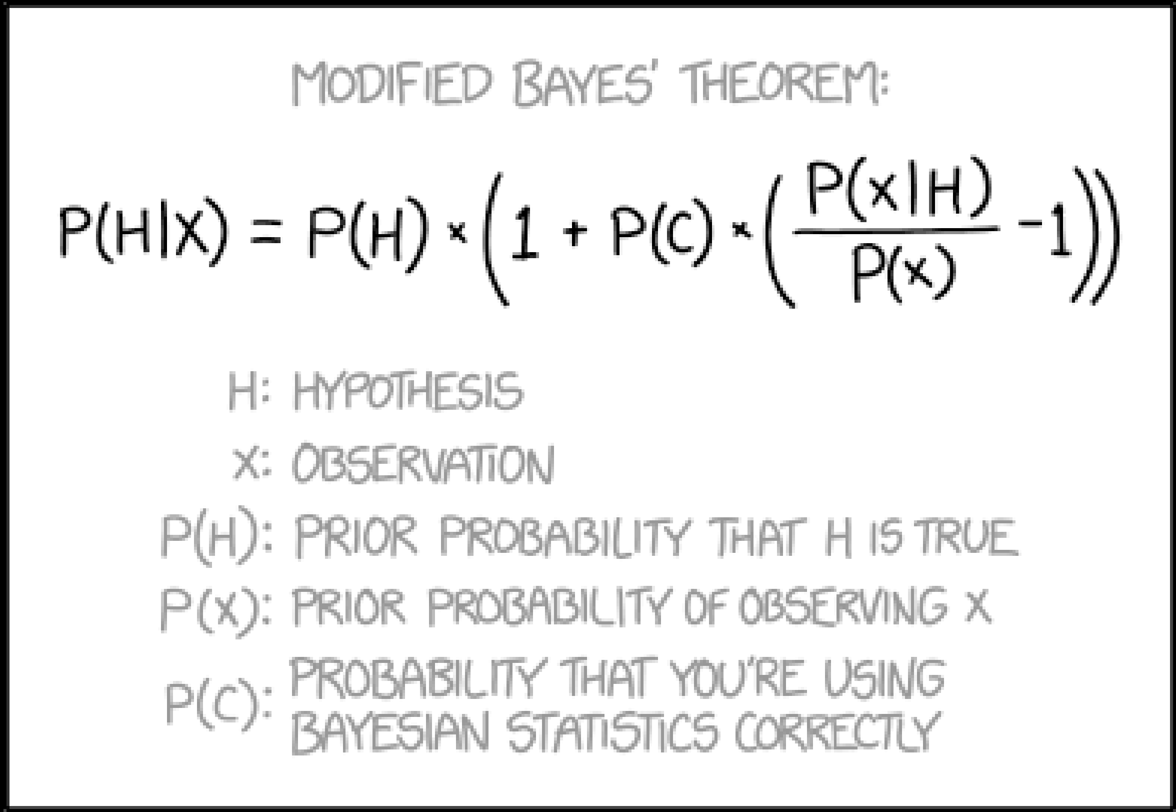

xkcd

INFO 251: Applied Machine Learning

Naïve Bayes

Announcements

- You're halfway done!
- Short quiz next Tuesday
- PS4 available

Key Concepts (last lecture)

- Logistic regression
- Simplified sigmoid cost function
- Odds ratios
- Overfitting revisited
- Ridge
- Lasso
- Support vector machines
- Hard vs. soft margins
- Kernel functions

Course Outline

- Causal Inference and Research Design
  - Experimental methods
  - Non-experiment methods
- Machine Learning
  - Design of Machine Learning Experiments
  - Linear Models and Gradient Descent
  - **Non-linear models**
  - Neural models
  - Unsupervised Learning
  - Practicalities, Fairness, Bias
- Special topics

Outline

- Mid-Semester Evaluation
- Rules and models
- Naïve Bayes Classifier
- Spam Example
- Smoothing
- Summary

This Lecture: Key Concepts

- Bayes' theorem
- Prior probability
- Conditional probability
- Posterior probability
- Log-Likelihood
- Spam classification
- Laplace smoothing

Rules vs. statistical models

- Rules
  - The first spam filters were manually curated blacklists
  - Hard to maintain; caught only 25% of spam (circa 2000)
- Models
  - Instead, treat words in email as evidence
  - Models allow us to combine prior knowledge with data
  - Paul Graham catches 99.5% of spam in 2002 with a simple statistical ("Bayesian") model
  - Our goal: understand how this model worked

Thomas Bayes

- Thomas Bayes
  - English mathematician and minister
  - 1701 - 1761
  - Wrote two books:
    - One tried to prove Newton's calculus correct
    - The other tried to prove that God is benevolent
  - But best known for his unpublished notes on probability

Is this really Bayes?

Bayes' Theorem

- Conditional probability: $P(A|B)$
  - Probability of A given that B is true
  - Recall: $P(A|B) = \dfrac{P(A \,\&\, B)}{P(B)}$
    - Why? Because $P(A\&B) = P(B)P(A|B)$
- Bayes Theorem:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Bayes' Theorem: Proof

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

1. $P(A|B) = \dfrac{P(A \,\&\, B)}{P(B)}$ (cond. probability)
2. $P(B|A) = \dfrac{P(B \,\&\, A)}{P(A)} = \dfrac{P(A \,\&\, B)}{P(A)}$ (cond. probability)
3. $P(A \,\&\, B) = P(B|A) * P(A)$ (rearrange 2)
4. $P\,(A|B) = \dfrac{P(B|A)\,P(A)}{P(B)}$ (combine 1 & 3)

Bayes' Theorem

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

- In practice we are typically dealing with hypothesis *h* and data *D*
  - h = "I have a cold"
  - D = "runny nose," "watery eyes," "coughing"
- $P(h|D) = \dfrac{P(D|h)\,P(h)}{P(D)}$
  - => Bayes' theorem is often thought of as "diagnostic"

Bayes' Theorem

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- P(*h*): **Prior probability** of *h*
  - What we know (or think we know) about *h* with no other evidence
  - Initial best guess that the hypothesis is true, based on no real data
- P(*D*|*h*): **conditional probability** of *D* given that *h* happened
  - Also called the "likelihood" of *D*
  - Probability that cause produces effect
  - E.g. probability that nose is runny if you *are* sick
- P(*D*): Probability of observing data, irrespective of any hypothesis,
  - Also called the "normalizing probability"
  - E.g. probability of a runny nose (for anyone, sick or healthy)
- P(*h*|*D*) is the **posterior probability** of *h* given *D*
  - This is what we want
  - Note that the posterior is influenced by the prior, so Bayes' has a GIGO liability

Bayes' Theorem

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- Who cares?
  - Idea: Update our belief about *h* given evidence *D*
- Why is this helpful?
  - Sometimes we can come up with estimates of P(*h*) and P(*D*|*h*) in situations where it's very hard to estimate P(*h*|*D*)

Bayes Rule: Diagnosis

- What is the probability that you have meningitis, given that your neck is stiff?
  - P(meningitis|stiff neck) = ?
- What we know
  - P(meningitis) = 1/50000 <== Prior probability, i.e., the "base rate"
  - P(stiff neck | meningitis) = 1/2 <== Conditional probability (effect | cause)
  - P(stiff neck) = 1/20 <== Normalizing probability
- What we can infer
  - $P(m|s) = P(m)P(s|m)/P(s)$

    $= (1/50000)(1/2)/(1/20)$

    $= (1/5000)$
- Got a stiff neck?
  - Chances of having meningitis increase by 10x. But still not so likely

Bayes Rule: Example Quiz Question

- Calculate the probability that an email is spam given that it contains the word "viagra"
- P(spam|"viagra") = ?
  - P(spam) = 0.4
  - P("viagra") = 0.05
  - P("viagra" | spam) = 0.06
- Answer: (.4)(.06)/(.05) = .48

Outline

- Mid-semester evaluations
- Rules and models
- **Naïve Bayes Classifier**
- Spam Example
- Practicalities
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Spam classification

- Input (X): an email corpus (collection)
  - X: $x_1,\ldots,x_n$ (the corpus x is comprised of n emails x_i)
  - x_i: $t_1,\ldots,t_k$ (each email x_i is comprised of k features t_j)
- Output (Y): [spam, ham]
- Goal: predict labels of new email
  - P(Y=spam|email) = ?
- P(spam|features)
  - words
  - urls
  - sender

> Dear Sir.
>
> First, I must solicit your confidence in this transaction, this is by virture of its nature as being utterly confidencial and top secret. …

> TO BE REMOVED FROM FUTURE MAILINGS, SIMPLY REPLY TO THIS MESSAGE AND PUT "REMOVE" IN THE SUBJECT.
>
> 99 MILLION EMAIL ADDRESSES FOR ONLY $99

> Ok, Iknow this is blatantly OT but I'm beginning to go insane. Had an old Dell Dimension XPS sitting in the corner and decided to put it to use, I know it was working pre being stuck in the corner, but when I plugged it in, hit the power nothing happened.

Naïve Bayes Classifier

- We are trying to learn a function f(x) that maps input data x_i, consisting of terms $t_1,\ldots,t_k$, to predicted class $\hat{Y}_i$
- Most likely class is:
  - $\hat{Y}_i = \underset{c_j \in C}{\operatorname{argmax}} P(c_j | t_1, \ldots, t_k)$

Why can't we estimate this directly?

$$= \underset{c_j \in C}{\operatorname{argmax}} \frac{P(t_1,\ldots,t_k | c_j) P(c_j)}{P(t_1,\ldots,t_k)}$$

(this is Bayes' rule in action)

$$= \underset{c_j \in C}{\operatorname{argmax}} P(t_1, \ldots, t_k | c_j) P(c_j)$$

(we can ignore $P(t_1, \ldots, t_k)$ - same for all c_j)

This is still a problem

Naïve Bayes Classifier

- Naïve Bayes (independence) Assumption:

$$P(t_1, \dots, t_k | c_j) = \prod_{1 \le i \le k} P(t_i | c_j)$$

- This allows us to complete the NB Classifier:

$$\hat{Y}_i = \underset{c_j \in C}{\operatorname{argmax}} P(c_j | t_1, \dots, t_k)$$

$$\dots$$

$$= \underset{c_j \in C}{\operatorname{argmax}} P(c_j) \prod_{1 \le i \le k} P(t_i | c_j)$$

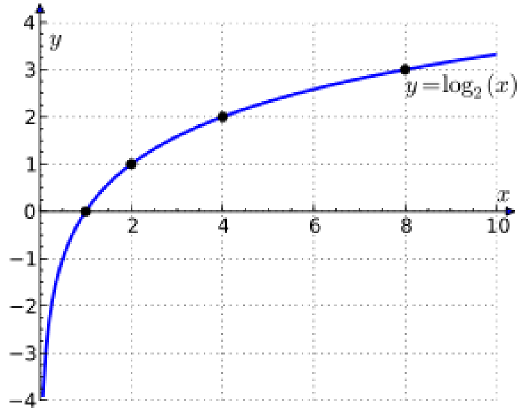
- Is this an appropriate generative model?

Classification

- For classification, we want the most likely class given the data. This is the "Maximum A-Posteriori (MAP) estimate"

$$\underset{c_j \in C}{\operatorname{argmax}} P(c_j) \prod_{1 \le i \le k} P(t_i | c_j)$$

- In practice, it's easier to work with logs:



$$\underset{c_j \in C}{\operatorname{argmax}} \left[\log P(c_j) + \sum_{1 \le i \le k} \log P(t_i | c_J) \right]$$

  - Why is it valid to work with logs?
  - What do we gain by working with logs?

Naïve Bayes Classifier: Recap

- Naïve Bayes Classifier:

$$\hat{Y}_i = \underset{c_j \in C}{\operatorname{argmax}} P(c_j | t_1, \dots, t_k)$$

$$\dots$$

$$= \underset{c_j \in C}{\operatorname{argmax}} P(t_1, \dots, t_k | c_j) \, P(c_j)$$

$$\dots$$

$$= \underset{c_j \in C}{\operatorname{argmax}} P(c_j) \prod_{1 \le i \le k} P(t_i | c_j)$$

$$\dots$$

$$= \underset{c_j \in C}{\operatorname{argmax}} \left[\log P(c_j) + \sum_{1 \le i \le k} \log P(t_i | c_j) \right]$$

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Spam classification parameters

- Example parameters:

$P(c_j)$	
ham	0.6
spam	0.4

$P(t_i \mid c_j=\text{spam})$	
the	0.0156
to	0.0153
and	0.0115
of	0.0095
you	0.0093
a	0.0086
with	0.0080
from	0.0075
...	

$P(t_i \mid c_j=\text{ham})$	
the	0.0210
to	0.0133
of	0.0119
2002	0.0110
with	0.0108
from	0.0107
and	0.0105
a	0.0100
...	

- Example test case:

Dear Sir.

First, I must solicit your confidence in this transaction, this is by virture of its nature as being utterly confidencial and top secret. ...

Spam classification example

- Prediction: $\underset{c_j}{\operatorname{argmax}} \log P(c_j) + \sum_i \log P(t_i | c_j)$

Feature	P(d_i|spam)	P(d_i|ham)	Tot Spam	Tot Ham
(prior)	0.4	0.6	-0.92	-0.51
dear	0.0013	0.0009	-7.56	-7.52
sir	0.0023	0.0004	-13.64	-15.35
,	0.0220	0.0241	-17.45	-19.07
first	0.0018	0.0023	-23.77	-25.15
I	0.0062	0.0119	-28.86	-29.57
must	0.0034	0.0028	-34.54	-35.45
solicit	0.0007	0.0002	**-41.08**	-43.97

Naive Bayes parameter estimation

- Naive Bayes is appealing because the parameters are so easy to estimate
  - All we need are counts
  - Easy to parallelize

Naïve Bayes Algorithm

Naive_bayes_train(*examples*):

For each target c_j:

$\hat{P}(c_j)$ <- estimate $P(c_j)$

For each attribute t_i:

$\hat{P}(t_i|c_i)$ <- estimate $P(t_i|c_i)$

Classify_new_instance(x):

$$c_x = \underset{c_j \in C}{\operatorname{argmax}} \left[\log \hat{P}(c_j) + \sum_{1 \le i \le k} \log \hat{P}(t_i | c_j) \right]$$

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Some nuances

- Probabilities may not add up to 1. Why?
  - Not normalizing by likelihood of data. In practice, doesn't matter
  - What matters is relative likelihood.
- Conditional independence often violated
  - So why does NB perform so well?
  - Model is bad, classifier is good

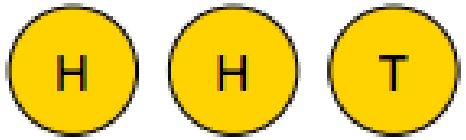
Smoothing

- Lots of words don’t occur in training set (because of power law behavior), even though those terms may occur in class documents outside the training set
  - Who cares?
- Just one of these drives the posterior estimate for that document/class to zero (or undefined), when in fact it may be a good choice
- You can avoid zero probability estimates by **smoothing**

Laplace smoothing

- Idea: Pretend we saw every outcome 1 more time than we actually did ("add-one smoothing")

- If we observed [hht]:
  - MLE P(heads) = 2/3
  - LAP1 P(heads) = 3/5
  - LAP100 P(heads) = 102/203 (larger k = stronger smoothing)



$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V}(T_{ct'} + 1)} = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

Additive Smoothing

- Additive smoothing generalizes this. In practice, it's worth trying smaller values of the additive smoothing parameter
  - Typically a positive value α< 1 is used

$$\hat{P}(t|c) = \frac{T_{ct} + \alpha}{\sum_{t' \in V}(T_{ct'} + \alpha)} = \frac{T_{ct} + \alpha}{(\sum_{t' \in V} T_{ct'}) + B\alpha}$$

- How do we choose α?
  - Cross-validation!

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Naïve Bayes Classifier: Recap

- Naïve Bayes Classifier:

$$\hat{Y}_i = \operatorname*{argmax}_{c_j \in C} P\left(c_j | t_1, \ldots, t_k\right)$$

$$\ldots$$

$$= \operatorname*{argmax}_{c_j \in C} P\left(t_1, \ldots, t_k | c_j\right) P\left(c_j\right)$$

$$\ldots$$

$$= \operatorname*{argmax}_{c_j \in C} P(c_j) \prod_{1 \le i \le k} P\left(t_i | c_j\right)$$

$$\ldots$$

$$= \operatorname*{argmax}_{c_j \in C} \left[\log P\left(c_j\right) + \sum_{1 \le i \le k} \log P\left(t_i | c_j\right)\right]$$

Naive Bayes summary

- Use Bayes Rule to get the probability of the label given the input
- The Naive Bayes assumption says that the features are independent conditional on the label
- Smoothing is important in real systems
- Use held-out data to tune features and hyper-parameters

Naïve Bayes: Pros and Cons

- Pros
  - Simple and fast. Depends only on term frequency data for the classes.
  - Very well-behaved numerically. Term weights depend only on term frequency
  - Easy to parallelize
  - Can work very well with sparse data, where combinations of dependent terms are rare
- Cons
  - Can't model more complex patterns in the data
  - Generative model not appropriate in many settings
  - Subject to bias when term probabilities are not independent (e.g. URL prefixes)

Spam: state of the art

- Gmail and Yahoo Mail are better than 99.9% accurate
  - huge amount of training data -> good estimates for a huge vocabulary
  - use OCR to convert images to text
  - some personalization
- Remains an active area of research
  - multilingual models
  - other varieties of spam require more creative features
    - review spam
    - youtube comments, twitter, …

For Next Class:

- Read:
  - Chapter 8 of Intro to Statistical Learning
  - Chapter 13 of Daume

Multinomial and Bernoulli NB

- Multinomial vs. Bernoulli
  - Bernoulli: Term presence is a binary variable
  - Multinomial models: distinct placements of terms are treated as independent terms with the same probability