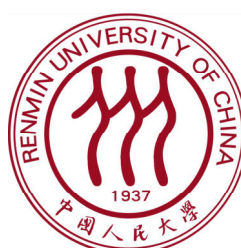# 2024 杭州热核及其相关问题研讨会

## Workshop on Heat Kernel and Related Topics in Hangzhou 2024

# 会议手册

## Conference Manual

浙江工业大学·理学院·应用数学系          中国人民大学·数学学院

**Zhejiang University of Technology, School of Science, Department of applied mathematics**
**Renmin University, School of Mathematics**

杭州 · 2024 年 3 月 22 日- 2024 年 3 月 25 日
**Hangzhou, March 22-25, 2024**

# 2024 杭州热核及其相关问题研讨会

## Workshop on Heat Kernel and Related Topics in Hangzhou 2024

## 会议时间┃Date：

2024.03.22—2024.03.25

## 会议地址┃Address：

杭州华北饭店┃Huabei Hotel

## 报到信息┃Registration Information：

The registration will open from 3.22, Friday 12:00 am, at the hall in first floor of East Building, Huabel Hotel ｜报告在华北饭店东楼一楼大厅，从 3 月 22 日周五中午 12 点整开始.

## 会议联系人┃Organizing committee：

| 曹　军┃ | CAO Jun | caojun1860@zjut.edu.cn, | 15168256040 |
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## 会议资助┃Funded by：

# Conference Agenda｜会议日程安排

**Day1, 3.23 (Saturday)** (Address: West lake Hall in 6th floor of East Building｜华北饭店东楼六楼西湖厅)

| | |
|---|---|
| **Morning｜上午　(08:30-12:00)** | |
| Host｜主持人：JIN Yongyang｜金永阳(Zhejiang University of Technology｜浙江工业大学) | |
| 08:30--09:15 | GRIGOR'YAN Alexander｜格里戈里安·亚历山大 (Bielefeld University) |
| Title | Tail estimates of heat kernels for jump processes |
| Host｜主持人：WU Huongxiong｜伍火熊(Xiamen University｜厦门大学) | |
| 09:20--10:05 | JIANG Renjin｜蒋仁进(Capital Normal Univeristy｜首都师范大学) |
| Title | Application of heat kernel estimates to Riesz transform on general manifolds |
| 10:05-10:25　Photo & Tea break｜照相与茶歇 | |
| Host｜主持人：CHEN Peng｜陈鹏(Sun Yat-Sen University｜中山大学) | |
| 10:25--11:10 | SUN Yuhua　｜孙玉华(Nankai University｜南开大学) |
| Title | Sharp criteria for nonlocal elliptic inequalities on manifolds |
| Host｜主持人：KONG Shilei｜孔诗磊(Sichuang University｜四川大学) | |
| 11:15--12:00 | GU Qingsong｜顾庆松(Nanjing Univeristy｜南京大学) |
| Title | Boundary value problems for harmonic functions on domains in p.c.f. self-similar sets |
| 12:00-14:00　Lunch (Wanghu Hall in third floor of East Building) & Rest｜午餐(东楼三楼望湖厅)与休息 | |
| **Afternoon｜下午　(14:00-16:20)** | |
| Host｜主持人：RUAN Huojun｜阮火军(Zhejiang University｜浙江大学) | |
| 14:00--14:45 | HU Jiaxin｜胡家信(Tsinghua University｜清华大学) |
| Title | Strong elliptic Harnack inequality and condition UJS |
| Host｜主持人：GUO Changyu｜郭常予(Shangdong University｜山东大学) | |
| 14:50--15:35 | QIU Hua｜邱华(Nanjing University｜南京大学) |
| Title | Dirichlet forms and Laplacians on fractals |
| 15:35-15:55　Tea break｜茶歇 | |
| Host｜主持人：RUAN Jianmiao｜阮建苗(Zhejiang International Studies University｜浙江外国语学院) | |
| 15:55--16:40 | HUANG Yi｜黄益(Nanjing Normal University｜南京师范大学) |
| Title | On gradient blowup profiles for nonlinear heat equations |
| Host｜主持人：FANG Qiquan｜房启全(Zhejiang University of Science and Technology｜浙江科技大学) | |
| 16:45--17:30 | ZHANG Junqiang｜张俊强(China University of Mining and Technology-Beijing 中国矿业大学) |
| Title | On odd-normal numbers |
| Host｜主持人：LI Hongliang｜李宏亮(Zhejiang International Studies University｜浙江外国语学院) | |
| 17:35--18:20 | QIU Yanqi｜邱彦奇(Hangzhou Institute for Advanced Study｜国科大杭州高等研究院) |
| Title | Mandelbrot Cascades: critical moments, Rajchman measures and Sobolov smoothness |
| 18:40　Banquet (Wanghu Hall in third floor of East Building)｜晚宴（东楼三楼望湖厅） | |

**Day 2, 3.24 (Sunday)** (Address: West lake Hall in 6<sup>th</sup> floor of East Building｜华北饭店东楼六楼西湖厅)

| | |
|---|---|
| **Morning｜上午　（08:30-12:00）** | |
| **Host｜主持人: HUANG Xueping｜黄学平**(Nanjing University Of Information Science & technology｜南京信息工程大学) | |
| 08:30--09:15 | **LIN Yong｜林勇(**Tsinghua University｜清华大学) |
| Title | Heat kernel and Green function on subgraphs of a complete graph |
| **Host｜主持人: ZHANG Chao｜张超**(Zhejiang Gongshang University｜浙江工商大学) | |
| 09:20--10:05 | **HU Eryan｜胡二彦**(Tianjin University｜天津大学应用数学中心) |
| Title | Lower estimates and Hölder regularity of the heat kernels for non-local Dirichlet form |
| 10:05-10:25　Tea break｜茶歇 | |
| **Host｜主持人：WANG Meng｜王梦**(Zhejiang University　浙江大学) | |
| 10:25--11:10 | **XI Yakun｜席亚昆**(Zhejiang University｜浙江大学) |
| Title | Weyl law, Kuznecov sum, and inverse spectral problems |
| **Host｜主持人：WU Yurong｜吴玉荣**(Zhejiang University of Technology｜浙江工业大学) | |
| 11:15--12:00 | **GAO Jin｜高晋**(Hangzhou Normal University｜杭州师范大学) |
| Title | Heat kernel-based p-energy norms on metric measure space |
| 12:00-14:00　Lunch (Wanghu Hall in third floor of East Building)& Rest｜午餐(东楼三楼望湖厅)与休息 | |
| **Afternoon** | |
| Free Discussion | |

## Titles and abstracts of talks | 报告题目与摘要

## Day 1 (3.23)

### Tail estimates of heat kernels for jump processes

GRIGOR'YAN Alexander ｜格里戈里安·亚历山大 (Bielefeld University)

**Abstract:** We present upper bounds of the heat kernel for a jump type Dirichlet form on an Ahlfors-regular metric measure space, where the off-diagonal term depends on the $L^q$ tail estimate of the jump kernel. This is a joint work with Jiaxin Hu and Eryan Hu.

### Application of heat kernel estimates to Riesz transform on general manifolds

JIANG Renjin｜ 蒋仁进 (Capital Normal Univeristy)

**Abstract:** In this talk, we shall report some progress on Riesz transform on general manifolds. In   particular we shall present the behavior of Riesz transform on guling manifolds, stability or instability result under the gluing operator are shown on manifolds with or without doubling volume property.

### Sharp criteria for nonlocal elliptic inequalities on manifolds

SUN Yuhua ｜孙玉华 (Nankai Univeristy)

**Abstract:** We investigate the existence and nonexistence to nonlocal differential inequalities on manifolds. This talk is based on joint work with Qingsong Gu, and Xueping Huang.

# Boundary value problems for harmonic functions on domains in p.c.f. self-similar sets

GU Qingsong｜顾庆松（Nanjing University）

**Abstract:** We study the boundary value problems for harmonic functions on open connected subsets of post-critically finite (p.c.f.) self-similar sets, on which the Laplacian is defined through a self-similar local regular Dirichlet form. For a p.c.f. self-similar set K, we prove that for any open connected subset $\Omega$ of K whose "geometric" boundary is a graph-directed self-similar set, there exists a finite number of matrices called flux transfer matrices whose products generate the hitting probability from a point in $\Omega$ to the "resistance" boundary $\partial\Omega$. The harmonic functions on $\Omega$ can be expressed by integrating functions on $\partial\Omega$ against the probability measures. Furthermore, we obtain a two-sided estimate of the energy of a harmonic function in terms of its values on $\partial\Omega$. This generalizes the known results on Sierpinski gasket to p.c.f. self-similar sets.

The talk is based on a joint work with Hua Qiu.

# Strong elliptic Harnack inequality and condition UJS

HU Jiaxin｜胡家信 (Tsinghua University)

**Abstract:** We study the strong elliptic Harnack inequality for a regular resurrected Dirichlet form on a metric space. Condition (UJS) plays a role, and we give an equivalent condition for (UJS). Joint with Zhenyu Yu.

# Dirichlet forms and Laplacians on fractals

QIU Hua｜邱华（Nanjing University）

**Abstract:** The analysis on fractals mainly investigates the theory of Laplacians on fractals, closely related to which are the Brownian motion (from a probabilistic perspective) and Dirichlet form (from an analytical perpective) theories on fractals. In this talk, I will introduce our recent progress on the existence and uniqueness problem of Laplacians on two types of fractals. This is based on some joint works with Shiping Cao.

# On gradient blowup profiles for nonlinear heat equations

HUANG Yi ｜ 黄益  (Nanjing Normal Univeristy)

**Abstract:** In a recent work, Duong, Ghoul and Zaag determined the gradient profile for blowup solutions of standard semilinear heat equation with power nonlinearities in the (supposed to be) generic case. Their method refines the constructive techniques introduced by Bricmont and Kupiainen and further developed by Merle and Zaag. In this paper, we extend their refinement to the problem about the reconnection of vortex lines with the boundary in a type-II superconductor under planar approximation, a physical model derived by Chapman, Hunton and Ockendon featuring the finite time quenching for the nonlinear heat equation

$$\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} + e^{-h} - \frac{1}{h^\beta}, \quad \beta > 0$$

subject to initial boundary value conditions
$$h(\cdot, 0) = h_0 > 0, \quad h(\pm 1, t) = 1.$$
We derive the intermediate extinction profile with refined asymptotics, and with extinction time T and extinction point 0, the gradient profile behaves as $x \to 0$ like

$$\lim_{t \to T} (\nabla h)(x,t) \quad \sim \quad \frac{1}{\sqrt{2\beta}} \frac{x}{|x|} \frac{1}{\sqrt{|\log|x||}} \left[ \frac{(\beta+1)^2}{8\beta} \frac{|x|^2}{|\log|x||} \right]^{\frac{1}{\beta+1} - \frac{1}{2}},$$

agreeing with the gradient of the extinction profile previously derived by Merle and Zaag. Our result holds with general boundary conditions and in higher dimensions.

# On odd-normal numbers

ZHANG Junqiang｜张俊强 (China University of Mining and Technology-Beijing)

**Abstract:** A real number x is considered normal in an integer base $b \geq 2$ if its digit expansion in this base is "equitable", ensuring that for each $k \geq 1$, every ordered sequence of k digits from $\{0, 1, \ldots, b-1\}$ occurs in the digit expansion of x with the same limiting frequency. Borel's classical result asserts that Lebesgue-almost every $x \in \mathbb{R}$ is normal in every base $b \geq 2$.

In this talk, we study the measure-theoretic properties of Lebesgue-null sets containing numbers that are normal only in certain bases. We consider the set $N(O, E)$ of reals that are normal in odd bases but not in even ones. This set has full Hausdorff dimension but zero Fourier dimension. The latter condition means that $N(O, E)$ cannot support a probability measure whose Fourier transform has power decay at infinity. Our main result is that $N(O, E)$ supports a Rajchman measure $\mu$, whose Fourier transform $\widehat{\mu}(\xi)$ approaches 0 as $|\xi| \to \infty$ by definiton, albeit slower than any negative power of $|\xi|$. Moreover, the decay rate of $\widehat{\mu}$ is essentially optimal, subject to the constraints of its support. The methods draw inspiration from the number-theoretic results of Schmidt and a construction of Lyons. As a consequence, $N(O, E)$ emerges as a set of multiplicity, in the sense of Fourier analysis. This addresses a question posed by Kahane and Salem in the special case of $N(O, E)$.

# Mandelbrot Cascades: critical moments, Rajchman measures and Sobolov smoothness

QIU Yanqi｜邱彦奇 （Hangzhou Institute for Advanced Study）

**Abstract:** We introduce a method for estimating weighted sum of random variables on trees. This method on the one hand will allow us to deal with the asymptotic order of moments of Mandelbrot Cascades at critical exponents, and on the other hand will allow us to establish Rajchman property of the random measure arising from the Mandelbrot Cascades, as well as the Sobolev smoothness of the self-convolution of the random measure. The talk is based on joint work with Xinxin Chen, Yong Han and Zipeng Wang.

## Day 2 (3.24)

## Heat kernel and Green function on subgraphs of a complete graph

LIN Yong｜林勇 (Tsinghua University)

**Abstract:** We derive expression for the subgraph heat kernel and compute the coefficients of the expansion. By using the heat kernel expansions for the Laplacian on a subgraph of a complete graph, we obtain formal expansions for the Green function of the Laplacian. These are based on the joint works with Ngai and Yau.

## Lower estimates and Hölder regularity of the heat kernels for non-local Dirichlet form on doubling spaces

HU Eryan｜胡二彦（Tianjing University）

**Abstract:** We study the heat kernel of a regular symmetric non-local Dirichlet form on a metric space with doubling measure, in particular, a connection between the properties of the jump measure and the long time behaviour of the heat kernel. Under the conditions: Poincaré inequality, generalized capacity condition, tail estimate of jump measure and so on, we obtain the Hölder regularity and lower estimates of the heat kernel. Moreover, we prove that these conditions are also necessary.

## Weyl law, Kuznecov sum, and inverse spectral problems

XI Yakun｜席亚昆（Zhejiang University）

**Abstract:** The study of (pointwise) Weyl asymptotics is one of the central problems in spectral geometry and harmonic analysis on Riemannian manifolds. The Kuznecov formula is a generalized version of the pointwise Weyl formula. We survey our recent works on pointwise Weyl law, (generalized) Kuzecov sum formula, and related inverse problems.

# Heat kernel-based p-energy norms on metric measure space

GAO Jin｜高晋（Hangzhou Normal University）

**Abstract:** We focus on heat kernel-based p-energy norms ($1<p<\infty$) on bounded and unbounded metric measure spaces, in particular, on nested fractals and their blowups. Under weak-monotonicity properties for different types of energies, we generalise the celebrated Bourgain-Brezis-Mironescu (BBM) convergence for $p\neq 2$. When there admits a heat kernel satisfying the two-sided estimates, we establish the equivalence of different p-energy norms and weak-monotonicity properties. Our paper's key result involves verifying various weak- monotonicity properties on nested fractals and their blowups. We achieve this by establishing the equivalence between discrete and integral-type weak-monotonicity. Consequently, many classical results on p-energy norms hold for such fractals, including the BBM convergence and Gagliardo-Nirenberg inequality. It is based on joint work with Zhenyu Yu and Junda Zhang.

# Participants｜参会名单

| 序号 | Name｜姓名 | Institution｜单位 |
|---|---|---|
| 1 | CHEN Jiao （陈焦） | Chongqing Normal Univeristy （重庆师范大学） |
| 2 | CHEN Peng （陈鹏） | Sun Yat-Sen University （中山大学） |
| 3 | FANG Chenglong （房成龙） | Renming University （中国人民大学） |
| 4 | FANG Qiquan （房启全） | Zhejiang University of Science and Technology （浙江科技大学） |
| 5 | FU Xing （付星） | Hubei University （湖北大学） |
| 6 | GAO Jin （高晋） | Hangzhou Normal University （杭州师范大学） |
| 7 | Grigor'yan Alexander （亚历山大·格里戈里安） | Bielefeld University(比勒费尔德大学) |
| 8 | GU Qingsong （顾庆松） | Nanjing Univeristy （南京大学） |
| 9 | GUO Changyu （郭常予） | Shangdong University （山东大学） |
| 10 | HE Ziyi （贺子毅） | Beijing University of Posts and Telecommunications （北京邮电大学） |
| 11 | HU Eryan （胡二彦） | Tianjin University （天津大学应用数学中心） |
| 12 | HU Jiaxin （胡家信） | Tsinghua University （清华大学） |
| 13 | HUANG Xueping （黄学平） | Nanjing University Of Information Science &technology （南京信息工程大学） |
| 14 | HUANG Yi （黄益） | Nanjing Normal University （南京师范大学） |
| 15 | JIANG Renjin （蒋仁进） | Capital Normal Univeristy （首都师范大学） |
| 16 | JIN Dalian （金大廉） | Renming University （中国人民大学） |
| 17 | KONG Shilei （孔诗磊） | Sichuang University （四川大学） |
| 18 | LI Bo （李波） | Jiaxin University （嘉兴大学） |
| 19 | LI Hongliang （李宏亮） | Zhejiang International Studies University （浙江外国语学院） |
| 20 | LIN Yong（林勇） | Tsinghua University （清华大学） |
| 21 | LIU Liguang （刘丽光） | Renming University （中国人民大学） |
| 22 | LIU Yao （刘瑶） | Nankai University （南开大学） |
| 23 | MIAO Changxing （苗长兴） | Institute of Applied Physics and Computational Mathematics （北京应用物理与计算数学研究所） |
| 24 | QIN Yanyu （秦颜玉） | Nankai University （南开大学） |
| 25 | QIN Yongheng （秦永恒） | Nankai University （南开大学） |
| 26 | QIU Hua （邱华） | Nanjing University （南京大学） |
| 27 | QIU Yanqi （邱彦奇） | Hangzhou Institute for Advanced Study （国科大杭州高等研究院） |

| 28 | RUAN  Huojun（阮火军） | Zhejiang University（浙江大学） |
|---|---|---|
| 29 | RUAN Jianmiao（阮建苗） | Zhejiang International Studies University（浙江外国语学院） |
| 30 | SUN Haorui（孙浩瑞） | Nankai University（南开大学） |
| 31 | SUN Yuhua（孙玉华） | Nankai University（南开大学） |
| 32 | TAO Jin （陶金） | Hubei University（湖北大学） |
| 33 | WANG Meng（王梦） | Zhejiang University（浙江大学） |
| 34 | WEN Yepei（温叶培） | Beijing Normal University（北京师范大学） |
| 35 | WU Huoxiong（伍火熊） | Xiamen University（厦门大学） |
| 36 | WU Suqing（吴素青） | Dalian Maritime University （大连海事大学） |
| 37 | XI Yakun （席亚昆） | Zhejiang University（浙江大学） |
| 38 | YAN Xianjie （闫现杰） | Henan University（河南大学） |
| 39 | YU Zhenyu（余振宇） | National University of Defense Technology（国防科技大学） |
| 40 | ZHANG Chao（张超） | Zhejiang Gongshang University （浙江工商大学） |
| 41 | ZHANG Junda（张俊达） | South China University of Technology （华南理工大学） |
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| 43 | ZHANG Lu（张璐） | Shanxi Normal University（陕西师范大学） |
| 44 | ZHANG Yuying （张郁英） | Renming University（中国人民大学） |
| 45 | ZHAO Shiliang（赵世良） | Sichuang University（四川大学） |
| 46 | ZHAO Yongqiang （赵永强） | Westlake University（西湖大学） |
| 47 | CAO Jun（曹军） | Zhejiang University of Technology （浙江工业大学） |
| 48 | DENG Chaohong（邓超红） | Zhejiang University of Technology （浙江工业大学） |
| 49 | FAN Yi （范毅） | Zhejiang University of Technology （浙江工业大学） |
| 50 | GAO Mengyao （高梦瑶） | Zhejiang University of Technology （浙江工业大学） |
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| 52 | SHEN Shoufeng（沈守枫） | Zhejiang University of Technology （浙江工业大学） |
| 53 | TANG Li （唐黎） | Zhejiang University of Technology （浙江工业大学） |
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| 55 | WU Yurong（吴玉荣） | Zhejiang University of Technology |

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| 56 | YU Zhuonan （俞卓楠） | Zhejiang University of Technology (浙江工业大学) |
| 57 | YU Nianxing（余念星） | Zhejiang University of Technology (浙江工业大学) |
| 58 | ZHANG Qishun(张旗顺) | Zhejiang University of Technology (浙江工业大学) |
| 59 | TANG Mingjie （谭明杰） | Hangzhou Institute for Advanced Study (国科大杭州高等研究院) |
| 60 | ZHOU Xianchao （周显潮） | Zhejiang Univeristy of Technology (浙江工业大学) |

## 浙江工业大学•理学院•应用数学系简介

浙江工业大学理学院应用数学系成立于 2000 年，同年信息与计算科学专业招生，2005 年数学与应用数学专业开始招生，2010 年获数学一级学科硕士点、浙江省重点学科，2013 年成立由汪徐家院士任学术负责人的校数学研究中心，2018 年数据科学与大数据技术专业开始招生（与计算机学院合办），2020 年信息与计算科学专业获国家一流本科专业，2021 年数学与应用数学专业获省一流本科专业，2021 年获数学一级学科博士点。

经过 20 年的建设，应用数学系已拥有 1 个一级学科博士点、1 个一级学科硕士点和 3 个本科专业（数学与应用数学、信息与计算科学、数据科学与大数据技术），形成了完整的本硕博培养体系，已具有基础数学、应用数学、非线性科学三个稳定的主干学科方向，并拥有分析与几何、微分方程与动力系统、数据科学、优化控制、可视化计算、金融数学与应用、大学数学教学等 7 个科学教学研究团队。

学科目前拥有专职教师 70 人，其中教授 12 人，副教授 33 人，讲师 26 人，硕士导师 28 人，博士生导师 13 人，博士（后）54 人，省杰青 1 人，省千人计划 1 人，省"151"人才 6 人，省中青年学科带头人 4 人，校杰青 1 人，校优青 7 人。数学学科以中、青年教师为主体，45 岁以下中青年教师占比 80%。经过多年的建设与发展已经形成研究方向明确、职称学历和年龄结构合理的师资队伍。

近五年，应用数学系共承担纵向科研项目 70 项，总经费高达 1307.46 万元，其中国家级纵向科研项目 30 余项，经费达 1113.68 万元。学科科研成果丰硕，近五年共发表论文 202 篇，其中不少重要成果发表在《Geom. Funct. Anal.》、《Memoirs of AMS》、《J. Differ. Geom.》、《Adv. Math.》、《Math. Ann.》、《J. Math. Pures Appl.》、《Transactions of AMS》、《ARMA》、《J. Funct. Anal.》、《SIAM J. Appl. Math.》、《SIAM J. Optim.》等国际顶级期刊。相关论文获得华人数学家大会联盟最佳论文奖（若琳奖），教育部自然科学二等奖 1 项，另有 ESI 高被引论文 11 篇。出版专著和教材 18 部。学科在注重理论研究同时，也积极注重学科交叉融合，并已在控制工程、经济数学、计算机视觉等多个交叉领域产生重要成果，获发明专利 11 项，呈现出显著的社会效益。

# 诚挚欢迎各位专家莅临指导！
# 诚挚邀请各位优秀博士加盟！

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