Prob Exercises

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The HAND-IN details:

- deadline is **begin 9.00 lecture Thu Sep 27th**
- I am perfectly happy for you to *write answers by hand on paper* – I know I would. However I am going to set up a Blackboard submission and besides giving me your handiwork on paper, I would ask you to make (a legible scan) and to submit that via blackboard. This is just in cases of piece of paper going astray.
- If you want to create your answer purely by digital means that is also absolutely fine.

the questions:

1. Consider the following equations (alternative conditions for independence)

   (i) $P(A \wedge B) = P(A) \times P(B)$
   (ii) $P(A|B) = P(A)$

   show that (i) implies (ii), and also that (ii) implies (i)

2. *Messi* plays for *Barcelona*. He 'often' scores a hat-trick, and also 'often' Barcelona win. Suppose the numbers actually are as follows

	bw	$\neg bw$
mh	28	2
$\neg mh$	140	30

   mh means 'Messi scored a hat-trick'
   bw means 'Barcelona won'

   (a) Calculate $P(bw \mid mh)$, the conditional prob that Barcelona won given that Messi scored a hat-trick and also indicate which counts are irrelevant to this calculation

   (b) Calculate $P(mh \mid bw)$, the conditional prob that Messi scored a hat-trick given that Barcelona won and also indicate which counts are irrelevant to this calculation

3. A sound-bite may or may not have been produced by JedWard. A sound-bite may or may not report contain the word *OMG*.

   You hear *OMG* and want to work out the probability that the speaker is Jedward

   Formalize with 2 discrete variables

- discrete $Speaker$, values in $\{Jedward, Other\}$
- discrete OMG, values in $\{true, false\}$

Let jed stand for $Speaker = Jedward$, omg stands for $OMG = true$

(a) Work out which of jed or $\neg jed$ is likelier, given omg, supposing the probabilities $p(jed) = 0.01$, $p(omg|jed) = 0.95$, $p(omg|\neg jed) = 0.01$

(b) Do the same assuming $p(jed) = 0.15$, $p(omg|jed) = 0.95$, $p(omg|\neg jed) = 0.01$

(c) Do the same assuming $p(jed) = 0.01$, $p(omg|jed) = 0.95$, $p(omg|\neg jed) = 0.001$

4. Consider someone who lives in a basement flat. Sometimes it is quite noisy in the flat, and sometimes not. Sometimes it is rather cool in the flat, and sometimes not. Let $noisy$ be a variable indicating whether it is rather noisy or not, on a given day, and let $cool$ be a variable indicating whether it is rather cool or not.

   Consider the frequency table

	$noisy:+$	$noisy:-$
$cool:+$	62	108
$cool:-$	38	292

(1)

   find $p(cool:+)$ and $p(cool:+|noisy:+)$

   and conclude from this whether or not $cool:+$ is independent of $noisy:+$

5. Unknown to the occupant of the flat there is ventilator fixture in the wall which can be opened and shut to let air from the street outside in, or keep it out. Unknown to the occupant a pet cat plays about with this at night-time, sometimes leaving it open and sometimes leaving it shut. The table (1) concerns 500 days. The two tables below split these into a group of 100 days where the cat has left the ventilator open (2), and 400 days where the cat has left it shut (3)

$open:+$	$noisy:+$	$noisy:-$
$cool:+$	54	36
$cool:-$	6	4

(2)

$open:-$	$noisy:+$	$noisy:-$
$cool:+$	8	72
$cool:-$	32	288

(3)

   With reference to the table (2), find $p(cool:+|open:+)$ and $p(cool:+|open:+, noisy:+)$ and conclude from this whether or not $cool:+$ is conditionally independent of $noisy:+$ given $open:+$.

   With reference to the table (3), find $p(cool:+|open:-)$ and $p(cool:+|open:-, noisy:+)$ and conclude from this whether or not $cool:+$ is conditionally independent of $noisy:+$ given $open:-$