Lecture 8

September 18, 2018

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- An obvious way to test this hypothesis is to look at the value of $|\hat{\beta}_1|$ and then to reject H_0 if $|\hat{\beta}_1| > 0$ is large.
- But how large?

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- Such a study requires some distributional assumptions on the errors e_1, \ldots, e_n .
- ► The most standard assumption on the errors is that e_1, \ldots, e_n are independently distributed according to the normal distribution with mean zero and variance σ^2 .
- This is written in multivariate normal notation as $e \sim N(0, \sigma^2 I_n)$.

A random vector $U = (U1, \ldots, U_p)^T$ is said to have the multivariate normal distribution with parameters $\mu(p \times 1 \text{ vector})$ and $\Sigma(p \times p \text{ matrix})$ if the joint density of U_1, \ldots, U_p is given by

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- ▶ Here |Σ| denotes the determinant of Σ.
- ▶ We use the notation $U \sim N_p(\mu, \Sigma)$ to express that U is multivariate normal with parameters μ and Σ .

An important example of the multivariate normal distribution occurs when U_1, \ldots, U_p are independently distributed according to the normal distribution with mean 0 and variance σ^2 . In this case, it is easy to show $U = (U_1, \ldots, U_p)^T \sim N_p(0, \sigma^2 I_p)$.

The most important properties of the multivariate normal distribution are summarized below:

▶ When p = 1, this is just the usual normal distribution.

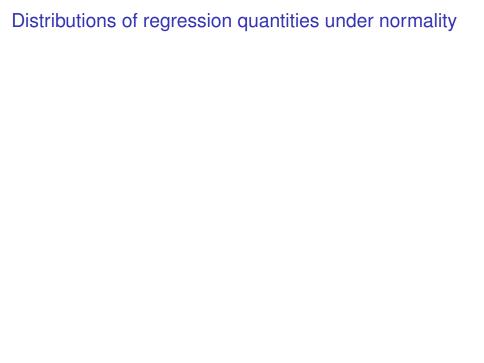
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- Every linear function is also multivariate normal: $a + AU \sim N(a + A\mu, A\Sigma A^T)$.
- Suppose $U \sim N_p(\mu, I)$ and A is a $p \times p$ symmetric and idempotent (symmetric means $A^T = A$ and idempotent means $A^2 = A$) matrix. Then $(U \mu)^T A(U \mu)$ has the chi-squared distribution with degrees of freedom equal to the rank of A.



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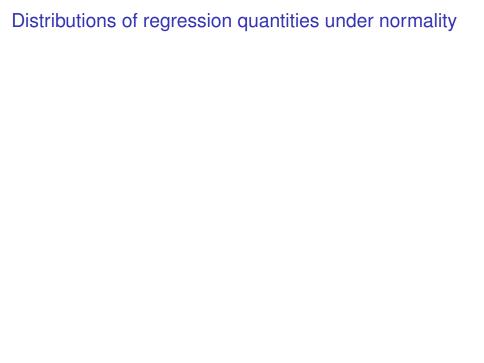
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- ▶ **Distribution of Fitted Values:** Y = HY. Thus $\mathbb{E} \hat{Y} = H\mathbb{E}(Y) = HX\beta = X\beta$. Also $Cov(\hat{Y}) = Cov(HY) = \sigma^2 H$. Therefore $\hat{Y} \sim N_n(X, \sigma^2 H)$.



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- Because $X^TH = (HX)^T = X^T$, we conclude that $\hat{\beta}$ and \hat{e} are independent.
- Also check that \hat{Y} and \hat{e} are independent.

► Recall

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▶ Because $e/\sigma \sim N_n(0, I)$ and I - H is symmetric and idempotent with rank n - p - 1, we have

$$\frac{RSS}{\sigma^2} \sim \chi^2_{n-p-1}.$$

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The are two equivalent ways of testing this hypothesis. One is the t-test which we shall study today. The other is the F-test which we shall look at in the next class.



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• Under the null hypothesis, when $\beta_j = 0$, we thus have

$$\frac{\hat{eta}_j}{\sigma\sqrt{v_i}}\sim N(0,1).$$

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Moreover, the numerator and the denominator are independent.

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Note that when n - p - 1 is large, the *t*-distribution is almost the same as a standard normal distribution.