Algorithm

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1 Initialization step

1) Check that:
• D is connected
• $g(x)$ is continuous and differentiable everywhere in D
• $h(x) = ln(g(x))$ is concave everywhere in D

2) Initialize T_k (vector with k elements):
• $T_k = \{xi; i = 1, ..., k\}$, where $x_1 \leq \ldots \leq x_k$ are the k abscissae in D where we will evaluate $h(x)$ and $h'(x)$
• If D unbounded on the left, chose x_1 s.t. $h'(x_1) > 0$
• If D unbounded on the right, chose x_k s.t. $h'(x_k) < 0$

3) Evaluate $h(x)$ and $h'(x)$ on T_k and store these as two-length k vectors, say h_x and h_prime_x

4) Calculate z (vector with k + 1 elements):
• z_0 = lower bound of D (or $-\infty$ if D is not bounded below)
• For $j = 1, \ldots k - 1$,

$$z_j = \frac{h(x_{j+1}) - h(x_j) - x_{j+1}h'(x_{j+1}) + x_j h'(x_j)}{h'(x_j) - h'(x_{j+1})}$$

(these are the points at which the tangents to $h(x)$ at x_j and x_{j+1} intersect)
• z_k = upper bound of D (or ∞ if D is not bounded above)

5) Find u_k (this is the piecewise linear upper hull formed by tangents to $h(x)$ at T_k):
• $u_k(x) = h(xj) + (x - x_j)h'(x_j)$, where $x \in [z_{j-1}, z_j]$ and $j = 1, ..., k$
• NB: $\exp(u_k(x))$ is the rejection envelope on T_k

6) Find s_k:

• $s_k(x) = \frac{exp(u_k(x))}{\int \exp(u_k(x))dx}$

7) Find l_k (this is the piecewise linear lower hull formed by connecting adjacent points on $h(x)$ where T_k is evaluated)
• $l_k(x) = \frac{(x_{j+1}-x)h(x_j)+(x-x_j)h(x_{j+1})}{x_{j+1}-x_j}$ where $x \in [x_j, x_{j+1}]$ and $j = 1, ..., k-1$
• For $x < x_1$ or $x > x_k$, define $l_k(x) = -\infty$
• NB: $\exp(l_k(x))$ is the squeezing function on T_k

2 Sampling step

1) Sample a value $x^*$from $s_k(x)$
2) Sample a value w independently from Unif(0, 1)
3) Perform the test:
• If $w \leq \exp(l_k(x^*) - u_k(x^*))$:
– Accept x^*
• Else:
– Evaluate $h(x^*)$ and $h'(x^*)$
– If $w \leq h(x^*) - u_k(x^*)$:
-Accept x^*
– Else:
-Reject x^*

3 Updating Step

If $h(x^*)$ and $h'(x^*)$ were evaluated in Sampling Step:
– Include x^*in T_k to form T_{k+1}
– Relabel the x_i in T_k in ascending order
– Construct new functions $u_{k+1}(x)$, $s_{k+1}(x)$, and $l_{k+1}(x)$
– Increment k
– Return to Sampling Step if n points have not been sampled yet
• Else:
No update necessary, repeat Sampling Step if n points have not been sampled yet 2.