Model Predictive Control using MATLAB
5: LMPC - Set point tracking

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Overview

1. LMPC: Set-point tracking
2. LMPC: Simulation results

LMPC: Set-point tracking

- Two major problems in MPC
  1. **Stabilization problem**: in which the reference $\mathbf{x}_r = 0$ and the objective is to drive the state to origin.
  2. **Set-point tracking problem**: in which the reference $\mathbf{x}_r \neq 0$, and the objective is to track the nonzero set point.
- For $\mathbf{x}_r \neq 0$, the steady state control input $\mathbf{u}_r \neq 0$. In steady state we have $\mathbf{x}_{k+1} = \mathbf{x}_k = \mathbf{x}_r$. Substituting this in the state equation gives

$$\begin{aligned} \mathbf{x}_r &= \mathbf{A}\mathbf{x}_r + \mathbf{B}\mathbf{u}_r \\ \implies \mathbf{u}_r &= \mathbf{B}^{-1}(\mathbf{I} - \mathbf{A})\mathbf{x}_r \end{aligned} \tag{1}$$

  where $\mathbf{B}^{-1}$ is the pseudo-inverse.

LMPC: Set-point tracking

- The set-point tracking can be transferred to a stabilization problem by using the error state and control vectors $\mathbf{x}_{e_k} = \mathbf{x}_k - \mathbf{x}_r, \mathbf{u}_{e_k} = \mathbf{u}_k - \mathbf{u}_r$.
- From state equation the error dynamics is obtained as

$$\begin{aligned} \mathbf{x}_{e_{k+1}} &= \mathbf{x}_{k+1} - \mathbf{x}_r = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k - \mathbf{x}_r \\ &= \mathbf{A}\mathbf{x}_k - \mathbf{A}\mathbf{x}_r + \mathbf{B}\mathbf{u}_k - \mathbf{x}_r + \mathbf{A}\mathbf{x}_r \\ &= \mathbf{A}[\mathbf{x}_k - \mathbf{x}_r] + \mathbf{B}[\mathbf{u}_k - \mathbf{B}^{-1}(\mathbf{I} - \mathbf{A})\mathbf{x}_r] = \mathbf{A}\mathbf{x}_{e_k} + \mathbf{B}\mathbf{u}_{e_k} \end{aligned} \tag{2}$$

- Using error state and control vectors the constraints can be rewritten as

$$\begin{aligned} \mathbf{F_x x} \le \mathbf{g_x} &\implies \mathbf{F_x}(\mathbf{x}_{e_k} + \mathbf{x}_r) \le \mathbf{g_x} \implies \mathbf{F_x x}_{e_k} \le \mathbf{g_x} - \mathbf{F_x x}_r \\ \mathbf{F_u u} \le \mathbf{g_u} &\implies \mathbf{F_u}(\mathbf{u}_{e_k} + \mathbf{u}_r) \le \mathbf{g_u} \implies \mathbf{F_u u}_{e_k} \le \mathbf{g_u} - \mathbf{F_u u}_r \end{aligned} \tag{3}$$

LMPC: set-point tracking

- Now the matrices $\mathbf{F_X}, \mathbf{g_X}, \mathbf{F_U}, \mathbf{g_U}$ can be defined as in stabilization case in which $\mathbf{g_x}, \mathbf{g_u}$ are replaced by $\mathbf{g}_x - \mathbf{F_x}\mathbf{x}_r, \mathbf{g}_u - \mathbf{F_u}\mathbf{u}_r$.
- Define $\mathbf{z} = \begin{bmatrix} \mathbf{X}_{e_k} \\ \mathbf{U}_{e_k} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_k - \mathbf{X}_r \\ \mathbf{U}_k - \mathbf{U}_r \end{bmatrix}$ and the optimization problem becomes

$$\begin{aligned} &\underset{\mathbf{z}}{inf} \;\; \mathbf{z}^T \mathbf{H} \mathbf{z} \quad \textit{subject to} \\ &\mathbf{F}\mathbf{z} \leq \mathbf{g} \\ &\mathbf{F}_{eq}\mathbf{z} = \mathbf{g}_{eq} \end{aligned} \tag{4}$$

- The MPC control input for the set-point tracking problem is

$$\mathbf{u}_k = [\mathbf{U}^*_{e_k}]_1 + \mathbf{u}_r \tag{5}$$

LMPC: Simulation results

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- System parameters

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ -1 & 1.5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix} \tag{6}$$

- Simulation parameters

$$N_T = 50, N = 5, \mathbf{Q} = \mathbf{I}_2, \mathbf{R} = 1, \mathbf{x}_0 = \begin{bmatrix} 10 \\ 5 \end{bmatrix}, \mathbf{x}_r = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, u_r = 0.59 \tag{7}$$

- Constraint set parameters

$$\mathbf{F_x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \mathbf{g_x} = \begin{bmatrix} 10 \\ 10 \\ 10 \\ 10 \end{bmatrix} \quad \mathbf{F_u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \mathbf{g_u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{8}$$

LMPC: set-point tracking

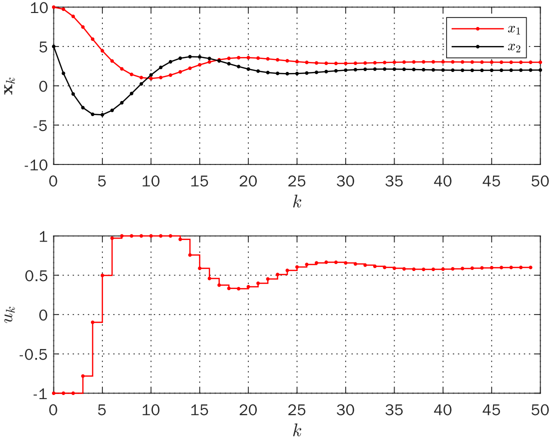

Figure 1: LMPC: set-point tracking

Thank you