Sliding Mode Controller Design for Spacecraft with Manipulator

Sung-mo Kang†, Yun-tae Kim†, and Hyo-sung Ahn†

Abstract— **In this paper, we consider a sliding mode controller for the spacecraft which has a manipulator. The developed controller controls the pose of spacecraft and the position of end-effector. It is assumed that the joint value, the position of end-effector and the pose of spacecraft can be measured, but there are uncertainties in the kinematic and dynamic parameters. The controller also estimates uncertain parameters. With the help of Lyapunov stability analysis, we show that the proposed control approach ensures stability. Simulation results are included to illustrate the theoretical results.**

I. INTRODUCTION

In recent years, the space development is a trend around the world. Many spacecrafts and satellites are launched and many astronauts are working in the space. The astronaut who is in the space does many things such as scientific experiments, repair the equipment, and so on. As works in the space increase, the astronaut is increasingly exposed to the danger. For safety of astronaut, it is necessary to replace the human by a robot. For this reason, more and more research on the space robot is conducted actively in recent days.

The space robot is a spacecraft which has the robot manipulators. The space robot can move like a spacecraft, but, because it has the manipulators, the space robot can perform to move the object, repair the equipment, assemble of parts, and so on.

The base of spacecraft can move freely because there is no constraint in the spacecraft. The spacecraft, however, responds freely to dynamic reaction forces that are produced by the manipulators motion because the manipulators are mounted at the base of space robot. Therefore, it is essential to consider the dynamic coupling between the spacecraft and manipulators. In order to analyze the dynamic coupling, we have to obtain the kinematics and the dynamic model of space robot [1].

If the manipulator of space robot has more degrees of freedom than dimensions of task space, then it is called kinematics redundant. In this case that since the system has redundant manipulators, there are several problems such as not invertible Jacobian matrix, and so on. To solve these problems, we use augmented Jacobian matrix [2].

The previous research considers only the attitude of rigid spacecraft (see, e.g., [3], [4]) or end-effector position of spacecraft with manipulator (see, e.g., [5]–[7]). Furthermore, the most of previous research considers the system which has no uncertainty (see, e.g., [9]–[12]). In this paper, we design sliding mode controller for both the position and the attitude of space robot. We use adaptive method for estimating the kinematic parameters.

The outline of this paper is as follows. In Section II, we show the background and preliminaries such as the kinematics and the dynamics of space robot. We propose the control law for the pose of space robot and the estimation of kinematic parameter in Section III. Our result through the simulations is shown in Section IV. The conclusion and the future works are described in Section VI.

II. BACKGROUND AND PRELIMINARIES

A. The kinematics of spacecraft with manipulator

The end-effector position $x_e \in R^3$ is expressed as,

$$x_e = h(q) \tag{1}$$

where h is generally a nonlinear transformation function which describes the relation between task space x_e and joint space $q = \begin{bmatrix} q_p^T & q_a^T & q_m^T \end{bmatrix}^T \in R^{6+n}$, $q_p = \begin{bmatrix} q_{px} & q_{py} & q_{pz} \end{bmatrix}^T \in R^3$ is the position of spacecraft, $q_a = \begin{bmatrix} q_{ax} & q_{ay} & q_{az} \end{bmatrix}^T \in R^3$ is the attitude of spacecraft, $q_m = \begin{bmatrix} q_{m1} & \cdots & q_{mn} \end{bmatrix}^T \in R^n$ is the joint angle of manipulator, and n is the number of joint.

The end-effector velocity $\dot{x}_e \in R^3$ is related to the Jacobian and the joint space velocity. It can be also expressed with a vector of kinematic parameters $a_{ek} = \begin{bmatrix} a_{ek1} & \cdots & a_{eki} \end{bmatrix}^T$ as,

$$\dot{x}_e = J_e(q)\dot{q} = J_p\dot{q}_p + Y_{ek}(q,\dot{q})\,a_{ek} \tag{2}$$

where $J_e(q) = \begin{bmatrix} J_p & J_a & J_m \end{bmatrix} = \partial h/\partial q \in R^{3\times(n+6)}$ is the Jacobian matrix, $J_p \in R^{3\times 3}$, $J_a \in R^{3\times 3}$, $J_m \in R^{3\times n}$, and $Y_k(q,\dot{q}) \in R^{3\times i}$ is called the kinematic regressors matrix.

If the manipulator of space robot has more degrees of freedom(DOF) than dimensions of task space, then it is called kinematics redundant. For example, in the three-dimensional task space, a space robot with four DOF manipulator or more DOF manipulator is redundant for achieving any end-effector position. Because the Jacobian matrix of redundant manipulator is not square, it can't be inversed. Therefore, we need to convert the position of end-effector into the generalized position. The generalized position $x \in R^n$ is defined as

$$x = \begin{bmatrix} x_e^T & \phi^T \end{bmatrix}^T \tag{3}$$

where $\phi = \begin{bmatrix} \phi_1 & \cdots & \phi_{n-3} \end{bmatrix}^T$ is task-related kinematic functions. From (3), the differential kinematic model is

†School of Mechatronics, Gwangju Institute of Science and Technology (GIST), 261 Cheomdan-gwagiro, Buk-gu, Gwangju, 500-712 Korea. E-mail: {smkang0521, yuntae1987, hyosung}@gist.ac.kr

978-1-4673-5769-2/13/$31.00 ©2013 IEEE

obtained as

$$\dot{x} = J(q)\,\dot{q} = J_p\dot{q}_p + Y_k(q,\dot{q})\,a_k \tag{4}$$

where

$$J(q) = \begin{bmatrix} J_e(q) \\ J_\phi(q) \end{bmatrix} = \begin{bmatrix} \partial x_e/\partial q \\ \partial\phi/\partial q \end{bmatrix} \tag{5}$$

is the $n\times n$ augmented Jacobian matrix. The ϕ is defined as a function which lets each of the rows and columns of Jacobian matrix become independent because the Jacobian matrix must satisfy the condition for invertible.

B. The dynamics of spacecraft with manipulator

The dynamic equation of n DOF space robot is expressed as,

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} = \tau \tag{6}$$

where $M(q) \in R^{(n+6)\times(n+6)}$ is the inertia matrix, $C(q,\dot{q}) \in R^{(n+6)\times(n+6)}$ is the centripetal and Coriolis matrix, $\tau = \begin{bmatrix} F_p^T & \tau_a^T & \tau_m^T \end{bmatrix}^T \in R^n$ is the external force and torque applied on spacecraft and joints of the manipulator.

In (6), the state of q is changed by the value of τ because τ is the external force and torque. Therefore, we consider τ as the control input. We rewrite the (6) as,

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}\ddot{q} + \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}\dot{q} = u \tag{7}$$

where $u = \begin{bmatrix} u_p^T & u_a^T & u_m^T \end{bmatrix}^T \in R^{n+6}$ is the control input.

Property 1: The inertia matrix $M(q)$ is positive definite and symmetric matrix.

Property 2: The $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric matrix.

Property 3: The pose of spacecraft is coupled with movement of the manipulator.

C. The uncertainty in the spacecraft with manipulator

We assume that there are uncertainties at the kinematic parameters and the dynamic parameters. Therefore, the estimated kinematics equation (4) and the estimated dynamics equation (6) are written as

$$\hat{\dot{x}} = \hat{J}(q)\,\dot{q} = J_p\dot{q}_p + Y_k(q,\dot{q})\,\hat{a}_k \tag{8}$$

$$\hat{M}(q)\,\ddot{q} + \hat{C}(q,\dot{q})\,\dot{q} = \tau \tag{9}$$

where $\hat{J}(q)$, $\hat{a}_k$, $\hat{M}(q)$ and $\hat{C}(q,\dot{q})$ are the estimates of $J(q)$, a_k, $M(q)$ and $C(q,\dot{q})$, respectively.

III. MAIN RESULTS

The pose of spacecraft and the generalized position of end-effector have to track the desired trajectory q_{pr}, q_{ar} and x_d, where $q_{pr} \in R^3$ is the desired value of spacecraft position, $q_{ar} \in R^3$ is the desired value of spacecraft attitude, and $x_d \in R^n$ is the desired value of the generalized end-effector position.

Let us define a task space reference velocity $\dot{x}_r \in R^n$ as

$$\dot{x}_r = \dot{x}_d - \alpha e \tag{10}$$

where α is a positive constant number and $e = x - x_d$ is the tracking error of the end-effector position. We define a joint space reference velocity $\dot{q}_{mr} \in R^n$ as,

$$\dot{q}_{mr} = \hat{J}_m^{-1}\left(\dot{x}_r - \hat{J}_p\dot{q}_p - \hat{J}_a\dot{q}_a\right) \tag{11}$$

where $\hat{J}_p$, $\hat{J}_a$ and $\hat{J}_m$ are the estimates of J_p, J_a and J_m, respectively.

Let us define the equation of state as

$$\begin{cases} z_1 = q - q_r \\ z_2 = \dot{q} - \dot{q}_r \end{cases} \tag{12}$$

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = M^{-1}(u - Cz_2 - C\dot{q}_r) - \ddot{q}_r \end{cases} \tag{13}$$

where $q_r = \begin{bmatrix} q_{pr}^T & q_{ar}^T & q_{mr}^T \end{bmatrix}^T \in R^{n+6}$ is the reference vector.

We define the sliding surface as

$$s = kz_1 + z_2 \tag{14}$$

Then,

$$\dot{s} = k\dot{z}_1 + \dot{z}_2 = kz_2 + M^{-1}(u - Cz_2 - C\dot{q}_r) - \ddot{q}_r \tag{15}$$

Now we propose the sliding mode control law,

$$u = -(A + \beta_0)\,\mathrm{sat}(s) + Y_d\hat{a}_{d_0} \tag{16}$$

where β_0 is positive arbitrary number, $Y_d \in R^{(n+6)\times j}$ is the dynamic regressor matrix and $\hat{a}_{d_0} \in R^j$ is the initial estimated value of dynamic parameter $a_d \in R^j$. Y_d, a_d and $\hat{a}_{d_0}$ are obtained from

$$M(-k\dot{z}_1 + \ddot{q}_r) + C(-kz_1 + \dot{q}_r) = Y_d a_d \tag{17}$$

$$\hat{M}(-k\dot{z}_1 + \ddot{q}_r) + \hat{C}(-kz_1 + \dot{q}_r) = Y_d\hat{a}_{d_0} \tag{18}$$

We assume that $Y_d\tilde{a}_d$ is bounded and A is the constant maximum value of $Y_d\tilde{a}_d$, $\tilde{a}_d = \hat{a}_d - a_d$.

The kinematic parameter estimate $\hat{a}_k$ is updated by

$$\dot{\hat{a}}_k = -\frac{1}{2}PY_k^T\left(\hat{\dot{x}} - \dot{x}\right) \tag{19}$$

where P is positive definite matrix and updated by

$$\frac{d}{dt}P^{-1} = -\lambda_0\left(1 - \|P\|/k_0\right) \tag{20}$$

λ_0 and k_0 are positive arbitrary number.

Let us define the Lyapunv function as

$$V = \frac{1}{2}s^T Ms + \frac{1}{2}\tilde{a}_k^T P^{-1}\tilde{a}_k \tag{21}$$

Then,

$$\dot{V} = s^T M\dot{s} + \frac{1}{2}s^T\dot{M}s + \tilde{a}_k^T P^{-1}\dot{\tilde{a}}_k + \frac{1}{2}\tilde{a}_k^T\left(\frac{d}{dt}P^{-1}\right)\tilde{a}_k \tag{22}$$

We would like to separate (22) into two parts for simplifying the calculation as

$$\dot{V}_1 \;=\; s^T M\dot{s} + \frac{1}{2}s^T\dot{M}s \tag{23}$$

$$\dot{V}_2 \;=\; \tilde{a}_k^T P^{-1}\dot{\tilde{a}}_k + \frac{1}{2}\tilde{a}_k^T\left(\frac{d}{dt}P^{-1}\right)\tilde{a}_k \tag{24}$$

From Property2, (15), (16) and (17),

$$\dot{V}_1 = s^T\left(-\left(A+\beta_0\right)\mathrm{sat}\left(s\right)+Y_d\tilde{a}_d\right) \leq -\beta_0\left|s\right| \quad (25)$$

From (4), (8), (19) and (20),

$$\dot{V}_2 = -\frac{1}{2}\left(\dot{\hat{x}}-\dot{x}\right)^T\left(\dot{\hat{x}}-\dot{x}\right)-\frac{1}{2}\lambda_0\left(1-\|P\|/k_0\right)\tilde{a}_k^T P^{-1}\tilde{a}_k \quad (26)$$

From (25) and (26),

$$\dot{V} = \dot{V}_1+\dot{V}_2 < 0 \quad (27)$$

Theorem 3.1: The sliding mode control law (16) and updating laws (19) for the spacecraft with manipulators (4) and (6) lead to the convergence of the space robot motion tracking errors. That is, $(q_p - q_{pr}) \to 0$, $(q_a - q_{ar}) \to 0$ and $(x - x_d) \to 0$ as $t \to \infty$. Further more, the kinematic parameter errors also converge to zero.

We use several lemma for proving Theorem 3.1. We would like to state lemmas.

Lemma 3.1: We consider the Lyapunov candidate function V which is positive definite function. If the derivative of the Lyapunov candidate function is negative definite function, then the system is stable and convergent. That is, the system is stable and convergent if $\dot{V} < 0$ where $V > 0$. This is called by Lyapunov stability.

Lemma 3.2: In (13) and (14), $\dot{z}_1 = -kz_1$ when $s = 0$. Therefore, if sliding surface s become to be zero, then the state of (13) converges to zero. That is, $z_1 \to 0$, $z_2 \to 0$ as $t \to \infty$.

In (27), the derivative of Lyapunov function is negative definite. Using Lemma 3.1, the sliding surface and the kinematic parameter error converge to zero, that is, $s \to 0$, $\tilde{a}_k \to 0$ as $t \to \infty$. And because of $s \to 0$, $(q - q_r)$ converges to zero according to Lemma 3.2. That is, the pose of spacecraft and the generalized position of end-effector track the desired trajectory. These prove Theorem 3.1.

IV. SIMULATION RESULTS

In this section, we examine the simulation result of the proposed controller. The simulation is performed about a spacecraft which has a three-DOF manipulator as shown in Fig.1. This system consist of a spacecraft and three links which are connected by revolute joint. The physical parameters of the space robot are listed in Table.I, where m_i and I_i (i =0, 1, 2 and 3) are the mass and the moment of inertia of the ith rigid body, respectively. l_i and r_i are shown as Fig.1.

TABLE I
THE PARAMETERS

Body	$m_i\,(kg)$	$I_i\,(kg \cdot m^2)$	$l_i\,(m)$	$r_i\,(m)$
0	40	6.667	0.5	0.5
1	4	0.333	0.5	0.5
2	4	0.333	0.5	0.6
3	4	0.333	0.5	0.6

In the simulation, the desired values of the pose of spacecraft and the position of end-effector are given in (28), (29), (30), where q_{pr}, q_{ar} and x_{ed} are the desired position of spacecraft, the desired attitude of spacecraft and the desired position of end-effector, respectively.

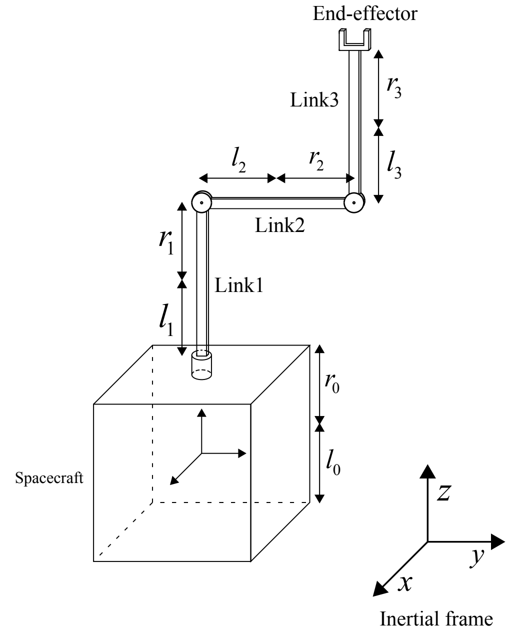

Fig. 1. A spacecraft with tree-DOF manipulator

$$q_{pr} = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \quad (28)$$

$$q_{ar} = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \quad (29)$$

$$x_{ed} = \begin{bmatrix} 0.6+0.1\sin(t) \\ 1.3+0.1\sin(t) \\ 2.6+0.1\sin(t) \end{bmatrix} \quad (30)$$

The initial state of the space robot is $q(0) = 0$ and the initial value of the end-effector position is $x_e(0) = \begin{bmatrix} 0 & 1 & 2.5 \end{bmatrix}^T$. All of the linear and angular velocities are zero because the space robot is initially at rest. The initial estimated values of kinematic parameters are defined as $\hat{a}_k(0) = \begin{bmatrix} 1.52 & 0.98 & 0.98 \end{bmatrix}^T$. In this system, the real kinematic parameters are $a_k = \begin{bmatrix} r_0+l_1+r_1 & l_2+r_2 & l_3+r_3 \end{bmatrix}^T = \begin{bmatrix} 1.5 & 1.1 & 1.1 \end{bmatrix}^T$ from Fig.1. And the gains of controller are defined as $k = 2$, $\alpha = 2$, $\beta_0 = 2$, $A = 50$.

The results of simulation are shown in Fig.2, Fig.3, Fig.4 and Fig.5. The estimates of kinematic parameters are shown in Fig.2 and it proves that the difference between estimated values and real values converges to zero. The position of end-effector converges to desired values, this is shown in Fig.3. The position and attitude of spacecraft also converge to desired values as we can see in Fig.4 and Fig.5, respectively.

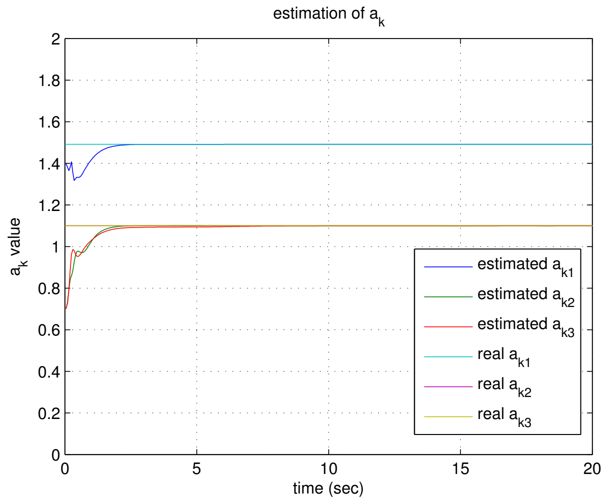

Fig. 2. Simulation Result(estimates of kinematic parameter)

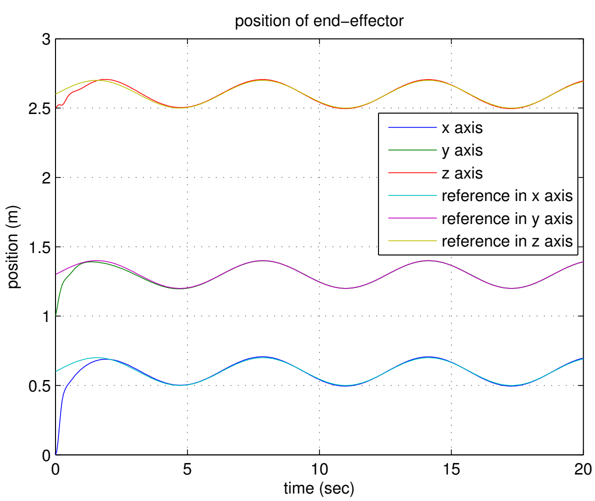

Fig. 3. Simulation Result(the position of end-effector)

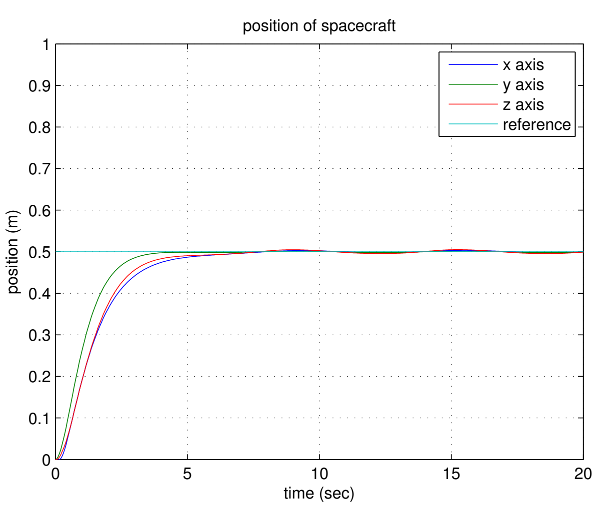

Fig. 4. Simulation Result(the position of spacecraft)

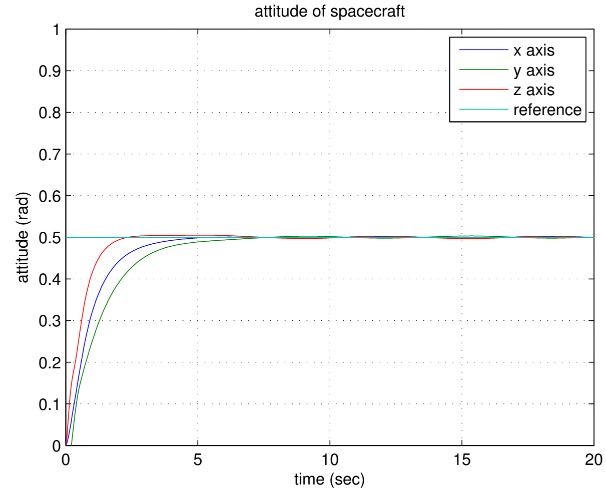

Fig. 5. Simulation Result(the attitude of spacecraft)

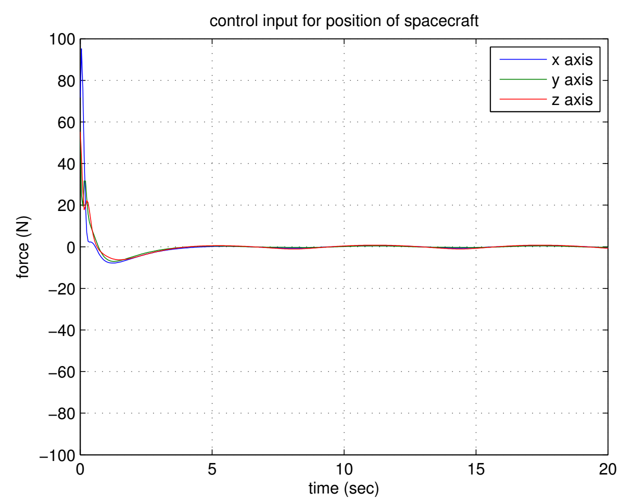

Fig. 6. Simulation Result(the control input for position of spacecraft)

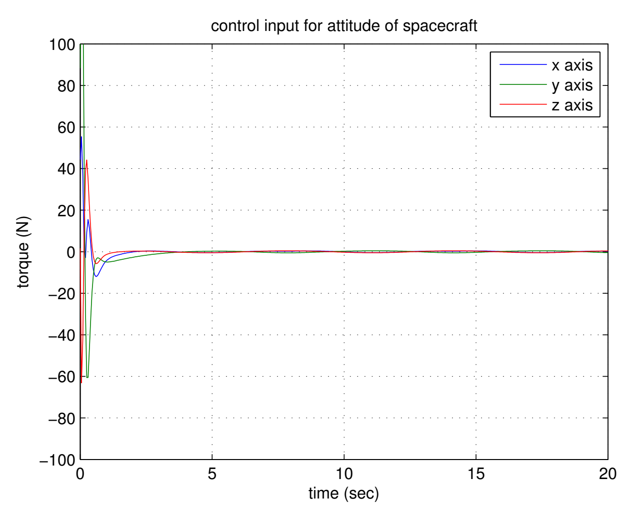

Fig. 7. Simulation Result(the control input for attitude of spacecraft)

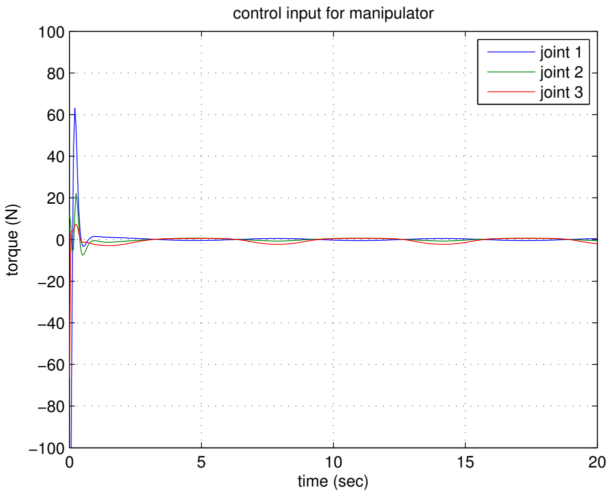

Fig. 8. Simulation Result(the control input for manipulator)

V. CONCLUSIONS AND FUTURE WORKS

In this paper, we have designed a robust controller for the space robot which is the spacecraft with manipulators. We specifically designed a controller for the position of end-effector and the pose of spacecraft by using the sliding mode control method. The controller also estimates the uncertain kinematic parameters by using the adaptive method. Using Lyapunov stability analysis, we showed that the tracking errors of pose and the errors of estimated kinematic parameters converge to zero. By performing the simulation, it is proven that the controller is working as designed.

VI. ACKNOWLEDGEMENT

The research of this paper was supported by Gwangju Institute of Science and Technology, and by the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology (No.2012-033597)

REFERENCES

[1] S. Ali A. Moosavian, “Dynamics and Control of Free-Flying Manipulators Capturing Space Objects”, *Faculty of Graduate Studies and Research*, 1996.

[2] H. Seraji, “Configuration Control of Redundant Manipulators: Theory and Implementation”, *IEEE Transactions on Robotics and Automation*, Vol. 5, No.4 1989.

[3] K. Lu, Y. Xia, M. Fu “Controller Design for Rigid Spacecraft Attitude Tracking with Actuator Saturation”, *Information Sciences*, Vol 220, 343-366, 2013.

[4] K. Lu, Y. Xia, Z. Zhu, M. V. Basin, “Sliding mode attitude tracking of rigid spacecraft with disturbances”, *Journal of The Franklin Institutes*, Vol.349, no.2 pp. 413-110, 2012.

[5] H. Wang, Y. Xie, “Stability Analysis of Adaptive Jacobian Controller for Free-Floating Space Manipulators”, *IEEE Conference on Decision and Control and Chinese Control Conference*, 2001.

[6] F. Bacconi, “Spacecraft Attitude Dynamics and Control”, *Anno Accademico*, 2005-2006.

[7] K. Yoshida, “Space Robot Dynamics and Control: To Orbit, From Orbit, and Future”, Robotics Research, The Ninth International Symposium, Eds, Hollerbach, J.M, and Koditschek, E.D, pp.449-456, Springer, 2000.

[8] A. Ibeas, M. de la Sen, “Robust Sliding Control of Robotic Manipulators Based on a Heuristic Modification of the Sliding Gain”, *Springer Science Business Media*, 2007.

[9] J. Y. S. Luh, M. W. Walker, and R. P. C. Paul, Resolved-acceleration control of mechanical manipulators, *IEEE Trans. Automat. Contr.*, vol. AC-25, no. 3, pp. 468-474, 1980.

[10] J. Y. S. Luh, M. W. Walker, and R. P. C. Paul, On-line computational scheme for mechanical manipulator, *J. Dynam. Syst., Meas., Contr.*, vol. 102, pp. 69-76, 1980.

[11] J. M. Hollerbach, A recursive Lagrangian formulation of manipulator dynamics and a comparative study of dynamics formulation complexity, *IEEE Trans. Syst., Man, Cybernetics*, vol. SMC-10, no. 11, pp. 730-736, 1980.

[12] J. J. Craig, *Introduction to Robotics: Mechanics and Control.* Third Edition, New York: Prentice Hall, 2005.