Model Predictive Control using MATLAB
9: Feasibility, Stability and Optimality

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Overview

1. Feasibility
   - Feasible set
2. Stability
   - Lyapunov Approach
3. Optimality

Feasibility

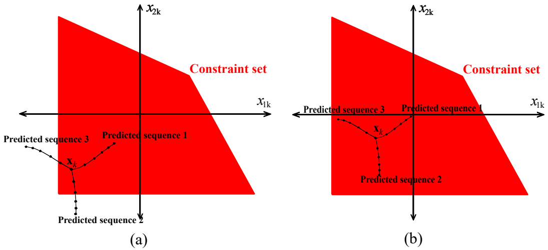
Feasible set

- **Feasible set of control sequences**: $\mathbb{U}_{fk} = \mathbb{U}_f(\mathbf{x}_k)$ is defined as

$$\mathbb{U}_{fk} = \{\mathbf{U}_k \in \mathbb{U}^N : \mathbf{X}_k(\mathbf{x}_k, \mathbf{U}_k) \in \mathbb{X}^{N+1}\} \tag{1}$$

- The number of elements in $\mathbb{U}_{fk} \subseteq \mathbb{U}^N$ decreases when $\mathbf{x}_k$ is closer to the boundary of $\mathbb{X}$.
- The MPC problem is said to be feasible for $\mathbf{x}_k \in \mathbb{X}$, if $\mathbb{U}_{fk}$ is nonempty.
- **Feasible set of states**: $\mathbb{X}_{fk} \subseteq \mathbb{X}$ is defined as

$$\mathbb{X}_{fk} = \{\mathbf{x}_k \in \mathbb{X} : \mathbb{U}_{fk} \neq \phi\} \tag{2}$$



Feasibility

- Let $\mathbb{X}_{fk}$ and $\mathbb{U}_{fk}$ are the feasible set of states and control sequences during time instant k.
- Then the MPC control law is computed by solving the optimization problem

$$\begin{aligned} \underset{\mathbf{U}_k \in \mathbb{U}_{f_k}}{inf} \quad & J_k(\mathbf{x}_k, \mathbf{U}_k) \qquad subject\ to \\ & \mathbf{x}_{i+1|k} = \mathbf{f}(\mathbf{x}_{i|k}, \mathbf{u}_{i|k}), \quad k \in \mathbb{T}, i = k, ..., k+N-1. \end{aligned} \tag{3}$$

- **Persistent feasibility**: feasibility of initial state $\mathbf{x}_0$ guarantees the feasibility of future states $\mathbf{x}_k, k = 1, 2, ..., N_T$,
i.e. $\mathbb{U}_{f0} \neq \phi \implies \mathbb{U}_{fk} \neq \phi, \forall k = 1, 2, ..., N_T$.
- Persistent feasibility depends on the system dynamics, prediction horizon N, and the constrained sets $\mathbb{X}, \mathbb{U}$.

Stability

Stability

- **Lyapunov approach**: design the control scheme in such a way that the optimal cost function becomes a Lyapunov function, i.e. $V_k = J_k^*$ and it satisfies

$$\Delta V = J_{k+1}^*(\mathbf{x}_{k+1}) - J_k^*(\mathbf{x}_k) < 0 \tag{4}$$

- In general for stabilizable LTI systems, by properly selecting the terminal weighting matrix and constraints the value function for the MPC scheme can be made as a Lyapunov function.
- The terminal weighting matrix $\mathbf{Q}_N$ and terminal constraints $\mathbf{F}_{\mathbf{x}_N}, \mathbf{g}_{\mathbf{x}_N}$ can be easily incorporated in the MPC algorithm by adding them in $\mathbf{Q}_\mathbf{X}, \mathbf{F}_\mathbf{X}, \mathbf{g}_\mathbf{X}$.

Optimality

Optimality

- MPC usually results in suboptimal solution.
- As the prediction horizon increases the MPC control law becomes more optimal.
- In general as $N \to N_T$ the control law becomes optimal.
- N is selected based on a trade off between optimality and computation.

Thank you