Homework 5 - Berkeley STAT 157

Your name: XX, SID YY (Please add your name, and SID to ease Ryan and Rachel to grade.)

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Handout 2/19/2019, due 2/26/2019 by 4pm in Git by committing to your repository.

In this homework, we will model covariate shift and attempt to fix it using logistic regression. This is a fairly realistic scenario for data scientists. To keep things well under control and understandable we will use [Fashion-MNIST (http://d2l.ai/chapter_linear-networks/fashion-mnist.html)](http://d2l.ai/chapter_linear-networks/fashion-mnist.html) as the data to experiment on.

Follow the instructions from the Fashion MNIST notebook to get the data.

In [1]:
```
%matplotlib inline
from mxnet import autograd, gluon, init, nd
from mxnet.gluon import data as gdata, loss as gloss, nn, utils
import numpy as np
from matplotlib import pyplot as plt
import d2l

mnist_train = gdata.vision.FashionMNIST(train=True)
mnist_test = gdata.vision.FashionMNIST(train=False)
```

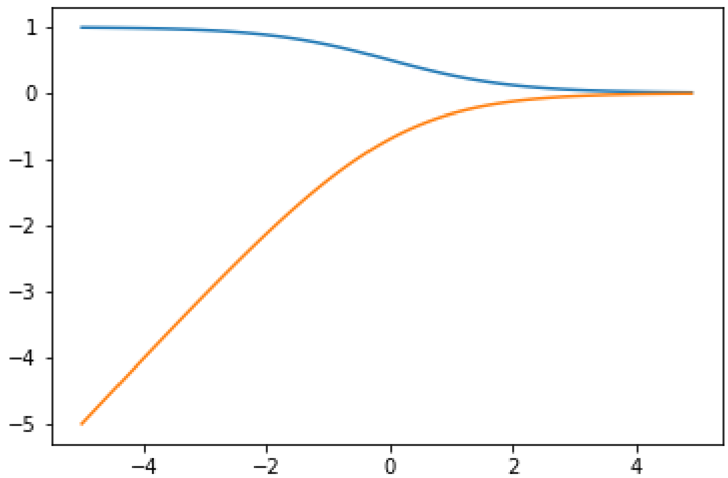
1. Logistic Regression

1. Implement the logistic loss function $l(y, f) = -\log(1 + \exp(-yf))$ in Gluon.
2. Plot its values and its derivative for $y = 1$ and $f \in [-5, 5]$, using automatic differentiation in Gluon.
3. Generate training and test datasets for a binary classification problem using Fashion-MNIST with class 1 being a combination of `sneaker` and `pullover` and class -1 being the combination of `sandal` and `shirt` categories.
4. Train a binary classifier of your choice (it can be linear or a simple MLP such as from a previous lecture) using half the data (i.e. $12,000$ observations mixed as abvove) and one using the full dataset (i.e. $24,000$ observations as arising from the 4 categories) and report its accuracy.

Hint - you should encapsulate the training and reporting code in a callable function since you'll need it quite a bit in the following.

In [2]:
```
#1
def logisticLoss(f, y):
    return -nd.log(1 + nd.exp(-y * f))
#2
y = nd.ones(1)
f = nd.arange(-5, 5, 0.1)
f.attach_grad()
with autograd.record():
    l = logisticLoss(f, y)
l.backward()

plt.plot(f.asnumpy(), f.grad.asnumpy())
plt.plot(f.asnumpy(), l.asnumpy())
plt.show()
```



Label Description 0 T-shirt/top 1 Trouser 2 Pullover 3 Dress 4 Coat 5 Sandal 6 Shirt 7 Sneaker 8 Bag 9 Ankle boot

In [3]:
```
#3
def extract(data):
    dataset = []
    for i in range(len(data)):
        feature, label = data[i]
        label = d2l.get_fashion_mnist_labels([label])[0]
        if label == "sneaker" or label == "pullover":
            dataset.append((data[i][0].astype('float32').reshape(1, 784
), nd.array([1])))
        elif label == "sandal" or label == "shirt":
            dataset.append((data[i][0].astype('float32').reshape(1, 784
), nd.array([-1])))
    return dataset

new_train = extract(mnist_train)
new_test = extract(mnist_test)
```

In [4]:
```
def getnet():
    net = nn.Sequential()
    net.add(nn.Dense(1))
    net.initialize(init.Normal(sigma=0.01))
    return net
```

In [5]:
```
def predict(y):
    return 1 if y[0].asscalar() > 0 else -1

def get_accuracy(data_iter, net):
    acc_sum, n = nd.array([0]), 0
    for X, y in data_iter:
        y = y.astype('float32')
        acc_sum += (predict(net(X)) == y).sum()
        n += y.size
    return acc_sum.asscalar() / n
```

In [6]:
```
def train(train_iter, test_iter, loss, num_epochs, batch_size, lr=None):
    net = getnet()
    trainer = gluon.Trainer(net.collect_params(), 'adam', {'learning_rat
e': lr})
    for epoch in range(num_epochs):
        train_l_sum, train_acc_sum, n = 0.0, 0.0, 0
#         print(len(train_iter))
        for X, y in train_iter:
#             print(X.shape)
            with autograd.record():
                y_hat = net(X)
                l = loss(y_hat, y).sum()
            l.backward()
            trainer.step(batch_size)
            y = y.astype('float32')
            train_l_sum += l.asscalar()
            train_acc_sum += (predict(y_hat) == y).sum().asscalar()
            n += y.size
        test_acc = get_accuracy(test_iter, net)
        print('epoch %d, loss %.4f, train acc %.3f, test acc %.3f'
              % (epoch + 1, train_l_sum / n, train_acc_sum / n, test_acc
))
    return net
```

In [15]:
```
#4
batch_size = 256
loss = gloss.LogisticLoss()
num_epochs, lr = 3, 0.5
halved = new_train[:12000]
trained_half = train(halved, new_test, loss, num_epochs, batch_size, lr)
```

```
epoch 1, loss 6832.9489, train acc 0.795, test acc 0.830
epoch 2, loss 6994.4781, train acc 0.822, test acc 0.616
epoch 3, loss 7087.2501, train acc 0.832, test acc 0.646
```

In [11]:
```
num_epochs, lr = 3, 0.5
# training takes too long, so one epoch is enough.
trained_full = train(new_train, new_test, loss, num_epochs, batch_size,
lr)
```

```
epoch 1, loss 6985.5531, train acc 0.811, test acc 0.873
epoch 2, loss 7380.3410, train acc 0.832, test acc 0.873
epoch 3, loss 7317.9732, train acc 0.836, test acc 0.880
```

Our logistic Regression model can perfectly separate two categories. We got same accurary on train and test because two dataset come from same distribution

2. Covariate Shift

Your goal is to introduce covariate shit in the data and observe the accuracy. For this, compose a dataset of $12,000$ observations, given by a mixture of `shirt` and `sweater` and of `sandal` and `sneaker` respectively, where you use a fraction $\lambda \in \{0.05, 0.1, 0.2, \ldots 0.8, 0.9, 0.95\}$ of one and a fraction of $1 - \lambda$ of the other datasets respectively. For instance, you might pick for $\lambda = 0.1$ a total of 600 `shirt` and $5,400$ `sweater` images and likewise 600 `sandal` and $5,400$ `sneaker` photos, yielding a total of $12,000$ images for training. Note that the test set remains unbiased, composed of $2,000$ photos for the `shirt + sweater` category and of the `sandal + sneaker` category each.

1. Generate training sets that are appropriately biased. You should have 11 datasets.
2. Train a binary classifier using this and report the test set accuracy on the unbiased test set.

In [7]:
```
#1
def get_data(frac, indices_map):
    num_per_class = len(indices_map[2])
    size_class_one = int(frac * num_per_class)
    size_class_two = int((1 - frac) * num_per_class)
    pullover_indices = np.random.choice(indices_map[2], size=size_class_
one, replace = False)
    sneaker_indices = np.random.choice(indices_map[7], size=size_class_t
wo, replace = False)
    shirts_indices = np.concatenate((pullover_indices, sneaker_indices),
axis=0)

    sandal_indices = np.random.choice(indices_map[5], size=size_class_tw
o, replace = False)
    shirt_indices = np.random.choice(indices_map[6], size=size_class_one
, replace = False)
    shoes_indices = np.concatenate((sandal_indices, shirt_indices), axis
=0)
    processed = []
    for i in np.concatenate((shirts_indices, shoes_indices), axis=0):
        feature, label = mnist_train[i]
        if label == 2 or label == 7:
            processed.append((mnist_train[i][0].astype('float32').reshap
e(1, 784), nd.array([1])))
        elif label == 5 or label == 6:
            processed.append((mnist_train[i][0].astype('float32').reshap
e(1, 784), nd.array([-1])))
    return processed

def indices(labels):
    indices_map = {}
    for i in labels:
        indices_map[i] = list()
    for i in range(len(mnist_train)):
        _, label = mnist_train[i]
        if label in indices_map:
            indices_map[label].append(i)
    return indices_map

# 2 = pullover, 5 = sandal, 6 = shirt, 7 = sneaker.
labels = [2, 5, 7, 6]
indices_map = indices(labels) # indices map stores indices of each of th
e classes.
```

In [8]:
```
splitted_data = []
frac = [0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95]
for f in frac:
    splitted_data.append(get_data(f, indices_map))
```

In [9]:
```
trained = []
num_epochs, lr = 3, 0.5
loss = gloss.LogisticLoss()
for i, datum in enumerate(splitted_data):
    print("fraction = ", frac[i])
    trained.append(train(datum, new_test, loss, num_epochs, 256, lr))
    print()
```

```
fraction =  0.05
epoch 1, loss 9.2170, train acc 1.000, test acc 0.500
epoch 2, loss 275.5560, train acc 0.999, test acc 0.500
epoch 3, loss 558.4436, train acc 0.999, test acc 0.500

fraction =  0.1
epoch 1, loss 8.3322, train acc 1.000, test acc 0.500
epoch 2, loss 295.8321, train acc 0.999, test acc 0.500
epoch 3, loss 628.8992, train acc 0.998, test acc 0.500

fraction =  0.2
epoch 1, loss 5.8576, train acc 1.000, test acc 0.500
epoch 2, loss 256.1671, train acc 0.999, test acc 0.500
epoch 3, loss 521.1045, train acc 0.998, test acc 0.500

fraction =  0.3
epoch 1, loss 2.4011, train acc 1.000, test acc 0.500
epoch 2, loss 400.2403, train acc 0.999, test acc 0.500
epoch 3, loss 614.5046, train acc 0.998, test acc 0.500

fraction =  0.4
epoch 1, loss 6.6500, train acc 1.000, test acc 0.500
epoch 2, loss 360.8222, train acc 0.999, test acc 0.500
epoch 3, loss 578.8242, train acc 0.998, test acc 0.500

fraction =  0.5
epoch 1, loss 1.9005, train acc 1.000, test acc 0.500
epoch 2, loss 210.3912, train acc 0.999, test acc 0.500
epoch 3, loss 358.5376, train acc 0.998, test acc 0.500

fraction =  0.6
epoch 1, loss 7.6018, train acc 1.000, test acc 0.500
epoch 2, loss 273.3808, train acc 0.999, test acc 0.500
epoch 3, loss 483.0471, train acc 0.999, test acc 0.500

fraction =  0.7
epoch 1, loss 0.0208, train acc 1.000, test acc 0.500
epoch 2, loss 224.7195, train acc 0.999, test acc 0.500
epoch 3, loss 492.1620, train acc 0.998, test acc 0.500

fraction =  0.8
epoch 1, loss 4.3827, train acc 1.000, test acc 0.500
epoch 2, loss 237.5205, train acc 0.999, test acc 0.500
epoch 3, loss 357.2230, train acc 0.998, test acc 0.500

fraction =  0.9
epoch 1, loss 12.6584, train acc 1.000, test acc 0.500
epoch 2, loss 265.6203, train acc 0.998, test acc 0.500
epoch 3, loss 245.8700, train acc 0.999, test acc 0.500

fraction =  0.95
epoch 1, loss 0.0138, train acc 1.000, test acc 0.500
epoch 2, loss 416.3008, train acc 0.999, test acc 0.500
epoch 3, loss 330.0973, train acc 0.999, test acc 0.500
```

3. Covariate Shift Correction

Having observed that covariate shift can be harmful, let's try fixing it. For this we first need to compute the appropriate propensity scores $\frac{dp(x)}{dq(x)}$. For this purpose pick a biased dataset, let's say with $\lambda = 0.1$ and try to fix the covariate shift.

1. When training a logistic regression binary classifier to fix covariate shift, we assumed so far that both sets are of equal size. Show that re-weighting data in training and test set appropriately can help address the issue when both datasets have different size. What is the weighting?
2. Train a binary classifier (using logistic regression) distinguishing between the biased training set and the unbiased test set. Note - you need to weigh the data.
3. Use the scores to compute weights on the training set. Do they match the weight arising from the biasing distribution λ?
4. Train a binary classifier of the covariate shifted problem using the weights obtained previously and report the accuracy. Note - you will need to modify the training loop slightly such that you can compute the gradient of a weighted sum of losses.

1. The idea is that we want to estimate p(y|x) with labeled data(xi,yi). However, samples are drawn from q(x) rather than p(x). We can show that we can reweight the data to correct covariate shift.

   $$\int p(x)f(x)dx = \int q(x)f(x)\frac{p(x)}{q(x)}dx = \int q(x)f(x)\alpha(x)dx$$

   $$\alpha(x) = \frac{p(x)}{q(x)} = \exp(f(x))$$

   the weighting is

   $$\exp(f(x))$$

In [20]:
```
#2
def train_with_weight(weight, train_iter, test_iter, loss, num_epochs, b
atch_size, lr=None):
    net = getnet()
    trainer = gluon.Trainer(net.collect_params(), 'adam', {'learning_rat
e': lr})
    for epoch in range(num_epochs):
        train_l_sum, train_acc_sum, n = 0.0, 0.0, 0
        for X, y in train_iter:
            with autograd.record():
                y_hat = net(X)
                l = loss(y_hat, y).sum() * weight
            l.backward()
            trainer.step(batch_size)
            y = y.astype('float32')
            train_l_sum += l.asscalar()
            train_acc_sum += (predict(y_hat) == y).sum().asscalar()
            n += y.size
        test_acc = get_accuracy(test_iter, net)
        print('epoch %d, loss %.4f, train acc %.3f, test acc %.3f'
              % (epoch + 1, train_l_sum / n, train_acc_sum / n, test_acc
))
    return net

biased_dataset = splitted_data[1]

train_test_mix = []
for i in range(len(biased_dataset)):
    train_test_mix.append((biased_dataset[i][0].astype('float32').reshap
e(1, 784), nd.array([-1])))
for i in range(len(new_test)):
    train_test_mix.append((new_test[i][0].astype('float32').reshape(1, 7
84), nd.array([1])))

num_epochs, lr = 3, 0.05
loss = gloss.LogisticLoss()
result = train_with_weight(1/3, train_test_mix, new_test, loss, num_epoc
hs, 256, lr) #2000/6000 = 1/3
```

```
epoch 1, loss 0.4407, train acc 1.000, test acc 0.500
epoch 2, loss 33.5446, train acc 0.999, test acc 0.500
epoch 3, loss 35.8785, train acc 0.999, test acc 0.500
```

we mixed biased traning dataset with unbiased test dataset. We got 99% accuracy on training dataset, which means that we can distinguish p(x) from q(x).

In [25]:
```
#3
def train_with_weight2(model, train_iter, test_iter, loss, num_epochs, b
atch_size, lr=None):
    net = getnet()
    trainer = gluon.Trainer(net.collect_params(), 'adam', {'learning_rat
e': lr})
    for epoch in range(num_epochs):
        train_l_sum, train_acc_sum, n = 0.0, 0.0, 0
        for X, y in train_iter:
            with autograd.record():
                y_hat = net(X)
#                 print("exp(x)", nd.exp(model(X)))
#                 l = nd.exp(model(X)).sum().asscalar() * loss(y_hat,
 y).sum()
                l = min(nd.exp(nd.sigmoid(model(X))).sum().asscalar(), 1
00) * loss(y_hat, y).sum()
                #Above: we are reweighting by βi  = min(exp(f(xi)),100)
 to correct the covariate shift.
            l.backward()
            trainer.step(batch_size)
            y = y.astype('float32')
            train_l_sum += l.asscalar()
            train_acc_sum += (predict(y_hat) == y).sum().asscalar()
            n += y.size
        test_acc = get_accuracy(test_iter, net)
        print('epoch %d, loss %.4f, train acc %.3f, test acc %.3f'
              % (epoch + 1, train_l_sum / n, train_acc_sum / n, test_acc
))
    return net
```

In [26]:
```
#4
num_epochs, lr = 5, 0.5
train_with_weight2(result, splitted_data[1], new_test, loss, num_epochs,
256, lr)
```

```
epoch 1, loss 28.3563, train acc 1.000, test acc 0.500
epoch 2, loss 723.1970, train acc 0.999, test acc 0.500
epoch 3, loss 1469.8216, train acc 0.998, test acc 0.500
epoch 4, loss 1109.3656, train acc 0.998, test acc 0.500
epoch 5, loss 2033.0729, train acc 0.997, test acc 0.500
```

Out[26]:
```
Sequential(
  (0): Dense(784 -> 1, linear)
)
```

In [ ]: