Position Feedback Pinning Control for Nonlinear Multi-agent Systems

Ming-Can Fan, Zhiyong Chen and Hai-Tao Zhang

***Abstract*— This paper addresses a velocity consensus control problem for multi-agent systems in a complicated scenario where the agent velocities are not measurable and the agent dynamics are intrinsically nonlinear. It is proved that the proposed position feedback controller with sufficiently large but explicitly designed gains is able to deal with system nonlinearities when the region of attraction is semi-globally specified. The results are also verified in numerical simulation.**

***Index Terms*— Cooperative control, Consensus, Nonlinear systems, Output feedback**

I. Introduction

The leader-follower configuration has been extensively explored among the large volume of consensus control methods for multi-agent systems. In natural systems, a virtual leader may represent the location of a food resource, a rendezvous target, a predator, or a habitat for biological populations such as bee swarms, fish schools/shoals, pigeon flocks, ant colonies and so on. In literature, a flocking algorithm was proposed in [1] that a virtual leader is able to guide agents to an α-lattice formation without any regular fragmentation. Another promising way to stabilize virtual leader-follower flocks is pinning control. For example, control methods were proposed to drive complex network states to an equilibrium point by pinning a small proportion of nodes (see, e.g., [2], [3], [4], [5].) However, most of the aforementioned consensus controllers (and many other controllers for multi-agents of second order dynamics, e.g., [6], [7]) are based on both position and velocity feedback from the neighborhood. Actually, in many biological collective motion observations, the communication frequency is usually not high [8], which implies that some feedback information like velocity observation may be redundant. From engineering point of view, the communication cost can be substantially saved if the controller requirement on velocity feedback is eliminated. More significantly, the velocity-free control method is of special benefit for multi-agent systems not equipped with velocity sensors or the situation where the precise measurement of velocities is unavailable. The velocity-free consensus control problem has attracted many researchers in recent years, e.g., in [9], [10], [11], [12], [13]. These works were mainly on a second-order consensus problems and various observers were constructed to estimate the unmeasurable velocities. However, the research in this direction is within the setup for agents of linear dynamics. In this paper, we are interested in a more complicated situation in the presence of system nonlinearities. The consensus problem for agents of nonlinear dynamics has been studied in relevant literature, e.g., [14], [15]. The major novelty of the present result, compared with the existing results, is twofold: (i) As mentioned above, the measurement output feedback or velocity-free scenario is studied in this paper. (ii) The technique dealing with system nonlinearities is novel. In particular, we do not assume the nonlinearities satisfy a certain globally Lipschitz condition (or bounded by a linear growth rate) as used in literature. As the cost for removing the global Lipschitz condition, the desired consensus is expected to occur semi-globally, that is, the initial velocities and the reference velocity trajectory are assumed within a specified region. It is worth mentioning that the specified region can be arbitrarily large but it is predefined for controller design. In summary, we aim to develop a velocity-free control algorithm for multi-agent systems of nonlinear intrinsic dynamics with a semi-global region of attraction.

M.-C. Fan and H.-T. Zhang are with Department of Control Science and Engineering and the State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan, 430074, P.R. China. zht@mail.hust.edu.cn, mingcan.fan@gmail.com.

Z. Chen is with the School of Electrical Engineering and Computer Science, The University of Newcastle, Callaghan, NSW 2308, Australia. zhiyong.chen@newcastle.edu.au. His work is partially supported by the Australian Research Council under Grant DP130103039.

Throughout the paper, we denote $I_n \in \mathrm{R}^{n\times n}$ an n-dimensional identity matrix. For a square matrix M, let $\langle M\rangle_l$ be the minor matrix of M by removing its first l row-column pairs. The positive (or, negative) definite property of the matrix M is represented by $M > 0$ (or, $M < 0$). When M is a Hermitian matrix, its eigenvalues are real of which the maximal one is denoted by $\lambda_{\max}(M)$. The symbol $\mathbf{1}$ represents a column vector with entries of one whose dimension is appropriate from context.

II. Problem description

We consider a multi-agent system consisting of n autonomous agents, each of which has the following nonlinear double integrator dynamics:

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= f(t, v_i) + u_i,\ i = 1, \cdots, n \end{aligned} \tag{1}$$

where $x_i, v_i \in \mathbb{R}^m$ are the agent states and $u_i \in \mathbb{R}^m$ is the control force. A typical scenario is that x_i represents the position of agent i and v_i the velocity. Specifically, v_i is generated by nonlinear dynamics $\dot{v}_i = f(t, v_i)$. The local control u_i aims to achieve the velocity consensus of the group of agents with the agreed velocity trajectory determined by a reference model. The reference model has the following autonomous dynamics:

$$\begin{aligned} \dot{x}_\gamma &= v_\gamma, \\ \dot{v}_\gamma &= f(t, v_\gamma), \end{aligned} \tag{2}$$

978-1-4673-5769-2/13/$31.00 ©2013 IEEE

where $x_\gamma, v_\gamma \in \mathbb{R}^m$ are the position and velocity, respectively. For conciseness, we denote $x_{ij} = x_i - x_j$, $x_{i\gamma} = x_i - x_\gamma$, $v_{ij} = v_i - v_j$, and $v_{i\gamma} = v_i - v_\gamma$. Moreover, it is assumed that the nonlinear function $f(t, v_i)$ is piecewise continuous in t and locally Lipschitz in v_i uniformly in t.

The network topology is described as follows. Let $l(1 \leq l \leq n)$ agents of the system (1) have the access to the reference model (2). Without loss of generality, we assume the l agents are labelled $1, \cdots, l$. These l agents are called the pinned agents (or leaders) and the remaining $n-l$ agents are called the followers. The network topology of the n agents is represented by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with a node set $\mathcal{V} = \{1, \ldots, n\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An undirected edge $(i, j) \in \mathcal{E}$ means that the nodes i and j have the access to each other. Then, the neighborhood of the node i is defined by $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$. The measurement output is defined as the agent position x_i. As a result, the available feedback information from the group for the agent i is

$$e_i = \{x_{ij} | j \in \mathcal{N}_i\} \cup \{x_{i\gamma} | i \leq l\}.$$

Therefore, we propose the following controller for the agent i:

$$\begin{aligned} u_i &= \kappa_i(e_i, z_i) \\ \dot{z}_i &= \varkappa_i(e_i, z_i) \end{aligned} \tag{3}$$

where κ_i and $\varkappa_i$ are two functions to be designed and z_i is a compensator state. Now, the main objective is to solve the semi-global velocity consensus problem which is precisely formulated below.

Definition 2.1: *The multi-agent system (1)-(2) is said to achieve semi-global velocity consensus if, for any $R > 0$, there exist controllers u_i, $i \in \mathcal{V}$ of the form (3) such that $\lim_{t\to\infty} \|v_i(t) - v_\gamma(t)\| = 0$, $i \in \mathcal{V}$, for any reference trajectory satisfying $\|v_\gamma(t)\| \leq R$, $\forall t \geq 0$, and any initial conditions satisfying $\|v_i(0)\| \leq R$, $i \in \mathcal{V}$.*

There are two major technical challenges in the aforementioned problem, i.e., measurement output feedback and nonlinearity. The desired consensus is in the sense of agent velocity, however, the velocities are unmeasurable in the setup. Therefore, an observer based on the measurement output, i.e., the position, is required for velocity estimation and hence velocity consensus. The observer design, represented by the z_i dynamics in (3), is challenging in a consensus problem. In literature, e.g., [9], [10], [11], [12], [13], researchers have proposed methods for such observers for linear systems. In this paper, the situation is more complicated in the presence of nonlinearities. Not like many existing results in literature, e.g., [14], [15], we do not assume the nonlinearities satisfy a certain globally Lipschitz condition (or bounded by a linear growth rate). As the cost for removing the global Lipschitz condition, the desired consensus is expected to occur semi-globally, that is, the initial velocities and the reference velocity trajectory are assumed within a region characterized by R. It is worth mentioning that the region size R can be arbitrarily large but it is pre-defined for controller design.

To close this section, we introduce two technical lemmas which will be used in the proof of the main result. The first lemma is cited from [16] and the second one is a variant of the result in [3] .

Lemma 2.1: (Schur complement [16]) The linear matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S^\intercal(x) & R(x) \end{bmatrix} > 0,$$

with $Q(x) = Q^\intercal(x)$ and $R(x) = R^\intercal(x)$, is equivalent to either of the following conditions:

1) $Q(x) > 0$, $R(x) - S(x)Q^{-1}(x)S(x) > 0$;
2) $R(x) > 0$, $Q(x) - S(x)R^{-1}(x)S(x) > 0$.

Lemma 2.2: *Consider symmetric matrices $M, D \in \mathbb{R}^{n\times n}$ of the forms*

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^\intercal & \langle M \rangle_l \end{bmatrix},\ D = diag\{[d_1, \ldots, d_l, 0, \ldots, 0]\}$$

with $d_i > 0$, $i = 1, \ldots, l(1 \leq l \leq n)$. The following inequality holds

$$k_1 M + \varrho I_n - k_2 D < 0 \tag{4}$$

for $k_1, k_2, \varrho > 0$ if there exists $\rho > \varrho$ such that

$$k_1 \lambda^* + \rho < 0,\ \ \lambda^* = \lambda_{\max}(\langle M \rangle_l) < 0 \tag{5}$$

and

$$\begin{aligned} k_2 d^* &> k_1 \sigma^*(\rho/k_1), \\ \sigma^*(s) &= \lambda_{\max}\left(M_{11} - M_{12}\langle M + sI_n\rangle_l^{-1} M_{12}^\intercal\right) + s, \\ d^* &= \min\{d_i | i = 1, \cdots, l\}. \end{aligned} \tag{6}$$

III. Main results

In this section, we present a specific pinning control algorithm for the nonlinear multi-agent system (1) to achieve the semi-global velocity consensus. First, we define a symmetric weighted adjacency matrix $A = [a_{ij}]_{n\times n}$ with nonnegative elements. The elements are defined such that $a_{ij} > 0$ for $(i, j) \in \mathcal{E}$, $a_{ij} = 0$ for $(i, j) \notin \mathcal{E}$, and $a_{ii} = 0$ for all $i \in \mathcal{V}$. Then, we define a weighted adjacency matrix $D = \text{diag}\{d_1, \cdots, d_n\}$ with $d_i > 0$, for $i = 1, \ldots, l$, and $d_i = 0$ otherwise. In other words, $D = \text{diag}\{d_1, \ldots, d_l, 0, \ldots, 0\}$. Obviously, the nonzero elements in the matrices A and D are consistent with the network edges.

When the matrices A and D are selected, we can define the following quantities. The Laplacian matrix $L = [l_{ij}]_{n\times n}$ of $\mathcal{G}$ is defined as $l_{ij} = -a_{ij}$, $i \neq j$ and $l_{ii} = \sum_{k=1,k\neq i}^{n} a_{ik}$. Obviously, we have $L\mathbf{1} = 0$. Let

$$M = -L = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^\intercal & \langle M \rangle_l \end{bmatrix},$$

and denote two constants and one function as follows,

$$\begin{aligned} \lambda^* &= \lambda_{\max}(\langle M \rangle_l),\ d^* = \min\{d_i | i = 1, \cdots, l\}, \\ \sigma^*(s) &= \lambda_{\max}\left(M_{11} - M_{12}\langle M + sI_n\rangle_l^{-1} M_{12}^\intercal\right) + s. \end{aligned} \tag{7}$$

In terms of network topology, we made the following assumption.

Assumption 3.1: The adjacency matrices A and D are such that $\lambda^* < 0$ and $d^* > 0$, respectively.

Remark 3.1: In Assumption 3.1, the adjacency matrix D satisfying $d^* > 0$ simply requires that the l leaders of the group have the access to the reference model. The selection of A satisfying $\lambda^* < 0$ is interpreted as follows. For a network with the adjacency matrix A, e.g., $\dot{x}_i = -\sum_{j=1,i\neq j}^{n} a_{ij}x_{ij}$, its dynamics can be written as $x = -Lx$. The assumption $\lambda^* = \lambda_{\max}(\langle M\rangle_l) < 0$ actually requires that $\langle M\rangle_l = \langle -L\rangle_l$ be Hurwitz. As a result, the subnetwork composed of the $n-l$ followers is stable. First, under the assumption, there exists at least one leader, i.e., $l \geq 1$. Otherwise, $l = 0$ implies that $\langle -L\rangle_l = -L$ is Hurwitz, which contradicts the fact $L\mathbf{1} = 0$. Secondly, under the assumption, there exists at least one edge connecting the leaders and the followers, that is, the matrix $L_{12} \neq 0$ with

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{12}^{\intercal} & \langle L\rangle_l \end{bmatrix}.$$

Otherwise, $L_{12} = 0$, together with the fact $L\mathbf{1} = 0$, gives $\langle L\rangle_l\mathbf{1} = 0$, which is a contradiction with the requirement that $\langle -L\rangle_l$ is Hurwitz.

Now, the dynamic controller can be explicitly constructed as follows

$$\begin{aligned} u_i &= \dot{z}_i - \beta z_i + \bar{u}_i \\ \dot{z}_i &= -\delta z_i + \bar{u}_i, \;\; z_i(0) = 0 \end{aligned} \tag{8}$$

with the two positive parameters β and δ to be designed. The auxiliary controller $\bar{u}_i$ is selected as

$$\bar{u}_i = -k_1 \sum_{j=1}^{n} a_{ij}(x_{ij} - x_{ij}(0)) - k_2 d_i(x_{i\gamma} - x_{i\gamma}(0)). \tag{9}$$

The two positive controller gains k_1 and k_2 are to be designed, which represent the connectivity strength.

Before the analysis on the controller (8)-(9), we define some quantities which characterize the nonlinear term $f(t, v_i)$. These quantities will be used in convergence analysis of consensus. For a positive number $R > 0$, there exists $\ell > 0$ such that

$$\begin{aligned} &\|f(t,v_1) - f(t,v_2)\| \leq \ell\|v_1 - v_2\|, \\ &\qquad \forall \|v_1 - v_2\| \leq 4\sqrt{n}R, \; \|v_2\| \leq R. \end{aligned} \tag{10}$$

Pick three constants μ_1, μ_2, and μ_3 satisfying $0 < \mu_1 < 4/\ell$, $\mu_2 > 0$, and $\mu_3 > 0$, and hence $\alpha > (1+\mu_2/4)\ell/(1-\mu_1\ell/4)$. Then, define two constants and one function as follows:

$$\begin{aligned} \beta^* &= 2\alpha, \\ \delta^* &= \mu_3 + \ell/\mu_2, \\ \rho^*(\beta,\delta) &= \max\{2(\beta^2 - 2\alpha\beta + 2\alpha^2), \\ &\quad \left[(\alpha\beta - \beta\delta + \alpha\delta)^2/(4\mu_3) + \alpha\ell/\mu_1\right]/(\beta - 2\alpha)\}. \end{aligned} \tag{11}$$

Now, the main theorem is summarized below.

Theorem 3.1: *Consider the closed-loop multi-agent system composed of the agent dynamics (1)–(2) and the controller (8)–(9) under Assumption 3.1. In particular, the constants λ^* and d^* and the function $\sigma^*(s)$ are defined by (7). For a positive number $R > 0$, the constants β^* and δ^* and the function $\rho^*(\beta,\delta)$ are defined by (11).*

Assume the parameters β and δ in (8) are such that

$$\begin{aligned} \beta &> \beta^*, \\ \delta &> \delta^* + \beta. \end{aligned}$$

Also, assume the parameters k_1 and k_2 in (9) are such that

$$\begin{aligned} k_1 &> -\rho/\lambda^*, \\ k_2 &> k_1\sigma^*(\rho/k_1)/d^*, \;\; \rho \geq \rho^*(\beta,\delta). \end{aligned}$$

Then, the semi-global velocity consensus problem is solved, that is,

$$\lim_{t\to\infty} \|v_i(t) - v_\gamma(t)\| = 0, \; \forall i \in \mathcal{V} \tag{12}$$

for any reference trajectory $\|v_\gamma(t)\| \leq R$, $\forall t \geq 0$ and any initial conditions satisfying $\|v_i(0)\| \leq R$, $\forall i \in \mathcal{V}$, Moreover, the position formation of the multi-agent system (1)-(2) converges to the initial setup, i.e.,

$$\lim_{t\to\infty} \|x_{ij}(t) - x_{ij}(0)\| = 0, \;\; \lim_{t\to\infty} \|x_{i\gamma}(t) - x_{i\gamma}(0)\| = 0, \quad \forall i,j \in \mathcal{V}. \tag{13}$$

Proof: The initial positions are $x_i(0)$ and $x_\gamma(0)$. Pick an arbitrarily reference position x_o and denote $\sigma_i = x_i(0) - x_o$ and $\sigma_\gamma = x_\gamma(0) - x_o$. Define $\chi_i(t) = x_i(t) - \sigma_i$ and $\chi_\gamma(t) = x_\gamma(t) - \sigma_\gamma$. It is easy to see that $x_{ij}(t) - x_{ij}(0) = \chi_{ij}(t)$ and $x_{i\gamma}(t) - x_{i\gamma}(0) = \chi_{i\gamma}(t)$. Now, the closed-loop system composed of (1), (8) and (9) can be rewritten as, for $i \in \mathcal{V}$,

$$\begin{aligned} \dot{\chi}_i &= v_i, \\ \dot{v}_i &= f(t,v_i) - k_1\sum_{j=1}^{n} a_{ij}\chi_{ij} - k_2 d_i\chi_{i\gamma} + \dot{z}_i - \beta z_i, \\ \dot{z}_i &= -\delta z_i - k_1\sum_{j=1}^{n} a_{ij}\chi_{ij} - k_2 d_i \chi_{i\gamma}. \end{aligned} \tag{14}$$

Also, the leader dynamics become

$$\begin{aligned} \dot{\chi}_\gamma &= v_\gamma, \\ \dot{v}_\gamma &= f(t, v_\gamma). \end{aligned} \tag{15}$$

Let $\hat{x}_i = \chi_i - \chi_\gamma$, $\hat{v}_i = v_i - v_\gamma$, and $\hat{y} = [\hat{x}^\intercal, \hat{v}^\intercal, z^\intercal]^\intercal$, with $\hat{x} = [\hat{x}_1^\intercal, \ldots, \hat{x}_n^\intercal]^\intercal$, $\hat{v} = [\hat{v}_1^\intercal, \ldots, \hat{v}_n^\intercal]^\intercal$, and $z = [z_1^\intercal, \ldots, z_n^\intercal]^\intercal$. From (14) and (15), one has

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{v}_i, \\ \dot{\hat{v}}_i &= f(t,v_i) - f(t,v_\gamma) - k_1\sum_{j=1}^{n} a_{ij}\hat{x}_{ij} - k_2 d_i\hat{x}_i \\ &\quad + \dot{z}_i - \beta z_i, \\ \dot{z}_i &= -\delta z_i - k_1\sum_{j=1}^{n} a_{ij}\hat{x}_{ij} - k_2 d_i\hat{x}_i. \end{aligned} \tag{16}$$

To design a Lyapunov function, we first construct a symmetric matrix Ω_ϵ as follows

$$\Omega_\epsilon = \begin{bmatrix} -k_1 M + k_2 D & -\alpha I_n & \beta I_n \\ -\alpha I_n & \epsilon I_n & -I_n \\ \beta I_n & -I_n & 2I_n \end{bmatrix} \tag{17}$$

depending on a parameter ϵ. We can show that Ω_ϵ is positive definite, i.e., $\Omega_\epsilon > 0$, for $3/4 \le \epsilon \le 1$. Indeed, by Lemma 2.1, $\Omega_\epsilon > 0$ is equivalent to

$$k_1 M + (\epsilon\beta^2 - 2\alpha\beta + 2\alpha^2)/(2\epsilon - 1) I_n - k_2 D < 0.$$

This inequality is true by using Lemma 2.2 and noting the facts by (11) that

$$\begin{gathered}(\epsilon\beta^2 - 2\alpha\beta + 2\alpha^2)/(2\epsilon - 1) \le \rho, \\ k_1\lambda^* + \rho < 0, \;\; k_2 d^* > k_1 \sigma^*(\rho/k_1).\end{gathered}$$

Now, we are ready to construct a Lyapunov function candidate as follows, with $\Omega = \Omega_1$ (i.e., $\epsilon = 1$),

$$V(\hat{y}) = \frac{1}{2}\hat{y}^\intercal(\Omega \otimes I_m)\hat{y}. \tag{18}$$

or, in a summation form,

$$\begin{aligned} & V(\hat{y}) \\ = \; & \frac{1}{2}\sum_{i=1}^{n} k_2 d_i \hat{x}_i^\intercal \hat{x}_i + \frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{n} k_1 a_{ij}\hat{x}_{ij}^\intercal \hat{x}_{ij} - \alpha\sum_{i=1}^{n}\hat{x}_i^\intercal \hat{v}_i \\ & +\beta\sum_{i=1}^{n}\hat{x}_i^\intercal z_i + \frac{1}{2}\sum_{i=1}^{n}(\hat{v}_i - z_i)^\intercal(\hat{v}_i - z_i) + \frac{1}{2}\sum_{i=1}^{n} {z_i}^\intercal z_i. \end{aligned}$$

Obviously, we have $\hat{x}_i(0) = \chi_i(0) - \chi_\gamma(0) = (x_i(0) - \sigma_i) - (x_\gamma(0) - \sigma_\gamma) = x_o - x_o = 0$ and $z_i(0) = 0$ by design. That is,

$$V(\hat{y}(0)) = \sum_{i=1}^{n}\|\hat{v}_i(0)\|^2/2 \le n(2R)^2/2.$$

If the time derivative of $V(\hat{y}(t))$ along the trajectory of (16) is non-positive, i.e., $dV(\hat{y}(t))/dt \le 0$, one has $V(\hat{y}(t)) \le V(\hat{y}(0))$, $\forall t \ge 0$. Also, we note that

$$\begin{aligned} V(\hat{y}) = \frac{1}{2}\hat{y}^\intercal(\Omega \otimes I_m)\hat{y} = \frac{1}{2}\hat{y}^\intercal(\Omega_{3/4} \otimes I_m)\hat{y} + \sum_{i=1}^{n}\|\hat{v}_i\|^2/8 \\ \ge \sum_{i=1}^{n}\|\hat{v}_i\|^2/8 \ge \|\hat{v}_i\|^2/8. \end{aligned}$$

Therefore, for any $t \ge 0$,

$$\|\hat{v}_i(t)\|^2/8 \le V(\hat{y}(t)) \le V(\hat{y}(0)) \le n(2R)^2/2,$$

and hence

$$\|\hat{v}_i(t)\| \le 4\sqrt{n}R.$$

Therefore, (10) gives

$$\|f(t, v_i) - f(t, v_\gamma)\| \le \ell\|\hat{v}_i\|.$$

What is left is to show $dV(\hat{y}(t))/dt \le 0$. In fact, taking the derivative of $V(\hat{y}(t))$ along the trajectory of (16) yields

$$\begin{aligned} & dV(\hat{y}(t))/dt \\ \le \; & (\alpha\ell/\mu_1)\hat{x}^\intercal I_n \hat{x} + (2\alpha - \beta)\,\hat{x}^\intercal\left[-k_1 M + k_2 D\right]\hat{x} \\ & + ((\alpha\mu_1/4 + 1 + \mu_2/4)\ell - \alpha)\,\hat{v}^\intercal I_n \hat{v} \\ & +(\ell/\mu_2 + \beta - \delta) z^\intercal I_n z + (\alpha\beta - \beta\delta + \alpha\delta)\hat{x}^\intercal I_n z \\ \le \; & \hat{x}^\intercal P_x \hat{x} + \hat{v}^\intercal P_v \hat{v} + z^\intercal P_z z, \end{aligned}$$

where

$$\begin{aligned} P_x &= \left[(\alpha\beta - \beta\delta + \alpha\delta)^2/(4\mu_3) + \alpha\ell/\mu_1\right] I_n \\ & \quad +(\beta - 2\alpha)\left[k_1 M - k_2 D\right], \\ P_v &= ((\alpha\mu_1/4 + 1 + \mu_2/4)\ell - \alpha)\, I_n, \\ P_z &= (\mu_3 + \ell/\mu_2 + \beta - \delta) I_n. \end{aligned}$$

In the above derivative, Young's inequality ($xy \le \mu x^2 + y^2/(4\mu)$, $\forall \mu > 0$, $x, y \in$ R) [17] is repeatedly used. From Lemma 2.2, one has $P_x < 0$ as $\beta - 2\alpha > 0$ and

$$\begin{aligned} \left[(\alpha\beta - \beta\delta + \alpha\delta)^2/(4\mu_3) + \alpha\ell/\mu_1\right]/(\beta - 2\alpha) < \rho, \\ k_1\lambda^* + \rho < 0, \; k_2 d^* > k_1\sigma^*(\rho/k_1). \end{aligned}$$

Also, it is easy to prove $P_v < 0$ and $P_z < 0$ by noting

$$\begin{gathered}(\alpha\mu_1/4 + 1 + \mu_2/4)\ell - \alpha < 0, \\ \mu_3 + \ell/\mu_2 + \beta - \delta < 0.\end{gathered}$$

Therefore, one has $dV(\hat{y}(t))/dt \le 0$. Finally, by using Lyapunov theorem, one has $\lim_{t\to\infty}\hat{v}_i(t) = 0$ and $\lim_{t\to\infty}\hat{x}_i(t) = 0$, which easily imply (12) and (13). The proof is thus complete.

Remark 3.2: In Theorem 3.1, the constants β^* and δ^* and the function $\rho^*(\beta, \delta)$ are defined by (11) for a given positive number R. In general, a larger R gives a larger ℓ which requires higher gains for the controller. In particular, if the nonlinearity $f(t, v_i)$ in the agent dynamics satisfies a global Lipschitz condition

$$\|f(t, v_1) - f(t, v_2)\| \le \ell\|v_1 - v_2\|, \; \forall v_1, v_2 \in \mathbb{R}^m \tag{19}$$

for a constant ℓ independent of R, i.e., the condition (10) holds for $R = +\infty$, then the controller developed in Theorem 3.1 achieves global velocity consensus. The moreover part of Theorem 3.1 is interesting for formation control of multi-agents in a certain circumstance when the initial formation is expected to be asymptotically maintained after the possible deformation during evolution. In other words, one can deliberately design the final formation for a group of agents by designing their initial distribution.

IV. A Numerical Example

We consider the multi-agent system (1) composed of $n = 10$ agents with network topology shown in Fig. 1. In particular, let the parameters $a_{ij} = 1$ or 0, depending on network topology, and $d_i = 1$ for $i = 1, \cdots, l$. For $l = 3$, Assumption 3.1 is satisfied with $\lambda^* = -0.47 < 0$

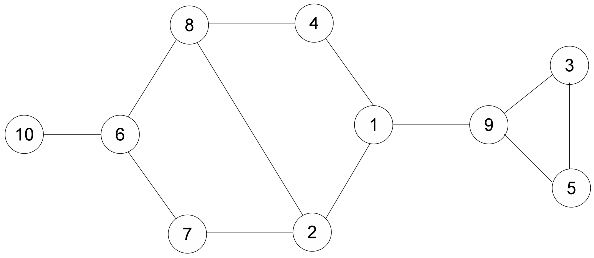

Fig. 1. A network of ten interacting agents.

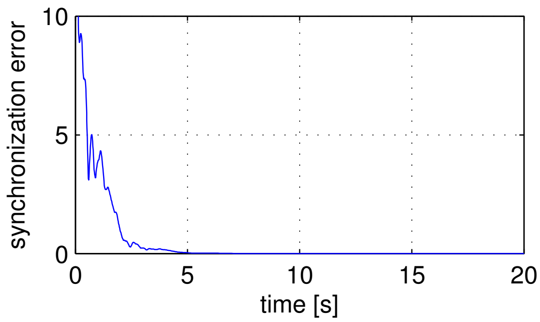

Fig. 2. The velocity synchronization error of the agents: $\sum_{k=1}^{10}\sum_{i=1}^{3}|v_{ki}(t)-v_{\gamma i}(t)|$.

and $d^* = 1 > 0$. The intrinsic vector field $f(t, v_i)$ in (1) is motivated from the Chua's circuit [18], i.e.,

$$f(t, v_i) = 0.1\begin{pmatrix} -v_{i1}+v_{i2}-s(v_{i1})v_{i1} \\ -v_{i2}+v_{i1}+v_{i3} \\ -v_{i2} \end{pmatrix} \tag{20}$$

where $v_i = [v_{i1}, v_{i2}, v_{i3}]^\intercal$ and $s(v) = -1.1 + 0.2|v|$ is a nonlinear function. The circuit gives a the chaotic attractor.

Next, we elaborate the parameter selection procedure in detail. First, we pick $R = 1.2$ and set $\|v_i(0)\| \le R$, $i = 1, \dots, 10$, and $\|v_\gamma(t)\| \le R$. A direct calculation shows that the inequality (10) holds for $\ell = 0.34$. Next, let $\mu_1 = 1.5 < 4/\ell$, $\mu_2 = 1.2$, $\mu_3 = 0.9$, and hence $\alpha = 0.6 > (1 + \mu_2/4)\ell/(1 - \mu_1\ell/4) = 0.51$. Then, pick $\beta = 2 > \beta^* = 2\alpha$ and

$$\begin{aligned} \delta &= 3.3 > \delta^* + \beta = \mu_3 + \ell/\mu_2 + \beta = 3.18, \\ \rho &= 4.7 > \rho^*(\beta, \delta) = 4.64. \end{aligned}$$

Finally, we choose the gains $k_1 = 12 > -\rho/\lambda^*$ and $k_2 = 50 > k_1 * \sigma^*(\rho/k_1)/d^*$ where $\sigma^*(\rho/k_1) = \sigma^*(0.39) = 3.72$.

The simulation result is illustrated in Fig. 2. It is observed that the velocities of the 11 agents quickly synchronize in about 5 seconds in terms of the synchronization error approaching zero. Therefore, the feasibility of Theorem 3.1 and the effectiveness of the developed multi-agent pinning controller (8)–(9) are verified.

V. Conclusion

The consensus problem has been solved for multi-agents of nonlinear dynamics in the leader-follower configuration. In particular, the velocity consensus is achieved while only the position information is available for feedback. It has been proved that the controller is able to deal with system nonlinearities when the region of attraction is semi-globally defined. The effectiveness of the design has been illustrated by numerical simulation on a group of agents whose dynamics are represented by Chua's circuits.

References

[1] R. Olfati-Saber. Flocking for multi-agent dynamics systems algorithms and theory. *IEEE Transactions on Automatic Control*, 51(3):401–420, 2006.
[2] X. F. Wang and G. R. Chen. Pinning control of scale-free dynamical networks. *Physica A*, 310:521–531, 2002.
[3] Q. Song, J. D. Cao, and W. Yu. Second-order leader-following consensus of nonlinear multi-agent systems via pinning control. *Systems and Control Letters*, 59:553–562, 2010.
[4] Y. G. Hong, J. P. Hu, and L. X. Gao. Tracking control for multi-agent consensus with an active leader and variable topology. *Automatica*, 42:1177–1182, 2006.
[5] F. Chen, Z. Q. Chen, L. Y. Xiang, Z. X. Liu, and Z. Z. Yuan. Reaching a consensus via pinning control. *Automatica*, 45:1215–1220, 2009.
[6] H.-T. Zhang, Z. Chao, and Z. Chen. A general alignment repulsion algorithm for flocking of multi-agent systems. *IEEE Trans. Automatic Control*, 56(2):430–435, 2011.
[7] Z. Chen and H.-T. Zhang. No-beacon collective circular motion of jointly connected multi-agents. *Automatica*, 47:1929–1937, 2011.
[8] T. Vicsek and A. Zafeiris. Collective motion. *Physcis Reports*, 517:71–140, 2012.
[9] W. Ren and E. Arkins. Distributed multi-vehicle coordinated control via local information exchange. *International Journal of Robust and Nonlinear Control*, 17(6):1002–1033, 2008.
[10] A. Abdessameud and A. Tayebi. On consensus algorithms for double-integrator dynamics without velocity measurements and with input constraints. *Systems and Control Letters*, 59:812–821, 2010.
[11] W. W. Yu, W. X. Zheng, G. Chen, W. Ren, and J. Cao. Second-order consensus in multi-agent dynamical systems with sampled position data. *Automatica*, 47(7):1496–1503, 2011.
[12] W. Ren. On consensus algorithms for double-integrator dynamics. *IEEE Transactions on Automatic Control*, 58(6):1503–1509, 2008.
[13] H. Su, X. F. Wang, and G. Chen. A connectivity-preserving flocking algorithm for multi-agent systems based only on position measurements. *International Journal of Control*, 82(7):1334–1343, 2009.
[14] W. W. Yu, G. Chen, M. Cao, and J. Kurths. Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics. *IEEE Transactions on Systems, Man and Cybernetics-B*, 40(3):881–891, 2010.
[15] H. Qing and M. H. Wassim. Distributed nonlinear control algorithms for network consensus. *Automatica*, 44:2375–2381, 2008.
[16] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan. *Linear matrix inequalities in system and control theory*. Philadelphia: SIAM, 1994.
[17] V. I. Arnold. *Mathematical Methods of Classical Mechanics (2nd Edition)*. Springer, 1989.
[18] T. Matsumoto, L. O. Chua, and M. Komuro. The double scroll. *IEEE Transactions on Circuits and Systems I- Regular paper*, 32(8):797–818, 1985.