An illustration of Conditional Independence

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Suppose you have some data on people concerning two possible variables sea, which is whether they live by the seaside, and hip which is whether they have hip problems:

	$sea:+$	$sea:-$
$hip:+$	31	54
$hip:-$	19	146

(1)

one of the formulations of independence is $P(X|Y) = P(X)$. Lets apply that to sea and hip, in fact to the '+' settings of these variables

$p(hip:+) = (31+54)/250 = 0.34$

$p(hip:+|sea:+) = 31/(31+19) = 0.62$

so $hip:+$ and $sea:+$ are not independent; in fact sea-side living seems to increase the chance of hip problems, which seems weird

suppose that digging into the data a little further you find there was one other variable: old for whether or not person was old. There were 50 old and 200 not old, and when the data is split into two sub-groups according to the value old you find:

old	$sea:+$	$sea:-$
$hip:+$	27	18
$hip:-$	3	2

$\neg old$	$sea:+$	$sea:-$
$hip:+$	4	36
$hip:-$	16	144

(2)

- we can show that **hip:+ is conditionally independent of sea:+ given old:+**
  $p(hip:+|old:+) = 45/50 = 9/10$
  $p(hip:+|old:+, sea:+) = 27/30 = 9/10$
- we can show that **hip:+ is conditionally independent of sea:+ given old:-**
  $p(hip:+|old:-) = 40/200 = 1/5$
  $p(hip:+|old:-, sea:+) = 4/20 = 1/5$
- so zeroing in old people, sea-side living does not to increase the chance of hip problems; zeroing in on young people, it doesn't either

once you have a conditional independence it means that you can use the chain rule and use the conditional independence to simplify. We will see this in other examples; in the current case you could do this to get relatively simple formula for $p(old, sea, hip)$

$$p(old, sea, hip) = p(hip|sea, old) \times p(sea|old) \times p(old) \quad (3)$$

$$= p(hip|old) \times p(sea|old) \times p(old) \quad (4)$$

(3) is just applying the chain rule and holds without any independence assumptions

(4) is the simplification which is possibly by putting in the conditional independence that $p(hip|sea, old) = p(hip|old)$