4CSLL5
IBM Translation Models

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October 4, 2018

IBM models intro

Outline

Lexical Translation

- How to translate a word → look up in dictionary

  Haus — *house, building, home, household, shell.*

- Multiple translations
  - some more frequent than others
  - for instance: *house*, and *building* most common
  - special cases: *Haus* of a *snail* is its *shell*

Collect Statistics

- Suppose a parallel corpus, with German sentences paired with English sentences, and suppose people inspect this marking how *Haus* is translated.

  ⋮

  das **Haus** *ist klein*   *the* **house** *is small*

  ⋮

- Hypothetical table of frequencies

Translation of Haus	**Count**
house	8,000
building	1,600
home	200
household	150
shell	50

Estimation of Translation Probabilities

- from this could use relative frequencies as estimate of translation probabilities $t(e|Haus)$
- technically this is a maximum likelihood estimate – there could be others
- outcome would be

$$tr(e|Haus) = \begin{cases} 0.8 & \text{if } e = \textit{house}, \\ 0.16 & \text{if } e = \textit{building}, \\ 0.02 & \text{if } e = \textit{home}, \\ 0.015 & \text{if } e = \textit{household}, \\ 0.005 & \text{if } e = \textit{shell}. \end{cases}$$

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- though originally developed as models of translation, these models are now used as models of alignment, providing crucial training input for so-called 'phrase-based SMT'

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- comments on notation in Koehn, J&M

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$$p(\mathbf{s}|\mathbf{o}) = \frac{count(\langle \mathbf{o}, \mathbf{s} \rangle \in \mathbf{d})}{\sum_{\mathbf{s}'} count(\langle \mathbf{o}, \mathbf{s}' \rangle \in \mathbf{d})}$$

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- but even in very large corpora the vast majority of possible $\mathbf{o}$ and $\mathbf{s}$ occur zero times. So this method gives uselessly bad estimates.

The Noisy-Channel formulation

- recalling Bayesian classification, finding **s** from **o**:

$$\arg\max_{\mathbf{s}} P(\mathbf{s}|\mathbf{o}) = \arg\max_{\mathbf{s}} \frac{P(\mathbf{s},\mathbf{o})}{P(\mathbf{o})} \tag{1}$$

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- The reason for the notation **s** and **o** is that (3) is the defining equation of Shannons 'noisy-channel' formulation of decoding, where an original 'source' **s** has to be recovered from a noisy observed signal **o**, the noisiness defined by $P(\mathbf{o}|\mathbf{s})$

Now have to start look at the details of the IBM models of $P(\mathbf{o}|\mathbf{s})$, starting with the very simplest models

What all the models have in common is that they define $P(\mathbf{o}|\mathbf{s})$ as a combination of other probability distributions

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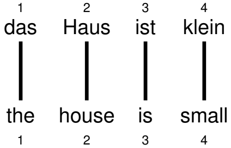

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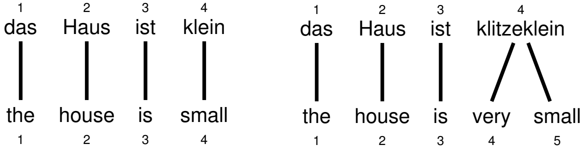

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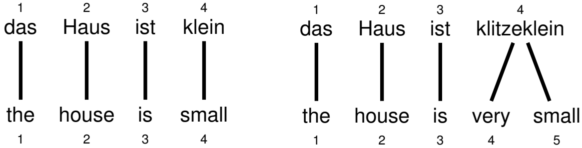

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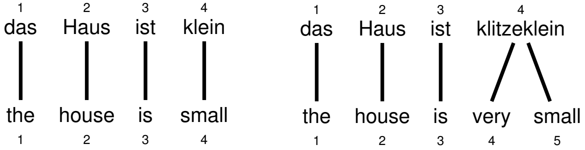

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- warning: there are quite a lot of varying formal definitions of alignment

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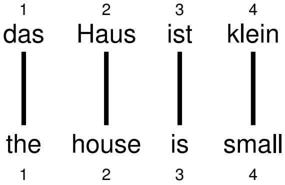

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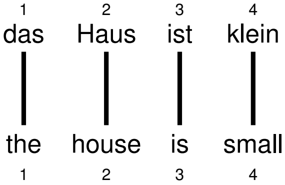

represents

$$a : \{1 \to 1, 2 \to 2, 3 \to 3, 4 \to 4\}$$

Some weirdness about directions

1	2	3	4
das	Haus	ist	klein
|	|	|	|
the	house	is	small
1	2	3	4

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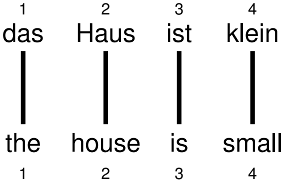

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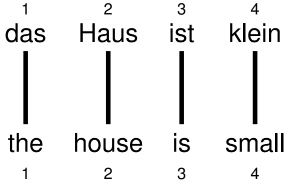

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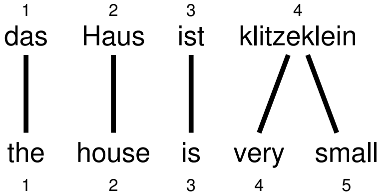
- the alignment goes up the page, English-to-German,
- they will be used though in a model of $P(\mathbf{o}|\mathbf{s})$, so down the page, German-to-English

Comparison to 'edit distance' alignments

in case you have ever studied 'edit distance' alignments . . .

- like edit-dist alignments, its a function:
  so can't align 1 **o** words with 2 **s** words
- like edit-dist alignments, some **s** words can be unmapped to
  (cf. insertions)
- like edit-dist alignments, some **o** words can be mapped to nothing
  (cf. deletions)
- unlike edit-dist alignments, order not preserved: so $j < j' \not\rightarrow a(j) < a(j')$

N-to-1 Alignment (ie. 1-to-N Translation)



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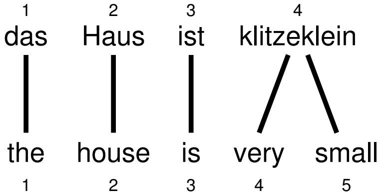

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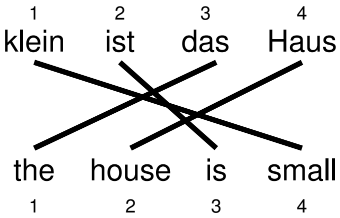
- N words of **o** can be aligned to 1 word of **s**

  (needed when 1 word of **s** translates into N words of **o**)

Reordering



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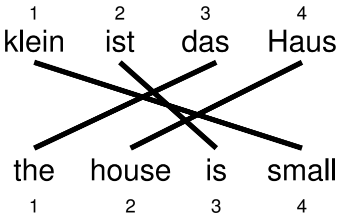

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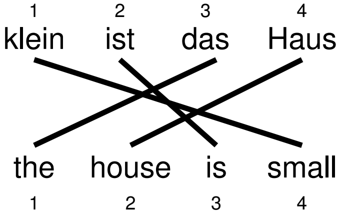

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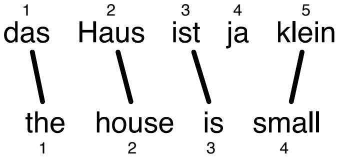
- alignment does not preserve **o** word order

  (needed when **s** words reordered during translation)

s words not mapped to (ie. dropped in translation)



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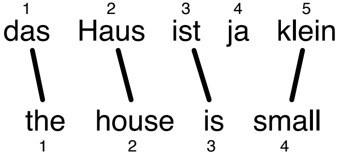
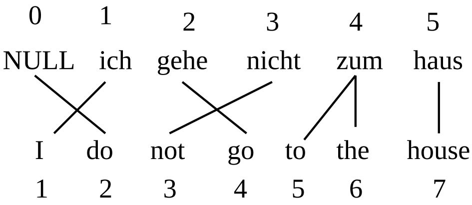


- $a : \{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 5\}$
- some **s** words are not mapped-to by the alignment

  (needed when **s** words are dropped during translation

  (here the German flavouring particle 'ja' is dropped)

- words mapped to nothing (ie. inserting in translation)



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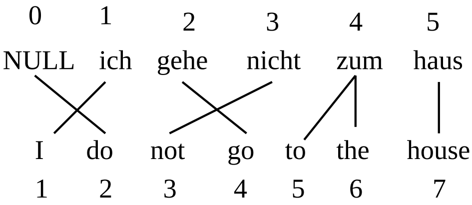

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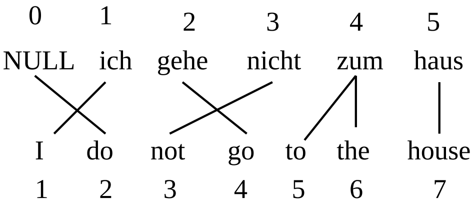

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- some **o** word are mapped to nothing by the alignment

  (needed when **o** words have no clear origin during translation)

  The is no clear origin in German of the English 'do'

  formally represented by alignment to special NULL token

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where L is length for $\mathbf{o}$ sentences, and a is an alignment from $\mathbf{o}$ sentences of length L to $\mathbf{s}$.
- $\mathbf{o}$, a, L are intended to be synchronized in the sense that if L is not the $\ell_{\mathbf{o}}$ the probability is zero. Similarly if a is not an alignment function from length L sequences to length $\ell_{\mathbf{s}}$ sequences, the probability is 0. So we will write

$$P(\mathbf{o}, a, \ell_{\mathbf{o}}, \mathbf{s})$$

Length dependency

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- first without any assumptions, via the chain rule:

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the IBM model1 assumptions are all about $P(\mathbf{o}, a, \ell_{\mathbf{o}}|\mathbf{s})$. The assumptions can be shown by a succession of applications of the chain rule concerning $(\mathbf{o}, a, \ell_{\mathbf{o}})$

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$$P(\mathbf{o}, a, \ell_{\mathbf{o}} | \mathbf{s}) = \frac{p(\ell_{\mathbf{o}} | \ell_{\mathbf{s}})}{(\ell_{\mathbf{s}} + 1)^{\ell_{\mathbf{o}}}} \times \prod_j [p(o_j | s_{a(j)})] \tag{7}$$

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3. for $j = 1$ to $j = \ell_{\mathbf{o}}$, choose o_j according to distribution $p(o_j|s_{a(j)})$

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[1] see p87 Koehn book

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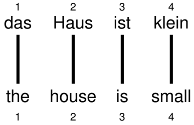

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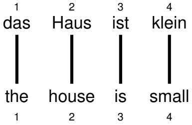

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- we will illustrate the value of $p(\mathbf{o}, a, \ell_{\mathbf{o}}|\mathbf{s})$ in this case, according to the formula (7)

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