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1. 解:

$\hat{\mu}_1=\bar{x}_1=\begin{pmatrix}4\\2\end{pmatrix}$　　$\hat{\mu}_2=\bar{x}_2=\begin{pmatrix}3\\-1\end{pmatrix}$

$\hat{\mu}=\frac{1}{2}(\hat{\mu}_1+\hat{\mu}_2)=\begin{pmatrix}3.5\\0.5\end{pmatrix}$　　$\hat{\Sigma}^{-1}=\frac{1}{2.99}\begin{pmatrix}8.4 & -1.1\\-1.1 & 0.5\end{pmatrix}$

故线性判别函数为

$$W(X)=a^T(X-\bar{\mu})=(\hat{\mu}_1-\hat{\mu}_2)'\Sigma^{-1}(X-\hat{\mu})$$
$$=1.7057x_1+0.1338x_2-6.0368$$

$W(X_0)=1.7057\times2+0.1338\times1-6.0368=-2.4916<0$

故判 $X_0\in G_2$

6.

解　$\bar{X}=\frac{1}{n}\sum_{i=1}^{6}n_i\bar{X}_i=\begin{pmatrix}2.6406\\1.2752\\0.6189\end{pmatrix}$

$B=\sum_{i=1}^{6}n_i(\bar{X}_i-\bar{X})(\bar{X}_i-\bar{X})^T=\begin{pmatrix}367.8307 & 177.7367 & 76.1685\\177.7367 & 105.1168 & 39.2294\\76.1685 & 39.2294 & 17.6680\end{pmatrix}$

$E=6\times\hat{\Sigma}=\begin{pmatrix}13.8050 & 1.5096 & 2.8452\\1.5096 & 3.6450 & 0.2148\\2.8452 & 0.2148 & 3.5706\end{pmatrix}$

$E^{-1}B=\begin{pmatrix}22.0306 & 9.8335 & 4.3482\\39.5553 & 24.6678 & 8.9059\\1.3977 & 1.6671 & 0.9476\end{pmatrix}$

求解 $|\lambda I-E^{-1}B|=0$　$\Rightarrow$ 特征值 $\lambda_1=43.6101$　$\lambda_2=3.5252$　$\lambda_3=0.5107$

对应单位特征向量为　$\alpha_1=\begin{pmatrix}0.4224\\0.9051\\0.0492\end{pmatrix}$　$\alpha_2=\begin{pmatrix}0.5002\\-0.8252\\-0.2621\end{pmatrix}$　$\alpha_3=\begin{pmatrix}-0.1308\\-0.1473\\0.9804\end{pmatrix}$

标准化 $\Rightarrow$ $a_1=(0.1626, 0.3480, 0.0189)^T$　$a_2=(0.2417, -0.3988, -0.1269)^T$

$a_3=(-0.0753, -0.0849, 0.5647)^T$

三、Fisher判别函数为：

$y_1=0.1626x_1+0.3484x_2+0.0189x_3$

$y_2=0.2417x_1-0.3988x_2-0.1269x_3$

$y_3=-0.0753x_1-0.0849x_2+0.5647x_3$

运用~~第~~第一判别函数y_1，Fisher判别规则：

对给定的样品x_0，若存在$l(1\le l\le 6)$，使

$|y_1-a_1^T\bar{x}_l|=\min\limits_{1\le l\le 6}|y_1-a_1^T\bar{x}_i|=\min\limits_{1\le l\le 6}|a_1^T(x_0-\bar{x}_l)|$ 成立，则判$x_0\in G_l$

对样品$x_0=(2,1,1)^T$

$G_1:|a_1^T(x_0-\bar{x}_i)|=0.3795$　　$G_2:|a_1^T(x_0-\bar{x}_i)|=0.2426$

$G_3:|a_1^T(x_0-\bar{x}_i)|=0.5834$　　$G_4:|a_1^T(x_0-\bar{x}_i)|=0.6485$

$G_5:|a_1^T(x_0-\bar{x}_i)|=0.3952$　　$G_6:|a_1^T(x_0-\bar{x}_i)|=0.5325$

$l=2$时　有$|a_1^T(x_0-\bar{x}_i)|=0.2426$最小

应判$x_0\in G_2$，即可诊断该患者为癌病类型