Formulas

Stat 151A, Fall 2018

October 29, 2018

1.
$$\frac{RSS}{\sigma^2} \sim \chi^2_{n-p-1}.$$

2. If $\hat{\beta}$ is the ols estimator then (under assumptions that you must know)
$$\frac{\hat{\beta}_j - \beta_j}{s.e(\hat{\beta}_j)} \sim t_{n-p-1}$$

3. For testing a reduced model m against the full model M:
$$T := \frac{(RSS(m) - RSS(M))/(p-q)}{RSS(M)/(n-p-1)} \ \sim \ F_{p-q,n-p-1}.$$

4. In one way ANOVA, if $\mu_1 = \cdots = \mu_t$, then
$$T = \frac{\sum_{i=1}^t n_i(\bar{y}_i - \bar{y})^2/(t-1)}{\sum_{i=1}^t \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2/(n-t)}$$
ha F-distribution with $t-1$ and $n-t$ degrees of freedom.

5.
$$cov(AZ) \ = \ A\, cov(Z)\, A^T$$

6.
$$\begin{aligned} \nabla_x(b^T x) \quad &= \quad b \\ \nabla_x(\tfrac{1}{2}x^T A x) \quad &= \quad A\, x. \end{aligned}$$

7.
$$\hat{\beta}_1 \ = \ \frac{\sum_{i=1}^n (x_i - \bar{x}) y_i}{\sum_{i=1}^n (x_i - \bar{x})^2}$$
and
$$\hat{\beta}_0 \ = \ \bar{y} - \hat{\beta}_1 \bar{x},$$

8.
$$x_0^T \hat{\beta} \pm t_{n-p-1}^{(\alpha/2)} \hat{\sigma} \sqrt{1 + x_0^T (X^T X)^{-1} x_0}.$$

9. Suppose Z is a random vector with mean μ and covariance matrix Σ. Then

$$\mathbb{E}(Z^T A Z) = tr(A\Sigma) + \mu^T A \mu. \tag{1}$$

10.

$$\hat{\beta}_{[i]} = \hat{\beta} - \frac{\hat{e}_i}{1 - h_i}(X^T X)^{-1} x_i. \tag{2}$$

11.

$$\hat{e}_{[i]} \; = \; \frac{\hat{e}_i}{1 \; - \; h_i}. \tag{3}$$

12.

$$t_i = \frac{\hat{e}_{[i]}\sqrt{1 - h_i}}{\sqrt{RSS_{[i]}/(n - p - 2)}} = \frac{\hat{e}_i}{\sqrt{RSS_{[i]}/(n - p - 2)}\sqrt{1 - h_i}}.$$

13.

$$r_i = \frac{\hat{e}_i}{\sqrt{RSS/(n - p - 1)}\sqrt{1 - h_i}}.$$

14.

$$C_i = r_i^2 \frac{h_i}{(1 - h_i)(p + 1)} = \frac{(\hat{\beta} - \hat{\beta}_{[i]})^T X^T X (\hat{\beta} - \hat{\beta}_{[i]})}{(p + 1)\hat{\sigma}^2}.$$

15.

$$cov(AZ) \; = \; A \, cov(Z) \, A^T$$

16.

$$AIC(m) = n \log\left(\frac{RSS(m)}{n}\right) + n \log(2\pi e) + 2(1 + p(m)))$$

17.

$$BIC(m) := n \log\left(\frac{RSS(m)}{n}\right) + (\log n)(1 + p(m)).$$

18.

$$C_p(m) := \frac{RSS(m)}{\hat{\sigma}^2} - (n - 2(1 + p(m))).$$

$$\mathrm{RSS}_{[i]} = \mathrm{RSS} - \frac{\hat{e}_i^2}{1 - h_i}$$