



Fuzzy Logic and Fuzzy Systems – Introduction

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FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

This course will focus on fuzzy logic and fuzzy control systems; there is a brief introduction to neural networks and (statistical) machine learning.

A knowledge of soft computing techniques will help you to work with folks involved with patient care, public administration for instance.



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

In this course you will learn:

1. how **imprecision** in concepts can be discussed using the basics of fuzzy sets;
2. the basic principles of **organizing** a fuzzy logic system
3. what is inside the **rule-base** of a fuzzy **control** system
4. about methods of **building** a fuzzy control system



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

In this course you will learn:

- About fuzzy sets that, though based on the notion of classical set theory, do not define the exclusive *belongingness* of an element to a set.
 - About fuzzy logic – a logic of vague concepts, some encountered every day – imprecision in measurement large/small, slow/fast - or uncertainty in expressing belief about an object, event, or state



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

In this course you will learn:

About fuzzy rule-based system that benefit from fuzzy expression of belief in statements like *IF A THEN B* by appending degree of precision in our measurement of A and B

About fuzzy control systems that have substantially improved the notion and implementation of (feedback) control system using fuzzy sets and fuzzy logic to describe a control system



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

In this course you will learn:

About neuro-fuzzy systems that have the capability to learn properties of fuzzy variables and fuzzy rules from sets of data presented to a system that learns using statistical learning and neural computing systems

About neural networks, one of the key systems used in machine learning, that learn in a manner more transparent than statistical learning



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Course Content

1. Terminology: Uncertainty, Approximations and Vagueness
2. Fuzzy Sets
3. Fuzzy Logic and Fuzzy Systems
4. Fuzzy Control
5. Neuro-fuzzy systems



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Course Assessment

Examination

One or more questions from each of the
following 5 topics. Attempt 3 out of 5
questions

1. Terminology: Uncertainty, Approximations and Vagueness
2. Fuzzy Sets
3. Fuzzy Logic and Fuzzy Systems
4. Fuzzy Control
5. Neuro-fuzzy systems



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Course Assessment
Coursework (20%)

A group project for building a fuzzy logic or fuzzy control system using Matlab or similar system. 2-3 page report

OR

A critical evaluation of an existing product using fuzzy logic/fuzzy control. 4-5 page report



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Soft computing is for the near future – next 5-10 years, and knowledge of the inclusive branches will help to work in almost every enterprise where computers are expected in helping with design, control and execution of complex processes.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Soft computing is used as an umbrella term for sub-disciplines of computing, including fuzzy logic and fuzzy control, neural networks based computing and machine learning, and genetic algorithms, together with chaos theory in mathematics.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

“More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. For example, the class of animals clearly includes dogs, horses, birds, etc. as its members, and clearly excludes such objects as rocks, fluids, plants, etc.”

(Zadeh 1965:338)



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

“However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals.

The same kind of ambiguity arises in the case of a number such as 10 in relation to the "class" of all real numbers which are much greater than 1.”



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

“Clearly, the "class of all real numbers which are much greater than 1," or "the class of [] tall women," or "the class of [] rude men," do not constitute classes or sets in the usual mathematical sense of these terms.

Yet, the fact remains that such imprecisely defined "classes" play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction."

(Zadeh 1965:338)



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Your perception of a moving vehicle.

Is it being driven slowly?
At a reasonable speed? Or
the driver is driving it too fast?

Question: where is the car?
Motorway? Inner City?
Residential block?

In a 40 km/h limit is it slow? Fast? Is 39.5 is slow?

Velocity	
Speed	Degree of Truth →+ve Medium
0	0
5	0
10	0
15	0
20	0
25	0.25
30	0.50
35	0.75
40	1
45	0.75
50	0.50
55	0.25
60	0
65	0
70	0



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Challenges to 19/20th century scientific orthodoxy

“One of the deepest traditions in science is”
to accord “respectability to what is
quantitative, rigorous, and categorically true.
[...] We [appear] to live in a world that is
pervasively imprecise, uncertain, and hard to
be categorical about. [...] We do pay] a steep
price for high precision and low uncertainty”.
(Zadeh 1994:48)

Zadeh, L. A. (1994). Soft computing and fuzzy logic. *IEEE Software*. Volume: 11, Issue: 6. Pages:
48 - 56

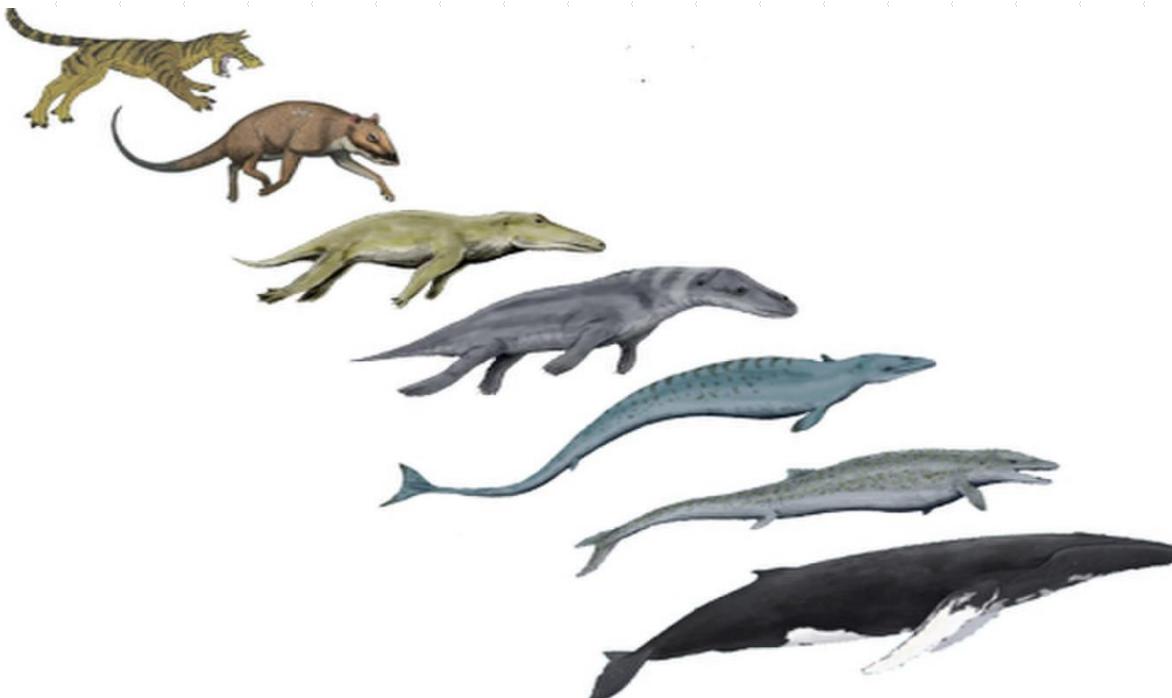


FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Challenges to 19/20th century scientific orthodoxy:

Darwin's Theory of Evolution



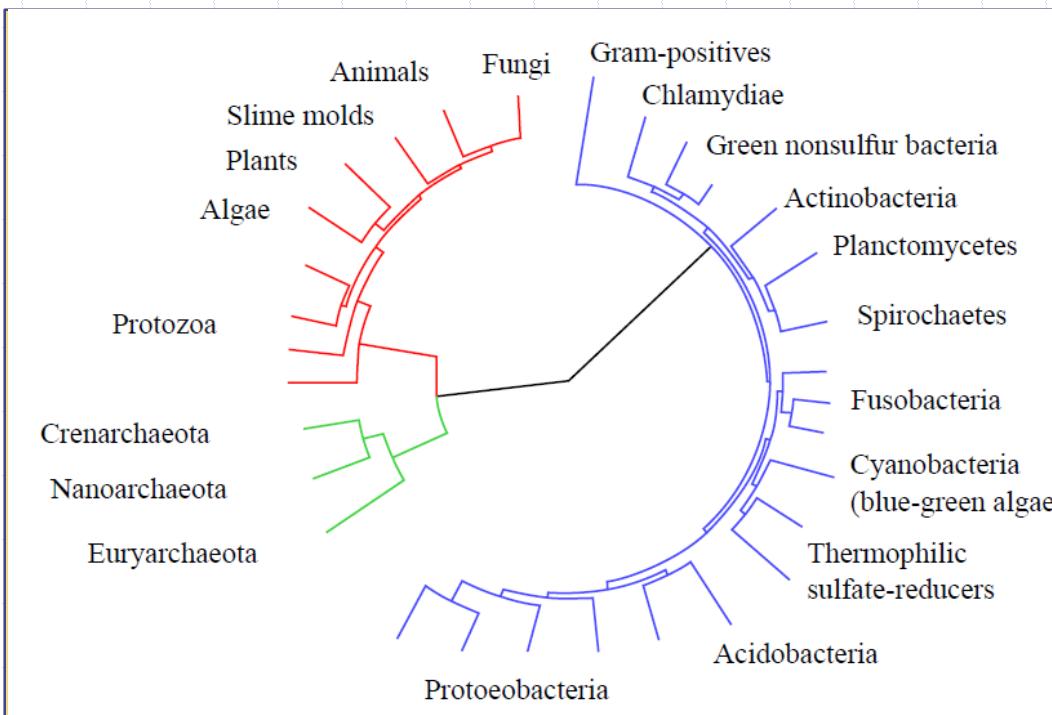
The last shore-dwelling ancestor of modern whales was *Sinonyx*, top left, a hyena-like animal. Over 60 million years, several transitional forms evolved: from top to bottom, *Indohyus*, *Ambulocetus*, *Rodhocetus*, *Basilosaurus*, *Dorudon*, and finally, the modern humpback whale.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

IMPRECISELY DEFINED "CLASSES" PLAY AN IMPORTANT ROLE IN HUMAN THINKING



The arcs represent progression

A 'tree of life' shows interconnectedness between various species: the interconnection blurs of *fuzzifies* the boundary between species. The notion of fuzzification has helped the evolutionary biologists to talk about *clades*

Diagram from <https://commons.wikimedia.org/wiki/File:CollapsedtreeLabels-simplified.svg>

LETUNIC, I. & BORK, P. 2006. Interactive Tree Of Life (iTOL): an online tool for phylogenetic tree display and annotation. *Bioinformatics*, 23, 127-128.

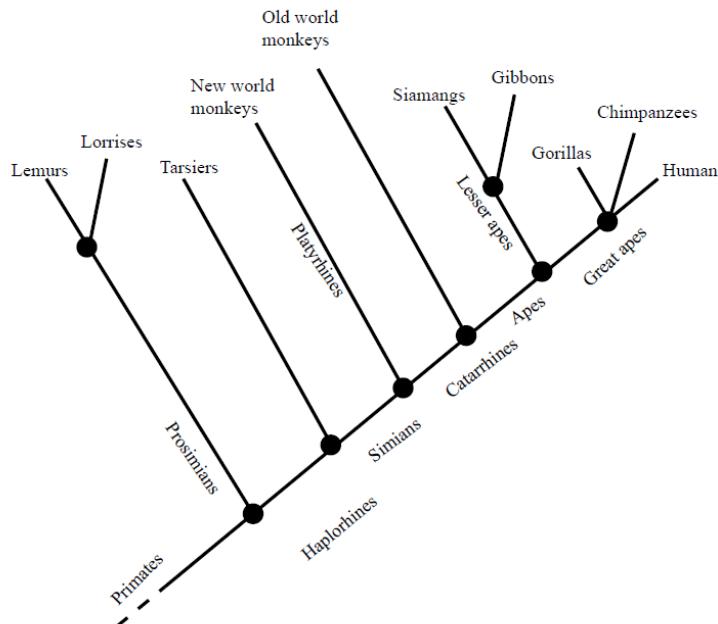


FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Challenges to 19/20th century scientific orthodoxy:

Darwin's Theory of Evolution – no more fixed species, but an uncertainty over time



This cladogram models the 'evolution' of our species – the *humans*. Note that vertical axis (at an angle shows time and the organisms are listed on perpendicular lines; some survive and others evolve. Our 'cousins' *gorillas* and *chimpanzees* descended from the Great Apes. (Haplorhines – *dry nosed* or *simple nosed* primates).



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Challenges to 19/20th century scientific orthodoxy:

Notes from Quantum Theory and Quantum Mechanics

“A measurement allows us to gain information about the state of a physical system. For example, when measuring the position of a particle, the measurement outcomes correspond to possible locations. Whenever the state of the system is such that we can predict this position with certainty, then there is only one measurement outcome that can occur.” (Oppenheim and Wehner, 2010:1072)

Oppenheim, J., & Wehner, S. (2010). The uncertainty principle determines the nonlocality of quantum mechanics. *Science*, 330(6007), 1072-1074.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Challenges to 19/20th century scientific orthodoxy:

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[In 1927] “Heisenberg [] observed that quantum mechanics imposes strict restrictions on what we can hope to learn—there are incompatible measurements such as position and momentum whose results cannot be simultaneously predicted with certainty. These restrictions are known as uncertainty relations.” (Oppenheim and Wehner, 2010:1072) (emphasis added).



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

<u>Year</u>	<u>Definition & Elaboration of the <i>adjective</i> fuzzy</u>
1616	1. Not firm or sound in substance; spongy.
1713	2. Frayed into loose fibres; covered with fuzz; fluffy, downy.
1778	3a. Blurred, indistinct.
1937	3b. Of thought, etc.: imprecisely defined; confused, vague. Also of persons: inexact in thought or expression.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Fuzziness is not Randomness

“Much of the decision-making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely.”

(Bellman and Zadeh 1970: B141)



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Uncertainty and Imprecision: Quantum Theory and beyond

Classical statistical mechanics is a 'crypto-deterministic' theory, where each element of the probability distribution of the dynamical variables specifying a given system evolves with time according to deterministic laws of motion; the whole uncertainty is contained in the form of the initial distributions.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Uncertainty and Imprecision: Quantum Theory and beyond

Classical statistical mechanics is a 'crypto-deterministic' theory, where each element of the probability distribution of the dynamical variables specifying a given system evolves with time according to deterministic laws of motion; the whole uncertainty is contained in the form of the initial distributions.

A [crypto-deterministic] theory based on such concepts could not give a satisfactory account of such non-deterministic effects as **radioactive decay or spontaneous emission**



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Fuzziness is not Randomness

“To deal quantitatively with imprecision, we usually employ the concepts and techniques of probability theory and, more particularly, the tools are provided by decision theory, control theory and information theory. In so doing, we are tacitly accepting the premise that imprecision – whatever its nature- can be equated with randomness. This, in our view, is a questionable assumption.” (Bellman & Zadeh: 1970:B141).
→(Inference) Randomness is not uncertainty.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Fuzziness is not Randomness

Assume that we are given a fuzzy goal G and a fuzzy, constraint C in a space of alternatives X . Then, G and C form a *decision*, D , which is a fuzzy set resulting from intersection of G and C : Symbolically,

$$D = G \cap C$$



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Fuzziness is not Randomness –

You can run a safe train service at very high speeds if you put your mind to it

Assume that we are given a fuzzy goal G and a fuzzy, constraint C in a space of alternatives X . Then, G and C to for a *decision*, D , which is a fuzzy set resulting from intersection of G and C : Symbolically,

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Abstract : A predictive fuzzy control including control rules to bring desirable conditions had been proposed and applied to an Automatic Train Operation (ATO) system. Capability of the newly developed Fuzzy ATO to control train operation as skillfully as experienced operators was confirmed by the trial run made in the Sendai Municipal Subway System. In this paper, control rules of the Fuzzy ATO and results of test run are described.

OSHIMA, H., YASUNOBU, S. & SEKINO, S.-I. Automatic train operation system based on predictive fuzzy control. Artificial Intelligence for Industrial Applications, 1988. IEEE AI'88., Proceedings of the International Workshop on, 1988. IEEE, 485-489.

FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Fuzziness is not Randomness –

You can run a safe train service at very high speeds if you put your mind to it

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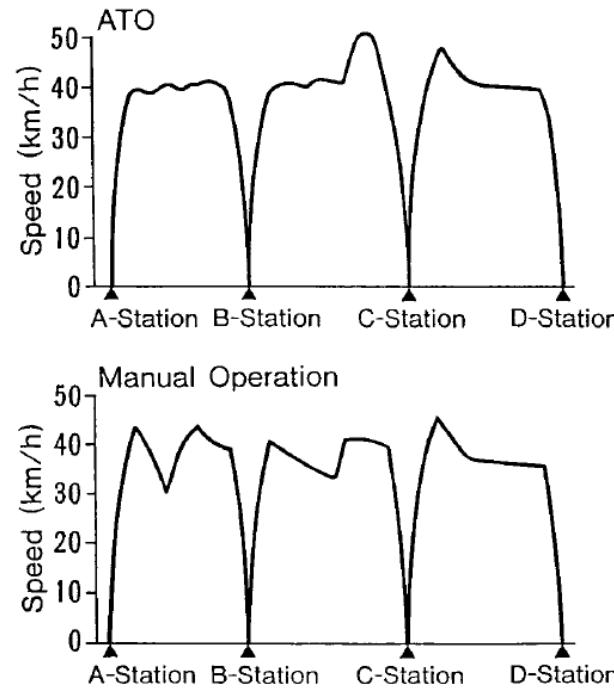
$$D = G \cap C$$


Fig. 8 Comparison between ATO Operation and Manual Operation



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

<u>Year</u>	<u>Definition & Elaboration of the <i>adjective</i> fuzzy</u>
1964	<p>3c.i <i>Computing and Logic.</i> [Fuzzy Set] (Of a set) defined so as to allow for imprecise membership criteria and for gradations of membership; pertaining or belonging to such a set;</p>
	<p>3c.ii fuzzy logic, the logic of fuzzy sets and fuzzy concepts; fuzzy matching, (the facility for) matching items which are similar but not identical.</p>



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

You have probably only encountered standard systems of logic – Boolean Logic, First-order logic. However, there are non-standard logics – future-conditional logic, logics used by von Neumann in the development of quantum mechanics (Boolean logic with quantum uncertainty). Fuzzy logic may be regarded as an alternative or *deviant* logic.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

You have probably only encountered classical control theory – the control and stabilization of a dynamical system by corrective action from an external control. However, such systems rely on the existence of mathematical and statistical formalisms (differential equations, Markov models), that are usually available for idealised situations.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

**Your perception is imprecise as well:
Consider the two images:**

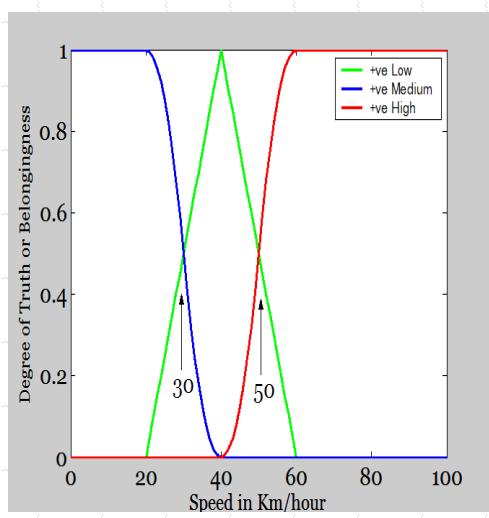


Is there a systematic difference between what you consider a tall/short woman and a tall short man?

FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Examples of velocity fuzzy membership function +ve Medium that may have been used by Ligier – the autonomous car



Velocity		Belongingness?
Speed	Degree of Truth →+ve Medium	
0	0	Definitely Not
5	0	Definitely Not
10	0	Definitely Not
15	0	Definitely Not
20	0	Definitely Not
25	0.25	Chances are less than even
30	0.50	Chances are about even
35	0.75	Chances are better than even
40	1	Definitely
45	0.75	Chances are better than even
50	0.50	Chances are about even
55	0.25	Chances are less than even
60	0	Definitely Not
65	0	Definitely Not
70	0	Definitely Not

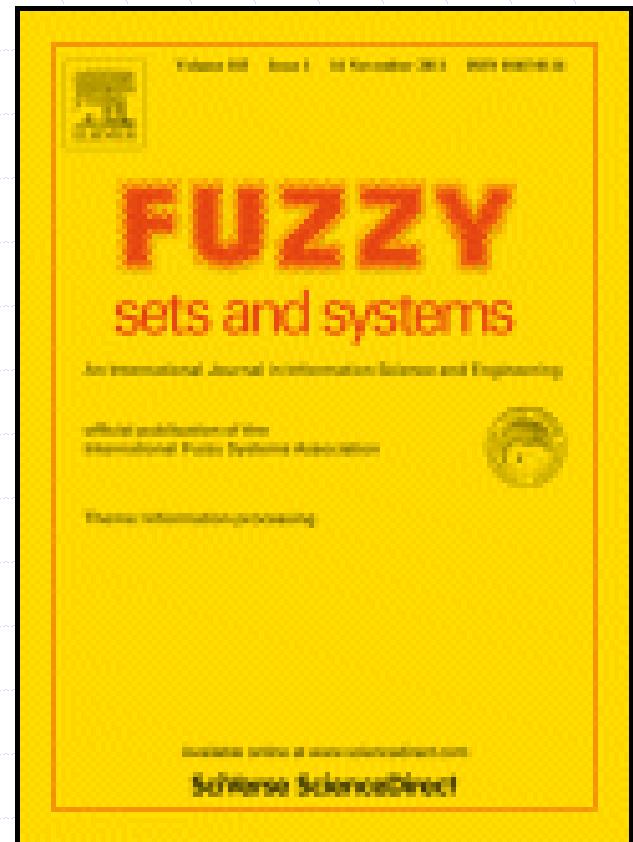


FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

What are fuzzy sets and systems

The theory of fuzzy sets now encompasses a corpus of basic notions including [...] aggregation operations, a generalized theory of relations, specific measures of information content, a calculus of fuzzy numbers.





FUZZY LOGIC & FUZZY SYSTEMS

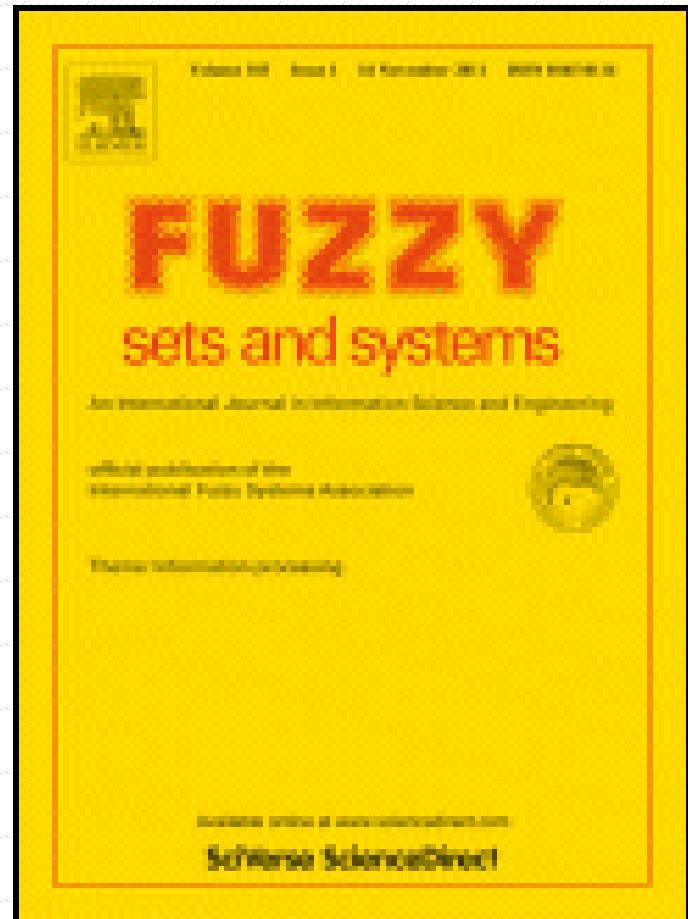
UNCERTAINTY AND ITS TREATMENT

What are fuzzy sets and systems

Fuzzy sets have led to

(1) a non-additive uncertainty theory [..possibility theory,]

(2)[a] tool for both linguistic and numerical modeling: fuzzy rule-based systems.

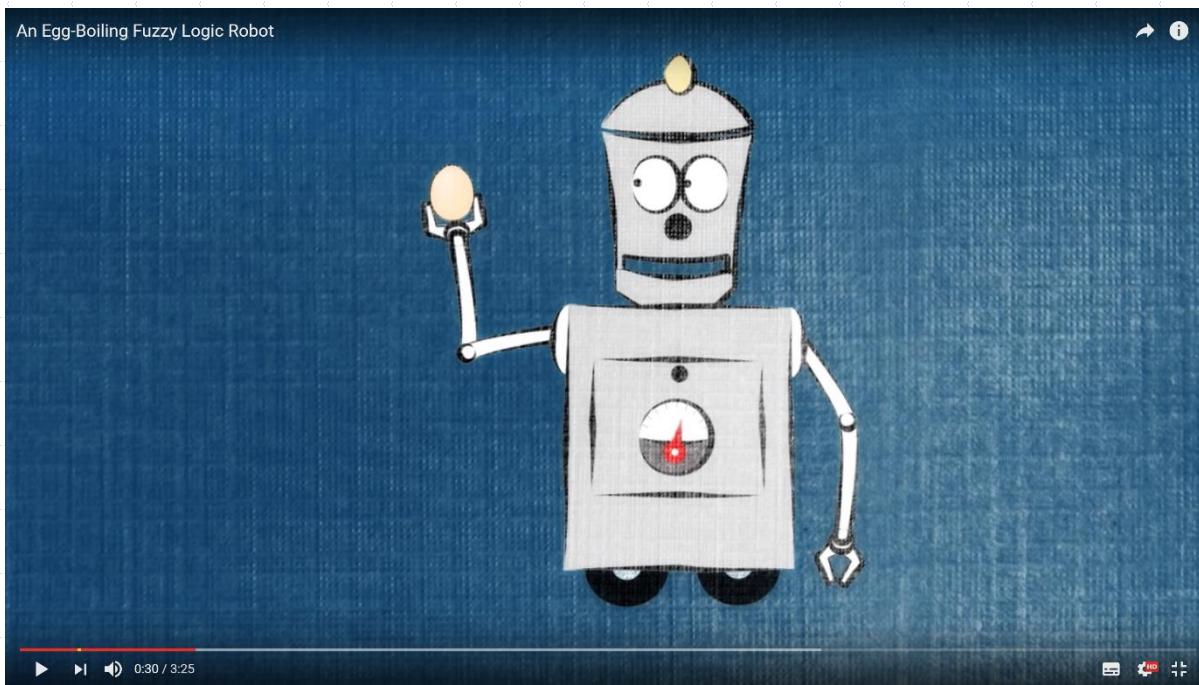




FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Let us hear about the *egg boiling* robot!



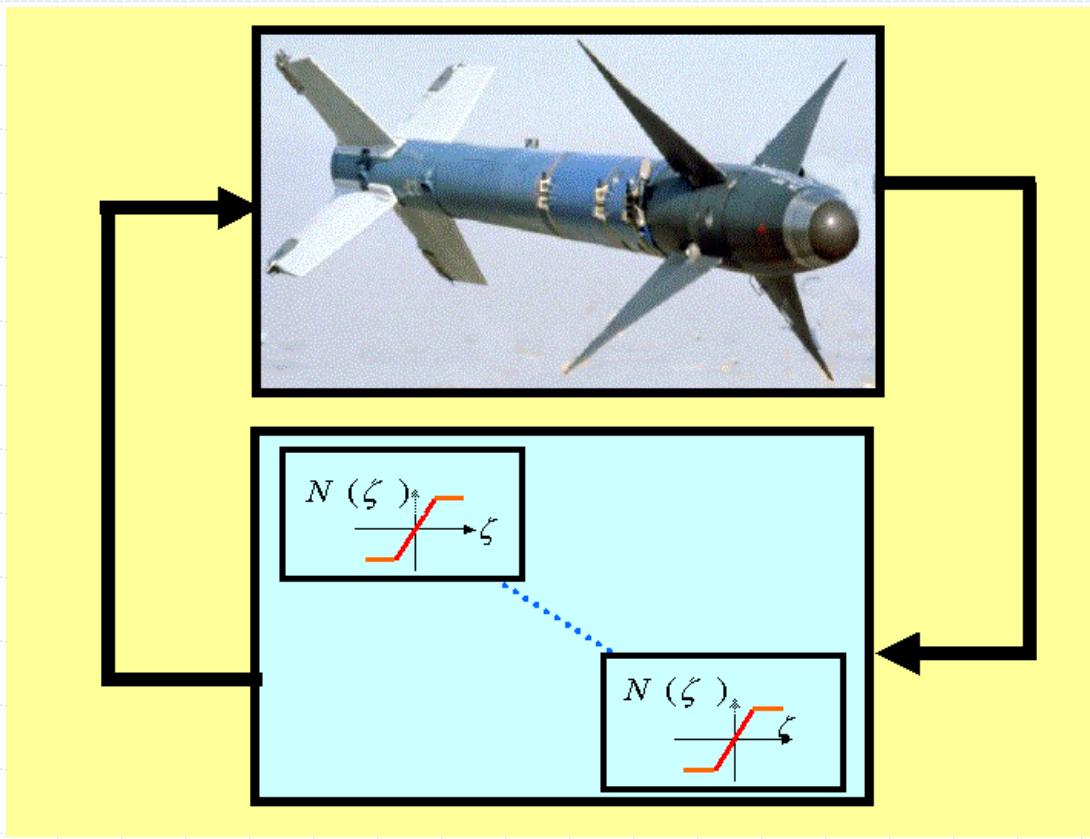
https://www.youtube.com/watch?v=J_Q5X0nTmrA



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

You are never far from a control system

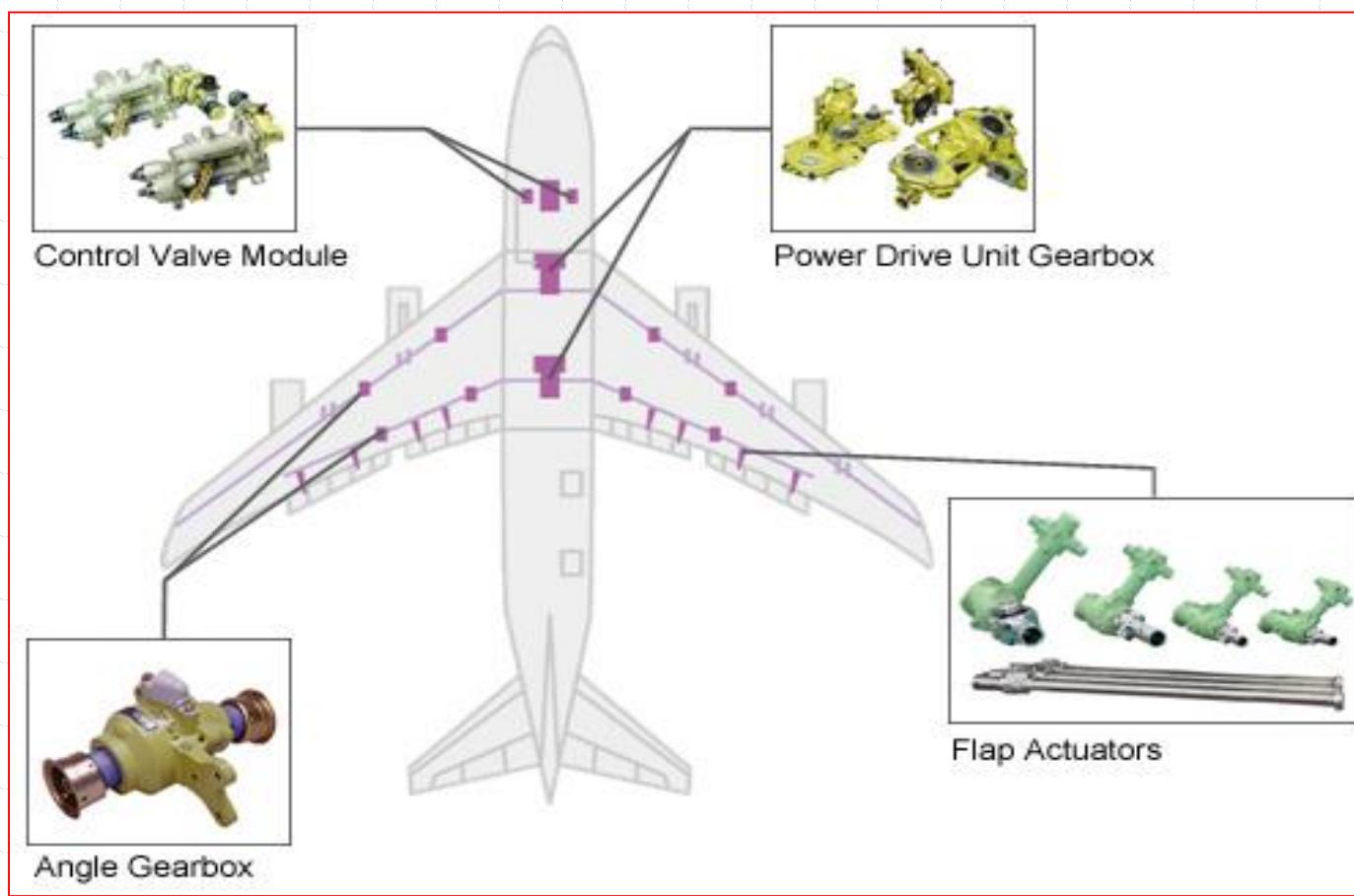




FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

You are never far from a control system



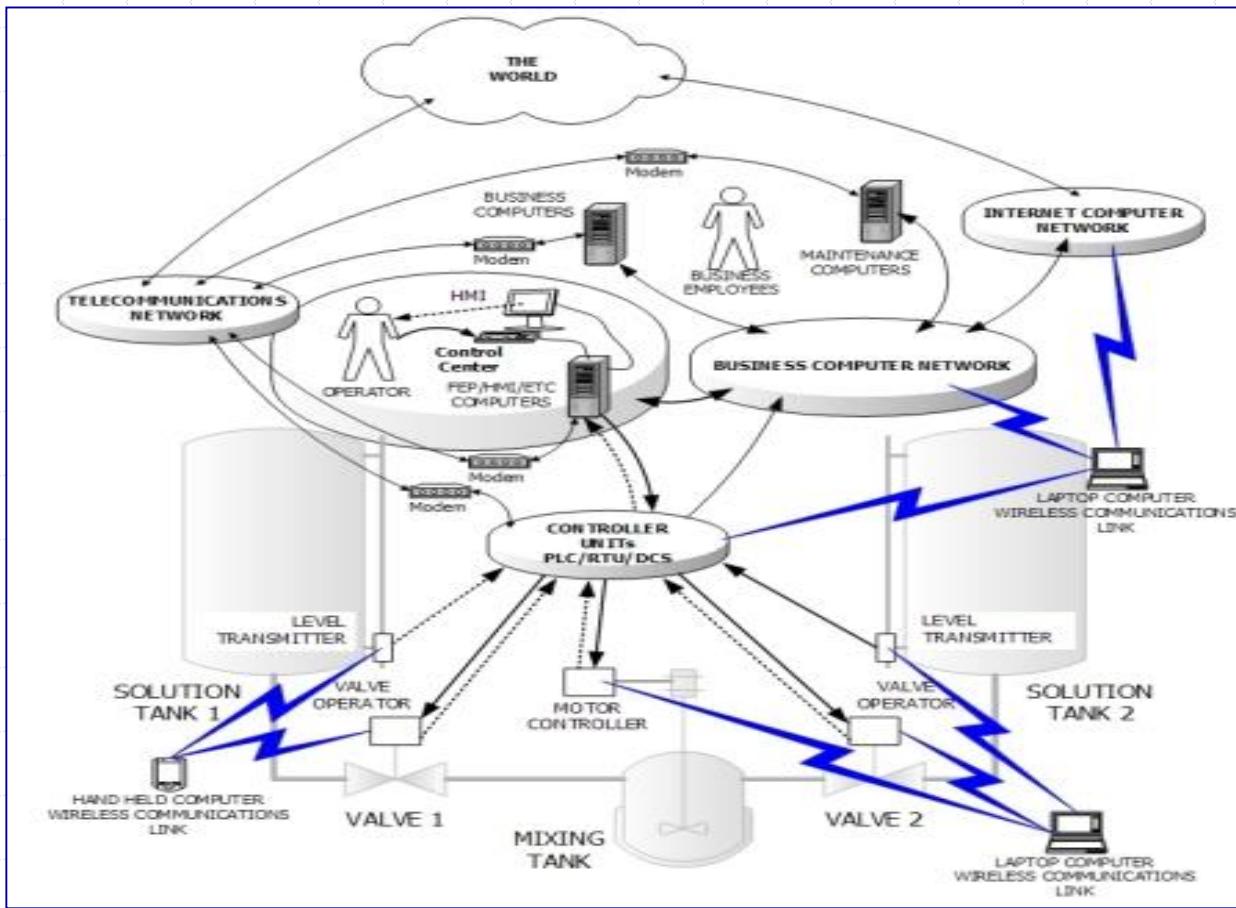
When you fly

FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

You are never far from a control system

<https://ics-cert.us-cert.gov/content/overview-cyber-vulnerabilities#under>



When you use any process control system



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

More complex the process, more sophisticated the control,
and more critical is the role of the controller!





FUZZY LOGIC & FUZZY SYSTEMS

Assessment

- 1. Assessment is by examination and by project work. Project work attracts a mark of up to 20% of the year end mark, and the examination makes up the remaining 80%.**

- 2. Project is conducted by each student individually. It encourages the design, writing and testing of programs as a means of appraising the theory and techniques discussed in the course.**



FUZZY LOGIC & FUZZY SYSTEMS

Assessment

The examination is two hours long, and students are required to answer three questions from a selection of five. Most questions will contain a short discursive component and a related question requiring the student to demonstrate problem-solving abilities related to that discursive component.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Fuzzy logic is being developed as a discipline to meet two objectives:

- 1. As a professional subject dedicated to the building of systems of high utility – for example fuzzy control**
- 2. As a theoretical subject – fuzzy logic is “symbolic logic with a comparative notion of truth developed fully in the spirit of classical logic [...] It is a branch of many-valued logic based on the paradigm of inference under vagueness.**



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Lotfi Zadeh introduced the *theory of fuzzy sets*: A fuzzy set is a collection of objects that might belong to the set to a degree, varying from 1 for **full belongingness** to 0 for full **non-belongingness**, through all intermediate values

Zadeh employed the concept of a membership function assigning to each element a number from the unit interval to indicate the **intensity of belongingness**. Zadeh further defined basic operations on fuzzy sets as essentially extensions of their conventional ('ordinary') counterparts.



Lotdfi Zadeh, Professor in the Graduate School, Computer Science Division
Department of Elec. Eng. and Comp Sciences, University of California Berkeley, CA 94720 -1776
Director, Berkeley Initiative in Soft Computing (BISC)

<http://www.cs.berkeley.edu/People/Faculty/Homepages/zadeh.html>

In 1995, Dr. Zadeh was awarded the IEEE Medal of Honor "For pioneering development of fuzzy logic and its many diverse applications." In 2001, he received the American Computer Machinery's 2000 Allen Newell Award for seminal contributions to AI through his development of fuzzy logic.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Fuzzy control provides a formal methodology for representing, manipulating, and implementing a human's heuristic knowledge about how to control a system.

**The heuristic information – information based on 'rules of thumb' come from two sources:
Operators running complex control systems and design engineers of such systems who have carried out mathematical analysis.**



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Zadeh also devised the so-called fuzzy logic: This logic was devised model 'human' reasoning processes comprising:

vague predicates:

e.g. *large, beautiful, small*

partial truths:

e.g. *not very true, more or less false*

linguistic quantifiers:

e.g. *most, almost all, a few*

linguistic hedges:

e.g. *very, more or less.*



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

The key point is this

We can be sure of the truth of the statement:

Jones/Jill is tall

But unsure of the truth of the statement:

Jones/Jill is 1.8297 metres tall



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

The key point is *vagueness*

Jones/Jill is tall: “Confidence in the truth of a vague assertion, just because of its vagueness” (Haack 1974:123).

Jones/Jill is 1.8297 m tall: “The laws of physics can acquire this minuteness of detail only by sacrificing some of the fixed absolute certainty of common sense laws. *There is a sort of balance between precision and certainty: one cannot be increased except to the detriment of the other*” (Pierre Duehm 1904; cited in Haack 1974:123).



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Fuzzy logic is not a vague logic system, but a system of logic for dealing with vague concepts.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

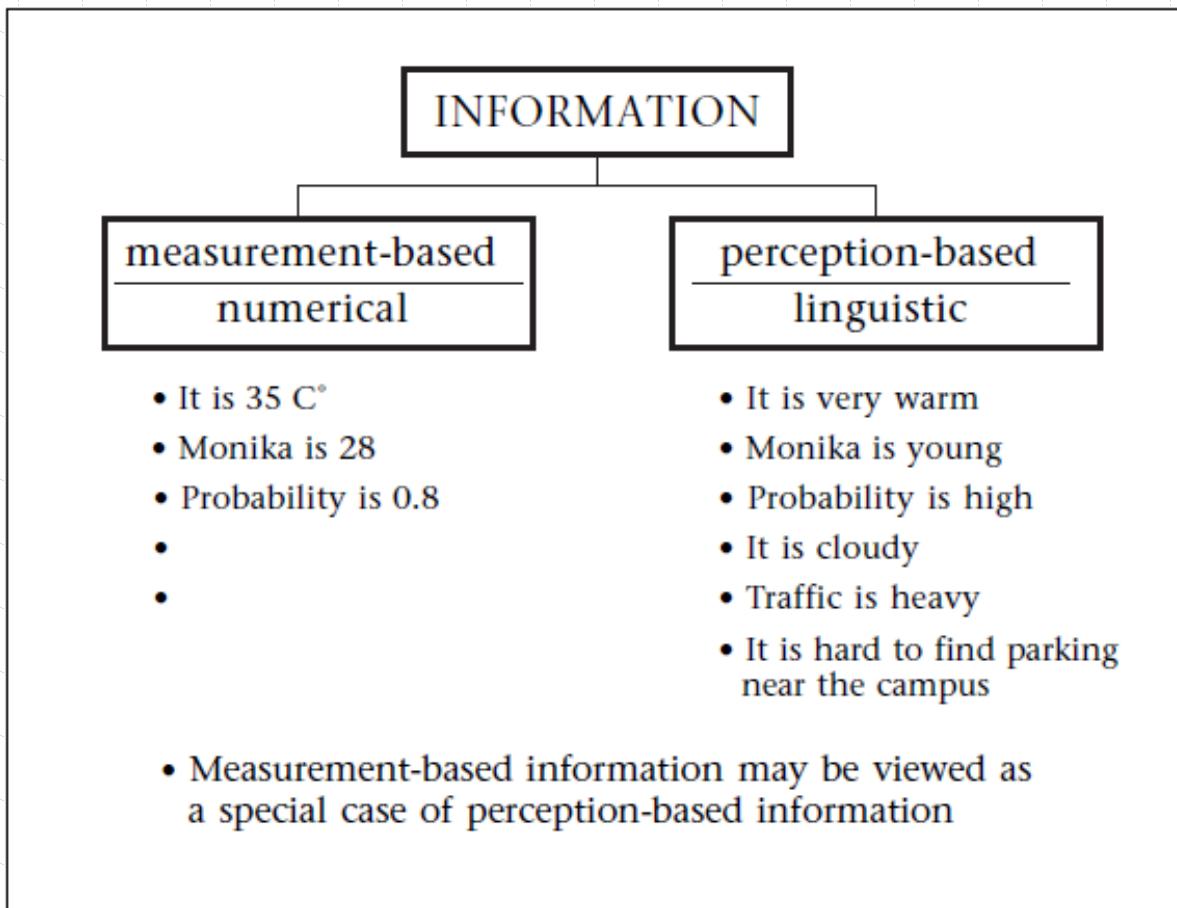
“Soft computing differs from conventional (hard) computing in that, unlike hard computing, it is tolerant of imprecision, uncertainty, partial truth, and approximation. In effect, the role model for soft computing is the human mind. The guiding principle of soft computing is: Exploit the tolerance for imprecision, uncertainty, partial truth, and approximation to achieve tractability, robustness and low solution cost.”

The above quotation is from <http://www.soft-computing.de/def.html>



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS





FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Fuzzy sets are sets whose elements have degrees of membership.

Fuzzy sets are an extension of the classical notion of set.

Taken from (Wikipedia) http://en.wikipedia.org/wiki/Fuzzy_set
on 7th October 2008



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition — an element either belongs or does not belong to the set.

Fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval [0, 1].



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval [0, 1].

Fuzzy sets generalize classical sets, since the indicator functions of classical sets are special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1

Taken from (Wikipedia) http://en.wikipedia.org/wiki/Fuzzy_set
on 7th October 2008



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Fuzzy logic is a form of multi-valued logic derived from fuzzy set theory to deal with reasoning that is approximate rather than precise.

Taken from (Wikipedia) http://en.wikipedia.org/wiki/Fuzzy_logic
on 7th October 2008



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

As in fuzzy set theory the set membership values can range (inclusively) between 0 and 1, in fuzzy logic the degree of truth of a statement can range between 0 and 1 and is not constrained to the two truth values {true, false} as in classic predicate logic.

Taken from (Wikipedia) http://en.wikipedia.org/wiki/Fuzzy_logicon 7th October 2008



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

The Originators:

Jan Lukasiewicz

Born: 21 Dec 1878 in
Lvov, Austrian Galicia
(now Ukraine); Died:
13 Feb 1956 in Dublin,
Ireland



Taken from <http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Lukasiewicz.html> on 7th October 2008



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

The Originators:

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Born: 21 Dec 1878 in Lvov, Austrian Galicia (now Ukraine); Died: 13 Feb 1956 in Dublin, Ireland.

Multi-valued logics
are logical calculi
in which there are
more than two
truth values.





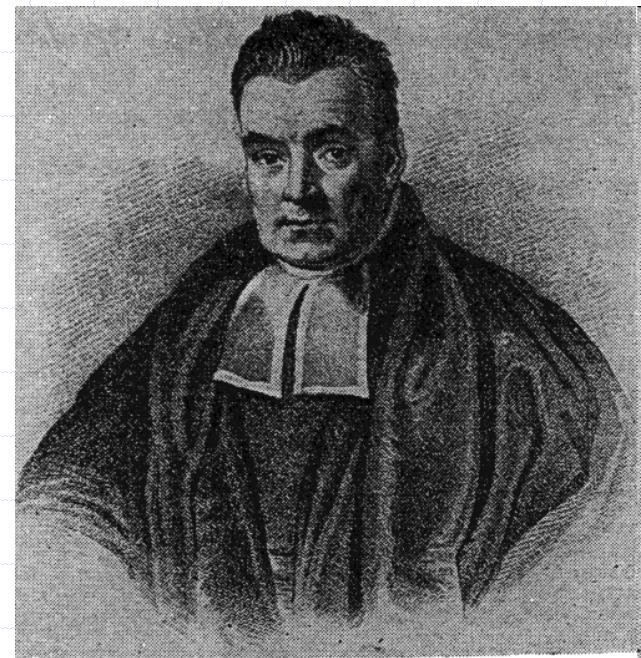
FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

The Originators:

Thomas Bayes
1702 – 1761

Bayesian probability is the name given to several related interpretations of probability, which have in common the notion of probability as something like a partial belief, rather than a frequency.



REV. T. BAYES



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

The Originators:

Lotfali Askar Zadeh

born February 4, 1921;
an Iranian-American
mathematician and
computer scientist, and a
professor of computer
science at the University
of California, Berkeley.





FUZZY LOGIC & FUZZY SYSTEMS

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FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

How is one to represent notions like:

large profit

high pressure

tall man

wealthy woman

moderate temperature.

Ordinary set-theoretic representations will require the maintenance of a crisp differentiation in a very artificial manner:

*high, high to some extent, not quite high,
very high*



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

What is 'fuzzy logic'?

Are there computers

that are inherently

fuzzy and do not apply
the usual binary logic?



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

The key point is this

The laws of physics can acquire this minuteness of detail only by sacrificing some of the fixed and absolute certainty of common-sense laws.

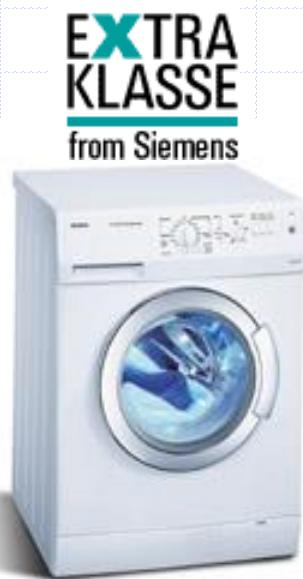
There is a sort of balance between precision and certainty: one cannot be increased except to the detriment of the other. (Haack citing the philosopher Pierre Duhem, 1974:123).



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

And more recently FUZZY Machines have been developed



The Extraklasse machine has a number of features which will make life easier for you.

Fuzzy Logic detects the type and amount of laundry in the drum and allows only **as much water** to enter the machine as is **really** needed for the loaded amount.

And **less water** will **heat up quicker** - which means **less energy consumption**.

Fuzzy Logic



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

And more recently FUZZY Machines have been developed

EXTRA
KLASSE
from Siemens



The Extraklasse machine has a number of features which will make life easier for you.

- **Foam detection**

Too much foam is compensated by an additional rinse cycle:

- **Imbalance compensation**

In the event of imbalance calculate the **maximum possible speed**, sets this speed and starts spinning.

- **Automatic water level adjustment**

Fuzzy automatic water level adjustment adapts water and energy consumption to the individual requirements of each wash programme, depending on the amount of laundry and type of fabric.

Fuzzy Logic



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Washing machines, blood pressure monitors, and obstacle avoiding cars, that claim to have built-in *fuzzy logic* demonstrate how fuzzy set theory, fuzzy logic and fuzzy control are used conjunctively to build the intelligent washing machine, the ‘wise’ monitors and the clever car.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Engineering & Technology

When confronted with a control problem for a complicated physical process, a control engineer generally follows a relatively systematic design procedure. A simple example of a control problem is an automobile “cruise control” that provides the automobile with the capability of regulating its own speed at a driver-specified set-point (e.g., 55 mph).



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Engineering & Technology

One solution to the automotive cruise control problem involves adding an electronic controller that can sense the speed of the vehicle via the speedometer and actuate the throttle position so as to regulate the vehicle speed as close as possible to the driver-specified value (the design objective). Such speed regulation must be accurate even if there are road grade changes, head winds, or variations in the number of passengers or amount of cargo in the automobile.



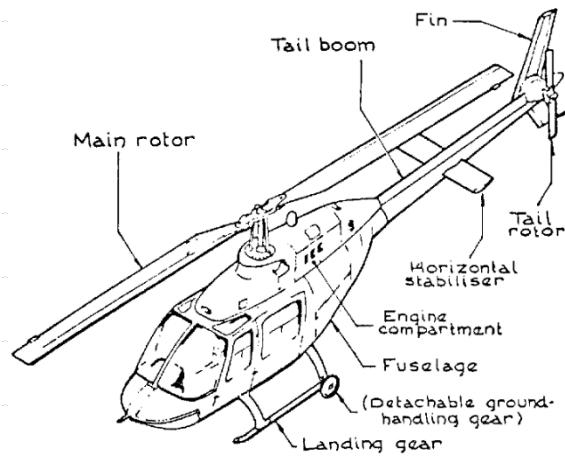
FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Engineering & Technology

Autonomous helicopter:
Control through approximate reasoning as most inputs are imprecise!

Collective Increment	Tail Collective Increment	Longitudinal Cyclic	Lateral Cyclic
Altitude	Yaw Angle	Longitudinal Position	Lateral Position
Vertical Velocity	Yaw Angle Velocity	Longitudinal Velocity	Lateral Velocity
		Pitch Angle	Roll Angle
		Pitch Angle Velocity	Roll Angle Velocity





FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Medicine

The diagnosis of disease involves several levels of uncertainty and imprecision, and it is inherent to medicine. A single disease may manifest itself quite differently, depending on the patient, and with different intensities. A single symptom may correspond to different diseases. On the other hand, several diseases present in a patient may interact and interfere with the usual description of any of the diseases.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Medicine

The best and most precise description of disease entities uses linguistic terms that are also imprecise and vague. Moreover, the classical concepts of *health* and *disease* are mutually exclusive and opposite. However, some recent approaches consider both concepts as complementary processes in the same continuum.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Medicine: Patient History

	Precise	Imprecise
Objective	Vital Signs, Lab Results, Diagnostic Tests	Patient behaving improperly during signs taking, diagnostic tests
Subjective	Mental disorder classification; Contraindications to drugs/therapy	Medical history supplied by patient/family



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Biology

- Phylogeny of mammals is the study of the connections between all groups of mammals as understood by ancestor/descendant relationships.
-
- In evolutionary biology there is a term a clade is group consisting of a species (extinct or extant) and all its descendants. Many properties of an ancestor are shared by the next generation and the descendants have newer properties.
- So we (the humans) have all descended from ‘monkeys’ but we are not quite monkeys! But some monkeys have remained more close to their ancestors – so they are more monkeys than we are?
- Species evolve is the clarion call in evolutionary biology; and this perhaps is the basis of a fuzzy classification scheme.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Biology

- Genome sequence data bases comprising sequences of primates, ranging from humans to lemurs, are being used to understand relationships between different organisms at the molecular level.
- Researchers at the US National Human Genome Research Institute after studying over 28 million gene sequences from 16 different species of primates.

We are sort of monkeys and not monkeys.

- There are three clades,, used by the researchers to describe the relationships between the primates. Strepsirrhini, or 'turning nose primates', Platyrrhini, or 'New World Monkeys', Catarrhini, or 'drooping nose primates'.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Biology

Primata (0)									
Strepsirrhini (1) → 'turning nose primates'		Platyrrhini (1) → 'New World Monkeys'			Catarrhini (1) → 'drooping nose primates'				
Lemur (2)			Not Dusky titi (2)		Cercopithecidae (2) → 'Old World Monkeys'		Non- Cercopithecidae		
				Not owl monkey (3)		Not Colobus (3)		Not Gibbon (3)	Not Orangutan (4)
Mouse lemur (3); Right tailed lemur (3)	Dusky titi (2)	Owl monkey (3)	Squirrel monkey (4); Marmoset (4)	Colobus Monkey (3)	Vervet (4)	Baboon (5); Macaque (5)	Gibbon (3)	Humans (5); chimps (5); gorilla (5)	Orangutan (4)
Galago (2)	(3)	(2)	(3)	(3)	(4)	(5)	(3)	(4)	(5)



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Economics, Finance & Politics

Many decisions are based on beliefs concerning the likelihood of uncertain events such as the outcome of an election, the guilt of a defendant, or the future value of the dollar. These beliefs are usually expressed in statements such as "I think that . . . , " "chances are . . . , " "it is unlikely that . . . , " and so forth.



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Economics, Finance & Politics

Occasionally, beliefs concerning uncertain events are expressed in numerical form as odds or subjective probabilities. What determines such beliefs? How do people assess the probability of an uncertain event or the value of an uncertain quantity?



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Economics, Finance & Politics

Occasionally, beliefs concerning uncertain events are expressed in numerical form as odds or subjective probabilities. What determines such beliefs? How do people assess the probability of an uncertain event or the value of an uncertain quantity?



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

How rice is cooked: Cooking **white** rice is a four-phase process.

- First, soak rice in water for a while;
- Second, bring the water to boil and keep the temperature to boiling point of water;
- Third, temperature increases now, tone down the heat;
- Fourth, few minutes afterwards, the rice is ready.



FUZZY LOGIC & FUZZY SYSTEMS

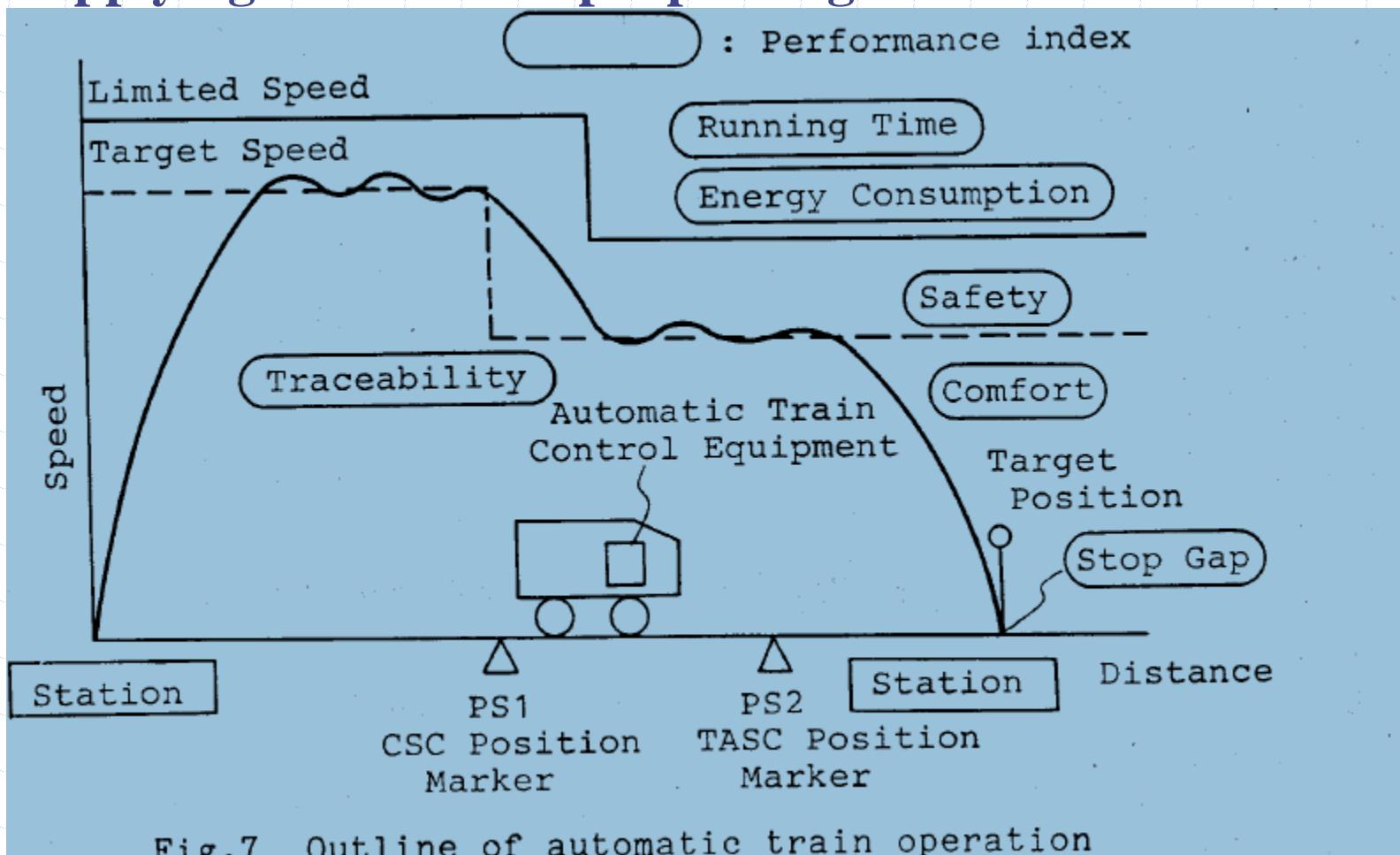
UNCERTAINTY AND ITS TREATMENT

How rice is cooked: Cooking white rice is a four-phase process. First, water is added to a pot that has ample capacity so the white rice sits in water. Then using a source of heat like a gas stove or electric plate, the mixture is heated until it is boiling and the white rice is absorbing water. The temperature remains at 212 degrees Fahrenheit, which is the boiling point of water. Part of the water turns into steam and escapes into the air. When all of the water is gone from the rice on the stove, the temperature increases. Now it is resting and there is a need to tone down the heat and to cut it off later. A few minutes afterwards, the rice is ready for serving. As we can see, there is a lot of important timing, especially at the latter phases.

FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Applying brakes to stop a passenger train:



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Applying brakes to stop a passenger train:

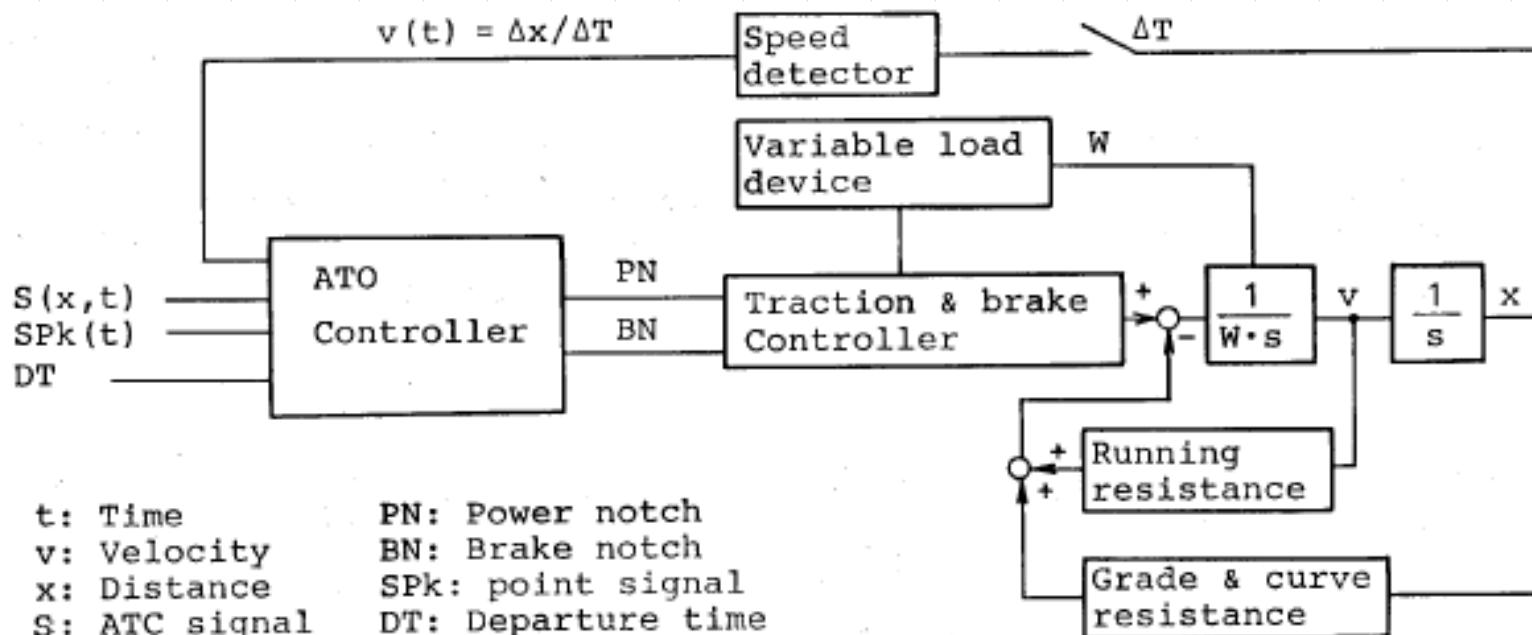


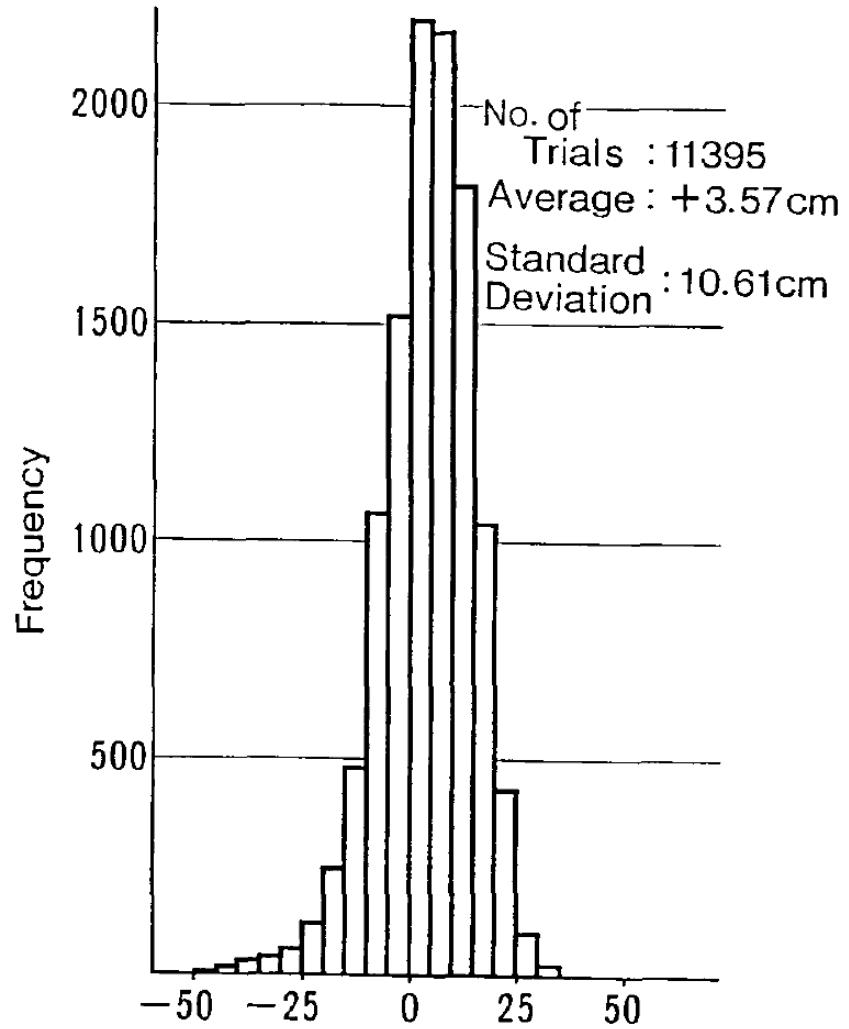
Fig.6 Block diagram of the ATO control system



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Applying
brakes to
stop a
passenger
train:





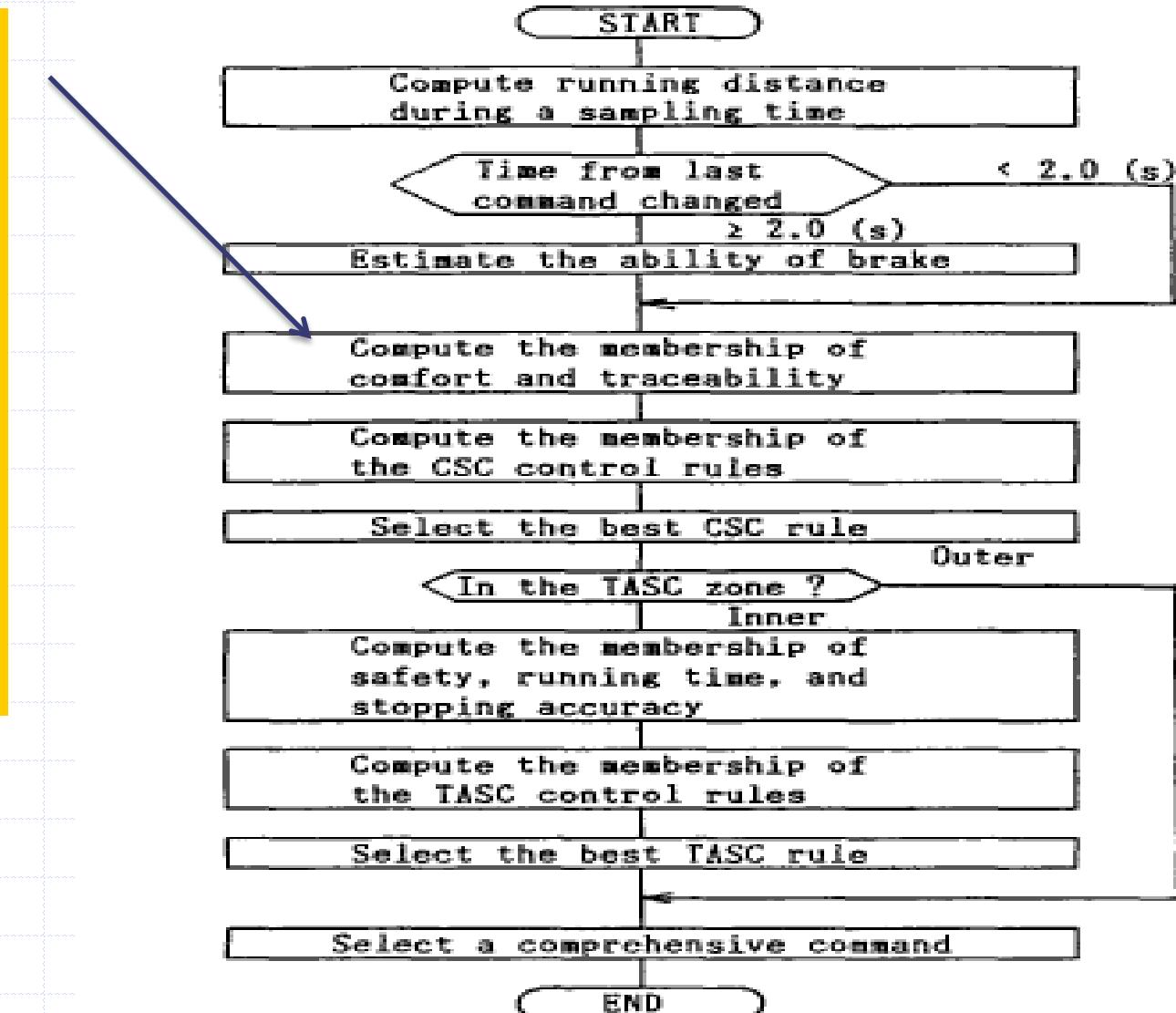
FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Oshirra, et al(1988)..

(1) CONSTANT SPEED CONTROL (CSC)

(2) TRAIN AUTOMATIC STOPPING CONTROL (TASC)



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Oshirra, et
al (1988)..

Fuzzy
Control
gives a
smoother
ride!

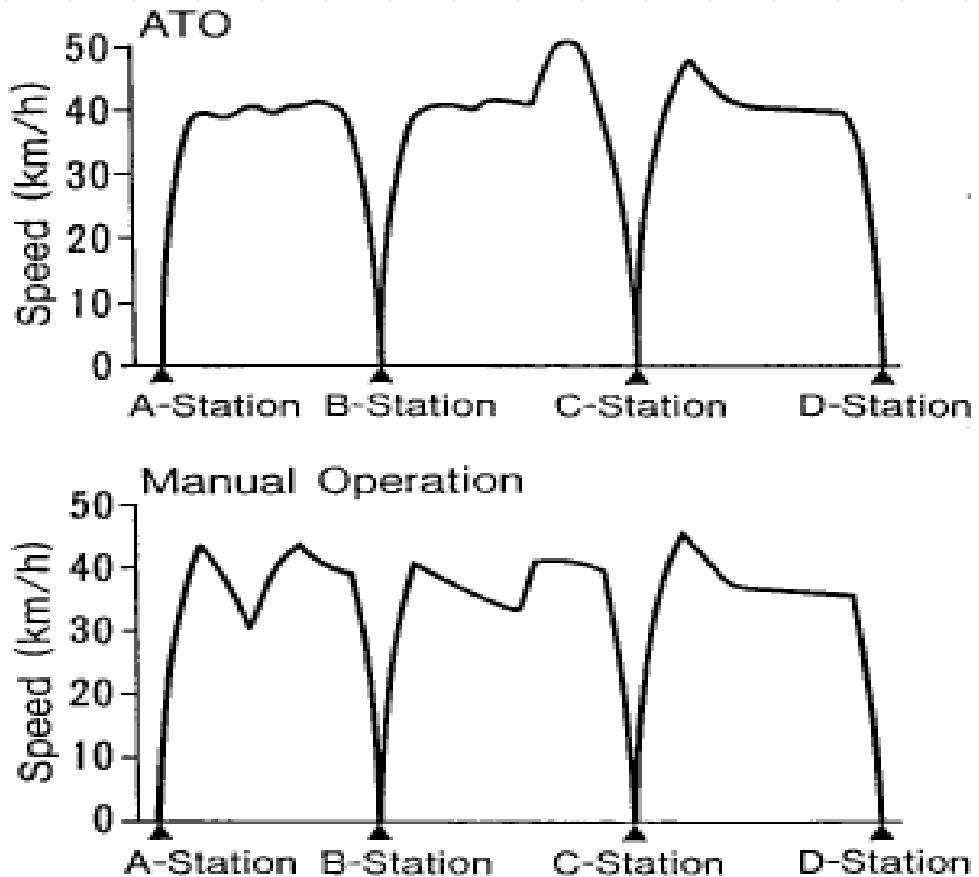


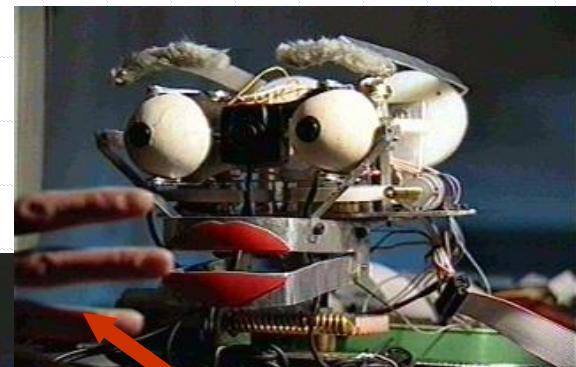
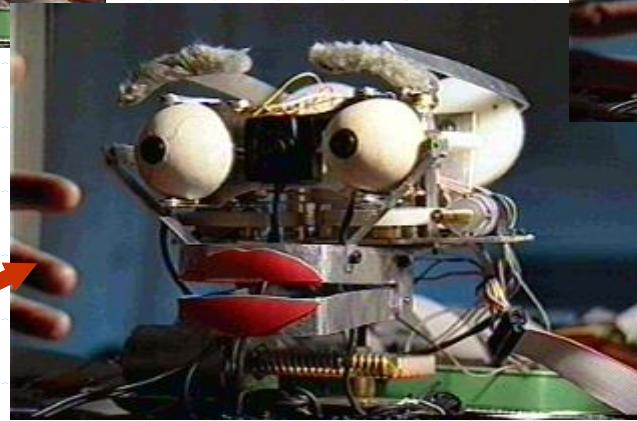
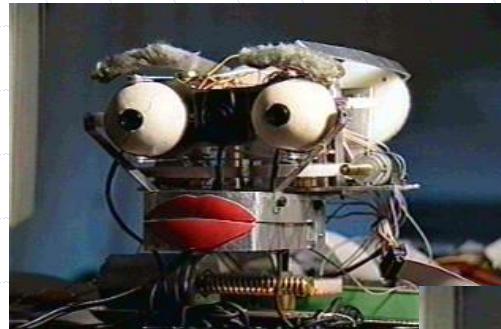
Fig. 8 Comparison between ATO Operation and Manual Operation



FUZZY LOGIC & FUZZY SYSTEMS UNCERTAINTY AND ITS TREATMENT

Reacting to *unexpected* and *expected* situations: A robot showing 'human emotions'. Somebody intruding in your space !

Relating PERCEPTION to EMOTION



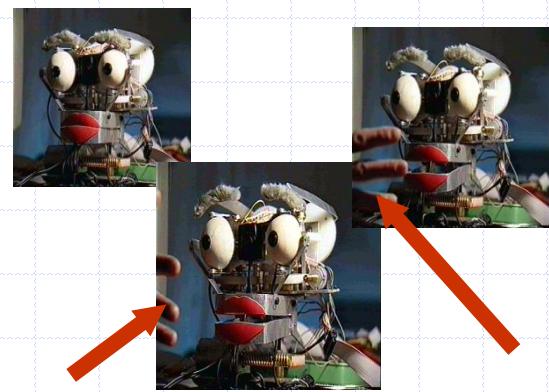


FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Reacting to *unexpected* and *expected* situations: A robot showing ‘human emotions’. Somebody intruding in your space !

Heuristics for *fear*, *anger* and *surprise* when somebody approaches you slowly or fast, (or you encounter a stationary object). The reaction to an intruder also depends on whether you are close to the intruder or some distance away.





FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Reacting to *unexpected* and *expected* situations: A robot showing ‘human emotions’. Somebody intruding in your space ! A single variable mapping – *SPEED* or *DISTANCE* + {*Emotion*}

<u>PERCEPTION</u>		<u>EMOTION</u>	
DISTANCE			
<i>IF</i>	the intruder is Far away	<i>THEN</i>	we have No Fear
<i>IF</i>	the intruder is Very Near	<i>THEN</i>	we are Not Surprised
SPEED			
<i>IF</i>	the intruder is Stationary	<i>THEN</i>	we have No Fear
<i>IF</i>	the intruder is moving Fast	<i>THEN</i>	we are Not Angry



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Reacting to *unexpected* and *expected* situations: A robot showing ‘human emotions’. Somebody intruding in your space ! A two variable mapping – *SPEED* and *DISTANCE* + {Emotion} → A Contingency Table

Distance	Speed	
	Stationary	Fast
Very Near	Very Angry, Not surprised, No Fear	Not Angry, Not surprised, Very Fearful
Far	Very Angry, Not surprised, No Fear	Not Angry, Very Surprised, No Fear



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Reacting to *unexpected* and *expected* situations: A robot showing ‘human emotions’. Somebody intruding in your space ! A two variable mapping – *SPEED* and *DISTANCE* + {*Emotion*}

		Speed		
		Stationary	Slow	Fast
Distance	Very Near	VA, NS, NF	A, NS, F	NA, NS, VF
	Near	A, NS, NF	NA, NS, NF	NA, S, F
Far		VA, NS, NF	A, S, NF	NA, VS, NF



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Reacting to *unexpected* and *expected* situations: A robot showing ‘human emotions’. Somebody intruding in your space ! A two variable mapping – *SPEED* and *DISTANCE* + {*Emotion*}

Emotional Linguistic Variable	Term Set
ANGER:	{VA → Very Angry; A → Angry; NA → Not Angry}
SURPRISE:	{VS → Very Surprised; S → Surprised; NS → Not Surprised}
FEAR:	{VF → Very Fearful; F → Fearful; NF → No Fear}



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Reacting to *unexpected* and *expected* situations: A robot showing ‘human emotions’. Somebody intruding in your space ! A two variable mapping – *SPEED* and *DISTANCE* + {*Emotion*}

Perception Linguistic Variable	Term Set
SPEED:	{F → Fast; SL → Slow; ST → Stationary}
DISTANCE:	{VN → Very Near; N → Near; F → Far Away }



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Perception and Gender

When we look at men and women, our perception of the heights is approximate and motivated by pre-conceptions of what it takes to be a tall man or short woman.

It appears that the very quantitative concept of height has an in-built uncertainty.



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Perception and Gender

MacVicar-Whelan (1978) conducted ‘an experimental and theoretical study of the categorization of human height is reported. Subjects of both sexes whose ages ranged from 6 to 72 were asked to class the height of both men and women using the labels VERY VERY SHORT, VERY SHORT, SHORT, TALL, VERY TALL, and VERY VERY TALL. The experimental results confirm Zadeh's contention **about the existence of fuzzy classification (the lack of sharp borders for the classes)**



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Perception and Gender: Term sets of heights were assigned different values by human observers in a controlled psychological experiment

Gender	Height in Centimetres						
	Very Very Short	Very Short	SHORT	TALL	Very Tall	Very Very Tall	
Men	138.7	143.1	156.8	179.4	189.5		197.7
Women	134.8	143.0	149.2	172.9	181.4		190.9



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Perception of Men's Height – figures in inches here

OBSERVER	Short			Tall		
	Very Very	Very	Just	Very Very	Very	Just
F1	59.2 (± 1)	61.8 (± 1)	67.0 (± 2)	80.5 (2.5)	77(± 2)	71.5(± 1)
F8	44.4 (± 1)	46.5(± 3)	57.2 (± 4.2)	75.2(± 6)	68.2 (± 10.8)	66.3 (± 12)
M11	53.0 (± 10)	54.0 (± 11.7)	55. (± 11.5)	80.4 (± 7.5)	78.0 (± 4)	75.2 (± 6.5)
AVERAGE (8 obs; 2-3 methods)	54.6 (4.5)	56.34 (± 5.58)	61.73 (5.75)	77.85 (± 6.27)	74.60 ± 5.32)	70.64 (± 5) 98



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Perception of Women's Height – figures in inches here

OBSERVER	Short			Tall		
	Very Very	Very	Just	Very Very	Very	Just
F1	55 (± 2)	57.5 (± 2)	60 (± 2)	78 (± 2)	74.5 (± 2)	70.4 (± 1)
F8	52.4 (± 5)	57.2 (± 4)	52.4 (± 9)	76.5 (± 9.5)	73.2 (± 10)	69.0 (± 6)
M11	49.5 (± 11)	51.5 (± 7)	54.8 (± 11.5)	79.0 (± 14.5)	76.4 (± 8.8)	75 (± 6.3)
AVERAGE (8 obs; 2-3 methods)	53.08 (7.4)	56.28 (± 4.62)	58.75 (5.75)	75.16 (± 6.75)	71.41 (± 5.32)	68.06($\pm 4.$ 9) 99



FUZZY LOGIC & FUZZY SYSTEMS

BACKGROUND & DEFINITIONS

Perception of Women's Height – figures in cms., here

OBSERVER	Short			Tall		
	"Very, Very"	Very	Just	"Very, Very"	Very	Just
F1	140±5	146±5	152±5	198±5	189±5	179±3
F8	133±13	145±10	133±23	194±24	186±25	175±15
M11	126±28	131±18	139±29	201±37	194±22	190±16
Average (8 observations)	135±19	143±12	149±15	191±17	181±14	173±12



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY

Perception and Gender: Term sets of heights were assigned different values by human observers in a controlled psychological experiment

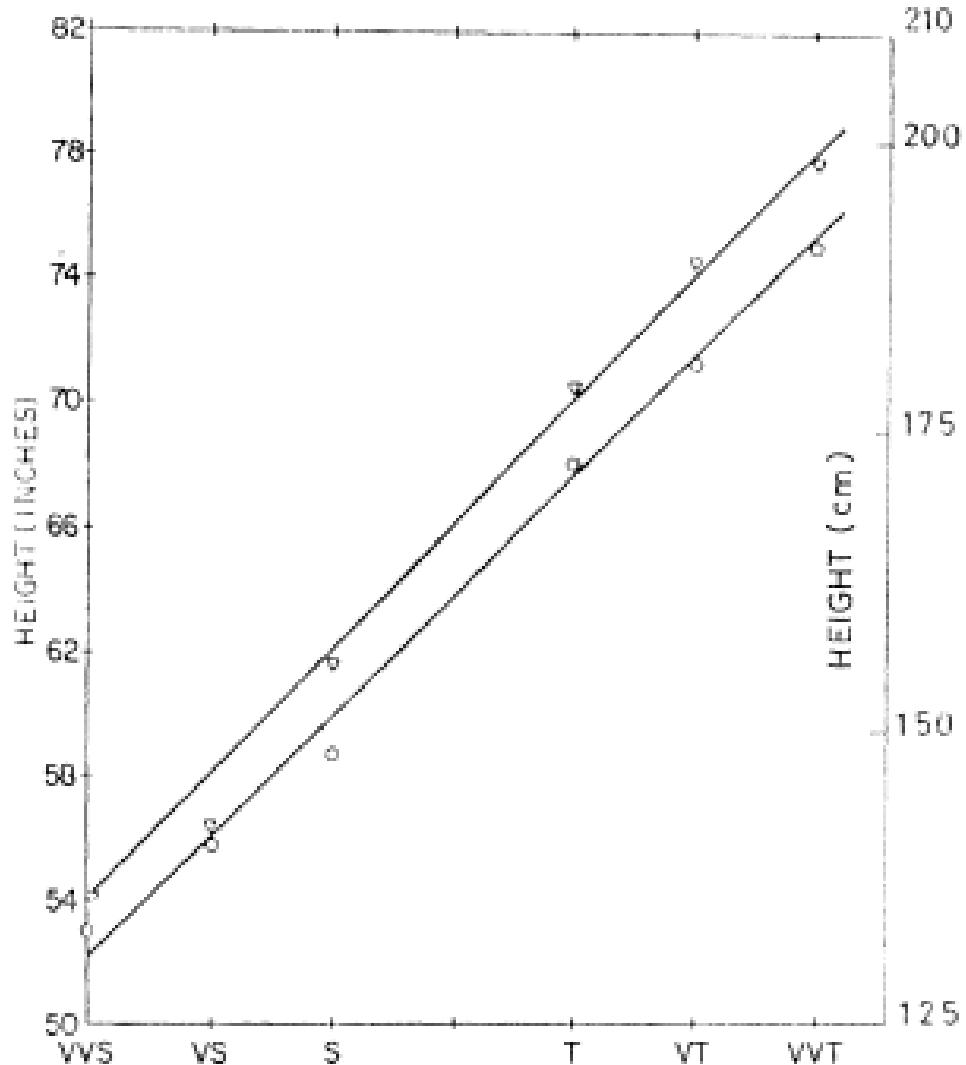


Fig. 4. Average values for neutral points of terms descriptive of height of men and women: VVS = VERY VERY SHORT; VS = VERY SHORT; S = SHORT; T = TALL; VT = VERY TALL; VVT = VERY VERY TALL; \circ = women; \triangle = men.



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Fuzzy logic is not a vague logic system, but a system of logic for dealing with vague concepts.



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Motion autonomy in Robotics may be defined as the ability for a robot to perform a given movement without any external intervention. It is a central problem in Robotics. Depending on the situation considered, motion autonomy is a goal more or less easy to reach.

Fraichard Th., & Garnier, Ph. (2001). "Fuzzy control to drive car-like vehicles," *Robotics and Autonomous Systems*, Vol. 34 (1) pp. 1-22, 2001. (available at <http://citeseer.ist.psu.edu/fraichard97fuzzy.html>)



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Easy and difficult motion autonomy

TASK DIFFICULTY	EXEMPLAR	REASON
Low	A manipulator arm operating on an assembly line	<ul style="list-style-type: none">• A priori known;• Carefully engineered;• Highly predictable workspace
High	A planetary rover	<ul style="list-style-type: none">• No detailed prior knowledge of application area;• Uncertain and little predictable environment

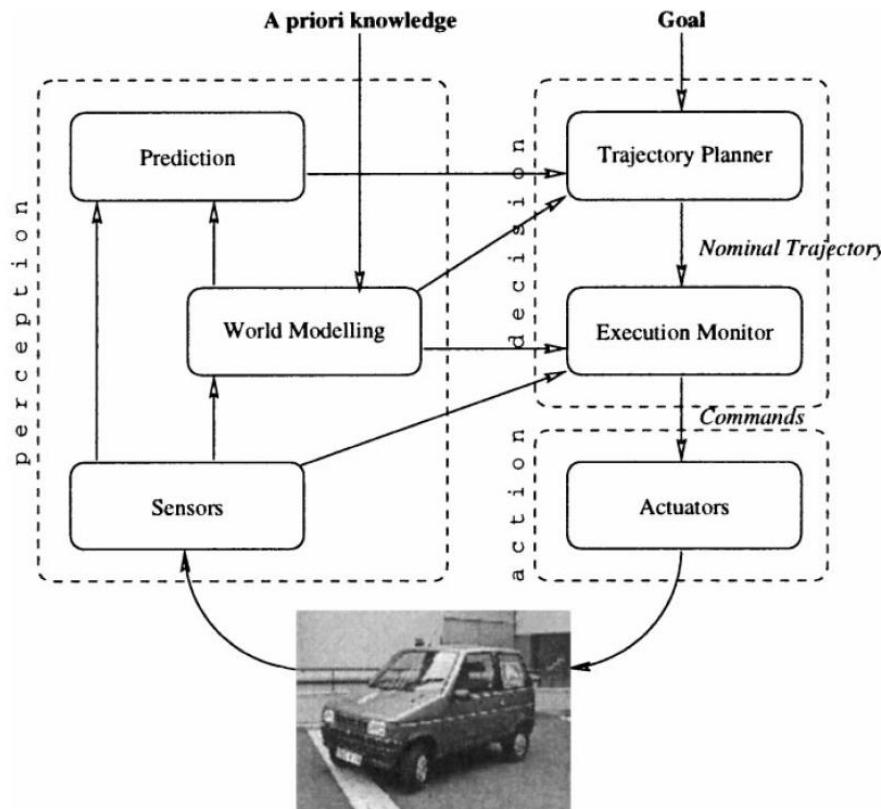


Fraichard Th., & Garnier, Ph. (2001). "Fuzzy control to drive car-like vehicles," *Robotics and Autonomous Systems*, Vol. 34 (1) pp. 1-22, 2001. (available at <http://citeseer.ist.psu.edu/fraichard97fuzzy.html>)

FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

An autonomously guided vehicle?



Fraichard Th., & Garnier, Ph. (2001). "Fuzzy control to drive car-like vehicles," *Robotics and Autonomous Systems*, Vol. 34 (1) pp. 1-22, 2001. (available at <http://citeseer.ist.psu.edu/fraichard97fuzzy.html>)



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Finally, been driven away by an autonomous car that successfully avoids obstacles on its own!



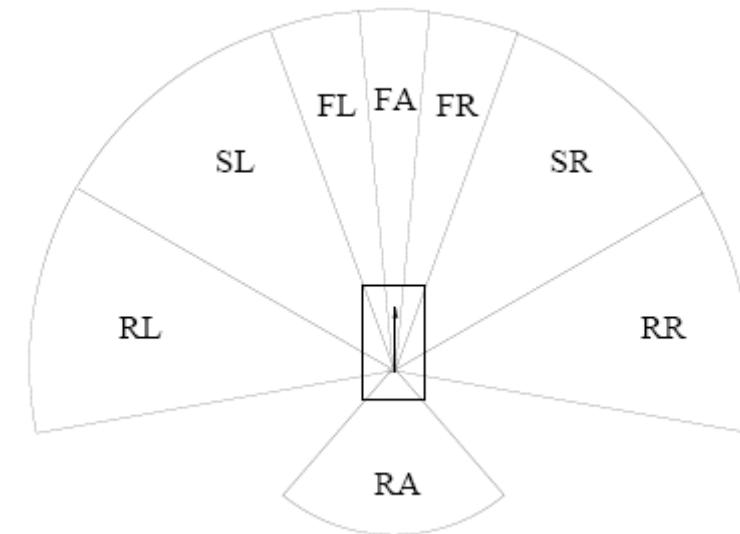
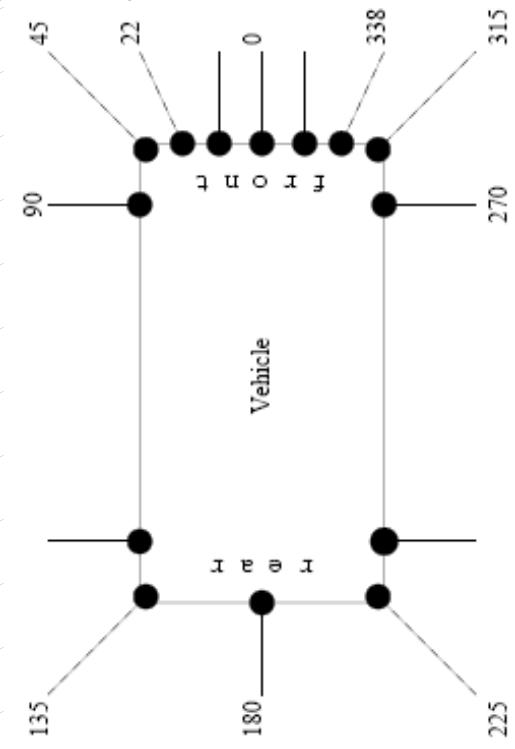
Obstacle avoidance requires a model of the actual environment of the vehicle. This model can be built from a priori information (maps of the stationary obstacles, etc.), or sensory data obtained on-line (information about the moving obstacles). This model is updated at each time step.

Fraichard Th., & Garnier, Ph. (2001). "Fuzzy control to drive car-like vehicles," *Robotics and Autonomous Systems*, Vol. 34 (1) pp. 1-22, 2001. (available at <http://citeseer.ist.psu.edu/fraichard97fuzzy.html>)

FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Finally, been driven away by an autonomous car that successfully avoids obstacles on its own!



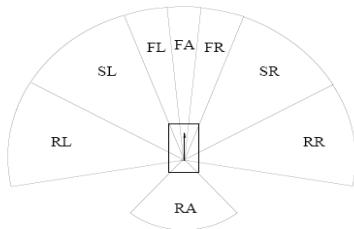
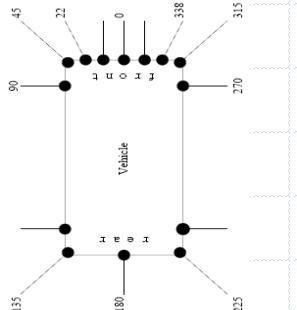
Forward Axle; Rear Axle; F. Left
F. Left; Side Left; Side Right; Rear Left; Rear Right



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Finally, been driven away by an autonomous car that successfully avoids obstacles on its own!



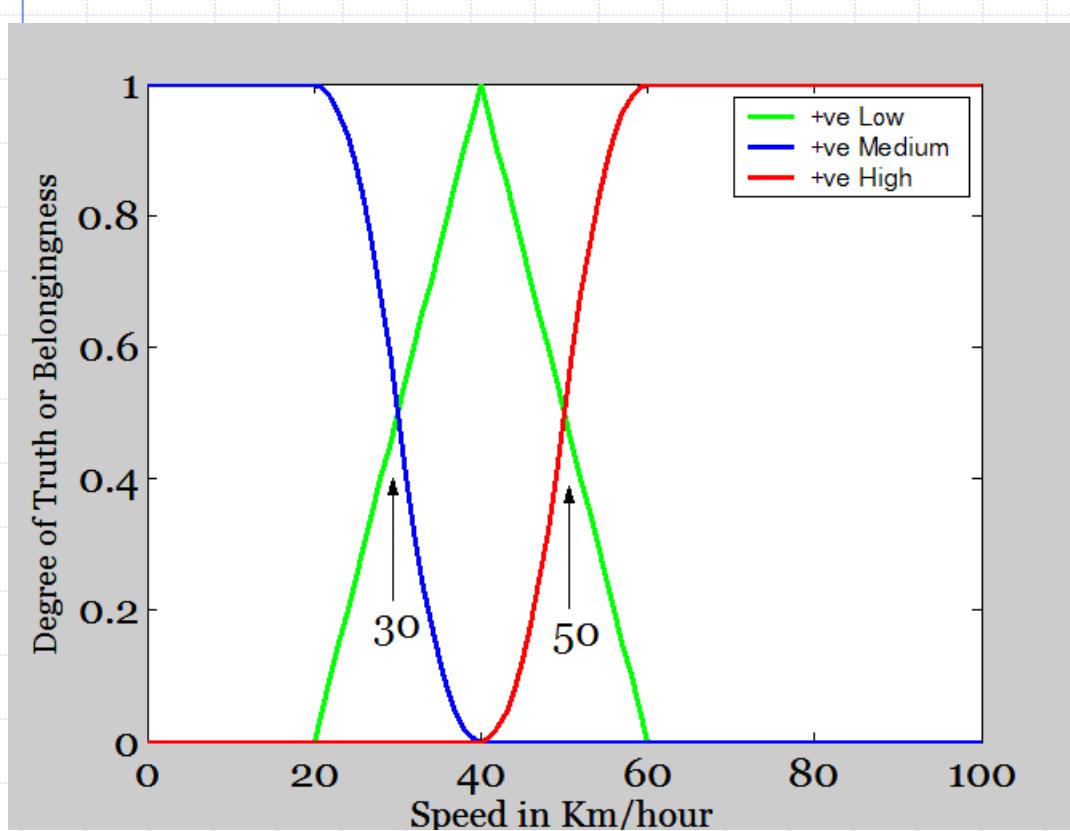
Forward Axle; Rear Axle; F. Left
F. Left; Side Left; Side Right; Rear Left;
Rear Right

A 'linguistic' rule

FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Examples of **velocity fuzzy membership functions (+ve Low, +ve Medium and +ve High, that may have been used by Ligier – the autonomous car**



– if (velocity is positive-high)
and (obstacle in FA is near)
and (obstacle in FL is near)
and (obstacle in FR is near)
then (acceleration is negative-high)

A 'linguistic' rule



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Twenty
linguistic
rules
drive a
Ligier



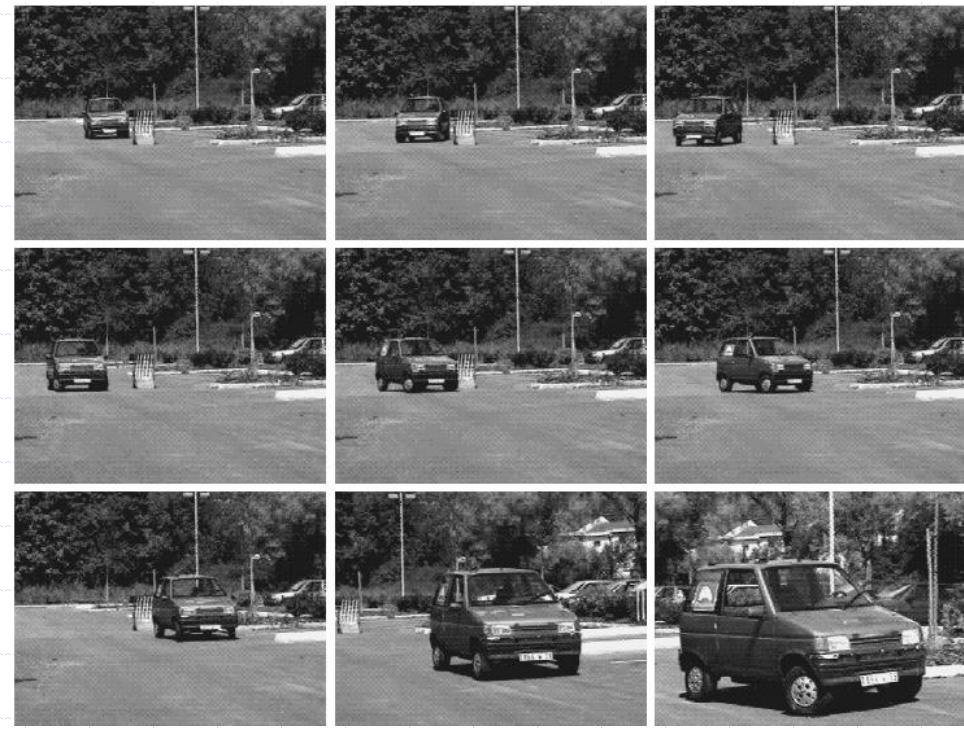


FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Finally, been driven away by an autonomous car that successfully avoids obstacles on its own!

Twenty
linguistic
rules drive
a *Ligier*





FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Books, Websites, Software

Recommended Texts

Kosko, Bart (1993). *Fuzzy Thinking: The New Science of Fuzzy Logic*. London: Harper Collins. (Available through Trinity Library → but have to wait for it to be called from Santry Collection);



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Books, Websites, Software

Companion Texts

Negnevitsky, Michael (2002). *Artificial Intelligence: A Guide to Intelligent Systems (1st Edition)*. Harlow: Pearson Education Ltd. (Chapter 4, pp 87-128). (Available at Hamilton Library Open-access Collection)

Kruse, Rudolf., Gebhardt, J., and Klawonn, F. (1994). *Foundations of Fuzzy Systems*. New York: John Wiley and Sons. (Chapter 2 for fuzzy sets and Chapter 4 for fuzzy control) (Available through Trinity Library → but have to wait for it to be called from Santry Collection)

Yager, Ronald R., and Filev, Dimitar P. (1994). *Essentials of Fuzzy Modeling and Control*. New York: John Wiley and Sons. (Chapter 4 for fuzzy control).



FUZZY LOGIC & FUZZY SYSTEMS

UNCERTAINTY AND ITS TREATMENT

Books, Websites, Software

Online Book

Passino, Kevin M. & Yurkovich, Stephen (1998). *Fuzzy Control*. Menlo Park (California):

Addison Wesley (<http://www.ece.osu.edu/~passino/FCbook.pdf#search=%22fuzzy%20control%22>, site visited 28th September 2015)

Milestone Papers:

Zadeh, L. (1965), "Fuzzy sets", *Information and Control*, Vol. 8, pp. 338-353.

Takagi, H., and Sugeno, M. (1985). 'Fuzzy Identification of Systems and its Applications to Modeling and Control'. *IEEE Transactions on Systems, Man, and Cybernetics. Volume 115*, pages 116-132.

Introductory Papers

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