

Formulas

Stat 151A, Fall 2018

December 4, 2018

1.

$$\frac{RSS}{\sigma^2} \sim \chi_{n-p-1}^2.$$

2. If $\hat{\beta}$ is the ols estimator then (under assumptions that you must know)

$$\frac{\hat{\beta}_j - \beta_j}{s.e(\hat{\beta}_j)} \sim t_{n-p-1}$$

3. For testing a reduced model m against the full model M :

$$T := \frac{(RSS(m) - RSS(M))/(p - q)}{RSS(M)/(n - p - 1)} \sim F_{p-q, n-p-1},$$

q is the number of variables in the reduced model without counting the intercept.

4. In one way ANOVA, if $\mu_1 = \dots = \mu_t$, then

$$T = \frac{\sum_{i=1}^t n_i (\bar{y}_i - \bar{y})^2 / (t - 1)}{\sum_{i=1}^t \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2 / (n - t)}$$

has F -distribution with $t - 1$ and $n - t$ degrees of freedom.

5. Suppose Z is a random vector with mean μ and covariance matrix Σ . Then

$$\mathbb{E}(Z^T A Z) = \text{tr}(A \Sigma) + \mu^T A \mu. \quad (1)$$

6.

$$\hat{\beta}_{[i]} = \hat{\beta} - \frac{\hat{e}_i}{1 - h_i} (X^T X)^{-1} x_i. \quad (2)$$

7.

$$\hat{e}_{[i]} = \frac{\hat{e}_i}{1 - h_i}. \quad (3)$$

8.

$$t_i = \frac{\hat{e}_{[i]} \sqrt{1 - h_i}}{\sqrt{RSS_{[i]} / (n - p - 2)}} = \frac{\hat{e}_i}{\sqrt{RSS_{[i]} / (n - p - 2)} \sqrt{1 - h_i}}.$$

9.

$$r_i = \frac{\hat{e}_i}{\sqrt{RSS/(n-p-1)}\sqrt{1-h_i}}.$$

10.

$$C_i = r_i^2 \frac{h_i}{(1-h_i)(p+1)} = \frac{(\hat{\beta} - \hat{\beta}_{[i]})^T X^T X (\hat{\beta} - \hat{\beta}_{[i]})}{(p+1)\hat{\sigma}^2}.$$

11.

$$\text{cov}(AZ) = A \text{cov}(Z) A^T$$

12.

$$f(x; \theta_i, \phi_i) := h(x, \phi_i) \exp \left(\frac{x\theta_i - b(\theta_i)}{a(\phi_i)} \right).$$

13.

$$\text{var}(y_i) = b''(\theta_i)a(\phi_i).$$

14.

$$\beta^{(m+1)} = \beta^{(m)} - (H\ell(\beta^{(m)}))^{-1} \nabla \ell(\beta^{(m)})$$

15. Logistic

$$\begin{aligned} \nabla \ell(\beta) &= \sum_{i=1}^n (y_i - p_i) (1, x_{i1}, \dots, x_{ip})^T \\ H\ell(\beta) &= - \sum_{i=1}^n p_i (1 - p_i) (1, x_{i1}, \dots, x_{ip})^T (1, x_{i1}, \dots, x_{ip}). \end{aligned}$$

16.

$$\begin{aligned} \beta^{(m+1)} &= (X^T W X)^{-1} X^T W Z \\ Z &= X \beta^{(m)} + W^{-1} (Y - p). \end{aligned}$$

17.

$$\begin{aligned} RSS(j, c) &= n_1 \bar{p}_1 (1 - \bar{p}_1) + n_2 \bar{p}_2 (1 - \bar{p}_2), \\ \text{Cross-entropy or Deviance} &: -2n_1 (\bar{p}_1 \log \bar{p}_1 + (1 - \bar{p}_1) \log(1 - \bar{p}_1)) \\ \text{Misclassification Error} &: n_1 \min(\bar{p}_1, 1 - \bar{p}_1). \end{aligned}$$

18.

$$\begin{aligned} \nabla_x (b^T x) &= b \\ \nabla_x (\tfrac{1}{2} x^T A x) &= A x. \end{aligned}$$