# Model free analysis and tuning of PID controller

Zhiqiang Zhu
State Key Laboratory of Robotics, Shenyang Institute of Automation, CAS, Shenyang, China
University of Chinese Academy of Sciences, Beijing 100049, China
zhuzhiqiang@sia.cn

Kun Liu
State Key Laboratory of Robotics, Shenyang Institute of Automation, CAS, Shenyang, China
Unit 93033; PLA of China; Shenyang 110141; China

Yuqing He, Juntong Qi and Jianda Han
State Key Laboratory of Robotics, Shenyang Institute of Automation, CAS, Shenyang, China

***Abstract*—In this paper, a new PID parameter tuning method is proposed. First, extensive analysis of the PID frequency properties is conducted. Based on the analysis results, the concept of characteristic frequency of the PID controller is proposed, which builds a relationship between the PID parameters and the oscillation characteristics of the closed loop response of the PID control system. Second, extensive study is made on how the PID parameters influence the closed loop system response characteristics. Third, based on the characteristic frequency of the PID controller and the corresponding analysis results, a set of tuning rules of the PID parameters are proposed, which are based on the characteristics of the closed loop response. The merits of these tuning rules are: only the characteristics of the closed loop response of the control system are required in the tuning process, while the system model of the controlled object is not required. Finally, the effectiveness of these tuning rules is verified by the simulation results on several typical models of the controlled objects.**

***Keywords—PID control; characteristic frequency; frequency characteristics; closed loop response; frequency zone analysis***

## I. INTRODUCTION

Due to the easiness in tuning and implementation, the PID control method has gain great popularity in the industrial applications, such as process control and mechatronic system control. Corresponding to its great influence in the engineering field, in the academic world, extensive and systematical study has been made on how to tune the PID controller effectively and conveniently. Abundant research results have been acquired [1]-[3]. Many famous and far-reaching methods can be found in the literature, such as the Ziegler-Nichols method [1] [2], the IMC method [3]-[5]. In the early Ziegler-Nichols method, the output of the system is analyzed, and then based on the shape of the output, the PID parameters are directly given according to a set of clear relationships. After the Ziegler-Nichols method, many other design methods were proponed, for example, the internal model based design method (IMC) [3]-[5], the loop shaping method [6] [7]. These methods are often based on certain characteristics of the plants or based on typical plant models. To apply these methods, the characteristics of the plants or the plant models are first identified, and then according to certain algorithm, the parameters of the PID controller are directly given. For example, the IMC method needs the model of the controlled object to build the internal model controller, and then through structural transformation and parameter approximation, the internal model controller is transformed into PID controller. In the loop shaping designing method, the system model is used to design the shape of the bode diagram of the open loop transfer function, and when the requirements of magnitude margin and phase margin are fulfilled, the PID controller designing is accomplished.

The above model based designing methods are useful in the optimization of the PID parameter. However, the original advantage of PID controller that the parameter tuning requires little system model information is lost, and the designing results are heavily dependent on the modeling precision. Thus, the designing result can be greatly affected by the modeling error and system identification error, and the popularizing of these designing methods can be difficult.

For the above reasons, in the practical engineering application, for a good PID tuning strategy, the required system model information should be very little, and the PID parameters tuning should be based directly on the closed loop system response characteristics [8] [9] [10].

The aim of this paper is to give a new PID tuning strategy which is based directly on the closed loop system response characteristics. The most significant characteristics of this tuning strategy is that the tuning process is based on information that can be easily and directly measured, or is known already, such as the vibration frequency of the closed loop system response, and the PID characteristic frequency which can be calculated from the existing PID parameters. In this way, the complicated analysis during the designing is avoided, and the PID designing becomes quite easy.

## II. NEW PARAMETERS FOR PID CONTROLLER

The transfer function of a typical PID controller is given as:

$$C(s) = K_i \frac{1}{s} + K_p + K_d s \tag{2.1}$$

To express the phase and magnitude of the PID controller in a clear and regular form, and make it convenient in the analysis and adjustment of the PID parameters. New parameters are proposed, and the PID controller is expressed as:

$$C(s) = \alpha + \beta\left(\frac{\omega_n}{s} + \frac{s}{\omega_n}\right) \tag{2.2}$$

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With:

$$\begin{cases}\alpha = K_p \\ \beta = \sqrt{K_i K_d} \\ \omega_n = \sqrt{K_i / K_d}\end{cases} \tag{2.3}$$

The phase and magnitude of the PID controller at frequency $\omega$ are given in(2.4).

$$\begin{cases}\arg C(j\omega) = \arctan(\frac{\beta}{\alpha}(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})) \\ |C(j\omega)| = \sqrt{\alpha^2 + \beta^2 (\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})^2}\end{cases} \tag{2.4}$$

Based on (2.4), the influence of the variation of $\omega_n$, $\alpha$ and $\beta$ on the phase and magnitude of $C(s)$ can be given in the form of partial derivative functions:

$$\begin{cases}\dfrac{\partial \arg C(j\omega)}{\partial \omega_n} = -\dfrac{(\frac{\omega}{\omega_n^2} + \frac{1}{\omega})}{\frac{\alpha}{\beta} + \frac{\beta}{\alpha}(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})^2} \\ \dfrac{\partial |C(j\omega)|}{\partial \omega_n} = -\dfrac{\beta^2 (\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})(\frac{\omega}{\omega_n^2} + \frac{1}{\omega})}{\sqrt{\alpha^2 + \beta^2 (\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})^2}}\end{cases} \tag{2.5}$$

$$\begin{cases}\dfrac{\partial \arg C(j\omega)}{\partial \alpha} = -\dfrac{\beta(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})}{\alpha^2 + \beta^2 (\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})^2} \\ \dfrac{\partial |C(j\omega)|}{\partial \alpha} = \dfrac{\alpha}{\sqrt{\alpha^2 + \beta^2 (\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})^2}}\end{cases} \tag{2.6}$$

$$\begin{cases}\dfrac{\partial \arg C(j\omega)}{\partial \beta} = \dfrac{\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega}}{\alpha + \frac{\beta^2}{\alpha}(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})^2} \\ \dfrac{\partial |C(j\omega)|}{\partial \beta} = \dfrac{\beta(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})^2}{\sqrt{\alpha^2 + \beta^2 (\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})^2}}\end{cases} \tag{2.7}$$

Thus the following conclusions can be obtained:

$$\frac{\partial \arg C(j\omega)}{\partial \omega_n} < 0 \tag{2.8}$$

$$\begin{cases}\dfrac{\partial |C(j\omega)|}{\partial \omega_n} > 0 \ (\omega < \omega_n) \\ \dfrac{\partial |C(j\omega)|}{\partial \omega_n} < 0 \ (\omega > \omega_n)\end{cases} \tag{2.9}$$

$$\begin{cases}\dfrac{\partial \arg C(j\omega)}{\partial \beta} > 0 \ (\omega < \omega_n) \\ \dfrac{\partial \arg C(j\omega)}{\partial \beta} < 0 \ (\omega > \omega_n)\end{cases} \tag{2.10}$$

$$\frac{\partial |C(j\omega)|}{\partial \beta} > 0 \ (\omega \neq \omega_n) \tag{2.11}$$

$$\begin{cases}\dfrac{\partial \arg C(j\omega)}{\partial \alpha} < 0 \ (\omega < \omega_n) \\ \dfrac{\partial \arg C(j\omega)}{\partial \alpha} > 0 \ (\omega > \omega_n)\end{cases} \tag{2.12}$$

$$\frac{\partial |C(j\omega)|}{\partial \alpha} > 0 \tag{2.13}$$

## III. THE ANALYSIS ON M-FIELD

To analyze the influence of the magnitude and phase of the open loop system on the magnitude of the closed loop system, the M-field is introduced and analyzed in this section.

### A. *The properties of M-field*

Equation **Error! Reference source not found.** is the equation of a circle with radius $M/(M^2-1)$ and with center at $(-M^2/(M^2-1), 0)$. This circle is defined as M-circle [14], sh

from **Error! Reference source not found.**we obtain:

$$\left(X + \frac{M^2}{M^2-1}\right)^2 + Y^2 = \frac{M^2}{\left(M^2-1\right)^2} \tag{2.14}$$

**Definition 3.1:** $\mathbb{M}_m$ $(m>0, m\neq 1)$ is a M-circle given by equation**Error! Reference source not found.**, with $M=m$. $\mathbb{M}_1$ is the straight line $X=-0.5$.

Equation **Error! Reference source not found.** is the equation of a circle with radius $M/(M^2-1)$ and with center at $(-M^2/(M^2-1), 0)$. This circle is defined as M-circle [14], shown in fig. 3.1.

In the complex plane of $G(s)$, according to **Error! Reference source not found.**, each point $X + j\cdot Y$ (excluding $-1+j\cdot 0$) corresponds to an unique value of $M$. Thus we have a scalar field: $M(X,Y)$, which is called the M-field.

**Lemma 3.1:** Given $\mathbb{M}_m$, for any point $X + j\cdot Y$ in the domain of $M(X,Y)$:

P1: When $m>1$, $M(X,Y) > m \Leftrightarrow X + j\cdot Y$ is inside $\mathbb{M}_m$;

P2: When $m<1$, $M(X,Y) > m \Leftrightarrow X + j\cdot Y$ is outside $\mathbb{M}_m$.

**Proof:** Given M-circle $\mathbb{M}_a$ $(a>1)$, suppose that $M(X,Y)=b$, $b>a$, and that the point $X + j\cdot Y$ is outside $\mathbb{M}_a$.

Consider the point $-b/(b+1)+0\cdot j$ in $\mathbb{M}_b$, we have:

$$\left(\frac{-b}{b+1} + \frac{a^2}{a^2-1}\right)^2 < \left(\frac{-a}{a+1} + \frac{a^2}{a^2-1}\right)^2 = \frac{a^2}{\left(a^2-1\right)^2} \tag{2.15}$$

Hence, $-b/(b+1)+0\cdot j$ is inside $\mathbb{M}_a$. According to definition 3.1, $X + j\cdot Y$ is in $\mathbb{M}_b$. This yields to the conclusion that $\mathbb{M}_b$ is partly inside $\mathbb{M}_a$ and partly outside $\mathbb{M}_a$. On the other hand, as the M-circles are all continuous and closed, $\mathbb{M}_b$ must cross $\mathbb{M}_a$, which means that $M$ owns two values at some certain points. This is impossible. So the hypothesis is not true. Thus we have the conclusion that if $M(X,Y) < a$, then

$X + j\cdot Y$ is outside $\mathbb{M}_a$. Similarly, if $M(X,Y) > a$, we can prove that $X + j\cdot Y$ is inside $\mathbb{M}_a$. Thus P1 is proved.

Using the same method, P2 can also be proved.

The scalar field $M(X,Y)$ is first order differentiable in its domain, so we can define its gradient vector: $\vec{V}(X,Y)$. The size and direction of $\vec{V}(X,Y)$ are respectively labeled as $V$ and $\vec{v}$, that is: $\vec{V}(X,Y) = V\cdot\vec{v}$.

**Theorem 3.1:** Given $\mathbb{M}_m$, and a point on $\mathbb{M}_m : X + j\cdot Y$, gradient vector $\vec{V}(X,Y)$ points to the inside (outside) of $\mathbb{M}_m$ along the normal line of $\mathbb{M}_m$ at $X + j\cdot Y$ if $m > 1$ $(m < 1)$.

**Proof:** Let $\vec{t}$ and $\vec{n}$ be the direction vector of the tangent line and the normal line of $\mathbb{M}_m$ at $X + j\cdot Y$. $\vec{V}(X,Y)$ can be given in the orthogonal decomposition form:

$$\vec{V}(X,Y) = T\cdot\vec{t} + N\cdot\vec{n} \tag{2.16}$$

According to the definition of M-circle, $M(X,Y)$ is constant on $\mathbb{M}_m$. So according to the definition of gradient:

$$\vec{V}(X,Y)\cdot\vec{t} = T = 0 \tag{2.17}$$

Thus we have: $\vec{V}(X,Y) = N\cdot\vec{n}$. According to lemma 3.1, $\vec{V}(X,Y)$ points to the inside (outside) of $\mathbb{M}_m$ if $m > 1$ $(m < 1)$. Thus theorem 3.1 is proved.

Fig. 3.2 shows the Nyquist curve of a system $G(s)$. Given a frequency value $\omega_1$, there is a corresponding point $P(\omega_1)$ in the Nyquist curve. Suppose the M-circle passing $P$ is $\mathbb{M}_m$, and the center of $\mathbb{M}_m$ is $o'$. Let $\angle o'Po$ be $\theta$, and the gradient vector of M-field at $P$ be $\vec{V}(\omega_1)$, $\vec{V}(\omega_1) = V(\omega_1)\cdot\vec{v}(\omega_1)$.

**Theorem 3.2:**

$$\frac{\partial M(\omega_1)}{\partial |G(j\omega_1)|} = -V(\omega_1)\cdot\cos\theta \tag{2.18}$$

**Proof:** Suppose due to the variation of the controller parameters, the magnitude of the open loop system at $\omega_1$ changed from $|G(j\omega_1)| = |oP|$ to $|G(j\omega_1)| = |oP'|$ while the phase at $\omega_1$ is kept unchanged, shown in fig. 3.2. Let $\vartriangle|G(j\omega_1)| = |PP'|$ be small enough that the variation of $\vec{V}(\omega_1)$ can be neglected when the point corresponding to $\omega_1$ is moved from $P$ to $P'$:

$$\begin{aligned}\vartriangle M &= M(P') - M(P) = \vec{V}\cdot\overrightarrow{PP'} \\ &= V\cdot|PP'|\cdot\cos(\pi-\theta) = -V\cdot|PP'|\cdot\cos\theta\end{aligned} \tag{2.19}$$

Hence:

$$\begin{aligned}\frac{\partial M(\omega_1)}{\partial |G(j\omega_1)|} &= \lim_{\vartriangle|G(j\omega_1)|\to 0}\frac{\vartriangle M(\omega_1)}{\vartriangle|G(j\omega_1)|} \\ &= \lim_{|PP'|\to 0}\frac{\vartriangle M(\omega_1)}{|PP'|} = -V\cdot\cos\theta\end{aligned} \tag{2.20}$$

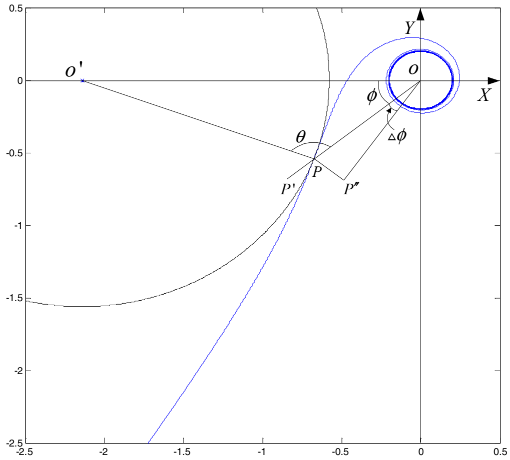


Fig. 3.2 the variation of the open loop magnitude and phase at a certain frequency

Fig. 3.2 shows the condition when $m > 1$, that is, $\mathbb{M}_m$ is in the left of $\mathbb{M}_1$, and $\vec{V}(P)$ points to the inside of $\mathbb{M}_m$. It can be proved that when $m < 1$, (2.19) still hold.

**Theorem 3.3:**

$$\frac{\partial M(\omega_1)}{\partial\phi} = -V(\omega_1)\cdot|G(j\omega_1)|\cdot\sin\theta \tag{2.21}$$

**Proof:** Suppose that due to the variation of the controller parameters, the point corresponding to $\omega_1$ moved from $P$ to $P''$: $|oP''| = |oP|$, $\angle o'oP = \varphi$, $\angle o'oP'' = \varphi + \vartriangle\varphi$, as shown in fig. 3.2. That is: the magnitude of the open loop system at $\omega_1$ is kept unchanged while the phase is changed from $\phi - \pi$ to $\phi + \vartriangle\phi - \pi$. Let $|PP''|$ be small enough that the variation of $\vec{V}(\omega_1)$ can be neglected when the point corresponding to $\omega_1$ is moved from $P$ to $P''$ and that $\vartriangle\phi$ can be regarded to be zero degree. In this condition, we have: $\angle oPP'' = \angle oP''P = \pi/2$, and:

$$\begin{aligned}\vartriangle M &= M(P'') - M(P) = \vec{V}\cdot\overrightarrow{PP''} \\ &= V\cdot|PP''|\cdot\cos(2\pi - \theta - \pi/2) \\ &= -V\cdot|PP''|\cdot\sin\theta\end{aligned} \tag{2.22}$$

Hence:

$$\begin{aligned}\frac{\partial M(\omega_1)}{\partial\phi} &= \lim_{\vartriangle\phi\to 0}\frac{\vartriangle M(\omega_1)}{\vartriangle\phi} = \lim_{\vartriangle|PP''|\to 0}\frac{\vartriangle M(\omega_1)}{|PP''|/|G(j\omega_1)|} \\ &= -V\cdot|G(j\omega_1)|\cdot\sin\theta\end{aligned} \tag{2.23}$$

Similar to (2.19), we can prove that (2.22) still hold when $M(P) \le 1$. Thus the theorem is proved.

To compare $\partial M(\omega_1)/\partial|G(j\omega_1)|$ with $\partial M(\omega_1)/\partial\phi$, we need to analyze the value of $\theta$ in the domain of M-field.

**Theorem 3.4:** Given a point $P$ in the domain of the M-field. Suppose the M-circle passing through $P$ is $\mathbb{M}_m$, with $m \ne 1$. Let the center of $\mathbb{M}_m$ be $o'$, and $U = -1 + j\cdot 0$. We have:

$$\angle o'Po = \theta \ (\theta \in [0,\pi]) \Leftrightarrow \angle PUo = \bar{\theta}$$

with:

$$\bar{\theta} = \begin{cases}\pi - \theta, & m > 1 \\ \theta, & m < 1\end{cases} \tag{2.24}$$

**Proof:** Let the position of $P$ be $(X,Y)$. When $\angle o'Po=\theta$, according to the equation of $\mathbb{M}_m$ and the cosine theorem:

$$\left(X+\frac{m^2}{m^2-1}\right)^2+Y^2=\frac{m^2}{\left(m^2-1\right)^2} \tag{2.25}$$

$$\frac{\left(X+\frac{m^2}{m^2-1}\right)^2+X^2+2Y^2-\frac{m^4}{\left(m^2-1\right)^2}}{2\sqrt{\left(X+\frac{m^2}{m^2-1}\right)^2+Y^2}\cdot\sqrt{X^2+Y^2}}=\cos\theta \tag{2.26}$$

From (2.25), we have:

$$X^2+Y^2=-\frac{m^2}{m^2-1}(2X+1) \tag{2.27}$$

Put (2.25) and (2.27) into (2.26), we have:

$$(1-m^2)(X+1)^2=(2X+1)(\cos\theta)^2 \tag{2.28}$$

Put (2.25) into (2.26)，(2.27) into the nominator of the left part of (2.26), we have:

$$m^2(X+1)^2=(X^2+Y^2)(\cos\theta)^2 \tag{2.29}$$

Add (2.28) to (2.29), we have:

$$(X+1)^2=(X^2+2X+1+Y^2)(\cos\theta)^2 \tag{2.30}$$

Hence:

$$(1-(\cos\theta)^2)\cdot(X+1)^2=Y^2\cdot(\cos\theta)^2 \tag{2.31}$$

Thus, when $(X,Y)\neq(-1,0)$, we have: $\theta=\pi/2 \Leftrightarrow X=-1$. This means that the necessary and sufficient condition for $\theta=\pi/2$ is: $P$ is in the line $X=-1$, as shown in Fig. 3.3.

When $\theta\neq\pi/2$, we have:

$$\frac{Y}{X+1}=\pm\tan\theta \tag{2.32}$$

That is: $P$ is in a line which passes through $U$, and the angle between this line and the real axis is $\theta$ or $\pi-\theta$.

When $m>1$, if $P$ (which is in $\mathbb{M}_m$) moves along $\mathbb{M}_m$ to the left (as shown in fig. 3.3), it is easy to prove that $|oP|$ increases. As $|oo'|$ and $|o'P|$ are constant, according to the cosine theorem, $\theta$ decreases. Hence, when $P$ is in the left of $X=-1$, $\theta=\angle oPo'<\pi/2$, however, it is obvious that $\bar{\theta}=\angle PUo>\pi/2$; when $P$ is in the right of $X=-1$, $\theta=\angle oPo'>\pi/2$, and $\bar{\theta}=\angle PUo<\pi/2$. So when $m>1$, $\bar{\theta}=\pi-\theta$.

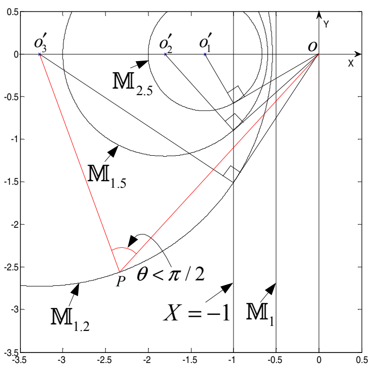


Fig. 3.3 The value of $\theta$ in the domain of M-field

When $m<1$, $\bar{\theta}=\angle PUo<\pi/2$, as $o$ is inside $\mathbb{M}_m$, $\theta=\angle oPo'<\pi/2$. So $\bar{\theta}=\theta$.

## IV. The Influence of the PID Parameters on the Magnitude of the Closed Loop System

If the behavior of the closed loop step response exhibits overshoot, it is typical that the system oscillates [2]. The oscillation reflects the fact that the magnitude of the closed loop system at and near a certain frequency is significantly higher than that at other frequency region.

This section discusses how to reduce the magnitude of the closed loop system at the oscillation frequency to reduce the oscillation. The following definitions are given:

$\omega_0$： the oscillation frequency of the closed loop system;

$\omega_u$： the ultimate frequency of the open loop system. That is: $\arg G(j\omega_u)=-\pi$;

$\omega_c$： the characteristic frequency, given in (2.3);

LFS (the low frequency section)：the frequency section where the frequency value is lower than $\omega_c$;

HFS (the high frequency section)：the frequency section where the frequency value is higher than $\omega_c$;

Region Ⅰ： the part of the complex plane left to $X=-1$;

Region Ⅱ： the part of the complex plane which is between $X=-1$ and $\mathbb{M}_1$ （$X=-0.5$）.

There are three cases of oscillations in PID system to be discussed:

### *A. Case 1: $\omega_0$ is in LFS*

Case1.1： $G(j\omega_0)$ is in region Ⅰ. In this region, $\theta<\pi/2$, according to theorem 3.2: $\partial M/\partial|G|=-V\cdot\cos\theta$, and theorem 3.3: $\partial M/\partial\phi=-V\cdot|G|\cdot\sin\theta$, both increasing open loop magnitude and open loop phase can decrease the closed loop magnitude at $\omega_0$. According to the discussion in section Ⅱ, of all the variations of a single parameter, only increasing $\alpha$ could increase the open loop magnitude and the open loop phase in LFS at the same time ((2.12): $\partial\arg C(j\omega)/\partial\alpha>0$ $(\omega<\omega_c)$; (2.13)： $\partial|C(j\omega)|/\partial\alpha>0$ ). So the parameter tuning method for case 1.1 is: increasing $\alpha$.

Case 1.2： $G(j\omega_0)$ is in region Ⅱ. In this region, $\theta>\pi/2$, according to theorem 3.2: $\partial M/\partial|G|=-V\cdot\cos\theta$, and theorem 3.3: $\partial M/\partial\phi=-V\cdot|G|\cdot\sin\theta$, both decreasing open loop magnitude and increasing open loop phase can decrease the closed loop magnitude at $\omega_0$. According to the discussion in section Ⅱ, of all the parameter variations, only decreasing $\omega_c$ could decrease the open loop magnitude and increase the open loop phase in LFS at the same time.((2.8)： $\partial\arg C(j\omega)/\partial\omega_c<0$; (2.9): $\partial|C(j\omega)|/\partial\omega_c>0$ $(\omega<\omega_c)$ ). So the parameter adjustment method for case 1.2 is: decreasing $\omega_c$.

However, there is a special case in case 1.2, that is, $\omega_0$ is very close to $\omega_c$, and the value of $\theta$ at the position of $G(j\omega_0)$ is very close to $\pi$. According to theorem 3.2: $\partial M/\partial\phi=-V\cdot|G|\cdot\sin\theta$, the closed loop magnitude at $\omega_0$ is insensitive to the variation of open loop phase. From(2.5), we have $\partial|C(j\omega)|/\partial\omega_c = 0\ (\omega=\omega_c)$. So when $\omega_0$ is very close to $\omega_c$, the variation of $\omega_c$ has little influence on the open loop magnitude. As a result, decreasing $\omega_c$ can't effectively reduce system oscillation. For this special case, the effective method is: decreasing $\alpha$.

*B. Case 2: $\omega_0$ is in HFS*

Case 2.1: $G(j\omega_0)$ is in region Ⅰ. In this case, $|G(j\omega_0)|$ is large, and the value of $\theta$ at the position of $G(j\omega_0)$ cannot be small (in region Ⅰ, $\theta$ is small means the phase is very close to $-\pi$, as $|G(j\omega_0)|$ is large, so the system can be unstable). According to theorem 3.3: $\partial M/\partial\phi = -V\cdot|G|\cdot\sin\theta$, the decreasing of open loop phase would significantly increase the closed loop magnitude at $\omega_0$. So, for case 2.1, increasing $\alpha$ cannot be used ((2.12): $\partial\arg C(j\omega)/\partial\alpha < 0\ (\omega>\omega_c)$ ). However, according to theorem3.2: $\partial M/\partial|G| = -V\cdot\cos\theta$, when the value of $\theta$ at the position of $G(j\omega_0)$ is close to $\pi/2$, the closed loop magnitude at $\omega_0$ is insensitive to the variation of open loop magnitude, so we can use the method of decreasing $\alpha$.

When decreasing $\alpha$ is ineffective or leads to oscillation in LFS, we need other method. $G(j\omega_0)$ is in region Ⅰ, so $\omega_0$ is far smaller than $\omega_u$, and $\omega_c$ is smaller than $\omega_0$. According to (2.5), when $\omega_c/\omega_u \ll 1$, $\partial|C(j\omega)|/\partial\omega_c$ is large, so decreasing $\omega_c$ would significantly increase $|G(j\omega_u)|$, and leads to high frequency oscillation. For the same reason, increasing $\beta$ cannot be used either. So we should increasing $\omega_c$, and the oscillation will be aggravated first and then be alleviated rapidly or the oscillation changes into case 1 or case 3.

Case 2.2: $G(j\omega_0)$ is in region Ⅱ. If $G(j\omega_0)$ is near the real axis, then $|G(j\omega_0)|$ is small and the value of $\theta$ is close to $\pi$. According to theorem 3.2: $\partial M/\partial\phi = -V\cdot|G|\cdot\sin\theta$, the closed loop magnitude at $\omega_0$ is insensitive to the variation of open loop phase. According to theorem 3.2: $\partial M/\partial|G| = -V\cdot\cos\theta$, the decreasing of open loop magnitude can effectively reduce the closed loop magnitude at $\omega_0$. So according to (2.5), increasing $\omega_c$ can effectively reduce the closed loop magnitude at $\omega_0$. If the value of $\theta$ at the position of $G(j\omega_0)$ is not close to $\pi$, then the influence of the variation of open loop magnitude cannot be ignored, according to (2.12) and (2.13), we have: $\partial\arg C(j\omega)/\partial\alpha < 0\ (\omega>\omega_c)$ and $\partial|C(j\omega)|/\partial\alpha > 0$, so the appropriate method for this case is: decreasing $\alpha$.

*C. Case 3: There are significant oscillation frequencies in both LFS and HFS*

Let $\omega_1$ and $\omega_2$ be the oscillation frequencies in LFS and HFS respectively. There is oscillation at $\omega_1$, so $\arg G(\omega_1)$ is small, and $\arg G(\omega_2)$ must be smaller. So $\arg G(\omega_2)$ should be close to $-\pi$. That is: $G(\omega_2)$ should be in region Ⅱ and close to the real axis. In this location, $\theta$ is close to $\pi$, and $|G(j\omega_2)|$ is small. According to theorem 3.3: $\partial M/\partial\phi = -V\cdot|G|\cdot\sin\theta$, so the closed loop magnitude at $\omega_2$ is insensitive to the variation of open loop phase. According to theorem 3.2: $\partial M/\partial|G| = -V\cdot\cos\theta$, so decreasing open loop magnitude can effectively reduce the closed loop magnitude at $\omega_2$.

$\omega_1$ is in LFS, so $|G(\omega_1)|$ is large and the corresponding $\theta$ should not be close to $\pi$ (nor 0), According to theorem 3.3: $\partial M/\partial\phi = -V\cdot|G|\cdot\sin\theta$, the increasing of open loop phase can effectively reduce the closed loop magnitude at $\omega_1$

According to the discussion in section Ⅱ, of all the variations of a single parameter, only decreasing $\beta$ could increase the open loop phase in LFS and decrease the open loop magnitude in HFS at the same time ((2.10): $\partial\arg C(j\omega)/\partial\beta < 0\ (\omega<\omega_c)$; (2.11): $\partial|C(j\omega)|/\partial\beta > 0\ (\omega\neq\omega_c)$ ). So, for case 3, the tuning method is: decreasing $\beta$.

*D. Parameter tuning rules*

The discussion in subsection A, B, C is concluded in table Ⅰ.

TABLE I. METHODS OF ALLEVIATING SYSTEM OSCILLATION

| Cases | Methods | Side conditions |
|---|---|---|
| Case 1 | Increasing $\alpha$ | $G(j\omega_0)$ is in region Ⅰ |
| | Decreasing $\omega_c$ | $G(j\omega_0)$ is in region Ⅱ |
| Case 2 | Increasing $\omega_c$ | $G(j\omega_0)$ is close to the real axis |
| | Decreasing $\alpha$ | |
| Case 3 | Decreasing $\beta$ | |

For a certain case, there are different tuning methods under different side conditions. However, from the system response, it is difficult to discern which side condition the system fulfills. So we need to try them one by one. As increasing the open loop magnitude can enhance disturbance suppression and quicken response. So the methods of increasing certain parameter should be tried first and those decreasing certain parameter should be tried last. Based on table 4.1, the following tuning rules are proposed:

**Case 1: First try increasing $\alpha$, if the oscillation cannot be effectively reduced, restore the parameters and decrease $\omega_n$.**

**Case 2: First try increasing $\omega_n$; if the oscillation cannot be effectively reduced, restore the parameters and reduce $\alpha$.**

**Case 3: Reduce $\beta$.**

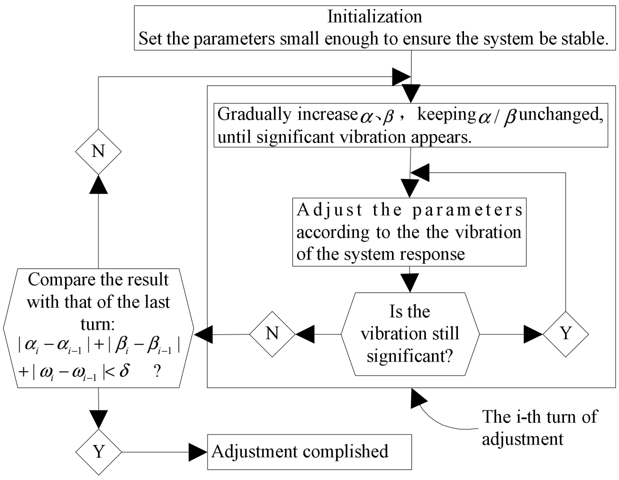


Fig. 4.2 The flow chart of multi-circles tuning.

However, the above tuning rules only tell which parameter should be changed (increased or decreased). But how much should the parameter be changed is unknown. So there are cases that the parameters are over tuned. To solve this problem, multi-circle tuning strategy is proposed, shown in fig. 4.2. $\alpha_i$, $\beta_i$ and $\omega_i$ are the values of $\alpha$, $\beta$, $\omega_c$ after the i-th circle of tuning is finished. $\delta$ is a pre-set small value. If the result of current circle of tuning and the result of the last circle of tuning fulfills the inequality in the box, that is: $|\alpha_i-\alpha_{i-1}|+|\beta_i-\beta_{i-1}|+|\omega_i-\omega_{i-1}|<\delta$, then the variation of parameters would be very small if tuning continues, so the tuning can be stopped. In the actual tuning process, other conditions can be used to judge if tuning can be stopped. Such as: in a circle of tuning, $\omega_c$ is not changed and both $\alpha$ and $\beta$ are reduced, so increasing $\alpha$ and $\beta$ once more is meaningless, and tuning can be stopped.

# V. SIMULATION

A typical model: $G_0(s)=\dfrac{1}{(20s+1)(2s+1)}e^{-1}$, choosing from [13] is used to verify the tuning rules proposed in section Ⅳ. The tuning result is compared with the design result using Ziegler-Nichols method and SIMC method in [13].

The corresponding curves are shown in fig. 5.1, in which the time unit is second. The tuning process is listed in table Ⅱ.

Fig. 5.1-i compares the system tuning with the proposed tuning method (curve 1) and systems designed using Ziegler-Nichols method (curve 2) and IMC method (curve 3) in servo tracking and disturbance suppression.

TABLE II. THE PID CONTROLLER TUNING PROCESS

| Curves | $\omega_c$ | $\alpha$ | $\beta$ | Cases | methods |
|---|---|---|---|---|---|
| a-1 | 1 | 0.1 | 0.1 | case 1 | increase $\alpha$ |
| a-2 | 1 | 2 | 0.1 | smooth | |
| b-1 | 1 | 8 | 0.4 | case 1 | increase $\alpha$ |
| b-2 | 1 | 10 | 0.4 | case 1 | roll back |
| b-1 | 1 | 8 | 0.4 | case 1 | decrease $\omega_c$ |
| c-1 | 0.3 | 8 | 0.4 | case 2 | decrease $\alpha$ |
| c-2 | 0.3 | 4 | 0.4 | smooth | |
| d-1 | 0.3 | 10 | 1 | case 2 | decrease $\alpha$ |
| d-2 | 0.3 | 6 | 1 | case 2 | decrease $\alpha$ |
| d-3 | 0.3 | 4 | 1 | case 1 | decrease $\omega_c$ |
| d-2 | 0.2 | 4 | 1 | case 1 | decrease $\omega_c$ |
| e-1 | 0.15 | 4 | 1 | smooth | |
| e-2 | 0.15 | 12 | 3 | smooth | |
| f-1 | 0.15 | 28 | 7 | case 2 | increase $\omega_c$ |
| f-2 | 0.17 | 28 | 7 | case 2 | roll back |
| f-1 | 0.15 | 28 | 7 | case 2 | decrease $\alpha$ |
| f-3 | 0.15 | 24 | 7 | case 3 | decrease $\beta$ |
| g-1 | 0.15 | 24 | 5 | case 2 | decrease $\alpha$ |
| g-2 | 0.15 | 22 | 5 | case 2 | decrease $\alpha$ |
| g-3 | 0.15 | 20 | 5 | | |

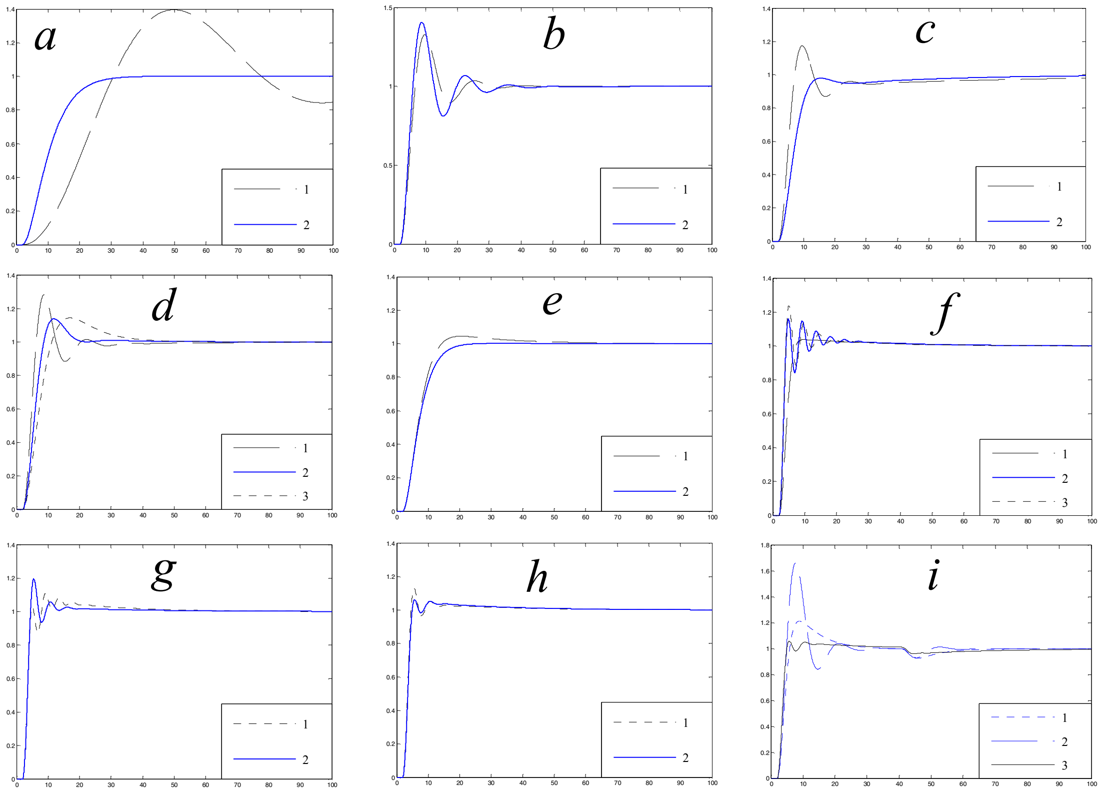


Fig 5.1 The tuning process of the PID control system and the comparison of the PID control systems designed by different methods (fig. i)

## VI. CONCLUSION

The main innovation point of this paper is a reference frequency value used to evaluate the oscillation frequency of the PID control system, and this reference frequency is the PID characteristic frequency, $\omega_c$. The time zone response of the PID control system usually contains oscillation. In the tuning methods proposed in this paper, the oscillation frequency of the PID system is first compared with $\omega_c$. Then, based on whether the oscillation frequency is higher or lower than $\omega_c$, corresponding tuning methods is given to tune the PID parameters. The PID characteristic frequency $\omega_c$ serves as a link between the PID parameters and the PID system frequency properties and the PID system response characteristics.

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