==2.31 == a) **ANSWER: ii)** Let A =

$$\begin{pmatrix} 1 & 5 & 4 \\ 2 & 3 & 6 \\ 1 & 1 & 1 \end{pmatrix}$$

det(A) = 1 * (3 * 1 - 6 * 1) - 5 * (2 * 1 - 6 * 1) + 4 * (2 * 1 - 3 * 1) = **13**

b) **ANSWER: i)** Let B =

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

det(K) = 6 * (11 * 16 - 12 * 15) - 7 * (10 * 16 - 12 *14) + 8 * (10 * 15 - 11 * 14)

Also let: K =

$$\begin{pmatrix} 6 & 7 & 8 \\ 10 & 11 & 12 \\ 14 & 15 & 16 \end{pmatrix}$$

det(K) = 6 * (11 * 16 - 12 * 15) - 7 * (10 * 16 - 12 * 14) + 8 * (10 * 15 - 11 * 14) = 0 L =

$$\begin{pmatrix} 5 & 7 & 8 \\ 9 & 11 & 12 \\ 13 & 15 & 16 \end{pmatrix}$$

det(L) = 5 * (11 * 16 - 12 * 15) - 7 * (9 * 16 - 12 * 13) + 8 * (9 * 15 - 11 * 13) = 0 M =

$$\begin{pmatrix} 5 & 6 & 8 \\ 9 & 10 & 12 \\ 13 & 14 & 16 \end{pmatrix}$$

det(M) = 5 * (10 * 16 - 12 * 14) - 6 * (9 * 16 - 12 * 13) + 8 * (9 * 14 - 10 * 13) = 0 N =

$$\begin{pmatrix} 5 & 6 & 7 \\ 9 & 10 & 11 \\ 13 & 14 & 15 \end{pmatrix}$$

det(L) = 5 * (10 * 15 - 11 * 14) - 6 * (9 * 15 - 11 * 13) + 7 * (9 * 14 - 10 * 13) = 0

det(B) = 1 * det(K) - 2 * det(L) + 3 * det(M) - 4 *det(N) = 0 - 0 + 0 - 0 = **0**

3.2

$$f(x) = x - 2e^{-x}$$

a) **ANSWER: ii)** 1st Iteration: a=0, b=1, x0=(0+1) / 2 = 0.5 f(0) = -2 f(0.5) = -0.71306 f(1) = 0.26424

f(0.5) and f(1) have opposite signs, and since f(x) is continuous, root lies between 0.5 and 1

2nd Iteration: a=0.5, b=1, x1=(0.5+1) / 2 = 0.75 f(0.5) = -0.71306 f(0.75) = -0.19473 f(1) = 0.26424

f(0.75) and f(1) have opposite signs, and since f(x) is continuous, root lies between 0.75 and 1

3rd Iteration: a=0.75, b=1, x2=(0.75+1) / 2 = 0.875 f(0.75) = -0.19473 f(0.875) = 0.04126 f(1) = 0.26424

f(0.75) and f(0.875) have opposite signs, and since f(x) is continuous, root lies between 0.75 and 0.875 x = (0.75+0.875)/2 = **0.8125**

b) **ANSWER: ii)** 1st Iteration: x1 = 0, x2 = 1: f(x1) = f(0) = -2 f(x2) = f(1) = 0.264

x3 = 0.883298

2nd Iteration: x2 = 1, x3 = 0.883298: f(x2) = f(1) = -2 f(x3) = f(0.883298) = 0.05646385

x4 = 0.8865

3rd Iteration: x3 = 0.883298, x4 = 0.8865: f(x3) = f(0.883298) = 0.05646385 f(x4) = f(0.8865) = 0.06231326

x5 = **0.85236**

c) **ANSWER: iv)**

$$f(x) = x - 2e^{-x}$$

$$f'(x) = 1 + 2e^{-x}$$

1st Iteration: x1 = 1 f(x1) = 0.264241 f'(x1) = 1.73576 x2 = 0.847766

2nd Iteration: x2 = 0.847766 f(x2) = -0.008975 f'(x2) = 1.85674 x3 = 0.8526

3rd Iteration: x3 = 0.8526 f(x3) = 0.00000000999 f'(x3) = 1.8526 x4 = 0.8526 x = **0.8526**

4.24 a) **ANSWER: i)**

$$\begin{pmatrix} -1 & 2 & 1 \\ 2 & 2 & -4 \\ 0.2 & 1 & 0.5 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R1 = R1 + 0.5 * R2 R3 = R3 - 0.1 * R2

$$\begin{pmatrix} 0 & 3 & -1 \\ 2 & 2 & -4 \\ 0 & 0.8 & 0.9 \end{pmatrix} \quad \begin{pmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & -0.1 & 1 \end{pmatrix}$$

R1 = R1 - 3.75 * R3 R2 = R2 / 2

$$\begin{pmatrix} 0 & 0 & -4.375 \\ 1 & 1 & -2 \\ 0 & 0.8 & 0.9 \end{pmatrix} \quad \begin{pmatrix} 1 & 0.875 & -3.75 \\ 0 & 0.5 & 0 \\ 0 & -0.1 & 1 \end{pmatrix}$$

R1 = R1 / (-4.375) R3 = R3 * (1.25)

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & -2 \\ 0 & 1 & 1.125 \end{pmatrix} \quad \begin{pmatrix} -8/35 & -0.2 & 6/7 \\ 0 & 0.5 & 0 \\ 0 & -0.125 & 1.25 \end{pmatrix}$$

R2 = R2 + 2 * R1 R3 = R3 - 1.125 * R1

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -8/35 & -0.2 & 6/7 \\ -16/35 & 0.1 & 12/7 \\ 9/35 & 0.1 & 2/7 \end{pmatrix}$$

R2 = R2 - R3

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -8/35 & -0.2 & 6/7 \\ -25/35 & 0 & 10/7 \\ 9/35 & 0.1 & 2/7 \end{pmatrix}$$

R1 --> R3 R2 --> R1 R3 --> R2

$$M^{-1} =$$

$$\begin{pmatrix} -25/35 & 0 & 10/7 \\ 9/35 & 0.1 & 2/7 \\ -8/35 & -0.2 & 6/7 \end{pmatrix}$$

$$M^{-1} =$$

$$\begin{pmatrix} -0.7143 & 0 & 1.4286 \\ 0.2571 & 0.1 & 0.2857 \\ -0.2286 & -0.2 & 0.8571 \end{pmatrix}$$

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b) **ANSWER: i)**

$$\begin{pmatrix} -1 & -2 & 1 & 2 \\ 1 & 1 & -4 & -2 \\ 1 & -2 & -4 & -2 \\ 2 & -4 & 1 & -2 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R1 = R1 + R2 R3 = R3 - R2

$$\begin{pmatrix} 0 & -1 & -3 & 0 \\ 1 & 1 & -4 & -2 \\ 0 & -3 & 0 & 0 \\ 2 & -4 & 1 & -2 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R3 = R3 / (-3) R4 = R4 + R2 R1 = R1 * (-1)

$$\begin{pmatrix} 0 & 1 & 3 & 0 \\ 1 & 1 & -4 & -2 \\ 0 & 1 & 0 & 0 \\ 3 & -3 & -3 & -4 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1/3 & -1/3 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

R2 = R2 - R1 R4 = R4 + 3 * R2

$$\begin{pmatrix} 0 & 1 & 3 & 0 \\ 1 & 0 & -7 & -2 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & -3 & -4 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1/3 & -1/3 & 0 \\ 0 & 2 & -1 & 1 \end{pmatrix}$$

R1 = R1 - R3 R4 = R4 - 3 * R2

$$\begin{pmatrix} 0 & 0 & 3 & 0 \\ 1 & 0 & -7 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 18 & 2 \end{pmatrix} \quad \begin{pmatrix} -1 & -4/3 & 1/3 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1/3 & -1/3 & 0 \\ -3 & -4 & -1 & 1 \end{pmatrix}$$

R2 = R2 + (7/3) * R1 R4 = R4 - 6 * R1

$$\begin{pmatrix} 0 & 0 & 3 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \quad \begin{pmatrix} -1 & -4/3 & 1/3 & 0 \\ -4/3 & -10/9 & 7/9 & 0 \\ 0 & 1/3 & -1/3 & 0 \\ 3 & 4 & -3 & 1 \end{pmatrix}$$

R1 = R1 / 3 R2 = R2 + R4 R4 = R4 / 2

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1/3 & -4/9 & 1/9 & 0 \\ 5/3 & 26/9 & -20/9 & 1 \\ 0 & 1/3 & -1/3 & 0 \\ 3/2 & 2 & -3/2 & 1/2 \end{pmatrix}$$

R1 --> R3 R2 --> R1 R3 --> R2

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 5/3 & 26/9 & -20/9 & 1 \\ 0 & 1/3 & -1/3 & 0 \\ -1/3 & -4/9 & 1/9 & 0 \\ 3/2 & 2 & -3/2 & 1/2 \end{pmatrix}$$

$$M^{-1} =$$

$$\begin{pmatrix} 5/3 & 26/9 & -20/9 & 1 \\ 0 & 1/3 & -1/3 & 0 \\ -1/3 & -4/9 & 1/9 & 0 \\ 3/2 & 2 & -3/2 & 1/2 \end{pmatrix}$$

$$M^{-1} =$$

$$\begin{pmatrix} 1.6667 & 2.8889 & -2.2222 & 1 \\ 0 & 0.3333 & -0.3333 & 0 \\ -0.3333 & -0.4444 & 0.1111 & 0 \\ 1.5 & 2 & -1.5 & 0.5 \end{pmatrix}$$