A Non-Communicating Multi-Robot System with Switchable Formations

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***Abstract*—We consider connected navigation of autonomous mobile robots with transitions in the group formation. The robots navigate using simple local steering rules without requiring explicit communication among themselves. The formations are achieved by designing proper cost functions and formation transitions are succeeded by switching among these cost functions. The resulting system is proven to be deadlock-free under certain conditions.**

***Keywords*—Autonomous motion, formation, connectivity**

I. INTRODUCTION

In this paper, we propose a methodology for the navigation of autonomous robot groups with switchable formations, while maintaining group connectivity. We assume that the robots have position sensors of limited range and with bounded measurement errors, but no communication capabilities. In studies related to connected navigation and the group behavior of mobile robots, many authors *assume* group connectivity or communication within the group during the period of motion as a prerequisite for the success of their methods. For example, graph theory or potential field techniques are employed in this way in [1]–[6].

Graph theoretic approaches to maintain the connectivity of mobile agents are mainly based on the maximization of the second smallest eigenvalue (Fiedler value) of the Laplacian matrix of the graph [3][4], [7]. Even if this maximization can be accomplished in a distributed manner as suggested in [3], this does not eliminate the necessity of communication between the robots. Only a few studies, however, have focused on maintaining connectivity without relying on information exchange or communication between robots [8][9]. The algorithmic methodologies in these studies assume that the robots are points, and are designed to work only in $\mathbb{R}^2$ with perfect measurements via sensors.

The approach proposed in a recent work by the authors [10] and [11] results in the navigation of a robot group having dynamic topology using only limited-range position sensors with guaranteed connectivity. In these works the sensors are subject to measurement errors and the robots may occult other robots in the group. It was also proven in [11] that the resulting navigation is deadlock-free as long as there is no restriction in the navigation space. However, as the group connectivity is the main concern, the connectivity level of the group is always non-decreasing in these works, unless occultation occurs. This gives rise to a ball-shaped formation of the group and creates difficulty in some tasks, such as passing through a narrow corridor. Hence, dynamically changing the group formation is inevitable in a multi-robot navigation system.

Group formation is a widely studied area in mobile robotics. While it can be achieved by a central mechanism and communication channels, decentralized formation is obviously much more challenging, where each agent decide on its own movements autonomously [12], [14]. The available approaches for the decentralized control of formation can be analyzed in three categories: Behavior-based approach [13], leader-follower models [12], and virtual structure techniques [14]. In this work we use cost functions which are basically virtual potential fields. Although a leader is present in the group of robots studied in this paper, its sole function is only to progress through the trajectory and cause the rest of the group to follow him. The leader has not a specific role in formation.

The agents in the group are described in the following section. In Section III, Local Steering Strategy and motion constraints are stated. Formations using cost functions, switching mechanism and a deadlock theorem are given in Section IV. The proposed methodology is tested by computer simulations in Section V. Finally, Section VI presents concluding remarks on the study.

II. PROBLEM FORMULATION

The group consists of identical robots that are omnidirectional and equipped with limited-range relative position sensors. The sensors provide continuous measurements of distances and relative angles within their range. The measurements can bear both angular and radial errors, which are bounded by the positive scalars $\Delta\theta$ and Δr, respectively.

Sensing other robots means obtaining information about the position of the robots in the neighborhood via relative position sensors. We refer to such a mutual visibility between robots as a *link*. However, such a link does not require any explicit communication or information exchange between the robots. Note that sensing the other robots does not imply recognizing a specific robot. In other words, the robots have no ID's or labels.

We denote a group of autonomous mobile robots with links based on their sensing neighborhood as $\mathcal{G}$ and the individual robots as R_i $(i = 1, \ldots, N)$. Note that the subscripts are

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arbitrary and for the sake of analysis only. Considering the robots $R_1, \ldots, R_N$ as the vertices and the links between them as the edges of an undirected graph, this graph is *connected* if there is a path from any robot to any other robot in the group through links [7]. Hence, without loss of any rigor, we can say that the group $\mathcal{G}$ is connected, whenever the graph corresponding to $\mathcal{G}$ is connected. Conversely, a group which has at least one pair of robots having no path between is *disconnected.*

Since we assume that the position sensing ranges of the robots are limited and the total number of robots in the group can be large, a robot may not sense all other robots in the group. We call the set of robots sensed by R_i as the subgroup $\mathcal{S}_i$. So, there are N such subgroups of $\mathcal{G}$ and, if $\mathcal{G}$ is connected, $\mathcal{S}_i$ $(i = 1, \ldots, N)$ are nonempty sets.

We denote the radius of the spherical region with R_i at its center and containing the robots in $\mathcal{S}_i$ as d_{max}. In other words, d_{max} is the maximum sensing distance for each robot. If d_{max} is large enough so that the robots can sense even the farthest member of the group, then $\mathcal{G}$ will be connected. However, one faces nontrivial and more interesting cases for robot groups of a large number of individuals having short sensing ranges and are spread over a relatively large area.

We assume that sensing is always mutual, that is, if a robot R_i senses any other robot R_j, then R_j has the position information of R_i too. In implementing position measurement, which might be performed using any kind of ultrasonic, laser or vision-based sensors, it is inevitable that some robots might occlude others. In such a case, occluded robots are not sensed by a robot, say R_1, (hence, they are not in $\mathcal{S}_1$) although their distances to R_1 are less than d_{max}. Consequently, whenever occlusion occurs, the positions of the occluded robots cannot be taken into account in the computation of the local movement at that time instant. Note that the mutuality of position sensing is also valid under occlusions.

Having these sensing limitations and assuming that a set of robots initially form a connected group, our objective in this work is to develop a decentralized steering methodology that allows navigation of the non-communicating group while preserving and adjusting its connectivity so that the group can change its formation for passing some static barriers.

Once connectivity is assured, the target or navigation trajectory of the mission need not be known by all group members. In fact, it suffices if only one robot has this information [9]. We call this robot the *leader* of the group and denote it as R_N. Nevertheless, the leader has the same physical properties and capabilities as the other robots. The only difference is that the trajectory to be followed by the group is given to R_N. In fact, the leadership of the group is *hidden*. None of the robots recognize the leader as a distinguished group member. In other words, if R_N is sensed by robot R_j, i.e. $R_N \in \mathcal{S}_j$, R_j can only see it as one of its neighbors and the leadership of R_N does not affect the local steering strategy of R_j. In the following part of the paper, we consider the group of N robots consisting of one leader, R_N, and $N-1$ *followers*, $R_1, \ldots, R_{N-1}$.

III. Local Steering Strategy

The goal is to develop a methodology for simple autonomous robots, such that a large group of them can move as a connected group. We assume that the robots update the position information about their neighbors at every Δt seconds. Also, to take measurement errors on the distance into account, we define a positive scalar d_m as

$$d_m \stackrel{def}{=} d_{max} - \Delta r$$

where Δr is the bound on the distance measurement error with $d_{max} > \Delta r > 0$. We denote the position of a robot R_i at time t, as $X_i(t)$, $i = 1, \ldots, N$. Since all robots in the group steer autonomously, we will set up local moving rules for each robot. While the leader R_N moves along a predefined trajectory, each follower robot R_j, $j = 1, \ldots, N-1$, determines a local target location for itself. This motion is most conveniently described in terms of a coordinate system attached to R_j. Obviously, R_j is at the origin of this local coordinate system. Let $x(t)$ denote the position vector in local coordinates. We will use a notation such that the superscripts in x relate the coordinate frame to a robot, and the subscripts in x indicate which robot's position it is. For example, x_k^j represents the position vector of R_k in the coordinate frame of R_j. For the robots in $\mathcal{S}_i$, $i = 1, \ldots, N$, we have

$$\|x_k^i(t)\| = \|X_k(t) - X_i(t)\| \leq d_m, \qquad k = 1, \ldots, M$$

where M is the number of robots in $\mathcal{S}_i$. Next, we propose the a steering strategy to be employed by each robot using the positions of other robots in its subgroup.

According to the notation given above, $x_i^i(t + \Delta t)$ is the location, which R_i is aiming at (for the time instant $t + \Delta t$), in R_i's own coordinate system at time t. For any $x_i^i(t + \Delta t)$, we define two complementary subsets of $\mathcal{S}_i$ as

$$\begin{aligned} \mathcal{S}_{ip} &= \left\{ R_p \in \mathcal{S}_i \mid [x_i^i(t + \Delta t)]^{\mathrm{T}} x_p^i(t) \leq 0 \right\} \\ \mathcal{S}_{iq} &= \left\{ R_q \in \mathcal{S}_i \mid [x_i^i(t + \Delta t)]^{\mathrm{T}} x_q^i(t) > 0 \right\}. \end{aligned}$$

If a displacement of R_i to $x_i^i(t + \Delta t)$ will take R_i closer to a robot, then this robot will appear in $\mathcal{S}_{iq}$. Otherwise, it will be a member of $\mathcal{S}_{ip}$. Using $\mathcal{S}_{ip}$ and $\mathcal{S}_{iq}$, we can state the following theorem on the group connectivity.

Theorem 1: Consider a group $\mathcal{G}$ of N autonomous mobile robots which are connected at $t = 0$. If the motion of the robots are subject to the constraints

$$\|x_i^i(t + \Delta t)\| \leq \frac{1}{2} \left(d_m - \max_{x_p^i(t) \in \mathcal{S}_{ip}} \|x_p^i(t)\| \right) \qquad (1)$$

and

$$\|x_i^i(t + \Delta t)\|^2 \leq \min_{x_q^i(t) \in \mathcal{S}_{iq}} \left\{ [x_i^i(t + \Delta t)]^{\mathrm{T}} x_q^i(t) \right\} \qquad (2)$$

for $i = 1, \ldots, N$, the group preserves its connectivity for $t > 0$.

Note that if a robot is occluded by another robot in $\mathcal{S}_i$, the number of robots in $\mathcal{S}_i$ might decrease. Nevertheless, this situation does not disturb the overall connectivity, as the existence of the occluding robot itself is the evidence of the connection between R_i and the occluded robot. Also, the fact that (1) and (2) restrict the maximum steering distances of the robots for each sampling period Δt brings the advantage of avoiding inappropriately large velocities.

As long as the constraints in (1) and (2) are satisfied, following a given navigation trajectory, formation control and other mission-oriented tasks can be accomplished by using potential function approaches or minimizing cost functions. Therefore, in view of Theorem 1, the following Local Steering Strategy assures the connectivity of the robot group, which is composed of follower robots and a leader in navigation.

Local Steering Strategy *Subject to the constraints (1) and (2),*

- *The follower robots R_i ($i = 1, \ldots, N-1$) move towards a target location $x_i^i(t+\Delta t)$, which minimizes a cost function, $J(x_i^i(t+\Delta t))$, related to the positions of the robots in $\mathcal{S}_i$.*
- *The leader R_N follows the navigation trajectory.*

At this point, one may raise the question whether these constraints can lead to a situation where none of the robots can move. Such a situation is called a *deadlock* and its avoidance is of crucial importance for the applicability of the proposed method in real-life implementations. In view of constraints (1) and (2), a deadlock occurs whenever $\|x_i^i(t+\Delta t)\| = 0$, $i = 1, \ldots, N$ [11].

Defining $z_j^i(t)$ as

$$z_j^i(t) = x_i^i(t+\Delta t) - x_j^i(t), \tag{3}$$

the following theorem characterizes the cost functions which assure that the group moves along its trajectory without any risk of deadlock.

Theorem 2: Let $\mathcal{G}$ be an initially connected group navigating freely in $\mathbb{R}^n$ and consisting of finite number of robots that move according to the Local Steering Strategy. Assume that $J(x_i^i(t+\Delta t)) = \tilde{J}(\|z_j^i(t)\|)$ is an increasing function of $\|z_j^i(t)\|$, at $\|z_j^i(t)\| = d_m$ for all j such that $R_j \in \mathcal{S}_i$, where $z_j^i(t) = x_i^i(t+\Delta t) - x_j^i(t)$ Then, for any robot $R_a \in \mathcal{G}$, we have

$$\max_k \|x_k^a(t)\| < d_m \qquad \text{as } t \to \infty \tag{4}$$

where x_k^a's are the position vectors of the robots in $\mathcal{S}_a$.

Theorem 2 presents an important basis for the guaranteed navigation of the group. That is, if one of the robots in the group is the leader and is given a trajectory to be followed, the leader will have the freedom to progress through its trajectory without breaking connectivity no matter how the trajectory is shaped. This is clear from the fact that

$$d_m - \lim_{t\to\infty} \max_k \|x_k^N(t)\| > 0$$

and the strict inequality assures a nonzero distance that the leader can move at each sampling time. The rest of the group will then follow the leader accordingly.

It should also be noted that Theorem 2 assumes that the group is navigating freely. In other words the motion of the robots are constrained only as in (1) and (2), and not by any obstacles around. Examples of deadlock where the navigation space is a proper subset of $\mathbb{R}^n$ are given in [11].

Several types of cost functions can be used in implementing the local steering strategy. Examples of cost functions that satisfy the requirement in Theorem 2 can be

$$J(x_i^i(t+\Delta t)) = \max_k \|z_k^i(t)\| \tag{5}$$

or

$$J(x_i^i(t+\Delta t)) = \sum_{k=1}^{M} \left(\|z_k^i(t)\| - d_0\right)^2. \tag{6}$$

The cost function in (5) makes the i^{th} robot try to decrease the distance to the farthest robot that it senses. On the other hand, (6) can be used to force the robots to keep their distances with the robots in their subgroups as close to a desired distance d_0 ($d_0 < d_m$) as possible.

Note that both (5) and (6) are defined in terms of local coordinates to ensure a distributed algorithm. Although they happen to be convex functions of $z_k^i(t)$, this is not a requirement from the point of view of connectivity. The choice of cost function depends on mission requirements. One can consider fixed as well as time-varying cost functions. They can incorporate the position information of all or only some of the neighbors. Further, the members of the group may minimize different cost functions to achieve a required group formation. In other words, choosing the cost function suitably can facilitate not only connected navigation but also a desired group formation. In the following section we employ such ideas to make the group switch between alternative formations during the navigation.

IV. Switching Group Formation

When applied with the cost function as given in (5) or (6), the Local Steering Strategy tries to strenghten and hence preserve the group connectivity during navigation. The group gains an amorphous shape, whose compactness is dependent on the value of d_0. Obviously, this is not suitable if the group is required to navigate around some obstacles or to pass through a relatively narrow corridor. In such cases, changing the formation to a specified one resolves the difficulty. Transitions in the formation can be managed by incorporating alternative cost functions in the system and switching among them whenever necessary.

A line formation may be most suitable when the group navigates in an environment narrowed by walls or obstacles. Although the group cannot be assumed as navigating freely whenever obstacles are around, nonetheless it is not difficult to see that a deadlock is not possible for a group moving in a line formation along the trajectory.

Let e_l be the unit vector in the direction of the desired line formation. A suitable cost function can be written as

$$\begin{aligned} J_f(x_i^i(t+\Delta t)) \quad &= \quad \frac{1}{2}\left[x_i^i(t+\Delta t) - \left((x_a^i(t) - d_1 e_l)\right)\right]^T \\ &\quad \times \left[x_i^i(t+\Delta t) - \left((x_a^i(t) - d_1 e_l)\right)\right] \end{aligned} \tag{7}$$

where

$$a = \underset{k|R_k\in\mathcal{S}_i,[x_k^i(t)]^T e_l \geq 0}{\arg\min} \|x_k^i(t)\|$$

and d_1 is the desired inter-robot distance of the line formation. Note that the cost function in (7) is a quadratic form of the difference between the local target of R_i and the point R_i should reach to fit in the formation. For the line formation, this point is at a distance of d_1 to R_a and located so that the direction from R_i to R_a is aligned with e_l.

Using (3), (7) can be rewritten as

$$\begin{aligned} J_f(x_i^i(t+\Delta t)) &= \frac{1}{2}\left[z_a^i(t)+d_1e_l\right]^T\left[z_a^i(t)+d_1e_l\right] \\ &= \frac{1}{2}\left\|z_a^i(t)+d_1e_l\right\|^2. \end{aligned} \tag{8}$$

Obviously, the term $z_a^i(t)$ is the distance between the local target of R_i and the closest robot whose position vector has a positive projection on e_l (Figure 1). As long as the robots minimize J_f, they will converge to and maintain a line formation. Different choices for e_l can be employed by individual robots to obtain more elaborate formations. For example, a $90°$ V-formation is achieved by applying J_f to half of the robots with e_{l1} and to the other half with e_{l2}, such that $[e_{l1}]^T[e_{l2}]=0$.

Denoting $\tilde{J}_f(\|z_a^i(t)\|)=J_f(x_i^i(t+\Delta t))$, we get

$$\begin{aligned} &\left.\frac{\partial \tilde{J}_f}{\partial\|z_a^i\|}\right|_{\|z_a^i(t)\|=d_m} \\ &= \frac{1}{2}\left[\frac{\partial}{\partial\|z_a^i\|}\left\|\|z_a^i(t)\|e_z(t)+d_1e_l\right\|^2\right]_{\|z_a^i(t)\|=d_m} \\ &= \left[\|z_a^i(t)\|+d_1e_l^Te_z(t)\right]_{\|z_a^i(t)\|=d_m} \\ &= d_m+d_1e_l^Te_z(t), \end{aligned} \tag{9}$$

where $e_z(t)=z_a^i/\|z_a^i\|$, that is, the unit vector in the direction of $\|z_a^i(t)\|$. Since $d_m>d_1>0$ and $|e_l^Te_z(t)|\leq 1$, we have

$$\left.\frac{\partial \tilde{J}_f}{\partial\|z_a^i\|}\right|_{\|z_a^i(t)\|=d_m} > 0.$$

This means that J_f satisfies the condition in Theorem 2 with respect to the robot R_a. In order to fulfill the condition with respect to all robots in $\mathcal{S}_i$, one can augment the cost function in (8) as

$$\begin{aligned} &J_1(x_i^i(t+\Delta t)) \\ &= J_f(x_i^i(t+\Delta t))+\beta\sum_{\substack{k=1\\k\neq a}}^{M}\left(\|z_k^i(t)\|-d_1\right)^2 \\ &= \frac{1}{2}\left\|z_a^i(t)+d_1e_l\right\|^2+\beta\sum_{\substack{k=1\\k\neq a}}^{M}\left(\|z_k^i(t)\|-d_1\right)^2 \end{aligned} \tag{10}$$

where $\beta>0$ is a weighting factor.

When the group converges to a line formation, each subgroup $\mathcal{S}_i$ consists of at most two robots, due to occlusions. This will greatly simplify the minimization of the cost function in (10).

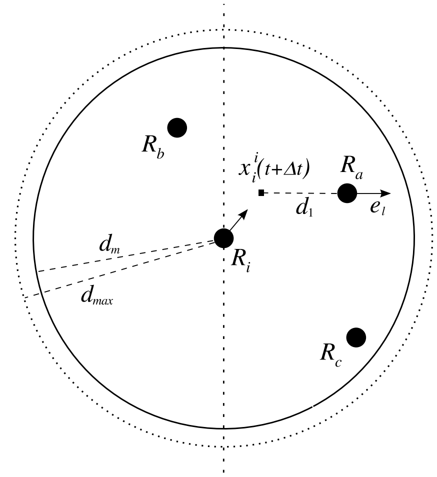

Fig. 1. Local target for the robot R_i in its subgroup. R_a is the closest robot to R_i, whose position vector has a positive projection on e_l.

V. Simulations

In the general case, for the ease computation, a gradient-based sub-optimal solution is invoked as described in [10] and [11]. That is,

$$x_i^i(t+\Delta t)=x_i^i(t)-\gamma\left.\frac{\partial J(x_i^i(t+\Delta t))}{\partial x_i^i(t+\Delta t)}\right|_{x_i^i(t+\Delta t)=x_i^i(t)} \tag{11}$$

where $\gamma>0$ is a positive gain, and x_i^i is the position vector of R_i in its local coordinates.

The application of (11) is much simpler than solving the system in (6) or (10). It gives the direction of the next movement and the movement in this direction is realized only if it satisfies the inequalities in (1) and (2). Hence, after simplifying the calculations, we illustrate the theoretical results of the previous sections with computer simulations.

A robot group, composed of disk-shaped robots having omni-directional motion capability, is assumed to navigate in $\mathbb{R}^2$. The sensor range (d_{max}) was 30 units. The bounds on the measurement errors were $\Delta\theta=12°$ for angle and $\Delta r=0.03d_{max}$ for distance measurement. Diameter of the robots was 1.5 units. For each robot in the group, this value and its position information in local coordinates were used to determine the occlusion cone caused by that robot with respect to the robot at the origin of that local coordinates. Any partially occluded robot was taken as if fully occluded. The following scenario was applied: The leader is given a trajectory and as the leader starts navigation, the rest of the group follows the leader under the Local Steering Strategy, which is implemented by the pseudo-code given in Figure 2. The group is pre-loaded with the locations where the cost function will be switched.

In the first simulation, a group consisting of 10 robots was initialized to the locations given in top of Figure 3. The snapshots of the simulation can be seen in Figure 3, where the trajectory of the leader is shown by the solid lines. As soon as the simulation started with the cost function given in (6), the Local Steering Strategy forced the robots to form an amorphous shape. There is no formation until the switching in the cost function. The first switching takes place where the trajectory of the leader points into a narrow corridor. The

```
read sensor data;

if leader,
  read given trajectory;
  assign  direction := value from trajectory;
          motion_size := predefined max value (physical limit);
else
  if specified time/location for J switch,
    switch  J := J_k;
  end if

  calculate J;
  assign  direction := -grad J;
          motion_size := value from evaluation of grad J;
end if

classify sensed robots as p and q using direction;

apply motion limits;
  motion_size := max value satisfying inequalities in Theorem 1;

if motion_size > predefined max value,
  assign  motion_size := predefined max value;
end if
```

Fig. 2. Pseudo-code for the algorithm used in simulations.

group, then switches to the cost function given in (10) for a horizontal line formation. After successfully passing through the corridor, the group switches back into the initial cost function to increase the connectivity. In the return path, the group faces another corridor and thus another switching in the cost function takes place. The last part of Figure 3 is a snapshot taken prior to completion of the line formation.

Figure 4 displays the connectivity of the group during the whole navigation. It can be easily distinguished that the connectivity dramatically drops after switching to line formation. But even in this case, the connectivity never goes below 9, which is the minimum number for a group of 10 robots. This means the group stays connected even in the worst scenario from connectivity point of view.

The second simulation was realized with 11 robots to show a V-formation. Initial positions of the robots are seen in top of Figure 5. Shortly after the start of navigation, a switching in the cost function occurs in order to go into a 120° V-formation. This is achieved by applying two different line-up vectors.

VI. CONCLUSIONS

This work is an extension to our previous works about the navigation of non-communicating robots without breaking the group connectivity. As the group navigates with the objective of preserving the connectivity in [10][11], the resulting amorphous shape of the group causes difficulty when the navigation space includes obstacles. In this work we showed a methodology to provide group formations by switching among several cost functions. The group connectivity is controlled and hence it can be decreased whenever necessary, but the group still stays connected after any such decrease in the number of links. Although we assume obstacles in the navigation space, the group navigates without any risk of deadlock with the assumption that the obstacles are far enough so that they never constrain the local targets.

Currently the switching mechanism depends on a priori information, since the robots have no communication capability at all. This can be done in a dynamic manner if each robot analyzes its environment and decides to switch its cost function autonomously, in future works.

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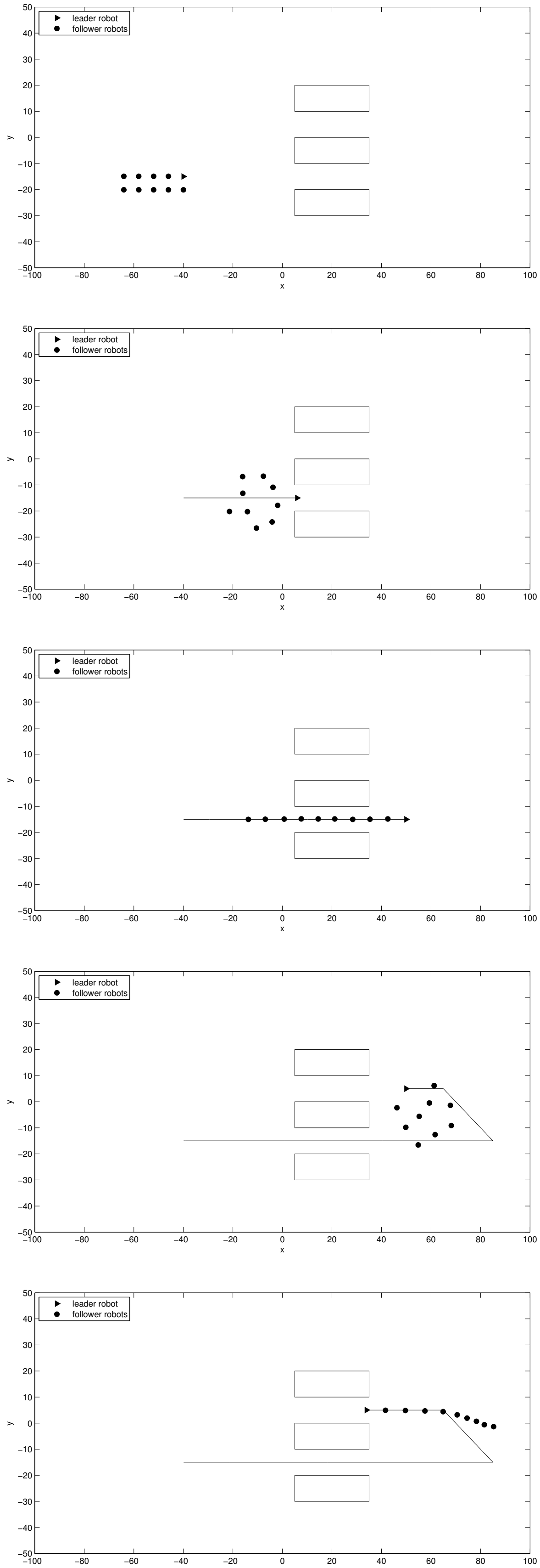

Fig. 3. Navigation of 10 robots switching to line formation before entering corridors.

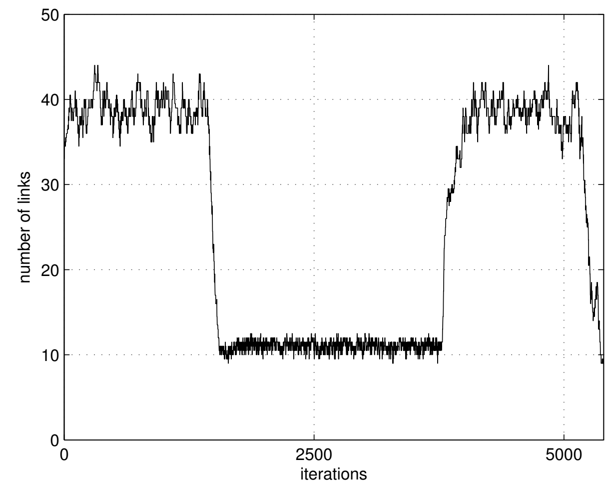

Fig. 4. Connectivity of the 10 robots during the whole navigation. The large drops in the number of links correspond to line formation which is active during corridor passing.

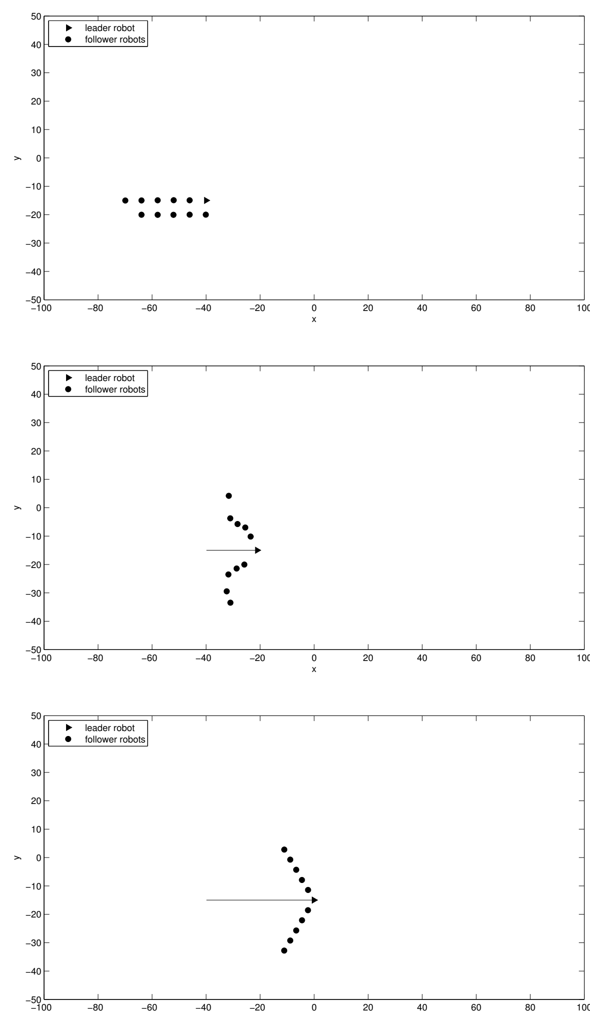

Fig. 5. V-formation by 11 robots.