

Control for Quadruped Robots in Trotting on Horizontal and Slanted Surfaces

Jeong Hoon Lee and Jong Hyeon Park

Department of Mechanical Engineering, Hanyang University, Seoul 133-791, Korea

Email: jongpark@hanyang.ac.kr

Abstract—The objective of this work is to make a quadruped robot trot on horizontal and slanted surfaces through a natural transition of motion. For this, control methods that are different from the ones devised for robot trotting on horizontal surface are required. The control methods proposed in this paper consist of two parts: natural movement of the center point of foot trajectory and the impedance control based on variable impedance parameters. The changes of the center point are generated for a gradual and continuous adaptive foot motion. And, the variable impedance control and leg controls are used to maintain the stability of locomotion as well as to reduce the yawing motion. Motion of a quadruped robot with the proposed control methods were simulated and their performance were verified through the simulations.

I. INTRODUCTION

A research on an robot locomotion is very closely connected with researches on an animal locomotion. We can come up with control methods for quadruped robots by studying efficient animal locomotion. The animal locomotion consists of fairly comprehensive and complicated mechanism because the situation and environment change frequently.

At first, the animal locomotion is changed by situation. For example, let's consider an animal approaching to prey for hunting. An animal does walking surreptitiously in order to approach to prey, and then does very fast galloping to catch prey that run away. Of course, An animal trots in middle phase with galloping and walking. Trot has been known that energy efficiency is the highest locomotion as well as the roll of middle phase with galloping and walking. An animal can maintain suitable locomotion instinctively in situation like this. Robots must learn this coping ability in the situation, and there were some achievements [1]–[4].

The animal locomotion is also changed by environment. The animal locomotion pattern differs from all the uneven terrain such as gravel road and mountainous terrain, special terrain such as stairs and sand beaches, extreme terrain such as snow-covered road or swamp. An animal handles by operating actively contraction and relaxation of muscles in these environment changes. But, even if the robot creates imitating an animal, it is difficult to show a motion same as an animal. The reason is that the animal locomotion consists of complicated mechanism from various situation and environment. Due to these complexity, expressing the robot locomotion by perfect kinetics is impossible actually [5]–[7].

Therefore, a suitable control method is required for robot to keep the balance fast just as an animal does and to do stable

locomotion. As a result, a lot of control methods such as fuzzy control, adaptive control, robust control, impedance control, and etc. were developed. Especially, Impedance control can control impact which is occurred by contact between a robot foot and an obstacle which makes the robot unstable [8], [9]. It compensates a desired trajectory changed by disturbance rapidly. Therefore, modified trajectory made by this compensation can withstand disturbance such as uneven terrain [10].

Also, another reason why an animal can maintain stable locomotion in several situations and environments is that an animal has intelligence. Of course, an animal does not hold excellent intelligence such as a human, but an animal has some intelligence to cope with repeated situations and environments by tracing back the past experience. Researcher that cannot describe complexity of gait by dynamics did not miss this fact that an animal has some intelligence, and then they developed various kinds of learning algorithms and optimization techniques [11]–[13].

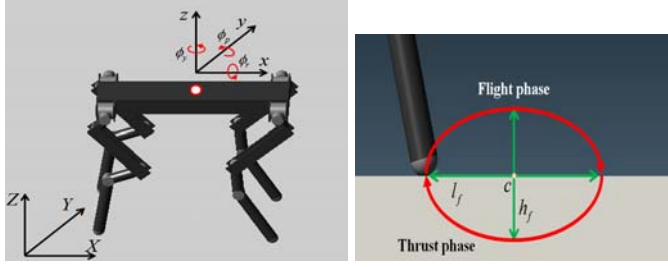
In this paper, controls for a natural movement of the center point of the foot trajectory and the variable impedance control are proposed for a quadruped robot to trot on horizontal and the slanted surfaces, and simulations were performed to verify the advantages of the proposed control methods. Methods for the movement of the foot center and the variable impedance modulation are proposed. The stability of locomotion and the performance of the control methods was confirmed through the simulations by comparing the results with different control schemes.

II. OPTIMAL GENERATION OF FOOT TRAJECTORY

First of all, verification was needed whether this robot can climb on a slope. For this, necessary control methods will be analyzed here. To perform this research, detailed observation was required for finding necessary actions for quadruped animal to climb on the slope. As a result, two typical action modes were observed. First, An animal bends legs and lowers posture on the slope. Because an animal tries to keep the balance of center of gravity (COG) on the slope as well as on the horizontal surface through these actions. Second, an animal generates a foot trajectory according to a slope angle understandably. In other words, an animal rotates the trajectory at the slope angle. For the moment, two action modes are confirmed through simulation whether the quadruped robot will climb on the slope.

A. Rotation of Ellipsoidal Trajectory

Quadruped animals such as dog and horse use an ellipsoidal trajectory by foot trajectory although it is not perfect form. The foot trajectory used for this robot is also an ellipsoidal trajectory. Advantages of the ellipsoidal trajectory are the followings. First, the trajectory is rhythmical and continuous just like an animal's one. Second, transforming the trajectory is easy. Here, the transform is not only changes of form of the trajectory according to shape of ground but also a movement of the center point of the trajectory. In this research, the trajectory is used by rotating at inclined angle of the slope.



(a) Robot model : Euler angles of yaw ϕ_y , pitch ϕ_p , roll ϕ_r (b) ellipsoidal trajectory

Fig. 1. Coordinate systems, body angles and ellipsoidal trajectory

The trajectory is generated with respect to the robot body, and each direction of axis (x-y-z) is same as shown in Fig. 1(a). The equations of the ellipsoid are

$$\bar{x}(t) = \frac{l_f}{2} \sin\left(\frac{2\pi}{T}t + \alpha\right) + c \quad \text{for } 0 \leq t \leq T \quad (1)$$

$$\bar{z}(t) = \frac{h_f}{2} \cos\left(\frac{2\pi}{T}t + \alpha\right) \quad \text{for } 0 \leq t \leq T, \quad (2)$$

where l_f is the stride, h_f is the maximum foot height, T is the one step period, α is the initial phase, and $(c, 0, 0)$ is the center point of the ellipsoid.

$$\begin{bmatrix} \cos \phi & 0 & -\sin \phi \\ 0 & 0 & 0 \\ \sin \phi & 0 & \cos \phi \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \bar{y}(t) \\ \bar{z}(t) \end{bmatrix} = \begin{bmatrix} \bar{x}'(t) \\ \bar{y}'(t) \\ \bar{z}'(t) \end{bmatrix}, \quad (3)$$

where ϕ is a angle of slope. In other words, $\bar{x}(t)$ and $\bar{z}(t)$ are rotated at angle of slope ϕ .

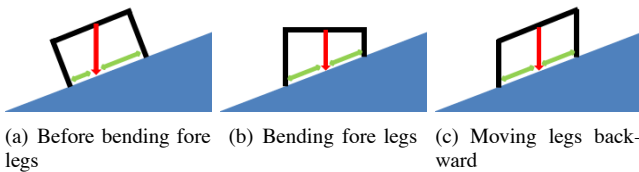


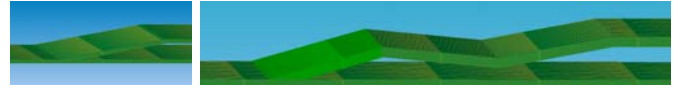
Fig. 2. Positioning of COG

B. Keeping the Balance: positioning the COG

When the robot climbs a slope without changing its posture used during the walk on a horizontal surface, its center of gravity (COG) shifts backward as shown in Fig. 2(a). If the robot trots based on this posture, the balance of the robot could become unstable and the robot may not able to walk successfully. To prevent this from happening, the robot can raise the lower body up and lower the front body down such that that the main body of the robot maintains a flat level as shown in Fig. 2(b). In this posture, the joint angles between the body and the legs are 90° as those during locomotion on a horizontal surface. The COG is positioned in the middle of the body, between the fore and hind feet, and thus the balance of COG is well maintained. This is the posture which a typical animal would use in climbing a slop. Of course, keeping the COG in the middle of the body can be achieved by taking the posture shown in Fig. 2(c), where both legs are turned backward relative to the main body. This posture will be dealt later in the paper.

C. Simulation (1)

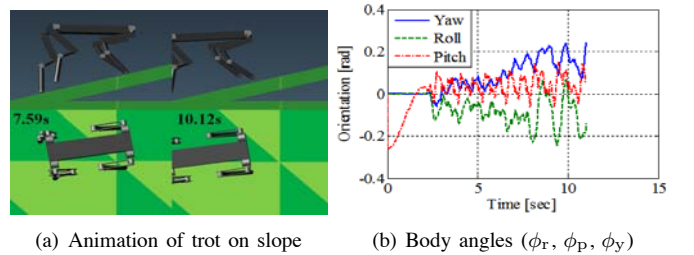
As shown in Fig. 1(a), the robot model is a model which simplified a hardware model HUNTER (Hanyang UNiversity Tetrapod Robot) developed by Hanyang University. The simulation model will be called HUNTER from now on. HUNTER is 0.5 m tall and 0.35 m wide. Its head-to-tail length is 0.6 m. It weights approximately 24 kg.



(a) Slope model (1) (b) Slope model (2) : horizontal surface \rightarrow slope (15°) \rightarrow slope (-5°) \rightarrow slope (10°) \rightarrow horizontal surface

Fig. 3. Simulation model

The model of the slope inclines to 15° -angle as shown in Fig. 3(a). Spring coefficient and damping coefficient between ground and robot foot are each 2000 kN/m and 1.0 kN·s/m. Dynamic friction coefficient and static friction coefficient are 0.6 and 0.9. l_f is 0.75 m, h_f is 0.5 m, and the one step period is 1 second. The simulation is processed by using Mathworks's Matlab and Functionbay's RecurDyn. Again, this simulation aims to confirm introduced two action modes.



(a) Animation of trot on slope (b) Body angles (ϕ_r , ϕ_p , ϕ_y)

Fig. 4. The result of simulation (1)

As a result, the robot cannot climb on the slope by increasing a yaw angle ϕ_y of a robot body as shown in Fig. 4.

The reason is that the trajectory is a 2D-trajectory on the x-z plane. Therefore, for the robot to climb on the slope, necessity reducing the yaw angle ϕ_y was risen very importantly. In this research, optimized ellipsoidal trajectory to reduce the yaw angle ϕ_y is generated by using learning algorithm based on evolutionary algorithm (EA).

D. Learning Algorithm Based on EA

TABLE I
DESIGN OF EA

(m)	l_f	h_f	initial position	bending effector*
upper boundary	0.085	0.06	0.45	0.14
lower boundary	0.065	0.03	0.44	0.11
crossover probability : 0.9 mutation probability : 0.1				
generation number : 30 population number : 16				

*bending effector is a degree of bending legs.

This learning algorithm follows as existing EA. Only, fitness is estimation of the robot behavior, that is, it is the score. In other words, a meaning of what fitness builds up is a meaning of what score is getting higher. TABLE I arranges all chromosomes used to EA. Scoring is based on score S :

$$S_t = S_\theta + S_{vmin} + S_{vmax} \quad (4)$$

where

$$S_\theta = \frac{1}{(w_1 P + w_2 R + w_3 Y) + 1} \times 100$$

$$S_{vmin} = \frac{\alpha_1}{|V_1|} \times 100$$

$$S_{vmax} = \alpha_2 V_2 \times 100,$$

where P , R and Y are maximum ϕ_p , ϕ_r and ϕ_y during the trot. w_1 , w_2 and w_3 are weights of ϕ_p , ϕ_r and ϕ_y . Therefore, S_θ is mainly to evaluate the stability of the robot behavior. $V_1 (< 0)$ is a forward minimum velocity of the robot during the trot. Therefore, S_{vmin} evaluates the robot climbing on the slope at minimum velocity without slip. $V_2 (\geq 0)$ is a forward maximum velocity of the robot during the trot so S_{vmax} evaluates maximum velocity of the robot. α_1 and α_2 are gains for controlling the scale of S_θ . And, if V_1 or V_2 does not exist, S_{vmin} and S_{vmax} become each zero.

E. Simulation (2)

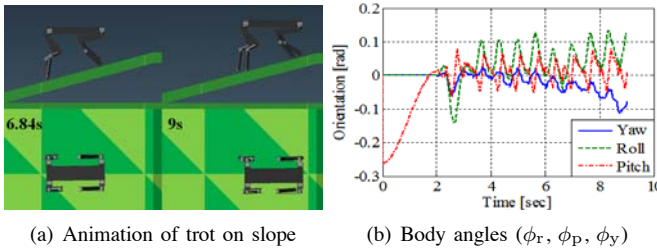


Fig. 5. The result of simulation (2)

As shown in Fig. 5, the robot can climb on the slope stably by controlling the yaw angle ϕ_y , and it means that the yaw angle ϕ_y can be controlled by properly changed form of the ellipsoidal trajectory.

III. CONTROL METHODS FOR TROTting AT EACH STAGE

Control methods of HUNTER are proposed to trot on horizontal surface \rightarrow slope (15°) \rightarrow horizontal surface on the basis of previous simulation result. For the robot to trot on the horizontal surface and the slope, each stage should be considered as shown in Fig. 6. Therefore, this paper proposes control methods at each stage.

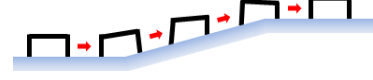


Fig. 6. Movement of HUNTER at each stage

A. Control for Trotting on Horizontal Surface

HUNTER trots using the ellipsoidal trajectory on the horizontal surface. But, other strategy is required at the transition of a motion (horizontal surface \rightarrow slope) as shown in Fig. 6. It is because of uncertainty that HUNTER does not know when and where the slope is happened to meet. Therefore, the robot trotting on the horizontal surface is always to prepare for meeting the slope. To do this, The robot recognizes pitch angle ϕ_p of the robot body by using gyro sensor in real time, and if the pitch angle ϕ_p exceeds reference pitch angle $\phi_{p,r}$, the trajectory will start to rotate at ϕ_p as shown in Fig. 7.



Fig. 7. Logic of recognizing the slope : $\phi_p = 0$ is a pitch angle of the robot body on the horizontal surface.

Also, a quadruped robot as well as a biped robot can use impedance module for stability of locomotion and in sudden disturbance [10]. The slope is a kind of disturbance. Therefore, HUNTER uses impedance control for climbing on the slope.

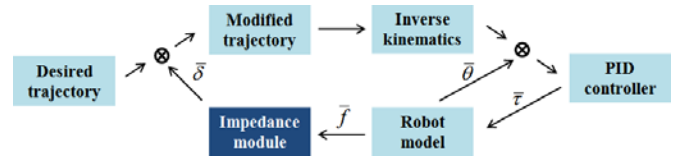


Fig. 8. Impedance control block diagram

As shown in Fig. 8, input of the impedance module is force \bar{f} , then output is position offset $\bar{\delta}$. Therefore, this impedance control is an admittance control.

$$\ddot{\bar{\delta}} = \frac{1}{M}[\bar{f} - B\dot{\bar{\delta}} - K\bar{\delta}], \quad (5)$$

where M , B and K are parameters of impedance module. M is the mass, B is the damping ratio and K is the stiffness. After integrating $\ddot{\bar{\delta}}$ of Eq. (5) twice, $\bar{\delta}$ is added to the desired trajectory as shown in Fig. 8.

Meanwhile, each value of these parameters can be find by heuristic method, but a lot of times are required to do so. Therefore, the approximate range of M , B and K determines by applying to EA. To reduce design variable, that is, chromosome, damping ratio is used to critical damping ratio $B = 2\sqrt{MK}$. It is method that can reduce overshoot most rapidly. The overshoot is a factor which occurred by impact distorting desired trajectory. S_θ is used only to evaluate the stability of the robot behavior.

As a result, the optimal parameters was founded in $M = 18.43$ kg, $K = 2857.54$ N/m, $B = 458.97$ N·s/m. These values must be tuning because they are only useful values when the robot trots on the horizontal surface. However, a lot of times can be saved by tuning in reduced range of the values by EA. Therefore, M is fixed at 18 kg and K carries out tuning by 100 step size from 2600 to 3400 N/m, and then K is selected to 3000 N/m.

B. Control for Trotting at Boundary between Horizontal Surface and Slope

In this stage, another control method as well as the rotation of the trajectory and using impedance control is required because of the transition of motion. It is just the movement of COG as what is already introduced. But, the robot must stop to move COG when it met the slope, then COG is moved by bending legs and it climbs on the slope again. The robot repeats same behaviors again and again. But, this method is not only obsolete but it is unfit for the purpose of this research. Therefore, a method for natural movement of COG needs so this paper proposes to move the center point of the trajectory using a polynomial expression gradually and continuously.

$$\bar{z}(t) = \frac{h_f}{2} \cos\left(\frac{2\pi}{T}t + \alpha\right) + \bar{p}(t), \quad (6)$$

which is a form that $\bar{p}(t)$ is added to Eq. (1). $\bar{p}(t)$ is a 3rd order polynomial equation. The trajectory is moved toward the robot body without cutting part as shown in Fig. 9(a).

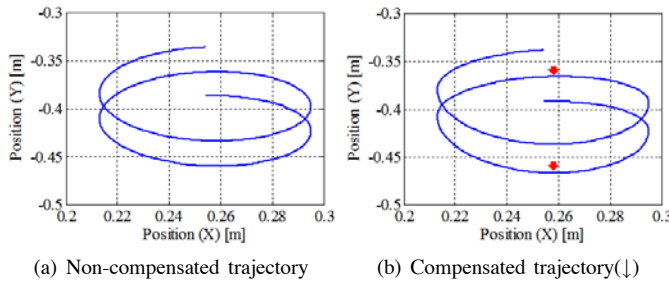


Fig. 9. Transforming ellipsoidal trajectory

But, slip is increased by the movement of the center point of the trajectory because a lower height of the trajectory gets short. On the other hand, a upper height of the trajectory gets longer, then the locomotion is unstable. Therefore, the height of the trajectory needs to be compensated appropriately as shown in Fig. 9(b).

$$h_{f,upper} = h_f + \frac{\bar{p}(t)}{\beta} \quad (7)$$

$$h_{f,lower} = h_f - \frac{\bar{p}(t)}{\beta}, \quad (8)$$

where $h_{f,upper}$ and $h_{f,lower}$ are compensated foot heights. β is a gain to prevent excessive compensation.

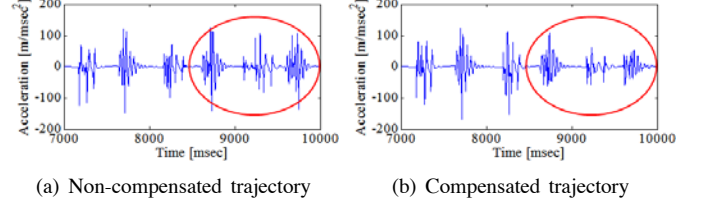


Fig. 10. Acceleration (x) of the robot foot

Fig. 10 shows the foot acceleration of x-direction and the degree of slip is judged by it at selected part. Reducing the slip is confirmed at compensated trajectory.

C. Control for Trotting on Slope

This stage as well, the ceter point of the trajectory is moved gradually and continuously. The ceter point is moved just as much as the robot keeps the balance as shown in Fig. 2(b).

TABLE II
PITCH ANGLE OF THE ROBOT BODY (ϕ_p)

Time(sec)	8s	8.5s	9s	9.5s	10s
Radian	0.3088	0.3186	0.3313	0.3364	0.3399

As shown in TABLE II, the pitch angle ϕ_p of the robot body was very irregular during the trot so the motion of the robot was unstable. It is because the trajectory is rotated at ϕ_p as shown in Fig. 7. The slope angle is 0.2618 rad for reference. But if the robot acquaint the angle of the slope in advance, it is also unfit for the purpose of this research. Therefore, a method for moving legs backward is needed as shown in Fig. 2(c). The trajectory does not need to rotate because it is generated with respect to the robot body. But, This is a inefficient method to the robot climbing on the slope. The robot can hardly move because the backward movement of the center point of the trajectory is offset by the forward movement of the robot.

Therefore, both what moves hind legs backward and what bends fore legs are chosen to control method. In other words, the movement of COG is shared. Although the trajectory may not be matched the angle of the slope, such a variable will be compensated by impedance control.

$$\bar{x}(t) = \frac{l_f}{2} \sin\left(\frac{2\pi}{T}t + \alpha\right) + c + \frac{\bar{p}(t)}{\gamma}, \quad (9)$$

which is a form that $\bar{p}(t)$ is added to Eq. (2). $\bar{p}(t)$ is a 3rd order polynomial equation same as Eq. (6). γ is the gain to prevent the excessive movement. Once the robot climbs on the slope, yaw error occurs. As confirmed through EA, the control of the yaw angle ϕ_y is possible by changing h_f . Sure, changing

l_f at the same time is more effective than changing h_f only, but in this case, an ellipsoidal trajectory has shortcoming not guaranteed a continuity.

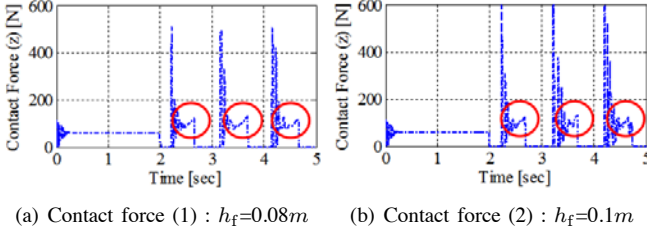


Fig. 11. Problem of transforming the trajectory

Therefore, in this paper, the yaw error is modified by controlling level of the trust and lifting of the trajectory come from changing h_f . But, a situation that impedance control compensates the trajectory at the thrust part is occurred. Fig. 11(a) and Fig. 11(b) show this situation. Even though the thrust is increased by lengthening h_f , the contact force of the selected part is almost unchanged. It is because contact force \bar{f} of Eq. (8) is changed suddenly in a state of being determined the parameters M , B , and K . Therefore, another method taking similar effect is needed so variable impedance control is used.

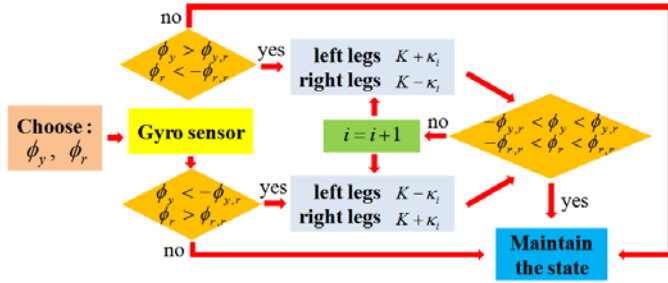


Fig. 12. Logic used variable impedance control : $\phi_{y,r}$ and $\phi_{r,r}$ are reference angles.

Fig. (12) shows how to apply variable impedance control. Here, K is fixed at 3000 N/m. κ_i is the stiffness of 100 step size from 100 to 400 N/m.

$$\ddot{\delta} = \frac{1}{M_i}[\bar{f} - B_i\dot{\delta} - K_i\delta], \quad (10)$$

where M_i , K_i and B_i are the impedance parameters of i_{th} state. i_{th} state means the state exceeding reference yaw angle $\phi_{y,r}$ and reference roll angle $\phi_{r,r}$. M_i is the mass, B_i is the critical damping ratio $B_i = 2\sqrt{M_i K_i}$ and K_i is the stiffness $K + \kappa_i$ or $K - \kappa_i$. Like the preceding, After integrating $\ddot{\delta}$ of Eq. (10) twice, position offset $\bar{\delta}$ is added to the desired trajectory as shown in Fig. 8.

Therefore, if the yaw angle ϕ_y of the robot body exceeds the reference angle $\phi_{y,r}$, the yaw angle ϕ_y will be reduced by changing the stiffness and damping ratio. As well as, a roll angle ϕ_r is also a important role to position control. Even though the yaw angle ϕ_y is controlled appropriately, sometimes the robot posture can be unstable. This is easy to

understand thinking about sliding down the robot on the slope without yawing. If the yaw angle ϕ_y is positive, the roll angle ϕ_r will be negative because of the body inclined to the left on the slope.

This method is not influenced by impedance control unlike the method for changing h_f directly because it is a control method for using impedance control autonomously. Therefore, it is shown a result same as changing the level of the h_f not using impedance control.

D. Control for Trotting on Slope \rightarrow Horizontal Surface

In this section, the moved center point of the trajectory is recovered to the original location. Therefore, $\bar{p}(t)$ term of Eq. (6) and Eq. (9) become negative. Existing control methods are stucked in other sections.

E. Simulation (1)

The data and one step period are same as previous simulations. Only, l_f and h_f are 0.082 m and 0.080 m considering the transition of motion.

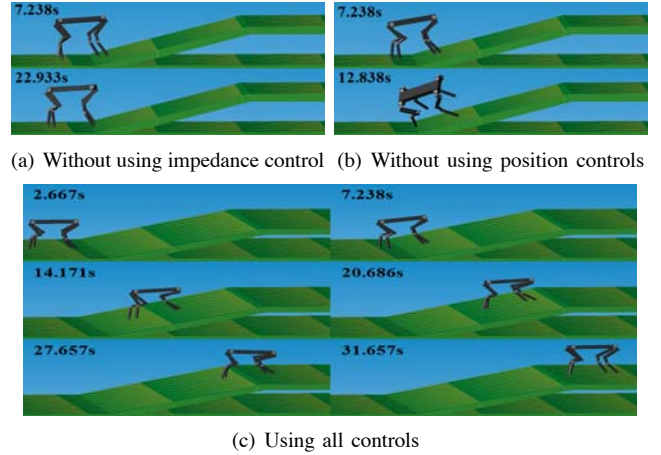
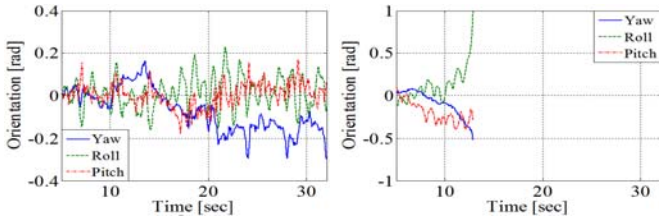


Fig. 13. The robot trots on horizontal surface \rightarrow slope \rightarrow horizontal surface

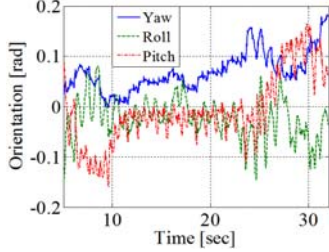
Fig. 13 shows simulation results depending on each situation. Not using impedance control, the robot cannot climb on the slope at all because of slip. Not using position controls, the robot collapses on the slope losing the balance. The stability of the robot is also confirmed by pitch ϕ_p , roll ϕ_r , and yaw angle ϕ_y as shown in Fig. 14.

F. Simulation (2)

Based on the result of simulation (1), simulation (2) giving various situations as shown in Fig. 3(b) is operated to confirm the validity and robustness of proposed control methods. Fig. 16(a) shows the result of Eq. (6) and Fig. 16(b) shows the result of Eq. (9). And, Figs. 17 and 18 show that yaw angle ϕ_y and roll angle ϕ_r are changing appropriately by variable impedance control.



(a) Without using impedance control (b) Without using position controls



(c) Using all controls

Fig. 14. Body angles (ϕ_r , ϕ_p , ϕ_y)

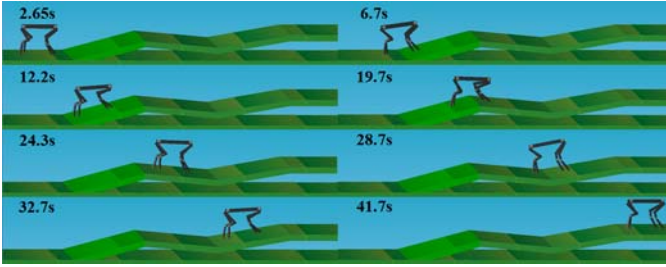
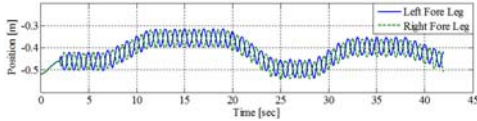
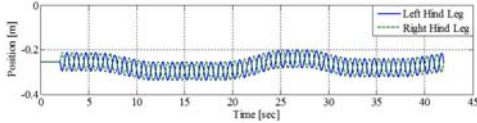


Fig. 15. The quadruped robot trotting on a horizontal and a slanted surfaces through a natural transition of motion

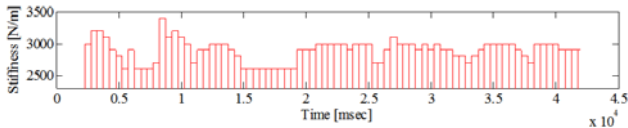


(a) $\bar{z}(t)$ of fore legs

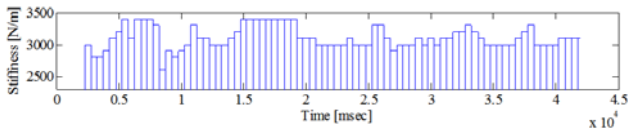


(b) $\bar{x}(t)$ of hind legs

Fig. 16. Foot trajectory



(a) Stiffness of right legs



(b) Stiffness of left legs

Fig. 17. Variable impedance control

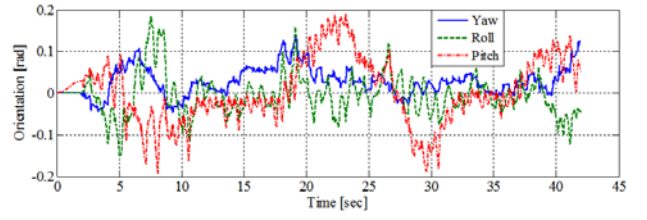


Fig. 18. Body angles (ϕ_r , ϕ_p , ϕ_y)

IV. CONCLUSION

In this research, for a quadruped robot to trot on horizontal and slanted surfaces through a natural transition of a motion, variable impedance control and position controls are proposed and simulations are processed to verify the advantage of the proposed control methods. The position controls moving the center point of the ellipsoidal trajectory and variable impedance control changing the impedance parameters according to yaw angle of the robot body have shown even better stability and performance than not using position controls and impedance control or using fixed impedance control in trotting on horizontal and slanted surfaces.

REFERENCES

- [1] J. Glower and U. Ozguner, "Control of a quadruped trot," in *Proceedings of IEEE International Conference on Robotics and Automation*, 1986, pp. 1496–1501.
- [2] R. Kurazume, S. Hirose, and K. Yoneda, "Feedforward and feedback dynamic trot gait control for a quadruped walking vehicle," in *Proceedings of IEEE International Conference on Robotics and Automation*, 2001, pp. 3172–3180.
- [3] K. Y. Kim, O. Kwon, J. S. Yeon, and J. H. Park, "Elliptic trajectory generation for galloping quadruped robots," in *Proceedings of IEEE International Conference on Robotics and Biomimetics*, 2006, pp. 103–108.
- [4] M. H. Raibert, "Running with symmetry," *International Journal of Robotics Research*, vol. 5, no. 4, pp. 3–19, Dec 1986.
- [5] L. Palmer and D. Orin, "Quadrupedal running at high speed over uneven terrain," in *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2007, pp. 303–308.
- [6] H. Kimura, Y. Fukuoka, and A. H. Cohen, "Adaptive dynamic walking of a quadruped robot on natural ground based on biological concepts," *International Journal of Robotics Research*, vol. 26, no. 5, pp. 475–490, May 2007.
- [7] S. Hirose, Y. Fukuda, and H. Kikuchi, "The gait control system of a quadruped walking vehicle," *Advanced Robotics*, vol. 1, no. 2, pp. 143–164, 1986.
- [8] J. Buchli, F. Stulp, E. Theodorou, and S. Schaal, "Learning variable impedance control," *International Journal of Robotics Research*, vol. 30, no. 7, pp. 820–833, June 2008.
- [9] J. H. Park, "Impedance control for biped robot locomotion," *IEEE Transactions on Automatic Control*, vol. 17, no. 6, pp. 870–882, 2001.
- [10] B. G. Son, J. T. Kim, and J. H. Park, "Impedance control for biped robot walking on uneven terrain," in *Proceedings of IEEE International Conference on Robotics and Biomimetics*, 2009, pp. 239–244.
- [11] R. S. Sutton and A. G. Barto, "Reinforcement learning," *Brain Research*, no. 9, 1998.
- [12] C. J. Chae and J. H. Park, "Galloping trajectory optimization and control for quadruped robot using genetic algorithm," in *Proceedings of IEEE International Conference on Robotics and Biomimetics*, 2007, pp. 1166–1171.
- [13] Y. Hu and S. Yang, "A knowledge based genetic algorithm for path planning of a mobile robot," in *Proceedings of IEEE International Conference on Robotics and Automation*, 2004, pp. 4350–4355.