Formulas

Stat 151A, Fall 2017

December 6, 2017

1.
$$\frac{RSS}{\sigma^2} \sim \chi^2_{n-p-1}.$$

2. If $\hat{\beta}$ is the ols estimator then (under assumptions that you must know)
$$\frac{\hat{\beta}_j - \beta_j}{s.e(\hat{\beta}_j)} \sim t_{n-p-1}$$

3. For testing a reduced model m against the full model M:
$$T := \frac{(RSS(m) - RSS(M))/(p-q)}{RSS(M)/(n-p-1)} \sim F_{p-q,n-p-1},$$
q is the number of variables in the reduced model without counting the intercept.

4. In one way ANOVA, if $\mu_1 = \cdots = \mu_t$, then
$$T = \frac{\sum_{i=1}^t n_i(\bar{y}_i - \bar{y})^2/(t-1)}{\sum_{i=1}^t \sum_{j=1}^{n_i}(y_{ij} - \bar{y}_i)^2/(n-t)}$$
ha F-distribution with $t-1$ and $n-t$ degrees of freedom.

5. Let $L \in \mathbb{R}^{q\times(p+1)}$ be a full rank matrix with rank $q \leq p+1$. If $L\beta = c$, for $c \in \mathbb{R}^q$, then
$$\frac{(L\hat{\beta} - c)^T \left[L(X^TX)^{-1}L^T\right]^{-1}(L\hat{\beta} - c)}{q\,\hat{\sigma}^2} \sim F_{q,n-p-1}.$$

6.
$$RSS_{[i]} = RSS - \frac{\hat{e}_i^2}{1-h_i}.$$

7. Suppose Z is a random vector with mean μ and covariance matrix Σ. Then
$$\mathbb{E}(Z^TAZ) = tr(A\Sigma) + \mu^TA\mu. \qquad (1)$$

8.
$$\hat{\beta}_{[i]} = \hat{\beta} - \frac{\hat{e}_i}{1-h_i}(X^T X)^{-1} x_i. \tag{2}$$

9.
$$\hat{e}_{[i]} \;=\; \frac{\hat{e}_i}{1\;-\;h_i}. \tag{3}$$

10.
$$t_i = \frac{\hat{e}_{[i]}\sqrt{1-h_i}}{\sqrt{RSS_{[i]}/(n-p-2)}} = \frac{\hat{e}_i}{\sqrt{RSS_{[i]}/(n-p-2)}\sqrt{1-h_i}}.$$

11.
$$r_i = \frac{\hat{e}_i}{\sqrt{RSS/(n-p-1)}\sqrt{1-h_i}}.$$

12.
$$C_i = r_i^2 \frac{h_i}{(1-h_i)(p+1)} = \frac{(\hat{\beta}-\hat{\beta}_{[i]})^T X^T X (\hat{\beta}-\hat{\beta}_{[i]})}{(p+1)\hat{\sigma}^2}.$$

13.
$$cov(AZ) \;=\; A\, cov(Z)\, A^T$$

14.
$$f(x;\theta_i,\phi_i) := h(x,\phi_i)\exp\left(\frac{x\theta_i - b(\theta_i)}{a(\phi_i)}\right).$$

15.
$$var(y_i) = b''(\theta_i)a(\phi_i).$$

16.
$$\beta^{(m+1)} = \beta^{(m)} - \left(H\ell(\beta^{(m)})\right)^{-1}\nabla\ell(\beta^{(m)})$$

17. Logistic
$$\begin{array}{rcl} \nabla\ell(\beta) & = & \sum_{i=1}^n (y_i - p_i)(1, x_{i1}, \ldots, x_{ip})^T \\ H\ell(\beta) & = & -\sum_{i=1}^n p_i(1-p_i)(1, x_{i1}, \ldots, x_{ip})^T(1, x_{i1}, \ldots, x_{ip}). \end{array}$$

18.
$$\begin{array}{lcl} \beta^{(m+1)} & = & (X^T W X)^{-1} X^T W Z \\ Z & = & X\beta^{(m)} + W^{-1}(Y-p). \end{array}$$

19.
$$\begin{array}{lcl} RSS(j,c) & = & n_1\bar{p}_1(1-\bar{p}_1) + n_2\bar{p}_2(1-\bar{p}_2), \\ \text{Cross–entropy or Deviance} & : & -2n_1\left(\bar{p}_1\log\bar{p}_1 + (1-\bar{p}_1)\log(1-\bar{p}_1)\right) \\ \text{Misclassification Error} & : & n_1\min(\bar{p}_1, 1-\bar{p}_1). \end{array}$$

20.
$$\begin{array}{lcl} \nabla_x(b^T x) & = & b \\ \nabla_x(\frac{1}{2}x^T A x) & = & A\,x. \end{array}$$