Model Predictive Control using MATLAB
8: NMPC - Set point tracking

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Overview

1. NMPC: Set-point tracking

2. NMPC: Simulation results

NMPC: Set-point tracking

NMPC: Set-point tracking

- **Set-point tracking problem**: in which the reference $\mathbf{x}_r \neq 0$.
- The reference value or steady state value of the control input $\mathbf{u}_r$ is computed by solving the steady-state equation

$$\mathbf{x}_r = \mathbf{f}(\mathbf{x}_r, \mathbf{u}_r) \tag{1}$$

- Using the error state and control vectors $\mathbf{x}_{e_k} = \mathbf{x}_k - \mathbf{x}_r, \mathbf{u}_{e_k} = \mathbf{u}_k - \mathbf{u}_r$ the inequality constraints can be rewritten as in LMPC.
- Similarly the equality constraint becomes

$$\mathbf{x}_{e_{k+1}} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) - \mathbf{x}_r = \mathbf{f}(\mathbf{x}_{e_k} + \mathbf{x}_r, \mathbf{u}_{e_k} + \mathbf{u}_r) - \mathbf{x}_r \tag{2}$$

NMPC: Set-point tracking

- Define $\mathbf{z} = \begin{bmatrix} \mathbf{X}_{e_k} \\ \mathbf{U}_{e_k} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_k - \mathbf{X}_r \\ \mathbf{U}_k - \mathbf{U}_r \end{bmatrix}$ and the optimization problem becomes

$$\begin{aligned} \underset{\mathbf{z}}{inf} \quad & \mathbf{z}^T \mathbf{H} \mathbf{z} \quad \textit{subject to} \\ & \mathbf{F}\mathbf{z} \leq \mathbf{g} \\ & \mathbf{f}_{eq}(\mathbf{z}) = 0 \end{aligned} \tag{3}$$

- The MPC control input for the set-point tracking problem is obtained as

$$\mathbf{u}_k = [\mathbf{U}^*_{e_k}]_1 + \mathbf{u}_r \tag{4}$$

NMPC: Simulation results

- State equation

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) = \begin{bmatrix} x_{1_k} + Tx_{2_k} \\ x_{2_k} + T(-\frac{g}{l} sin(x_{1_k}) - \frac{B}{Ml^2} x_{2_k} + \frac{1}{Ml^2} u_k) \end{bmatrix} \tag{5}$$

- The steady-state equation $\mathbf{x}_r = \mathbf{f}(\mathbf{x}_r, \mathbf{u}_r)$ gives

$$\begin{aligned} x_{1_r} &= x_{1_r} + Tx_{2_r} \implies x_{2_r} = 0 \\ 0 &= T(-\frac{g}{l} sin(x_{1_r}) + \frac{1}{Ml^2} u_r) \implies u_r = Mgl\, sin(x_{1_r}) \end{aligned} \tag{6}$$

- System parameters

$$M = 1, l = 1, B = 3, g = 9.8, T = 0.1 \tag{7}$$

- Simulation parameters

$$N_T = 50, N = 5, \mathbf{Q} = \mathbf{I}_2, \mathbf{R} = 1, \mathbf{x}_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \mathbf{x}_r = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, u_r = 4.69 \tag{8}$$

- Constraint set parameters

$$\mathbf{F_x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \mathbf{g_x} = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix} \quad \mathbf{F_u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \mathbf{g_u} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \tag{9}$$

Simple pendulum: set-point tracking

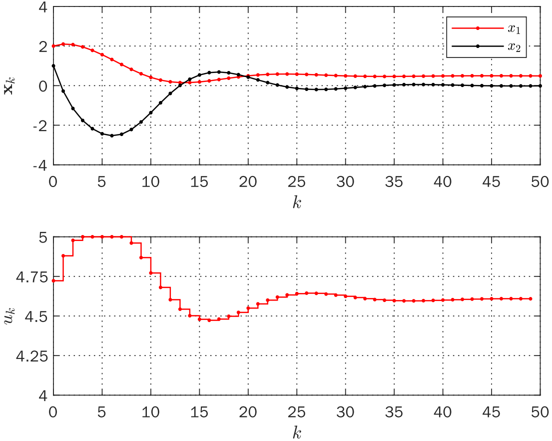

Figure 1: Simple pendulum: set-point tracking

Thank you