# Coordinated control of a four-area power system under structural perturbation

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Abstract—This paper discusses the procedure of coordinated controller design under system structural perturbation when new interconnections are adding into the overall system. With the concept of pair-wise decomposition, a group of adding matrices is defined to obtain the structure-changed transformation matrices used in permuted inclusion principle. By adjusting the former controllers structure corresponding to the information structure constraints variation, the new coordinated controller is capable of achieving high performance after this structural perturbation. A numerical simulation of automatic generation control (AGC) for a four-area power system under information structure constraints addition is provided by coordinated control.

#### I. Introduction

Coordinated control often serves as an ideal control strategy for the complex interconnected system with multi-overlapping parts, and it also has the ability to withstand the system structural perturbation. With the concept of pair-wise decomposition, coordinated control or overlapping decentralized control is widely applied, such as AGC for a four-area power system [1]–[4], decentralized LQG suboptimal longitudinal control of a platoon of automotive vehicles [4], [5], formation control of unmanned aerial vehicles [6], structural vibration control of tall buildings under seismic excitations [7], [8], speed and tension control in reversing cold-strip mill [9].

The results of [10], [11] indicate that real systems composed of a large number of interconnected dynamic elements will not always stay in one piece or hold the same dimension during long periods of time. With an interconnected system being decomposed into disjoint subsystems, the structural variation can be treated as deleting and adding interconnections among subsystems. When systems are subject to structural perturbations whereby subsystems are disconnected and again connected in various ways, the coordinated controller can present control performance in high quality as well.

This paper mainly pays attention on dealing with the situation when new connections are added in the overall system. In Section II, a pair of column and row group adding matrices is defined based on a brief introduction of pair-wise decomposition. Coordinated controller of the overall system is discussed in Section III, it have to be in the same structure with system matrix. In Section IV, numerical simulations in two different cases on adding a pair-wise subsystem and an individual subsystem are provided of a four-area power system (originated from [12], [13] in modeling), the robustness

characteristics are inherent in the control structure design. The last section gives conclusion of this paper.

#### II. ADDING MATRICES IN PAIR-WISE DECOMPOSITION

# A. Pair-wise decomposition

In a complex system, subsystems may share common parts remained to be decomposed. One of the general mathematical frameworks of decomposition is inclusion principle. Treating each pair of subsystem with information structure constraint as a basic unit of complex system interaction is the main feature of pair-wise decomposition. This decomposition can partition and arrange all pairs of subsystems of the system in a recurrent reverse order in the system expansion. Then a coordinated control can be applied, based on feedback controllers for each pair-wise subsystem carried over independently. Therefore, the coordinated control has the ability to adapt any information structure constraint of the system. Details on inclusion principle and pair-wise decomposition methodology can be found in [1], [4], [14], [15].

Consider an interconnected system S with N subsystems

$$\mathbf{S} = \{\mathbf{S}_i\},\$$

$$\mathbf{S}_i : \dot{x}_i = A_{ii}x_i + B_{ii}u_i + \sum_{\substack{j=1\\j \neq i}}^{N} A_{ij}x_j, \quad y_i = C_{ii}x_i, \quad (1)$$

$$i = 1, 2, \dots, N,$$

where  $x_i(t) \in \mathbf{R}^{n_i}$ ,  $u_i(t) \in \mathbf{R}^{m_i}$ ,  $y_i(t) \in \mathbf{R}^{l_i}$  are the state, input and output vectors of the subsystem  $\mathbf{S}_i$  at fixed time  $t \in \mathbf{R}$ . By choosing any pair of subsystems  $\mathbf{S}_i$  and  $\mathbf{S}_j$  which are connected from  $\mathbf{S}$ , call

$$\mathbf{S}_{ij}: \begin{cases} \dot{x}_{i} = A_{ii}x_{i} + A_{ij}x_{j} + B_{ii}u_{i}, & y_{i} = C_{ii}x_{i} \\ \dot{x}_{j} = A_{ji}x_{i} + A_{jj}x_{j} + B_{jj}u_{j}, & y_{j} = C_{jj}x_{j} \\ i = 1, 2, \dots, N, & i \neq j \end{cases}$$
(2)

a pair-wise subsystem. If each pair-wise subsystem is coordinated respectively, then the whole system can be coordinated so that it can achieve better performance.

System **S** in full network structure can be decomposed into expanded space of N(N-1)/2 pair-wise subsystems, and it is permuted in a reverse order [1]

$$\mathbf{S}_{ij}: \mathbf{S}_{12}, \mathbf{S}_{23}, \mathbf{S}_{13}, \mathbf{S}_{34}, \mathbf{S}_{24}, \mathbf{S}_{14}, \cdots, \\ \mathbf{S}_{(N-1),N}, \mathbf{S}_{(N-2),N}, \cdots, \mathbf{S}_{2N}, \mathbf{S}_{1N}.$$
 (3)

This non-natural order is convenient for representing the system information structure constraints variation, particularly when some subsystems are disconnected from or then reconnected to the overall system. A pair of permutation matrices composed of a series of basic permutation matrices is defined to transform pair-wise subsystems to this expected order. Permutation matrices of both the column and row group are provided in [1] as

$$P = \vec{\Pi}_{i=1}^{N-2} \vec{\Pi}_{j=1}^{N-i-1} \vec{\Pi}_{k=1+i(i-1)}^{N(N-j)-i(j+1)} p_{k(k+1)}$$

$$P^{-1} = \vec{\Pi}_{i=1}^{N-2} \vec{\Pi}_{j=1}^{N-i-1} \vec{\Pi}_{k=1+i(i-1)}^{N(N-j)-i(j+1)} p_{k(k+1)}^{T},$$

$$(4)$$

then the permuted matrices of expanded system  $\tilde{\mathbf{S}}_P$  can be obtained by

$$\begin{split} \tilde{A}_{P} &= V_{P}AU_{P} + M_{A}^{P} = P_{A}^{-1}VAUP_{A} + M_{A}^{P} \\ \tilde{B}_{P} &= V_{P}BQ_{P} + M_{B}^{P} = P_{A}^{-1}VBQP_{B} + M_{B}^{P} \\ \tilde{C}_{P} &= T_{P}CU_{P} + M_{C}^{P} = P_{C}^{-1}TCUP_{A} + M_{C}^{P}. \end{split}$$
 (5)

The system **S** is a typical restriction of the permuted system  $\tilde{\mathbf{S}}_P$ , if there is a triplet of full rank matrices  $(V_P, R_P, T_P)$  such that

$$\tilde{A}_P V_P = V_P A, \quad \tilde{B}_P R_P = V_P B, \quad \tilde{C}_P V_P = T_P C$$
 (6)

or

$$M_A^P V_P = 0, \quad M_B^P R_P = 0, \quad M_C^P V_P = 0.$$
 (7)

This restriction is one of the necessary and sufficient conditions for the permuted inclusion principle.

P is composed by some identity matrices like  $I_i$  and  $I_j$  with appropriate dimensions, whose positions in P represent the structure information of corresponding subsystem  $\mathbf{S}_{ij}$ . Suppose that system  $\mathbf{S}$  is in full network structure, then  $I_i$  and  $I_j$  are in ci and cj block column of P respectively as

$$ci = j(j-1) - 2(i-1) - 1$$

$$cj = j(j-1) - 2(i-1)$$

$$i = j - k, \quad j = 2, 3, \dots, N, \quad k = 1, 2, \dots, j-1.$$
(8)

Similarly, the corresponding relationship that  $I_i$  and  $I_j$  are in ri and rj block row of P respectively as

$$\begin{aligned} ri &= Ni - j + 1 \\ rj &= N(j-1) - i + 1 \\ i &= j - k, \quad j = 2, 3, \cdots, N, \quad k = 1, 2, \cdots, j - 1. \end{aligned} \tag{9}$$

In this way, the recurrent reverse order permutation matrix of system S can be constructed as

$$P((Ni-j+1), [j(j-1)-2(i-1)-1])_b = I_i P([N(j-1)-i+1], [j(j-1)-2(i-1)])_b = I_j,$$
(10)

the notation  $P(m,n)_b$  stands for the block position in P of identity matrix  $I_k$  corresponding to subsystem  $S_k$ .

# B. Adding Matrices under structural perturbation

Usually, the complex systems are subject to structural perturbations whereby subsystems, or groups of subsystems, are disconnected and then connected again during lifetime [10]. The deletion case for disconnected modes is described in literature [1], [4], a pair of deleting matrices is defined to delete the column and row groups corresponding to the disconnected parts in permutation matrices. This paper mainly focuses on the addition case, for both a pair-wise subsystem and an individual subsystem.

Taking a system matrix of a pair-wise subsystem  $S_{ij}$  as demonstration

$$A_{Dij} = \begin{bmatrix} A_{ii} & A_{ij} \\ A_{ji} & A_{jj} \end{bmatrix},$$

$$i = j - k, \quad j = 2, 3, \dots, N, \quad k = 1, 2, \dots, j - 1,$$
(11)

there are four basic interconnection modes of  $S_{ij}$ :

- (a) Full connection, a bidirectional information structure, if  $A_{ij} \neq 0, A_{ji} \neq 0$ ;
- (b) Half connection, a unidirectional information structure, if  $A_{ij} \neq 0, A_{ji} = 0$ ;
- (c) The same as (b), but if  $A_{ij} = 0, A_{ji} \neq 0$ ;
- (d) Disconnection, if  $A_{ij} = 0$ ,  $A_{ji} = 0$ .

Literature [11] shows that incorporate growth and preferential attachment are two key features of real networks. System growth is due to connecting a pair of disjointed subsystems  $S_{ij}$ into pair-wise, which means to set  $A_{ij} \neq 0$  or  $A_{ji} \neq 0$  or both from the viewpoint of matrix calculation. The modification of system matrix may have some effects on system inclusion, such as, the transformation matrices must be augmented to follow the increased dimension in expanded space, and so does the permutation matrix to reconstruct the expanded reverse order of pair-wise decomposition. A pair of adding matrices is defined to deal with the modification mentioned above. To add an information structure constraint of pair-wise subsystem means to copy the corresponding subsystems from the former expanded system. With the help of (8) & (9), those position data before and after the addition can be obtained. The afteraddition position data locates the position of subsystems to be added in permutation, and the before-addition one indicates the exact subsystems to be copied of former system. Eventually, the adding matrices are composed of those identify matrices located in proper positions. Since the addition of system information structure constraints involves reconstructing the interconnection between subsystems, it's necessary not only to add the relevant structure data of new interconnection into adding matrices, but also to complement the result into the expected form.

Suppose that a pair-wise subsystem  $S_{ij}$  is to be added to system S, and for description convenience, assume that S is in full network structure after this addition. In order to obtain the permutation and transformation matrices applied in pair-wise decomposition provided in (4) & (5), construct the column

group and row group adding matrices

$$I_{qa}^{c} = \begin{bmatrix} E_{1}^{1}, E_{2}^{2}, \cdots, E_{k}^{k}, \cdots, E_{sci}^{uci}, E_{scj}^{ucj}, \cdots, E_{N(N-1)-2}^{N(N-1)} \end{bmatrix}$$

$$I_{qa}^{r} = \begin{bmatrix} (E_{1}^{1})^{T}, (E_{2}^{2})^{T}, \cdots, (E_{sri}^{uri})^{T}, \cdots, (E_{k-1}^{k})^{T}, \cdots, (E_{srj}^{urj})^{T}, \cdots, (E_{N(N-1)-2}^{N(N-1)})^{T} \end{bmatrix}^{T},$$

$$(12)$$

so that

$$P_{Aqa} = I_{qa}^r P_A I_{qa}^c + M_P, \ V_{qa} = I_{qa}^r V, \ U_{qa} = U I_{qa}^{rT} I_{U_{qa}}.$$
(13)

In (12), set a standard basic block column matrix  $E_a^b = (0, \cdots, 0, I_k, 0, \cdots, 0)^T$  to indicate that the position of corresponding identity matrix  $I_k$  in  $I_{qa}^c$  is  $(a,b)_b$ , and  $(E_a^b)^T$  indicates the position  $(b,a)_b$  in  $I_{qa}^r$  with block dimensions considered, respectively. These adding matrices are defined under the concept of pair-wise decomposition, with the position data of each subsystem in the permutation inside. As a result, the superscripts and subscripts of E can be determined as below: superscripts in  $E_{sc}^{uc}$  for column group according to  $\mathbf{S}_{ij}$ 

$$uci = j(j-1) - 2(i-1) - 1$$
  
 $ucj = j(j-1) - 2(i-1);$  (14)

subscripts in  $E_{sc}^{uc}$  for column group according to  $S_{ij}$ 

$$sci = i(i-3) + 4$$

$$scj = \begin{cases} j(j-3) + 6, & j-i=1\\ j(j-3) + 4, & j-i>1, \end{cases}$$
(15)

specially, due to the permutation procedure, there are

$$\mathbf{S}_{12}: \left\{ \begin{array}{l} sci = N(N-1) - 3 \\ scj = N(N-1) - 5 \\ sci = N(N-1) - 3 \\ scj = k(k-3) + 4 \end{array} \right. \tag{16}$$

$$\mathbf{S}_{1N}: \left\{ \begin{array}{l} sci = (N-1)(N-2) - 1 \\ scj = N(N-3) + 4; \end{array} \right.$$

superscripts in  $(E_{sr}^{ur})^T$  for row group according to  $\mathbf{S}_{ij}$ 

$$uri = Ni - j + 1$$
  
 $urj = N(j - 1) - i + 1;$  (17)

subscripts in  $(E_{sr}^{ur})^T$  for row group according to  $\mathbf{S}_{ij}$ 

$$sri = (N-1)(i-1) + 1$$
  

$$srj = (N-1)(i-1).$$
(18)

A coefficient complementary matrix of  $U_{qa}$ 

$$I_{U_{qa}} = blockdiag \left[ \overbrace{I_{1}, I_{1}, \cdots, I_{1}}^{N}, \overbrace{I_{2}, I_{2}, \cdots, I_{2}}^{N}, \left( \frac{N-2}{N-1} \right) \left( \overbrace{I_{i}, I_{i}, \cdots, I_{i}}^{N} \right), \cdots, \left( \frac{N-2}{N-1} \right) \left( \overbrace{I_{j}, I_{j}, \cdots, I_{j}}^{N} \right), \cdots, \right]$$

$$\left( \frac{N-2}{N-1} \right) \left( \overbrace{I_{j}, I_{j}, \cdots, I_{j}}^{N} \right), \cdots,$$

$$\left( \frac{N-2}{N-1} \right) \left( \overbrace{I_{j}, I_{j}, \cdots, I_{j}}^{N} \right), \cdots,$$

$$\left( \frac{N-2}{N-1} \right) \left( \overbrace{I_{j}, I_{j}, \cdots, I_{j}}^{N} \right), \cdots,$$

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$$\left( \frac{N-2}{N-1} \right) \left( \overbrace{I_{j}, I_{j}, \cdots, I_{j}}^{N} \right), \cdots,$$

$$\left( \frac{N-2}{N-1} \right) \left( \overbrace{I_{j}, I_{j}, \cdots, I_{j}}^{N} \right), \cdots,$$

is constructed to satisfy the inclusion conditions, where  $I_k$  is identity matrix of subscript dimension. Moreover, use a complementary matrix  $M_P$  to complement the deviation between calculated result and expectation, there are two identify matrix blocks to be complemented for each  $\mathbf{S}_k$ 

$$\begin{cases} M_{P}(sri, sci)_{b} = -I_{i}, i = 1, j = N \\ M_{P}(sri, uci)_{b} = -I_{i}, i = 1, j \neq N \\ M_{P}(\max(uri, sri + 1), N(N - 1) - 1)_{b} = -I_{i}, i = 1 \\ M_{P}(\max(uri, sri + 1), sci)_{b} = -I_{i}, i \neq 1 \\ M_{P}(srj + 1, \max(ucj, scj + 2))_{b} = -I_{j} \\ M_{P}(\max(urj, srj + 2), \min(ucj, scj))_{b} = -I_{j} \end{cases}$$
(20)

while the other parts of  $M_P$  are zero block matrices.

The above addition procedure is also fit for  $\tilde{B}_P \Rightarrow \tilde{B}_{Pqd}$  and  $\tilde{C}_P \Rightarrow \tilde{C}_{Pqd}$  to calculate  $P_{Bqa}$ , R, Q and  $P_{Cqa}$ , T, S, and it can be extended to addition of arbitrary reconnected pair-wise subsystems.

The same method can be expanded on adding an individual new subsystem to the overall system. Assume that every subsystem of the whole will connect to this new one, then there are a series of pair-wise subsystems  $\mathbf{S}_{N,(N+1)}, \mathbf{S}_{(N-1),(N+1)}, \cdots, \mathbf{S}_{1,(N+1)}$  to be added. Their relevant positions in permutation are identified by

$$\begin{split} &P\left([(N+1)(k-1)+1],[(N+1)N-2(k-1)-1]\right)_b = I_k \\ &P\left([(N+1)N-k+1],[(N+1)N-2(k-1)]\right)_b = I_{N+1}, \\ &(21) \end{split}$$

where  $k=1,2,\cdots,N$ . Besides, the new subsystem matrices have larger orders than former ones, so the permutation and transformation matrices should be augmented to contain the identity matrix of subsystem  $\mathbf{S}_{N+1}$  as

$$P_{a} = \begin{bmatrix} P_{N} & 0 \\ 0 & I_{N+1} \end{bmatrix}, V_{a} = \begin{bmatrix} V_{N} & 0 \\ 0 & I_{N+1} \end{bmatrix}, U_{a} = \begin{bmatrix} U_{N} & 0 \\ 0 & I_{N+1} \end{bmatrix}.$$
(22)

# III. COORDINATED CONTROLLER

The normal centralized control methodologies usually treat the system model as a whole. When the information structure constraints varied, it is necessary to execute the controller design procedure again. One feature of coordinated controller is the ability to use interconnections of a system adequately. Its control law has the same structure of system state, so that coordinated control can adapt the structural perturbation by following the system structure.

Theoretically, any existing control technique can be applied to the coordinated control of pair-wise subsystem  $S_{ij}$ , and LQ control is imposed here as an easy paradigm in designing. The control gain can be calculated with (11) as

$$\tilde{K}_{Dij} = \begin{bmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{bmatrix}, \ \tilde{K}_D = (\tilde{K}_{Dij}), \ \tilde{K}_P = \tilde{K}_D + \Delta \tilde{K}.$$
(23)

According to inclusion principle, the coordinated controller of system S is contracted from the expanded space by satisfying

$$\tilde{K}_P V_P = R_P K. \tag{24}$$

This control gain matrix K in (24) has the same structure as system state matrix A. When the information structure constraints varied, use deleting matrices (introduced in [1], [4]) or adding matrices to adjust the permutation and transformation matrices for pair-wise decomposition, so that K can follow A's structural perturbation. That is to say, if pair-wise subsystem  $\mathbf{S}_{ij}$  is disconnected then set  $K_{ij}$  and  $K_{ji}$  equal to zero, or recover their former value when  $\mathbf{S}_{ij}$  is connected again.

Obviously,  $\tilde{K}_P$  may also have the same structure with the expanded system state matrix  $\tilde{A}_P$ . In (21), as  $\tilde{K}_D$  is actually formed by each pair-wise control gain  $\tilde{K}_{D_{ij}}$  arranged in reverse order diagonally,  $\Delta \tilde{K}$  is established as the coordinated compensator. Details can be found in [1], [4]. However, by treating this structural offset as uncertain disturbance [16],  $\Delta \tilde{K}$  can be ignored in some circumstance. Then this control gain  $K_d$  can be simplified as diagonal block controller like

$$K_d = Q_P \tilde{K}_D V_P. (25)$$

Though the expanded control gain may mismatch  $\tilde{A}_P$ 's structure in this way, both the control effect with or without  $\Delta \tilde{K}$  are usually similar. In another point of view, it may suppose the point that coordinated control is robust on the perturbation of system structure.

# IV. NUMERICAL SIMULATION ON AGC

Take a four-area power system as example, and apply automatic generation control (AGC) on it to recover frequencies and tie-line power exchanges in each area when one of the areas is disconnected from and then reconnected to others. Figure 1 shows the power system discussed in detail by [1]–[4], [12], [13], which are interconnected by tie-lines indicated by solid lines. Its assumed that area 1, 2 and 3 contain three reheat turbine type thermal units and area 4 contains a hydro unit, respectively. In these simulations the control gain matrices  $K_{ij}$ 's are calculated by LQ control scheme.

The main purpose of simulations in [1]–[4] is to illustrate the effect of coordinated control. Even with information structure constraints varied, that is one pair-wise subsystem or one individual subsystem is disjoint from the overall system, the coordinated controller has the ability to achieve high performance in stabilizing the system against load disturbance. And the response curves are quite similar with those obtained by the centralized optimal control. This paper here pays more attention to the dynamic responding outputs of system before and after the structural perturbation, and as an opposite procedure of former works, it considers the adding case. When the system structure changed, a group of adding matrices is constructed, so that the coordinated controller adjusted correspondingly to keep the same information structure constraints of the system.

Consider a step load increasing of  $\xi_1 = 0.01p.u$ . at area 1 in the power system. First, assume the area 1 and 2 are disjointed, which means the four areas are presented as a longitudinal structure in the order 4-1-3-2. Connect these two areas at t = 0.01p.u.

100s, and construct the adding matrices as

$$I_{qa}^{c} = \begin{pmatrix} 0 & I_{2} & I_{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{3} & 0 & 0 \\ I_{1} & 0 & 0 & 0 & 0 & 0 & I_{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{4} \end{pmatrix}, \tag{26}$$

$$I_{qa}^{r} = \begin{pmatrix} I_{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{1} & 0 & 0 & 0 & 0 \\ I_{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{2} & 0 & 0 & 0 \\ 0 & 0 & I_{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{4} \end{pmatrix} . \tag{27}$$

It can be found that the deviation of frequency  $f_i$  and tieline power  $P_{tie_i}$  are almost steady in Figure 2. The curves show that coordinated control has the ability to adapt the system information structure constraints variation effectively. Unlike the centralized optimal control discussed in [1], [3], [4], which needs to re-calculate control gain matrix each time the system structure changed, the decentralized structure of coordinated controller makes it possible to follow the information structure constraints variation by just adjusting its own structure correspondingly.

On another side, consider a positive step load perturbation in area 1 and a disturbance with the opposite sign in area 2 [13], that is  $\xi_1 = 0.01p.u.$ ,  $\xi_2 = -0.01p.u.$ . Assume that area 4 as an individual subsystem stays out of the system, which means area 1, 2 and 3 forms a circle structure and in system state matrix there stand  $A_{14} = A_{41} = 0_{6\times 6}$ , then at t = 100s connect area 1 and 4, so that area 4 joins the entire system, construct the adding matrices as

$$I_{qa}^{r} = \begin{bmatrix} I_{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I_{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_{4} \end{bmatrix}.$$
 (29)

The response curves are shown in Figure 3. Notice that the load perturbation influences aren't very strong in area 1, 2 and 3, except area 4 keeping damped oscillation until a period of time elapsed. Because area 4 is new to the power system, its obvious that area 4 needs to exchange information with other

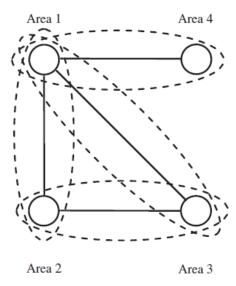


Fig. 1. Schematic diagram of the power system with four areas

areas, and its states need to be regulated to follow the entire system. Area 1, 2 and 3 remain the main part in the power system all time, which explains the steady response curves of theirs in Figure 3. Notice that the curves of area 4 after addition is somehow steadier than those of the same area from former works in fully connected case. The reason is, before adding area 4, the power system is already stabilized, and its much easier for the controller to stabilize one part than the whole system. This should be considered to support the point of view that the whole network may provide robustness to its component subsystems under disturbance. Figure 4 shows the curves in this case without coordinated compensator. Theres no significant difference whether the coordinated controller is compensated or not, and this result supports the structure-based robustness of this pair-wise coordinated control.

# V. CONCLUSION

In this paper, a pair of column and row group adding matrices is defined to add interconnections into a complex system, under the concept of pair-wise decomposition. In this way, a coordinated control procedure is applied. By using the adding matrices to calculate the permutation and transformation matrices after structural perturbation, the coordinated control gain matrix can be contracted from the expanded space with the former gain matrix. Simulations for AGC to a four-area power system are provided, and the results indicate that the coordinated control are effective to stabilize the interconnected system by adjusting the controller structure varied with the information structure constraints variation of system. Further research can be focused on the mechanism how network structure affects the system performance, and how to provide intelligence to the subsystems in building proper connections.

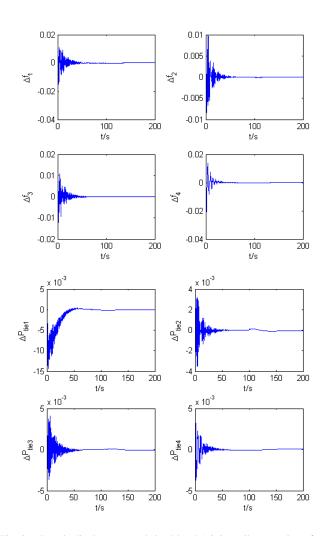


Fig. 2. Longitudinal structure chained by 4-1-3-2, until connection of area 1 and 2 is added

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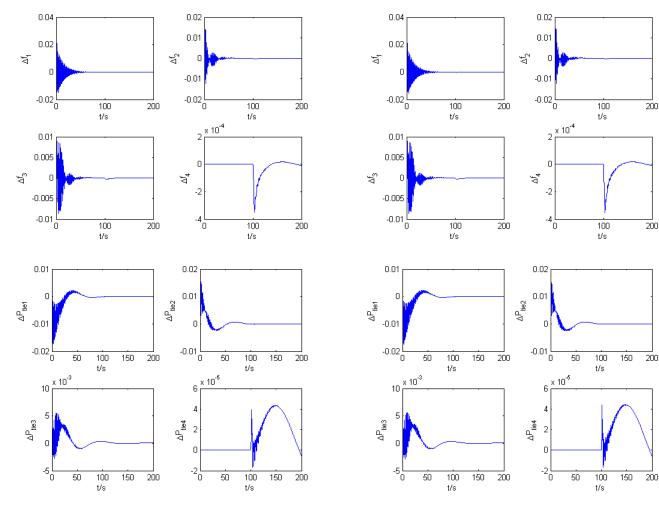


Fig. 3. Circle structure formed by area 1, 2, 3, then area 4 is added to the entire system

Fig. 4. The case of Figure 3 without coordinated compensator

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