曹建秋 20375177 201411

性质5 证明 $\mathrm{cov}(\hat{\beta}, e) = 0$

证明 $\mathrm{cov}(\hat{\beta}, e) = \mathrm{cov}((I_n - X(X'X)^{-1}X')Y, (X'X)^{-1}X'Y)$

$= [I_n - X(X'X)^{-1}X']\mathrm{var}(Y)[(X'X)^{-1}X']'$

$= \sigma^2 [I_n - X(X'X)^{-1}X'][(X'X)^{-1}X']' = 0$

4.

(1) $\hat{y} = 3.1006 - 2.1955x$

$\hat{\sigma}^2 = 0.00134$

(2) a 的 95% 置信区间为 $[3.0145, 3.1866]$

b 的 95% 置信区间为 $[-2.3307, -2.0603]$

(3) 检验回归效果的t检验的 $p < 0.0001$

而 $\alpha = 0.05 > p$ 故拒绝原假设 $H_0: b = 0$ 认为回归效果显著

(4) $\bar{x} = 0.6125$ $L_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 = 0.3638$

$\Rightarrow \delta(x) = \hat{\sigma} t_{1-\frac{\alpha}{2}}(n-2)\sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{L_{xx}}}$

$\Rightarrow$ y 的置信水平为 95% 的预测区间为

$\left[\hat{y} - 0.0816 \times \sqrt{1.0833 + \frac{(x-0.6125)^2}{0.3638}}, \ \hat{y} + 0.0816 \times \sqrt{1.0833 + \frac{(x-0.6125)^2}{0.3638}}\right]$

(5) $x_1^* = \frac{1}{\hat{b}}(y_1^* - \hat{a} + \hat{\sigma} z_{1-\frac{\alpha}{2}}) = 0.8877$

$x_2^* = \frac{1}{\hat{b}}(y_2^* - \hat{a} - \hat{\sigma} z_{1-\frac{\alpha}{2}}) = 0.6797$

$\hat{b} = -2.1955 < 0$ 故 x 应控制在 $[x_2^*, x_1^*] = [0.6797, 0.8877]$ 范围内

10. 解:

① $E(y)=a+b\ln x$ ② $E(y)=ax^b$ ③ $\frac{1}{E(y)}=a+\frac{b}{x}$

①: $\hat{y}=21.0058+19.5283\ln x$ $R_1^2=0.9479$

②: $\hat{y}=30.6827x^{0.3183}$ $R_2^2=0.4632$

③ $\hat{y}=\frac{x}{0.0109x+0.0378}$ $R_3^2=0.9465$

$R_1^2>R_2^2>R_3^2 \Rightarrow$ 选用 $E(y)=a+b\ln x$ 拟合

12. 解 $\hat{y}=-158.7676+4.8434x_1+5.2014x_2$

检验假设检验问题 $H_0: \beta_1=\beta_2=0$ $H_1: \beta_1, \beta_2$ 不全为零

F 检验法: $F=\frac{n-p-1}{p}\frac{U}{Q}$

$W=\{F \geq F_{1-\alpha}(p, n-p-1)\}$ $F_{0.95}(2,15)=3.68$

而实际值 $F=623.64>3.68$

拒绝原假设. 可认为回归方程显著

$\varepsilon^*=W^{-\frac{1}{2}}\varepsilon$

15. $Y=X\beta+\varepsilon \Rightarrow W^{-\frac{1}{2}}Y=W^{-\frac{1}{2}}X\beta+W^{-\frac{1}{2}}\varepsilon$ 令 $Y^*=W^{-\frac{1}{2}}Y$ $X^*=W^{-\frac{1}{2}}X$

则 $Y^*=X^*\beta+\varepsilon^*$ $Var(\varepsilon^*)=W^{-\frac{1}{2}}Var(\varepsilon)W^{-\frac{1}{2}}=\sigma^2 I$

$\Rightarrow \hat{\beta}=[(X^*)'X^*]^{-1}(X^*)'Y^*=(X'W^{-1}X)^{-1}X'W^{-1}Y$

又有 $E(\hat{\beta})=(X'W^{-1}X)^{-1}X'W^{-1}E(Y)=\beta$ $Q=(Y-X\hat{\beta})'W^{-1}(Y-X\hat{\beta})$

又由于 $E(W^{-\frac{1}{2}}(Y-X\hat{\beta}))=0$ $\Rightarrow E(Q)=tr\{Var[W^{-\frac{1}{2}}(Y-X\hat{\beta})]\}$

$=\sigma^2[tr(I_n)-tr(I_{p+1})]$

$=(n-1-p)\sigma^2$

故 σ^2 的无偏估计为

$\hat{\sigma}^2=\frac{Q}{n-p-1}=\frac{1}{n-p-1}(Y-X\hat{\beta})'W^{-1}(Y-X\hat{\beta})$