Model Predictive Control using MATLAB
4: LMPC - Reducing online computation

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Overview

1. LMPC: Reducing online computation
2. LMPC: Simulation results

LMPC: Reducing online computation

LMPC: Reducing online computation - method 1

- **Basic idea**: Reduce the number of variables in $\mathbf{z}$ by removing $\mathbf{X}_k$.
- $\mathbf{x}_k, \mathbf{x}_{i|k} \in \mathbb{R}^n$ contains n elements.
- $\mathbf{u}_k, \mathbf{u}_{i|k} \in \mathbb{R}^m$ contains m elements.
- $\mathbf{X}_k \in \mathbb{R}^{n(N+1)}$ contains $n(N+1)$ elements.
- $\mathbf{U}_k \in \mathbb{R}^{mN}$ contains mN elements.
- $\mathbf{z} = \begin{bmatrix} \mathbf{X}_k \\ \mathbf{U}_k \end{bmatrix} \in \mathbb{R}^{n(N+1)+mN}$ contains $n(N+1)+mN$ elements.
- Number of decision variables in $\mathbf{z}$: $n_{\mathbf{z}} = nN + mN.$

LMPC: Reducing online computation - method 1

- The state equation constraint

$$\mathbf{X}_k = \mathbf{A}_{\mathbf{X}}\mathbf{x}_k + \mathbf{B}_{\mathbf{U}}\mathbf{U}_k \tag{1}$$

- Substituting this in the cost function gives

$$\begin{aligned} J_k =& \left[\mathbf{A}_{\mathbf{X}}\mathbf{x}_k + \mathbf{B}_{\mathbf{U}}\mathbf{U}_k\right]^T \mathbf{Q}_{\mathbf{X}} \left[\mathbf{A}_{\mathbf{X}}\mathbf{x}_k + \mathbf{B}_{\mathbf{U}}\mathbf{U}_k\right] + \mathbf{U}_k^T \mathbf{R}_{\mathbf{U}} \mathbf{U}_k \\ =& \mathbf{U}_k^T \left[\mathbf{B}_{\mathbf{U}}^T \mathbf{Q}_{\mathbf{X}} \mathbf{B}_{\mathbf{U}} + \mathbf{R}_{\mathbf{U}}\right] \mathbf{U}_k + 2\mathbf{x}_k^T \left[\mathbf{A}_{\mathbf{X}}^T \mathbf{Q}_{\mathbf{X}} \mathbf{B}_{\mathbf{U}}\right] \mathbf{U}_k + \\ & \mathbf{x}_k^T \left[\mathbf{A}_{\mathbf{X}}^T \mathbf{Q}_{\mathbf{X}} \mathbf{A}_{\mathbf{X}}\right] \mathbf{x}_k \\ =& \mathbf{U}_k^T \mathbf{H} \mathbf{U}_k + \mathbf{q}_k^T \mathbf{U}_k + r_k \end{aligned} \tag{2}$$

where

$$\begin{aligned} \mathbf{H} &= \mathbf{B}_{\mathbf{U}}^T \mathbf{Q}_{\mathbf{X}} \mathbf{B}_{\mathbf{U}} + \mathbf{R}_{\mathbf{U}} \\ \mathbf{q}_k^T &= 2\mathbf{x}_k^T \mathbf{A}_{\mathbf{X}}^T \mathbf{Q}_{\mathbf{X}} \mathbf{B}_{\mathbf{U}} \\ r_k &= \mathbf{x}_k^T \mathbf{A}_{\mathbf{X}}^T \mathbf{Q}_{\mathbf{X}} \mathbf{A}_{\mathbf{X}} \mathbf{x}_k \end{aligned} \tag{3}$$

- Similarly the constraints becomes

$$\begin{aligned} &\mathbf{F_X}\left[\mathbf{A_X}\mathbf{x}_k + \mathbf{B_U}\mathbf{U}_k\right] \leq \mathbf{g_X} \implies \mathbf{F_X}\mathbf{B_U}\mathbf{U}_k \leq \mathbf{g_X} - \mathbf{F_X}\mathbf{A_X}\mathbf{x}_k \\ &\mathbf{F_U}\mathbf{U}_k \leq \mathbf{g_U} \end{aligned} \tag{4}$$

- Define $\mathbf{z} = \mathbf{U}_k$, $\mathbf{F} = \begin{bmatrix} \mathbf{F_X}\mathbf{B_U} \\ \mathbf{F_U} \end{bmatrix}$, $\mathbf{g} = \begin{bmatrix} \mathbf{g_X} - \mathbf{F_X}\mathbf{A_X}\mathbf{x}_k \\ \mathbf{g_U} \end{bmatrix}$ which results in the optimization problem

$$\begin{aligned} \underset{\mathbf{z}}{inf} \quad & \mathbf{z}^T\mathbf{H}\mathbf{z} + \mathbf{q}_k^T\mathbf{z} + r_k \quad subject\ to \\ & \mathbf{F}\mathbf{z} \leq \mathbf{g} \end{aligned} \tag{5}$$

- Here the parameters $\mathbf{q}_k, r_k$ and $\mathbf{g}$ are functions of $\mathbf{x}_k$.
- Number of decision variables $n_{\mathbf{z}} = mN$.

LMPC: Reducing online computation - method 2

- **Basic idea**: Reduce the number of optimization variables by using a control horizon N_C lesser than the prediction horizon N.
- Define the control sequence as $\mathbf{U}_k = \left(\mathbf{u}_{k|k}, ..., \mathbf{u}_{k+N_C-1|k}, \mathbf{0}, ..., \mathbf{0}\right)$.
- Number of decision variables $n_{\mathbf{z}} = mN_C$.
- N_C is usually chosen as 2.

LMPC: Simulation results

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- Consider an LTI system

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ -1 & 1.5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix} \tag{6}$$

- Simulation parameters

$$N_T = 50, N = 5, \mathbf{Q} = \mathbf{I}_2, \mathbf{R} = 1, \mathbf{x}_0 = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \tag{7}$$

- Constraint set parameters

$$\mathbf{F_x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \mathbf{g_x} = \begin{bmatrix} 10 \\ 10 \\ 10 \\ 10 \end{bmatrix} \quad \mathbf{F_u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \mathbf{g_u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{8}$$

LMPC: simulation results

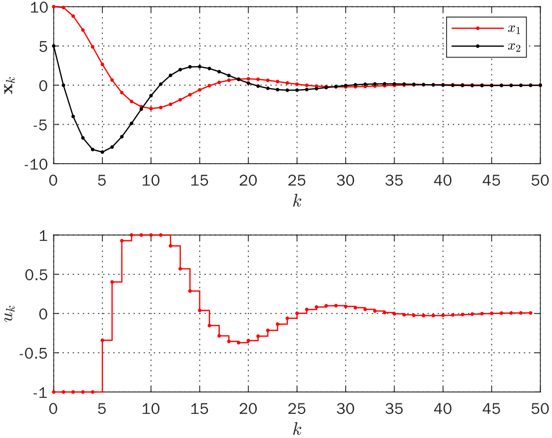

Figure 1: LMPC Response (Method 1)

LMPC: simulation results

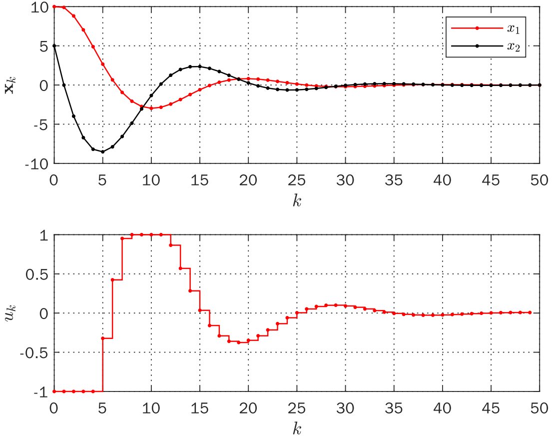

Figure 2: LMPC Response (Method 2 with $N_C = 2$)

Thank you