-\frac{7\tau}{2}$ -3τ $-\frac{5\tau}{2}$ -2τ $-\frac{3\tau}{2}$ $-\tau$ $-\frac{\tau}{2}$ O $\frac{\tau}{2}$ τ $\frac{3\tau}{2}$ 2τ $\frac{5\tau}{2}$ 3τ $\frac{7\tau}{2}$ t



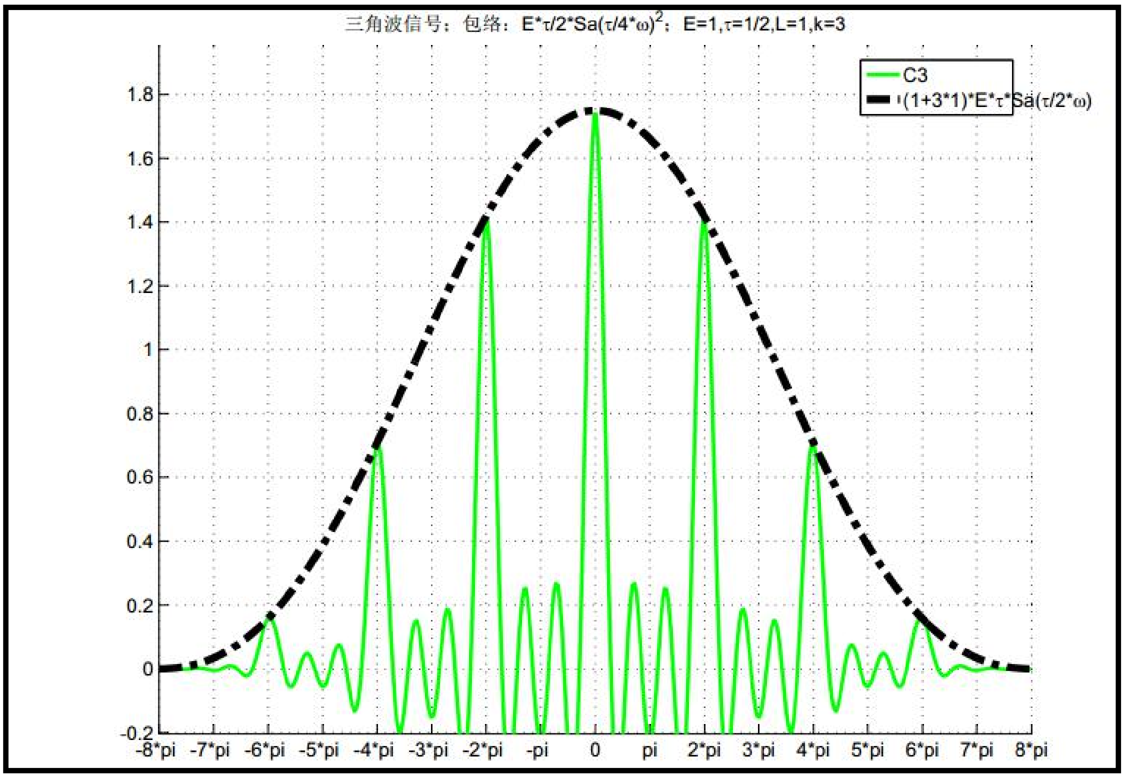
y(t)
E
O
t
-7τ/2 -3τ -5τ/2 -2τ -3τ/2 -τ -τ/2
τ/2 τ 3τ/2 2τ 5τ/2 3τ 7τ/2

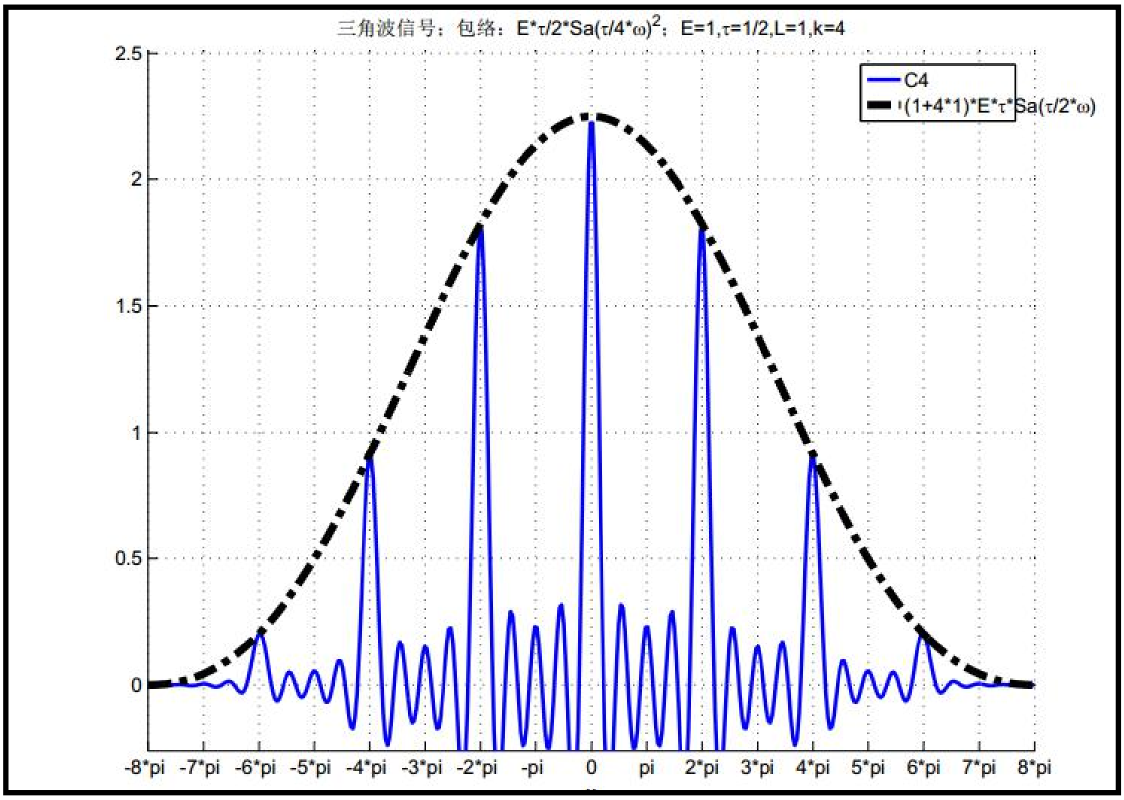


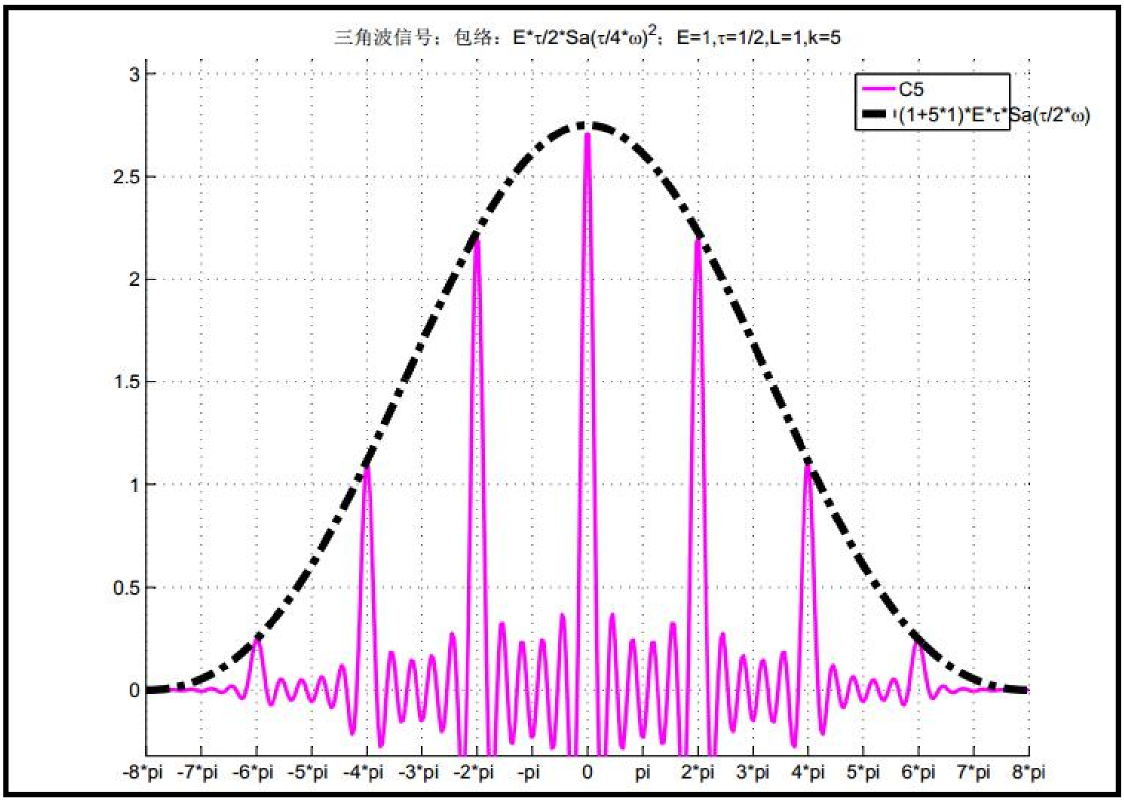
三角波信号：包络：E*τ/2*Sa(τ/4*ω)^2：E=1,τ=1/2,L=1,k=1
C1
(1+1*1)*E*τ*Sa(τ/2*ω)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi



三角波信号：包络：E*τ/2*Sa(τ/4*ω)^2：E=1,τ=1/2,L=1,k=2
C2
(1+2*1)*E*τ*Sa(τ/2*ω)
1.2
1
0.8
0.6
0.4
0.2
0
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi


三角波信号：包络：E*τ/2*Sa(τ/4*ω)^2；E=1,τ=1/2,L=1,k=3
C3
(1+3*1)*E*τ*Sa(τ/2*ω)
-8*pi
-7*pi
-6*pi
-5*pi
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi
4*pi
5*pi
6*pi
7*pi
8*pi


三角波信号：包络：E*τ/2*Sa(τ/4*ω)^2；E=1,τ=1/2,L=1,k=4
C4
(1+4*1)*E*τ*Sa(τ/2*ω)
-8*pi
-7*pi
-6*pi
-5*pi
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi
4*pi
5*pi
6*pi
7*pi
8*pi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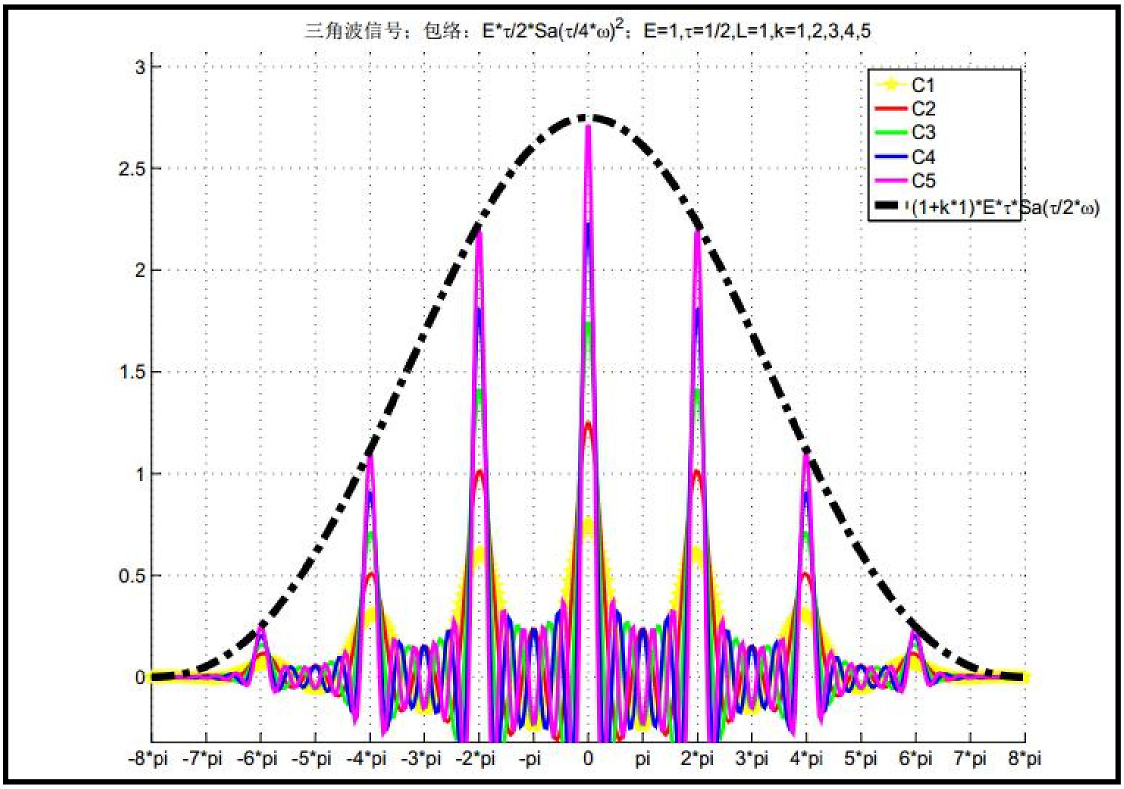
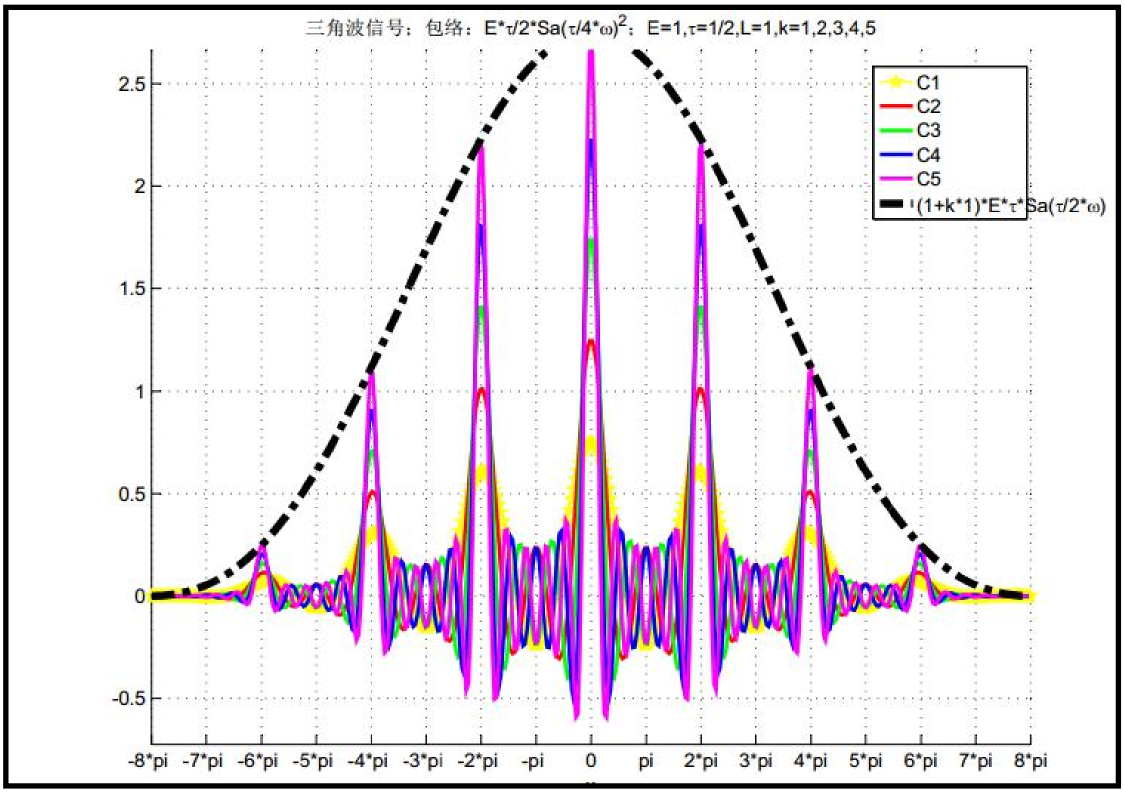
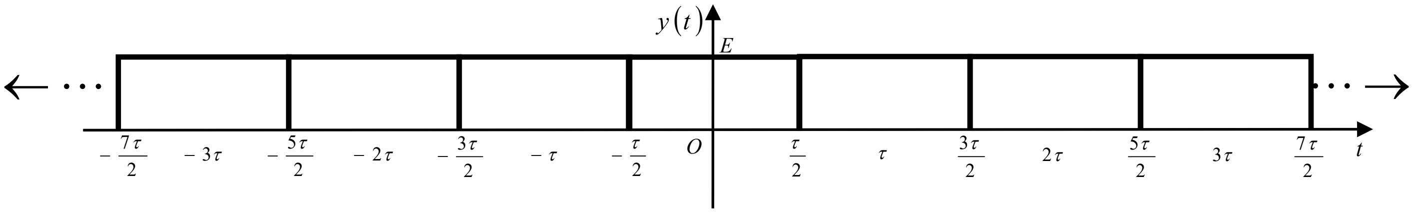
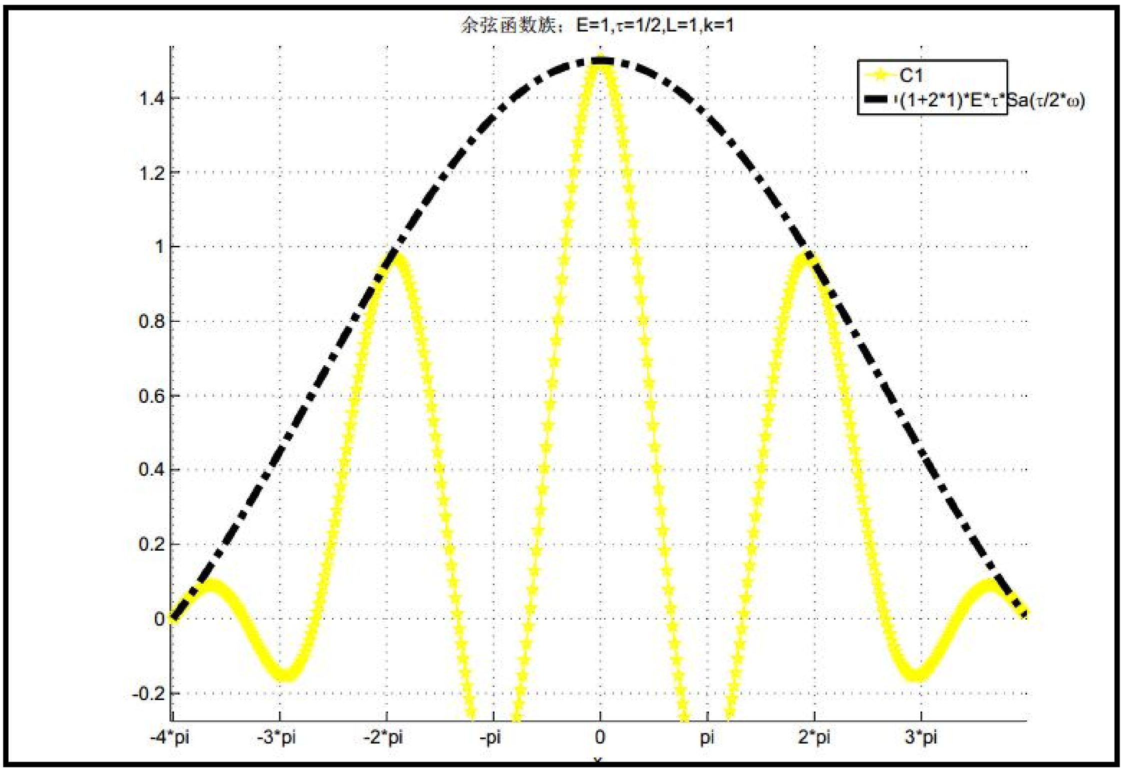
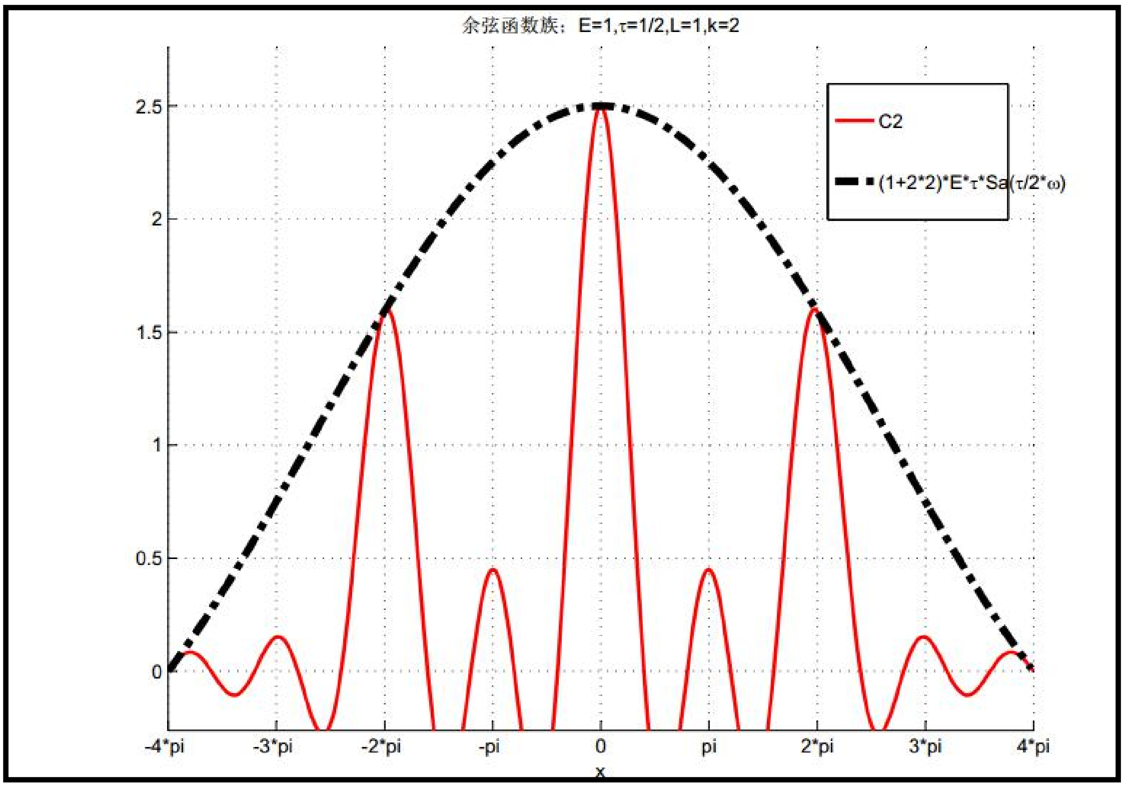
C5
(1+5*1)*E*τ*Sa(τ/2*ω)
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi
0 0.5 1 1.5 2 2.5 3

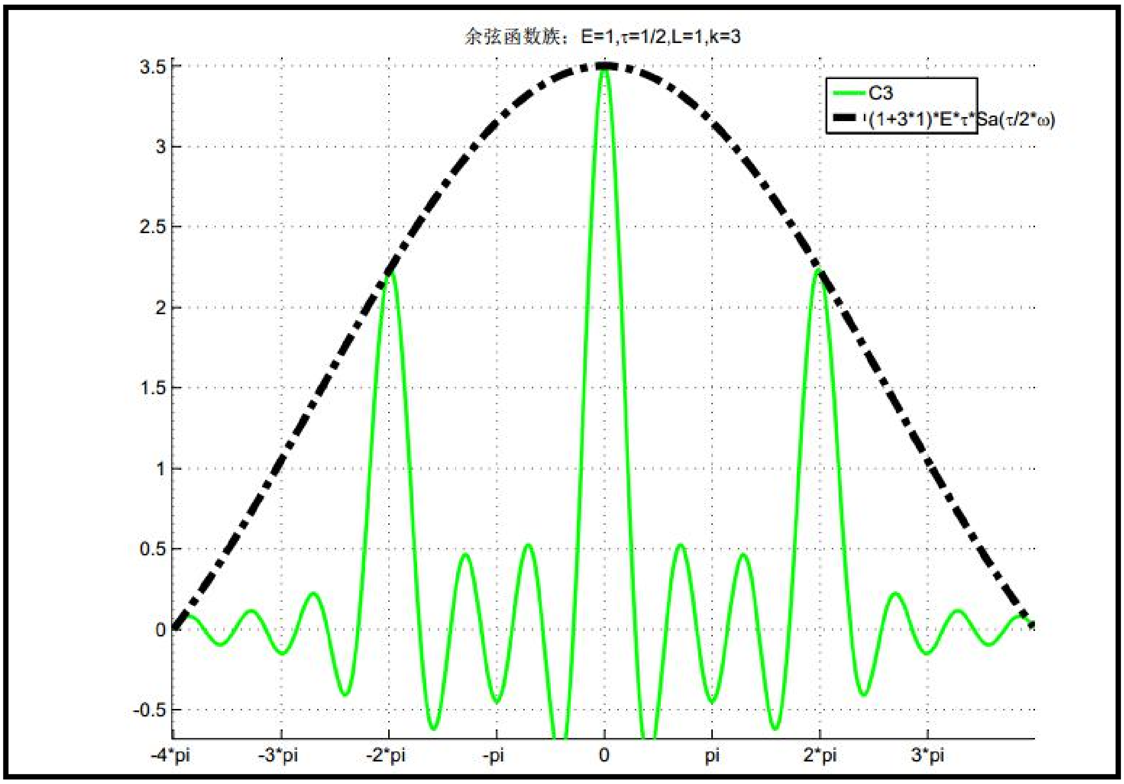

三角波信号：包络：E*τ/2*Sa(τ/4*ω)^2；E=1,τ=1/2,L=1,k=1,2,3,4,5
C1
C2
C3
C4
C5
(1+k*1)*E*τ*Sa(τ/2*ω)
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi
0 0.5 1 1.5 2 2.5 3


三角波信号：包络：E*τ/2*Sa(τ/4*ω)²；E=1,τ=1/2,L=1,k=1,2,3,4,5
C1
C2
C3
C4
C5
(1+k*1)*E*τ*Sa(τ/2*ω)
2.5
2
1.5
1
0.5
0
-0.5
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi

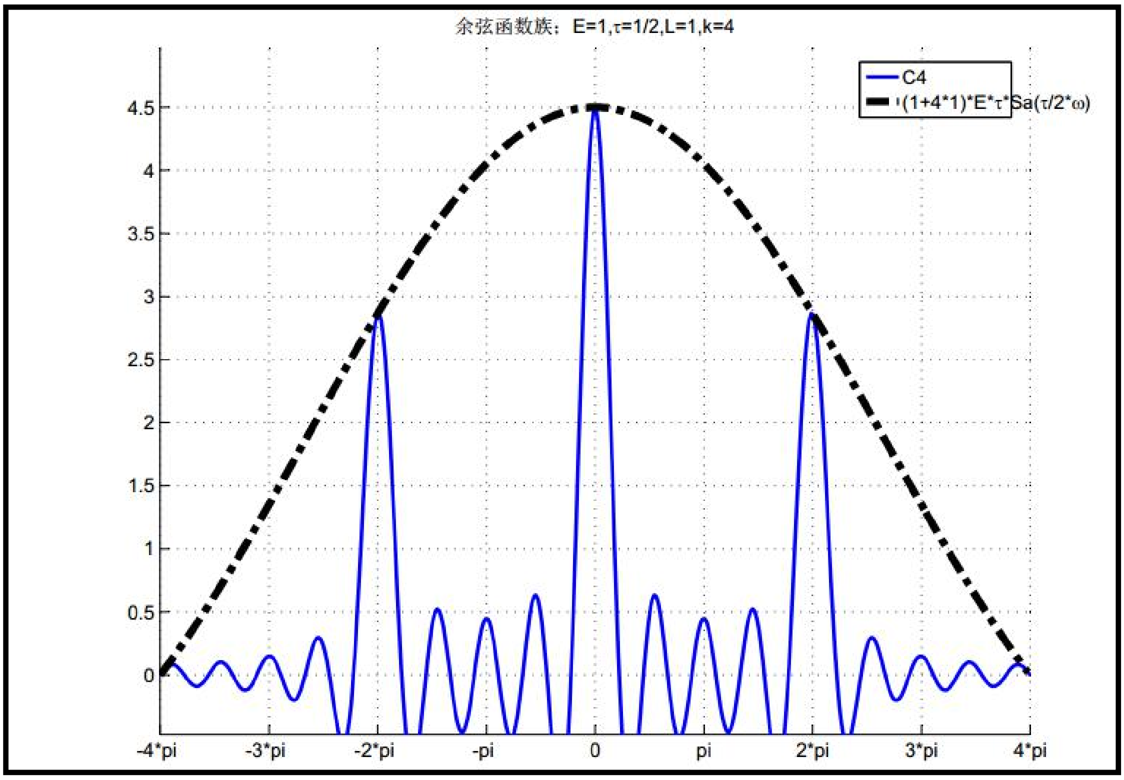

y(t)
E
O
t
−7τ/2
−3τ
−5τ/2
−2τ
−3τ/2
−τ
−τ/2
τ/2
τ
3τ/2
2τ
5τ/2
3τ
7τ/2


余弦函数族：E=1,τ=1/2,L=1,k=1
C1
(1+2*1)*E*τ*Sa(τ/2*ω)
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi
x

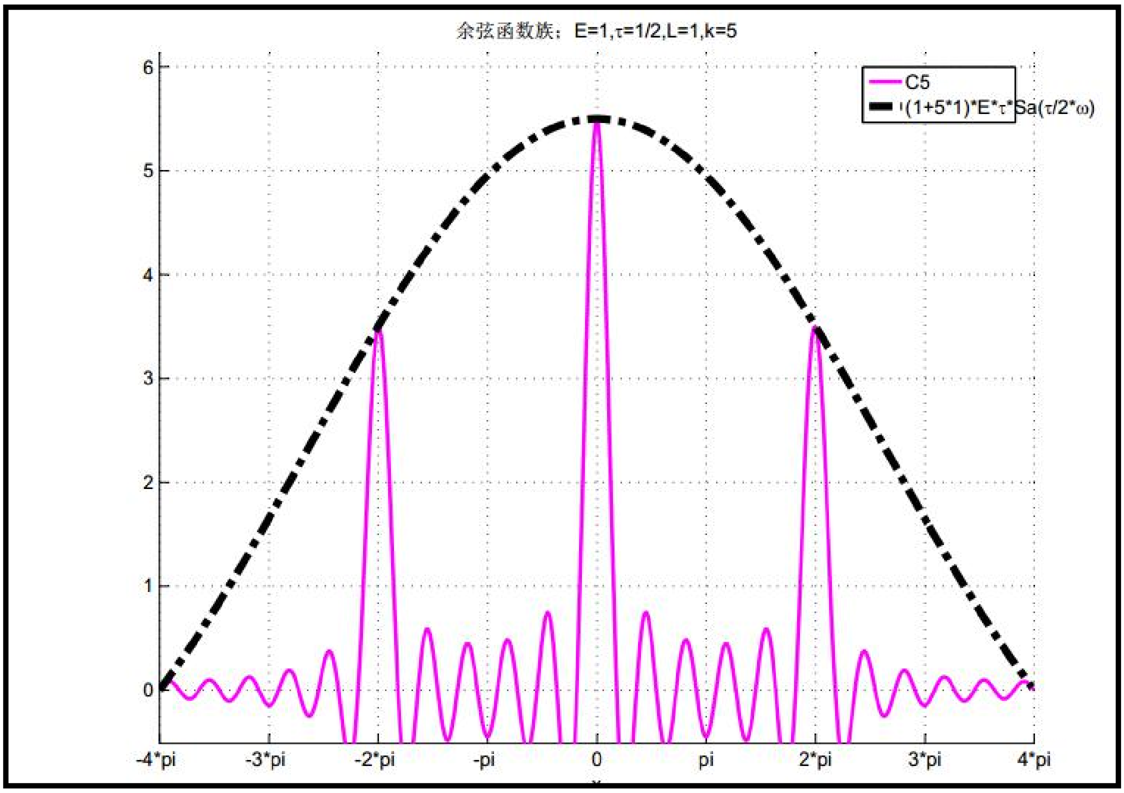

余弦函数族：E=1,τ=1/2,L=1,k=2
C2
(1+2*2)*E*τ*Sa(τ/2*ω)
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi
4*pi
x



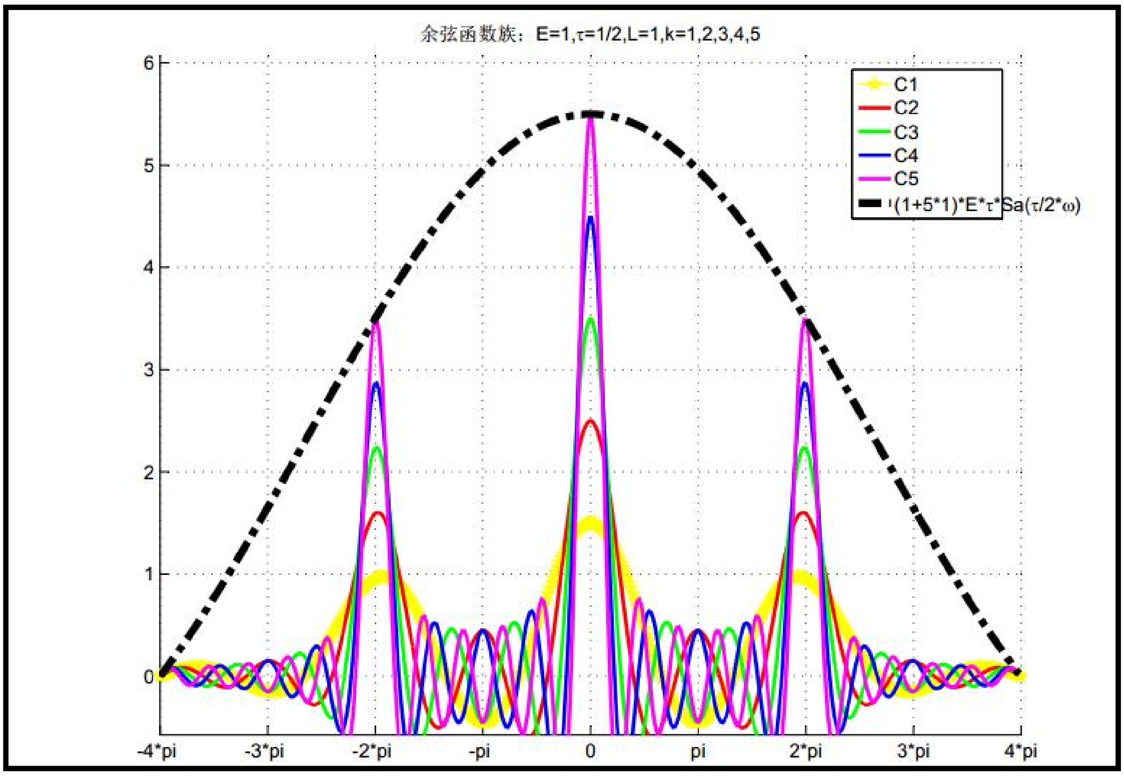
余弦函数族：E=1,τ=1/2,L=1,k=3
C3
(1+3*1)*E*τ*Sa(τ/2*ω)
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi



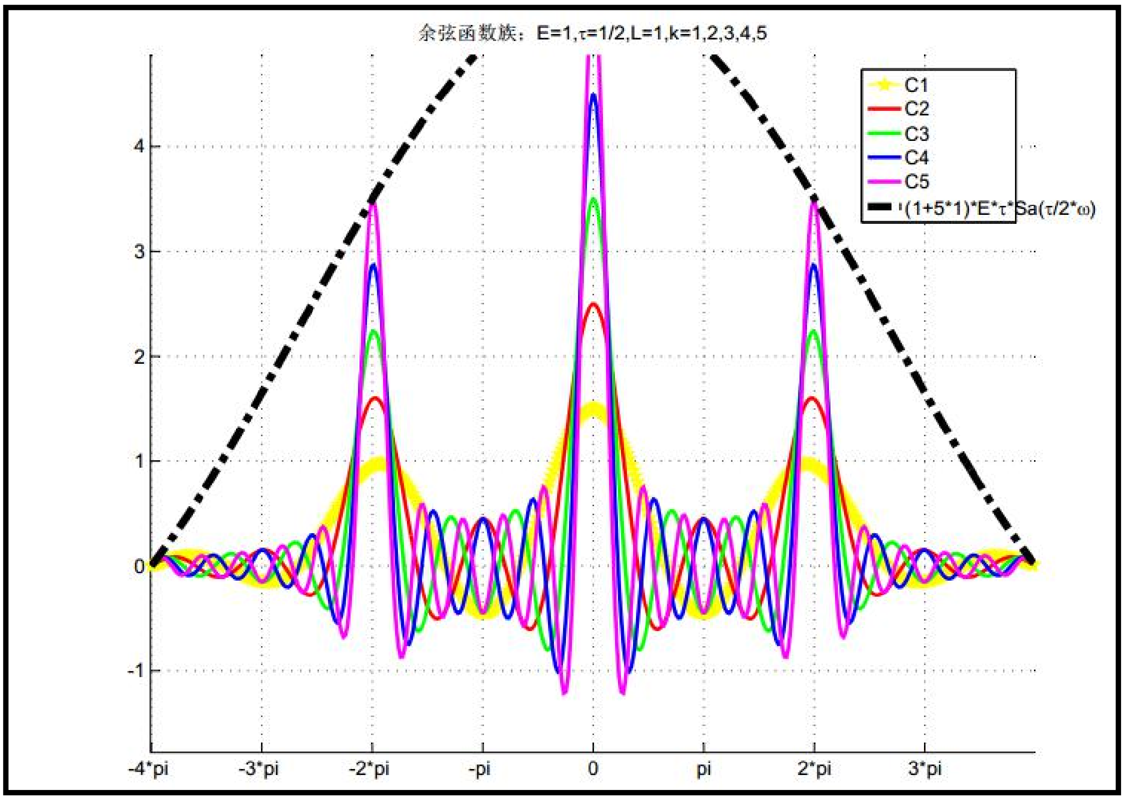
余弦函数族：E=1,τ=1/2,L=1,k=4
C4
(1+4*1)*E*τ*Sa(τ/2*ω)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi
4*pi

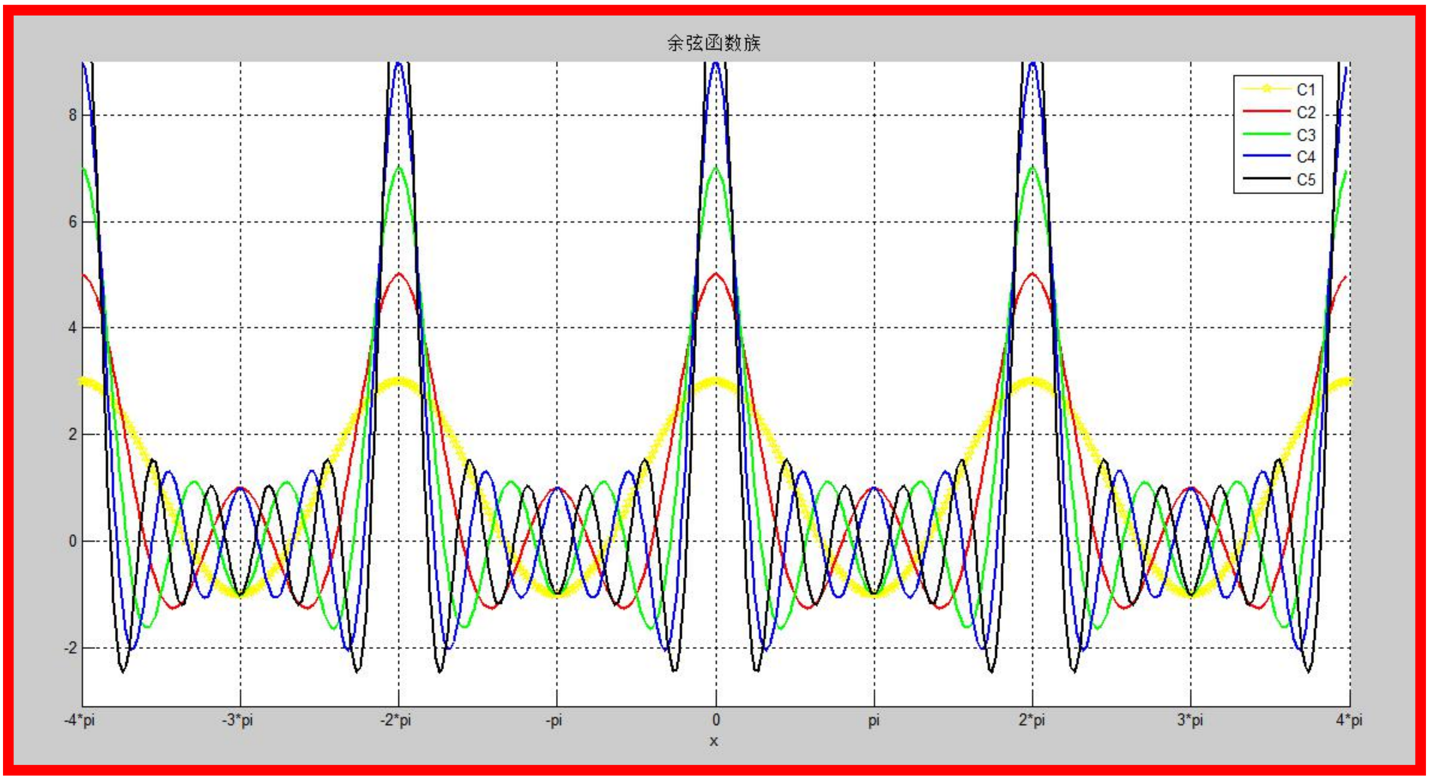


余弦函数族：E=1,τ=1/2,L=1,k=5
C5
(1+5*1)*E*τ*Sa(τ/2*ω)
6
5
4
3
2
1
0
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi
4*pi



余弦函数族：E=1,τ=1/2,L=1,k=1,2,3,4,5
C1
C2
C3
C4
C5
(1+5*1)*E*τ*Sa(τ/2*ω)
6
5
4
3
2
1
0
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi
4*pi


余弦函数族：E=1,τ=1/2,L=1,k=1,2,3,4,5
C1
C2
C3
C4
C5
(1+5*1)*E*τ*Sa(τ/2*ω)
4
3
2
1
0
-1
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi


余弦函数族
C1
C2
C3
C4
C5
8
6
4
2
0
-2
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi
4*pi
x

$Sa(c\cdot x)$函数图像的“周期”为$\frac{\pi}{c}$，$Sa(c\cdot x)$函数的零点在$x=\left(\frac{\pi}{c}\cdot n\right)$($n$为非零整数)处；

$[Sa(c\cdot x)\times\sin(c\cdot x)]$函数图像的“周期”为$\frac{\pi}{c}$，其零点仍在$x=\left(\frac{\pi}{c}\cdot n\right)$($n$为非零整数)处；

$[Sa(c\cdot x)\times\cos(c\cdot x)]$函数图像的“周期”为$\frac{\pi}{c}$，其零点仍在$x=\left(\frac{\pi}{c}\cdot n\right)$($n$为非零整数)处；

矩形脉冲(高度为E，长度为τ)$\Leftrightarrow(E\cdot\tau)\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right)$函数图像；

奇对称(II、IV象限)矩形脉冲(II象限内，高度为E，长度为$\frac{\tau}{2}$)$\Leftrightarrow(j2E)\cdot\left[Sa\left(\frac{\tau}{4}\cdot\omega\right)\cdot\sin\left(\frac{\tau}{4}\cdot\omega\right)\right]$函数图像；

偶对称(I、II象限)矩形脉冲(II象限内，高度为E，长度为$\frac{\tau}{2}$)$\Leftrightarrow(2E)\cdot\left[Sa\left(\frac{\tau}{4}\cdot\omega\right)\cdot\cos\left(\frac{\tau}{4}\cdot\omega\right)\right]$函数图像；

三角形脉冲(高度为E，长度为τ)$\Leftrightarrow\frac{E\cdot\tau}{2}\cdot Sa^2\left(\frac{\tau}{4}\cdot\omega\right)$函数图像。

单个典型波形(关于坐标轴对称[奇对称、偶对称])，以矩形脉冲矩形脉冲(高度为E，长度为τ)$G_\tau(t)$为例，

$$G_\tau(t)\Leftrightarrow G_0(\omega)=(E\cdot\tau)\cdot Sa\left(\frac{\tau}{2}\cdot\omega\right),\begin{cases}G_\tau(t)\to G_\tau\left[t+\left(-\frac{\tau}{2}\right)\right]\\ G_0(\omega)\to G_0(\omega)\cdot e^{+j\omega\cdot\left(-\frac{\tau}{2}\right)}\end{cases}\left\{\begin{cases}G_0(\omega)\to G_0\left[\omega+\left(-\frac{\Omega}{2}\right)\right]\\ G_\tau(t)\to G_\tau(t)\cdot e^{-jt\cdot\left(-\frac{\Omega}{2}\right)}\end{cases}\right.$$

以$z_1(t)=G_\tau\left[t+\left(-\frac{\tau}{2}\right)\right]$时域波形为基准，以间隔$L(L>\tau)$为周期，沿$t$轴正向顺序平移$n$次，

得到$(n+1)$个基准波形，令$z(t)=\sum_{k=0}^{n}z_1[t-(k\cdot L)]$，利用傅里叶变换的线性性质，

令$G_1(\omega)=G_0(\omega)\cdot e^{-j\omega\cdot\frac{\tau}{2}}$，$Z(\omega)=\sum_{k=0}^{n}\left[G_1(\omega)\cdot e^{-j\omega\cdot(k\cdot L)}\right]$，现在计算$Z(\omega)$和$G_0(\omega)$的关系。

$$Z(\omega)=\sum_{k=0}^{n}\left[G_1(\omega)\cdot e^{-j\omega\cdot(k\cdot L)}\right]=\sum_{k=0}^{n}\left\{\left[G_0(\omega)\cdot e^{-j\omega\cdot\frac{\tau}{2}}\right]\cdot[\cos([k\cdot L]\cdot\omega)-j\sin([k\cdot L]\cdot\omega)]\right\}$$

$$=G_0(\omega)\cdot\sum_{k=0}^{n}\left\{\left[\cos\left(\frac{\tau}{2}\cdot\omega\right)-j\sin\left(\frac{\tau}{2}\cdot\omega\right)\right]\cdot[\cos([k\cdot L]\cdot\omega)-j\sin([k\cdot L]\cdot\omega)]\right\}$$

$$=G_0(\omega)\cdot\left(\cos\left(\frac{\tau}{2}\cdot\omega\right)\cdot\sum_{k=0}^{n}[\cos([k\cdot L]\cdot\omega)]-\sin\left(\frac{\tau}{2}\cdot\omega\right)\cdot\sum_{k=0}^{n}[\sin([k\cdot L]\cdot\omega)]\right)$$

$$+j[-G_0(\omega)]\cdot\left(\cos\left(\frac{\tau}{2}\cdot\omega\right)\cdot\sum_{k=0}^{n}[\sin([k\cdot L]\cdot\omega)]+\sin\left(\frac{\tau}{2}\cdot\omega\right)\cdot\sum_{k=0}^{n}[\cos([k\cdot L]\cdot\omega)]\right)$$

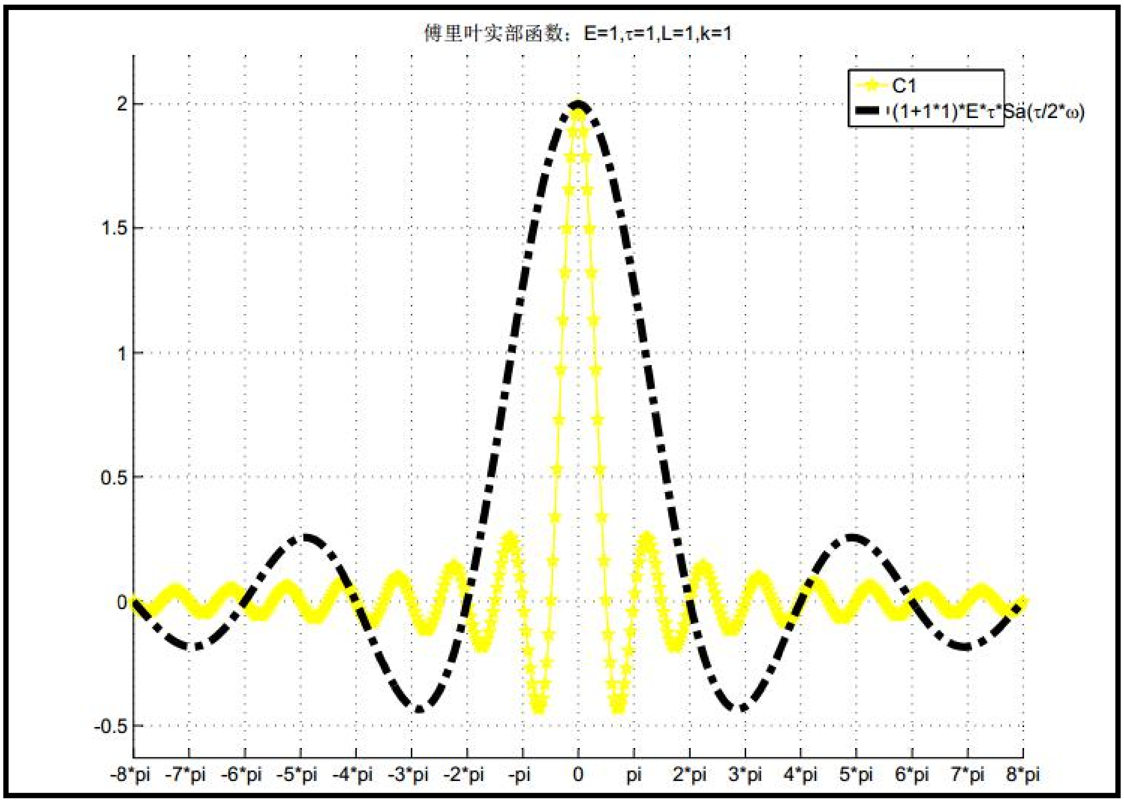
$$=G_0(\omega)\cdot\left(\cos\left(\frac{\tau}{2}\cdot\omega\right)\cdot\sum_{k=0}^{n}[\cos([k\cdot L]\cdot\omega)]-\sin\left(\frac{\tau}{2}\cdot\omega\right)\cdot\sum_{k=0}^{n}[\sin([k\cdot L]\cdot\omega)]\right)$$

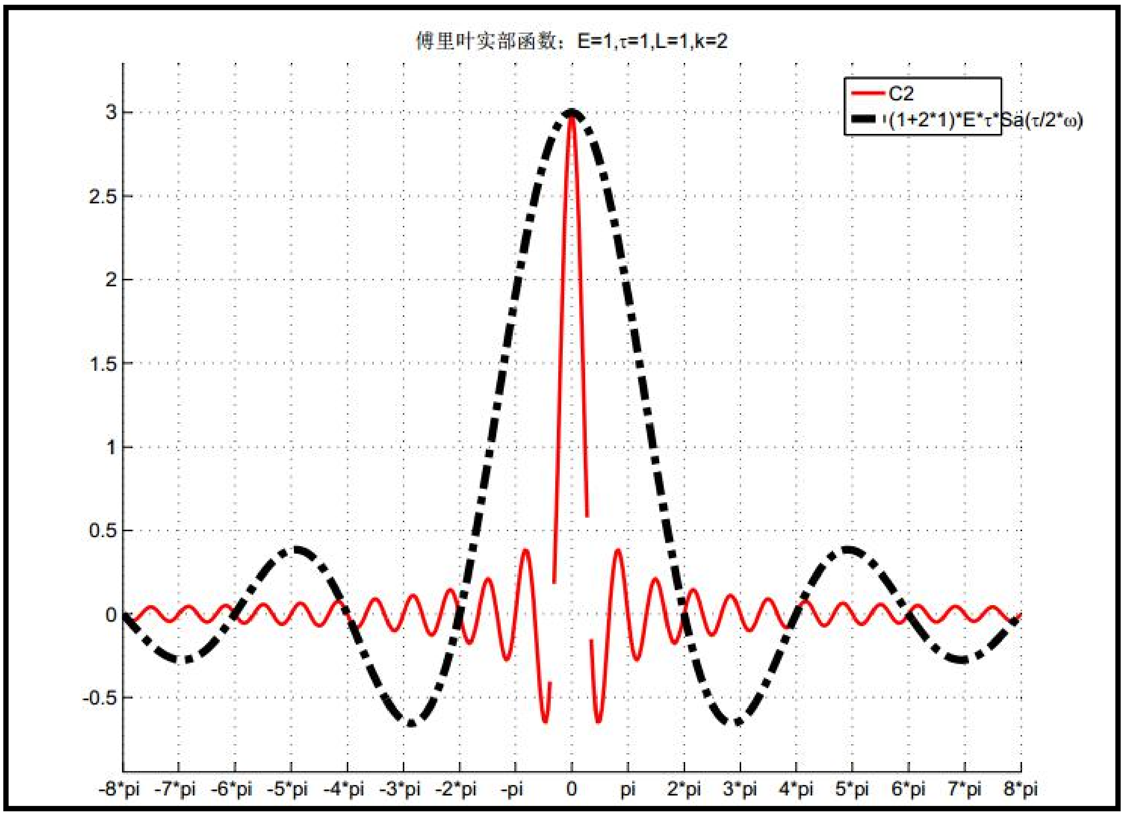
$$+j[-G_0(\omega)]\cdot\left(\cos\left(\frac{\tau}{2}\cdot\omega\right)\cdot\sum_{k=1}^{n}[\sin([k\cdot L]\cdot\omega)]+\sin\left(\frac{\tau}{2}\cdot\omega\right)\cdot\sum_{k=1}^{n}[1+\cos([k\cdot L]\cdot\omega)]\right)$$

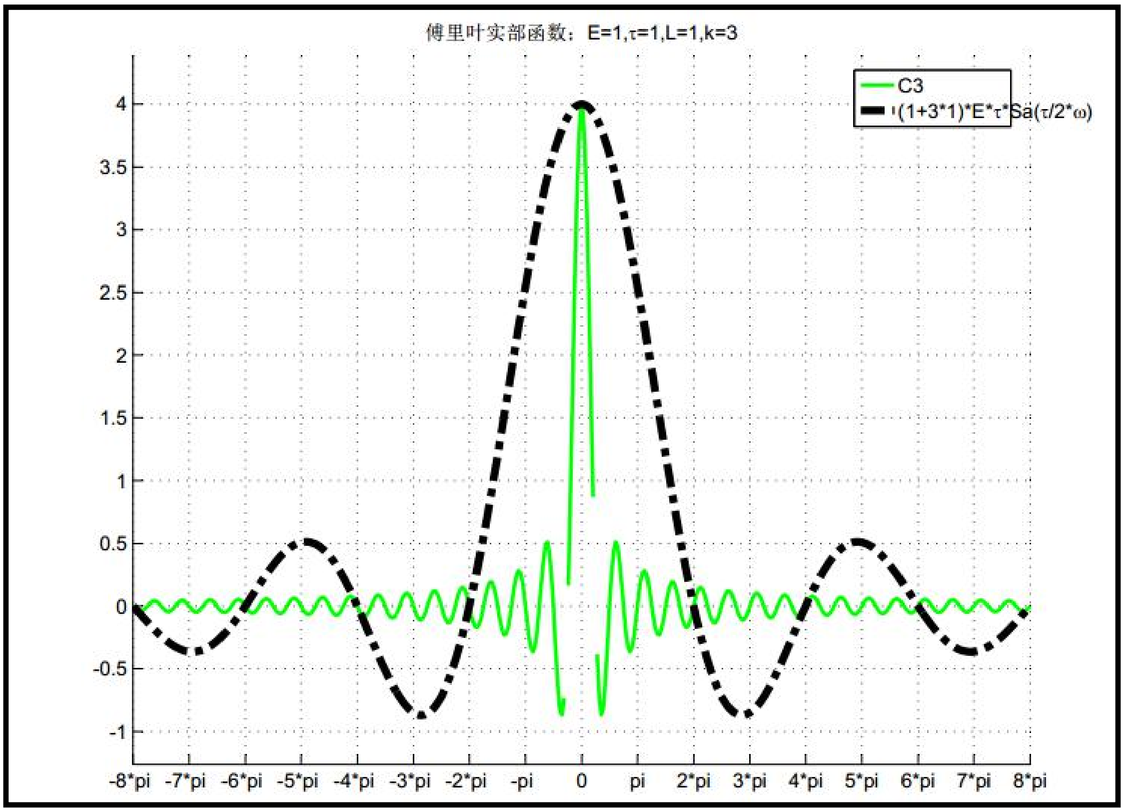
通过MATLAB画图，在$n\to\infty$过程中：

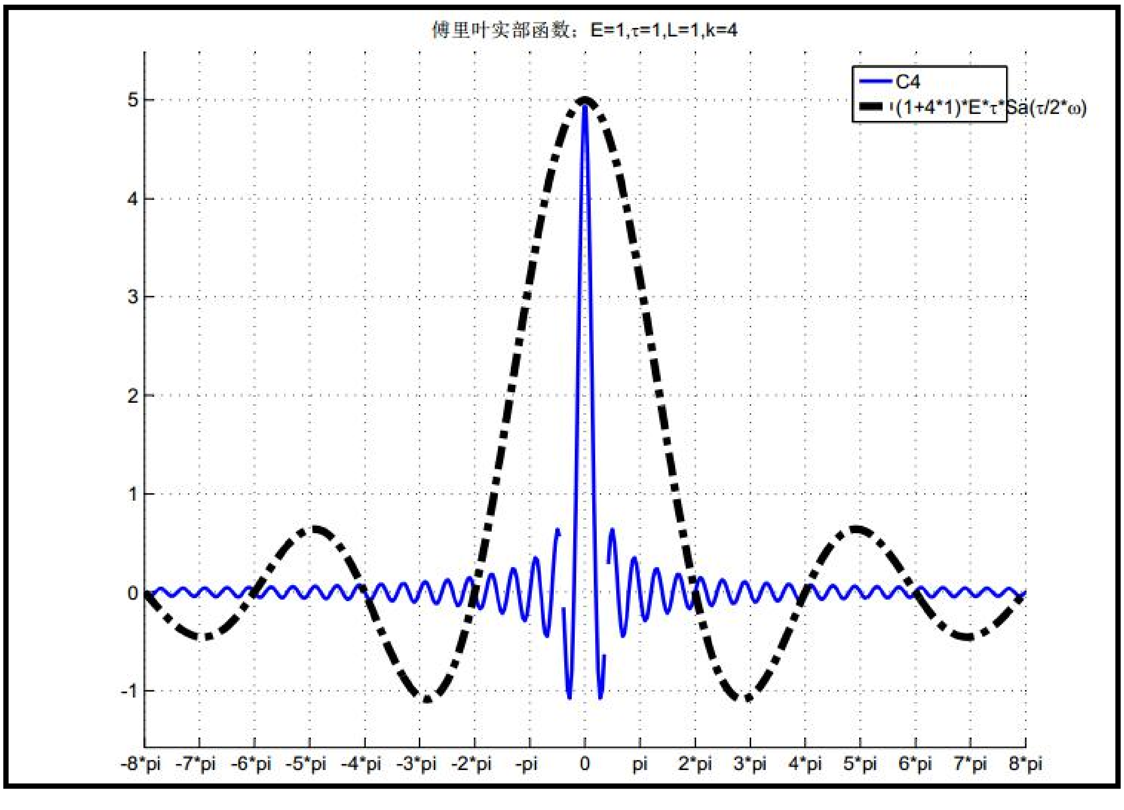
得到$Z(\omega)$的实部函数(ω的函数)图像形如$Sa(\omega)$函数变为$c\cdot\delta(\omega)$；

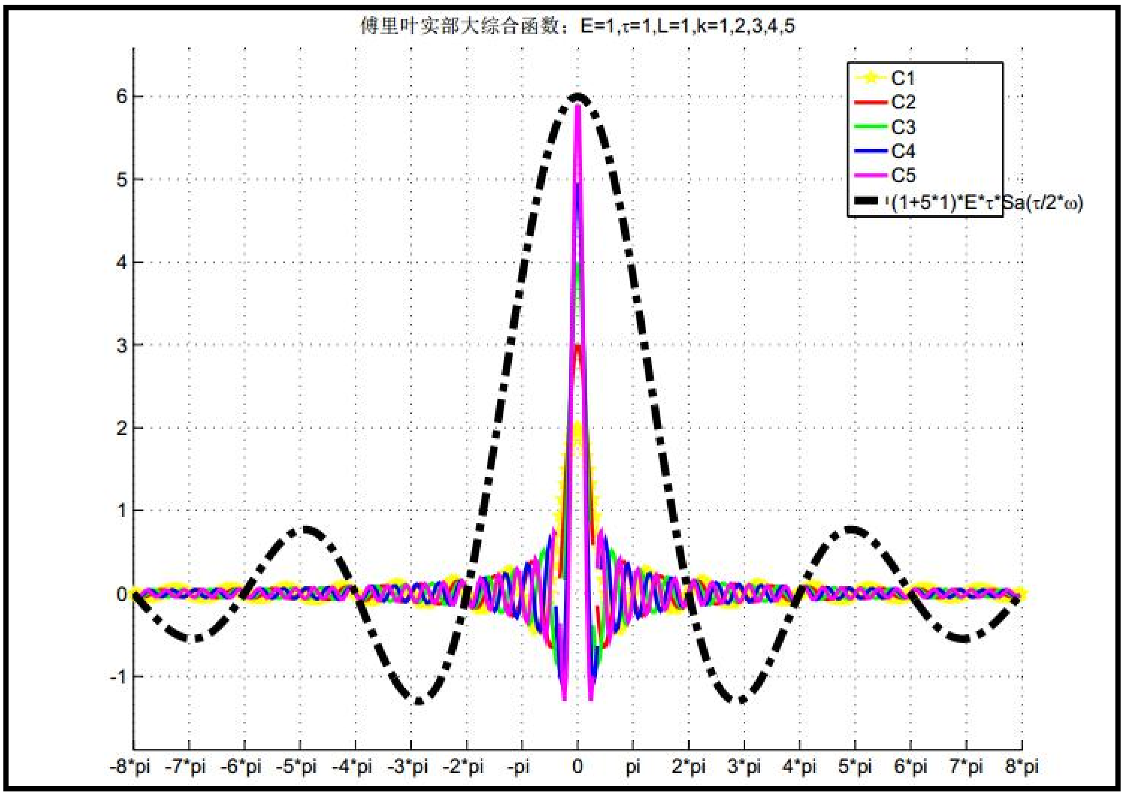
得到$Z(\omega)$的虚部函数(ω的函数)图像形如$Sa'(\omega)$函数变为$c\cdot\delta'(\omega)$。


傅里叶实部函数：E=1,τ=1,L=1,k=1
C1
(1+1*1)*E*τ*Sa(τ/2*ω)
2
1.5
1
0.5
0
-0.5
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi

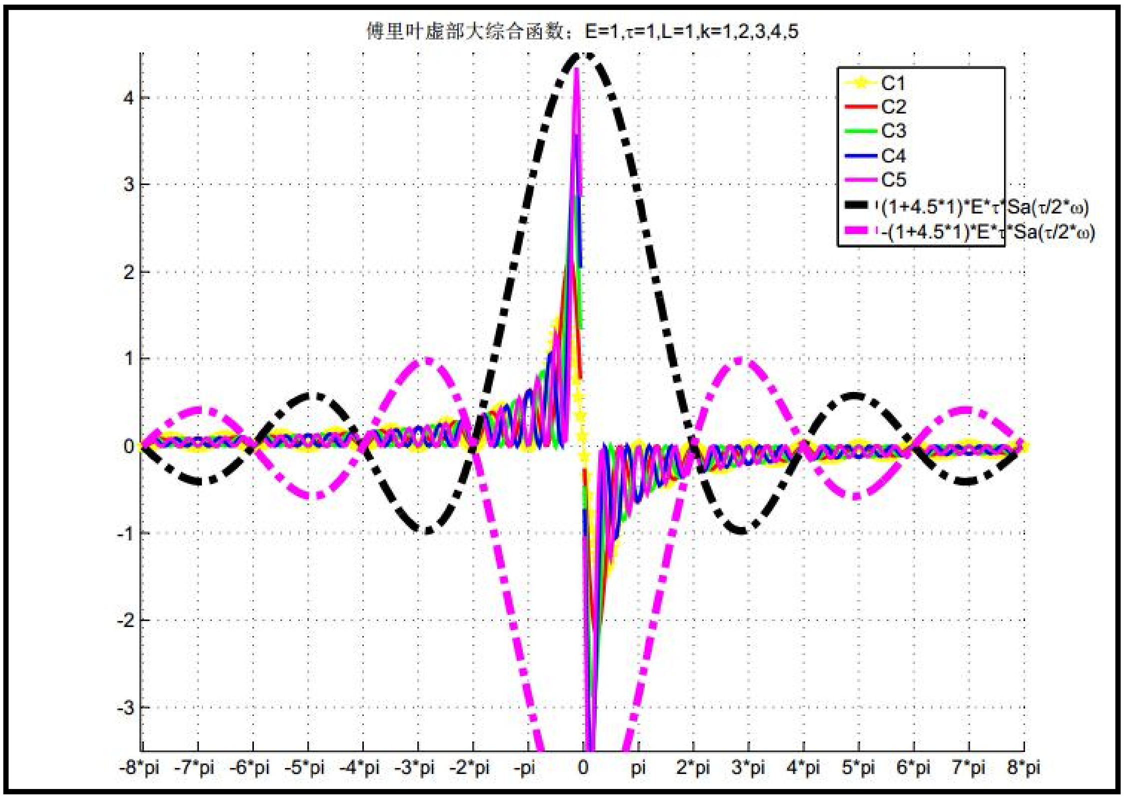

傅里叶实部函数：E=1,τ=1,L=1,k=2
C2
(1+2*1)*E*τ*Sa(τ/2*ω)
3
2.5
2
1.5
1
0.5
0
-0.5
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi


傅里叶实部函数：E=1,τ=1,L=1,k=3
C3
(1+3*1)*E*τ*Sa(τ/2*ω)
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi

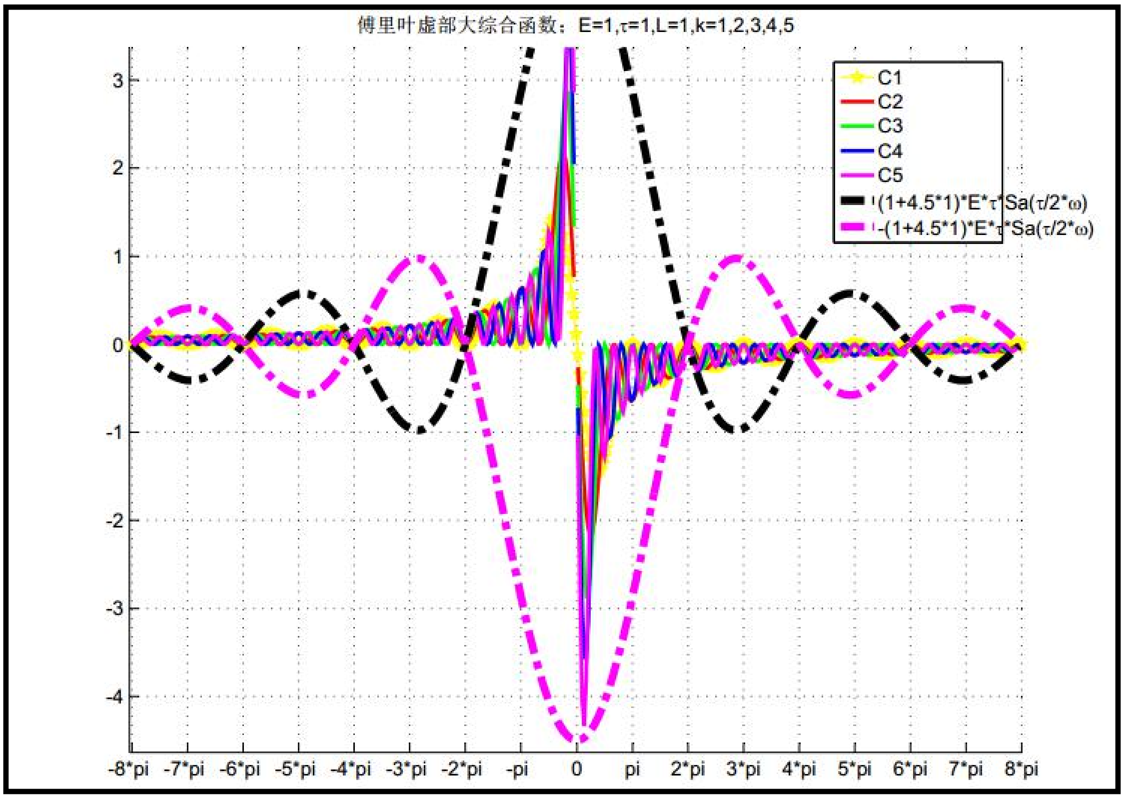

傅里叶实部函数：E=1,τ=1,L=1,k=4
C4
(1+4*1)*E*τ*Sa(τ/2*ω)
5
4
3
2
1
0
-1
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi



傅里叶实部大综合函数：E=1,τ=1,L=1,k=1,2,3,4,5
C1
C2
C3
C4
C5
(1+5*1)*E*τ*Sa(τ/2*ω)
6
5
4
3
2
1
0
-1
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi



傅里叶虚部大综合函数：E=1,τ=1,L=1,k=1,2,3,4,5
C1
C2
C3
C4
C5
(1+4.5*1)*E*τ*Sa(τ/2*ω)
-(1+4.5*1)*E*τ*Sa(τ/2*ω)
4
3
2
1
0
-1
-2
-3
-8*pi -7*pi -6*pi -5*pi -4*pi -3*pi -2*pi -pi 0 pi 2*pi 3*pi 4*pi 5*pi 6*pi 7*pi 8*pi



傅里叶虚部大综合函数：E=1,τ=1,L=1,k=1,2,3,4,5
C1
C2
C3
C4
C5
(1+4.5*1)*E*τ*Sa(τ/2*ω)
-(1+4.5*1)*E*τ*Sa(τ/2*ω)
3
2
1
0
-1
-2
-3
-4
-8*pi
-7*pi
-6*pi
-5*pi
-4*pi
-3*pi
-2*pi
-pi
0
pi
2*pi
3*pi
4*pi
5*pi
6*pi
7*pi
8*pi