Second-order Leader-following Consensus of Multi-agent Systems with Nonlinear Dynamics and Time Delay via Periodically Intermittent Pinning Control

Xiaoling Wang Bo Liu Housheng Su Xiaofan Wang

Abstract— **This paper investigates the second-order leader-following consensus of multi-agent systems with nonlinear dynamics and time delay by virtue of the periodically intermittent pinning control. All member agents and the virtual leader share the same nonlinear dynamics related to both the position information and the velocity information. Based on Lyapunov stability theory, some useful criteria are obtained to drive all the agents to achieve consensus. Finally, a numerical example is presented to illustrate the theoretical results.**

Keywords: **Second-order consensus, multi-agent systems, time delay, pinning control, intermittent control.**

I. Introduction

Over the past years, consensus of multi-agent systems has received great attention in variety of fields, such as biology, computer science and control engineering. Recently, many effective techniques have been devoted to make multi-agent systems achieve consensus, including feedback control [1], impulsive control [2], adaptive control [3], intermittent control [4]–[9] and pinning control [10]. In particular, the intermittent control is paid more attention and many excellent works have been presented because this discontinuous control widely exists in reality. In [4], [5], the authors investigated the stabilization of nonlinear systems and the synchronization of chaotic systems respectively by introducing the periodically intermittent control method. However, if the size of the network is very large, it is too difficult and costly to control all the nodes to achieve the objective. Therefore, the synchronization of complex dynamical networks are studied by techniques of the pinning control and the periodically intermittent control. In [6], Liu and Chen researched the cluster synchronization of linearly coupled networks without time delay by adding some intermittent pinning controllers, while references [7]–[9] discussed the synchronization of delayed networks via periodically intermittent pinning control with diverse methods.

Second-order consensus of multi-agent systems requiring all agents move with the same velocity and converge to the same destination has been widely researched [3], [11]–[16], especially the second-order multi-agent systems are governed by nonlinear dynamics such as [3], [11], [12]. In [12], the authors studied the second-order consensus of multi-agent dynamical systems with nonlinear dynamics by defining the generalized algebraic connectivity. References [13] and [14] considered the influence of time delays and investigated the second-order consensus of multi-agent systems via Lyapunov-Krasovskii functional method on general fixed directed topology and jointly-connected topologies, respectively. In [15], the authors introduced a novel algorithm to investigate the flocking of multi-agent systems with intermittent nonlinear velocity measurements, while [16] researched the second-order consensus of multi-agent system with directed topology and intermittent feedback control. In this paper, we will extend the results in [16] to the dynamical networks with nonlinear dynamics and time delay. Furthermore, the nonlinear intrinsic dynamics is related to the position and the delayed position, the velocity and the delayed velocity information, so that the dynamics is universal and rational.

In addition, the leader-following approach is employed diffusely in first-order dynamics [7], [9] and second-order dynamics [3], [11] and [17]–[19]. In this paper, we also adopt this method in order to make all agents reach the desired destination. Based on Lyapunov stability theory, we investigate the second-order leader-following consensus of multi-agent systems with nonlinear dynamics and time delay by using periodically intermittent pinning control, and gain some simple convergence conditions in the form of linear matrix inequalities (LMI).

The remainder of this paper is arranged as follows: Section II states the model we will study and some mathematical preliminaries. Section III establishes the main results on the second-order leader-following consensus. Section IV presents the simulation about the main results. Section V draws conclusions to this paper.

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Notation: Throughout this paper, A^T means the transpose of the matrix A, $\|.\|$ is the Euclidean norm and $\otimes$ represents the Kronecker product. $F(t,x(t),x(t-\tau),v(t),v(t-\tau))=(f(t,x_1(t),x_1(t-\tau),v_1(t),v_1(t-\tau)),\cdots,f(t,x_N(t),x_N(t-\tau),v_N(t),v_N(t-\tau)))^T$, and $1_N\in R^{N\times 1}$ is a vector with each entry being 1.

II. Preliminaries and model formulation

A. Model formulation

Consider a multi-agent system consisting of N agents and each agent updates itself according to the following dynamics:

$$\begin{cases} \dot{x}_i(t)=v_i(t) \\ \dot{v}_i(t)=f(t,x_i(t),x_i(t-\tau),v_i(t),v_i(t-\tau)) \\ \qquad +c\sum_{j\in N_i}a_{ij}(x_j(t)-x_i(t)) \\ \qquad +c\sum_{j\in N_i}a_{ij}(v_j(t)-v_i(t)) \\ \qquad +u_i(t), \quad i=1,\cdots,N, \end{cases} \tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\cdots,x_{in}(t))^T\in R^n$ $(i=1,2,\ldots,N)$ describes the position vector of agent i and $v_i(t)=(v_{i1}(t),v_{i2}(t),\cdots,v_{in}(t))^T\in R^n$ $(i=1,2,\ldots,N)$ is its velocity vector, $t\in[0,+\infty)$. $f:R^n\longrightarrow R^n$ is a continuous nonlinear function and describes the intrinsic dynamics of agents i, τ is the time delay. N_i is the neighboring set of agent i, constant c is the coupling strength of the position and the velocity. Matrix $A=(a_{ij})\in R^{N\times N}$ with $a_{ii}=-\sum_{j=1,j\neq i}^{N}a_{ij}$ depicts the topology of the system, if agent i can receive information from agent j, then $a_{ij}=a_{ji}=1$; otherwise, $a_{ij}=0$. The corresponding Laplacian matrix is $L=(l_{ij})_{N\times N}=-A$ (obviously, $L=L^T$).

Remark 1: Differing from the second-order algorithm with periodically intermittent pinning control in [16], algorithm (1) involves time delay, nonlinear dynamics as well as the virtual leader. Note also that all agents and the virtual leader share the same nonlinear dynamics which is related to the position and the delayed position, the velocity and the delayed velocity, respectively.

The virtual leader of system (1) is regarded as an external input described as:

$$\begin{cases} \dot{x}_0(t)=v_0(t) \\ \dot{v}_0(t)=f(t,x_0(t),x_0(t-\tau),v_0(t),v_0(t-\tau)), \end{cases} \tag{2}$$

where x_0 and v_0 are the position and the velocity of the virtual leader, respectively.

Aiming at the leader-following consensus, some periodically intermittent controllers are added to a small fraction of agents and designed as follows. Without loss of generality, assume that the first $l(1\leq l<N)$ agents can be pinning controlled, then

$$u_i(t)=-d(t)(x_i(t)-x_0(t))-d(t)(v_i(t)-v_0(t)),i=1,2,\cdots,N, \tag{3}$$

where

$$d(t)=\begin{cases} d_i,mT\leq t<mT+h, \quad 1\leq i<l, \\ 0,mT+h\leq t<(m+1)T, \quad l+1\leq i\leq N, \end{cases} \tag{4}$$

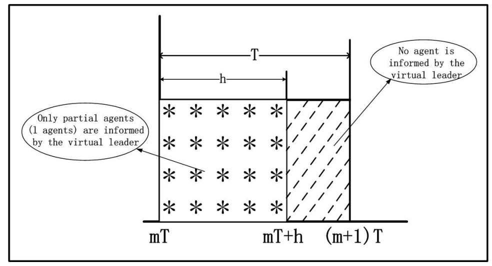

Fig. 1. The work schedule of the controllers during one period.

where d_i is the feedback gain, $T>0$ is the control period, $h>0$ is the control width and $m=0,1,2,\cdots$. These new controllers are explained as Figure 1.

Let $\tilde{x}_i(t)=x_i(t)-x_0(t)$ and $\tilde{v}_i(t)=v_i(t)-v_0(t)$. Then, the error dynamical network can be rewritten as:

$$\begin{cases} \dot{\tilde{x}}_i(t)=\tilde{v}_i(t), \quad t\geq 0, \quad 1\leq i\leq N, \\ \dot{\tilde{v}}_i(t)=\begin{cases} f(t,x_i(t),x_i(t-\tau),v_i(t),v_i(t-\tau)) \\ -f(t,x_0(t),x_0(t-\tau),v_0(t),v_0(t-\tau)) \\ +c\sum_{j\in N_i}a_{ij}(\tilde{x}_j(t)-\tilde{x}_i(t))+c\sum_{j\in N_i}a_{ij}(\tilde{v}_j(t)-\tilde{v}_i(t)) \\ -d_i(\tilde{x}_i(t)+\tilde{v}_i(t)), \quad mT\leq t<mT+h, \ 1\leq i\leq l; \\ f(t,x_i(t),x_i(t-\tau),v_i(t),v_i(t-\tau)) \\ -f(t,x_0(t),x_0(t-\tau),v_0(t),v_0(t-\tau)) \\ +c\sum_{j\in N_i}a_{ij}(\tilde{x}_j(t)-\tilde{x}_i(t))+c\sum_{j\in N_i}a_{ij}(\tilde{v}_j(t)-\tilde{v}_i(t)), \\ \qquad mT\leq t<mT+h, \ l+1\leq i\leq N; \\ f(t,x_i(t),x_i(t-\tau),v_i(t),v_i(t-\tau)) \\ -f(t,x_0(t),x_0(t-\tau),v_0(t),v_0(t-\tau)) \\ +c\sum_{j\in N_i}a_{ij}(\tilde{x}_j(t)-\tilde{x}_i(t))+c\sum_{j\in N_i}a_{ij}(\tilde{v}_j(t)-\tilde{v}_i(t)), \\ \qquad mT+h\leq t<(m+1)T, \ 1\leq i\leq N. \end{cases} \end{cases} \tag{5}$$

Let $D=diag\{d_1,d_2,\cdots,d_l,0,0,\cdots,0\}$, then (5) can be changed into the matrix form:

$$\begin{cases} \dot{\tilde{x}}(t)=\tilde{v}(t), \quad t\geq 0, \quad 1\leq i\leq N, \\ \dot{\tilde{v}}(t)=\begin{cases} F(t,x(t),x(t-\tau),v(t),v(t-\tau)) \\ -1_N\otimes f(t,x_0(t),x_0(t-\tau),v_0(t),v_0(t-\tau)) \\ -(cL+D)\tilde{x}(t)-(cL+D)\tilde{v}(t),mT\leq t<mT+h, \\ F(t,x(t),x(t-\tau),v(t),v(t-\tau)) \\ -1_N\otimes f(t,x_0(t),x_0(t-\tau),v_0(t),v_0(t-\tau)) \\ -cL\tilde{x}(t)-cL\tilde{v}(t),mT+h\leq t<(m+1)T. \end{cases} \end{cases} \tag{6}$$

B. Mathematical preliminaries

Next, we will introduce the following Lemmas and results.

Assumption 1: For the nonlinear function in (1), there exist four positive constant m_1, m_2, m_3 and m_4, such that

$$\begin{aligned} &\|F(t,x(t),x(t-\tau),v(t),v(t-\tau))-1_N\otimes f(t,x_0(t),x_0(t-\tau),v_0(t),v_0(t-\tau))\| \\ &\leq m_1\|x(t)-1_N\otimes x_0(t)\| \\ &+m_2\|v(t)-1_N\otimes v_0(t)\| \\ &+m_3\|x(t-\tau)-1_N\otimes x_0(t-\tau)\| \\ &+m_4\|v(t-\tau)-1_N\otimes v_0(t-\tau)\|, \quad \forall x,v\in R^N. \end{aligned}$$

Remark 2: Notice that Assumption 1 is a Lipschitz-like condition so that it is easy to find certain functions satisfying this condition such as the derivable functions.

Lemma 1: [11] (Schur complement) The following linear matrix inequality (LMI):

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

where $Q(x) = Q^T(x), R(x) = R^T(x)$, is equivalent to one of the following conditions:

$$(i): Q(x) > 0, R(x) - S^T(x)Q^{-1}(x)S(x) > 0;$$
$$(ii): R(x) > 0, Q(x) - S(x)R^{-1}(x)S^T(x) > 0.$$

Lemma 2: [5] For any vectors $x, y \in R^n$ and positive definite matrix $G \in R^{n\times n}$, the following matrix inequality holds:

$$2x^T y \leq x^T Gx + y^T G^{-1}y.$$

Lemma 3: [13] Suppose that a and b are vectors, then for any positive-definite matrix E, the following inequality holds:

$$-2a^T b \leq \inf_{E>0}\{a^T Ea + b^T E^{-1}b\}.$$

Lemma 4: [7] Let $\omega : [\mu - \tau, \infty] \to [0, \infty)$ be a continuous function such that

$$\dot{\omega}(t) \leq -a\omega(t) + b\ \max \omega_t$$

is satisfied for $t \geq \mu$. If $a > b > 0$, then

$$\omega(t) \leq [\max \omega_\mu] \exp\{-\gamma(t-\mu)\}, t \geq \mu,$$

where $\max \omega_t = \sup_{t-\tau\leq\theta\leq t} \omega(\theta)$, and $\gamma > 0$ is the smallest real root of the equation

$$a - \gamma - b\ \exp\{\gamma\tau\} = 0.$$

Lemma 5: [7] Let $\omega : [\mu - \tau, \infty] \to [0, \infty)$ be a continuous function such that

$$\dot{\omega}(t) \leq a\omega(t) + b\ \max \omega_t$$

is satisfied for $t \geq \mu$. If $a > 0, b > 0$, then

$$\omega(t) \leq \max \omega_t \leq [\max \omega_\mu] \exp\{(a+b)(t-\mu)\}, t \geq \mu,$$

where $\max \omega_t = \sup_{t-\tau\leq\theta\leq t} \omega(\theta)$.

III. Theoretical analysis and main results

In this section, we will choose appropriate h, T and D to drive the network (1) to achieve the second-order consensus.

Theorem 1 The position and the velocity of each agent of the network (1) can converge to those of the virtual leader asymptotically under the periodically intermittent pinning controllers (3), if Assumption 1 holds, $\tau \leq h$ and $\tau \leq T - h$, where $h = E_1 T$ and $\tau = E_2 T$; and if there exist constants $a_1 > 0$, $a_2 > 0$ and $d_i (i = 1, 2, \cdots, l)$, such that

(i) $\lambda_{min}\left\{cL + D, cL + \frac{a_2}{2}I_N\right\} > \frac{5}{4} + \frac{q+a_1}{2}$;

(ii) $\left[1 - \frac{3a_1}{2}\right](cL+D) - pI_N - \frac{a_1^2}{4}[2(cL+D) - (q+1+a_1)I_N]^{-1} > 0$;

(iii) $cL + \frac{3a_2}{2}(cL+D) - pI_N - \frac{3}{2}D^T D - \frac{a_2^2}{4}\left[2cL + a_2 I_N - \left(q+\frac{5}{2}\right)I_N\right]^{-1} > 0$;

(iv) $a_1 > k$;

(v) $\gamma(E_1 - E_2) - (a_2 + k)(1 - E_1) > 0$,

where $\gamma > 0$ is the smallest real root of the equation

$$a_1 - \gamma - b\ \exp\{\gamma\tau\} = 0.$$

Proof: Construct the Lyapunov function as

$$J(t) = \frac{1}{2}(\tilde{x}^T(t), \tilde{v}^T(t)) \begin{bmatrix} 3(cL+D) & I_N \\ I_N & 2I_N \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{v}(t) \end{bmatrix},$$

where

$$\Omega = \begin{bmatrix} 3(cL+D) & I_N \\ I_N & 2I_N \end{bmatrix} > 0.$$

By Lemma 1, $\Omega > 0$ is equivalent to $3(cL+D) - 2I_N > 0$. In term of condition (i), we can get $cL + D > \frac{2}{3}I_N$ which indicates that $\Omega > 0$. Further, this means that $J(t) \geq 0$ and $J(t) = 0$ if and only if $\tilde{x}(t) = \tilde{v}(t) = 0$. Based on the above analysis and Lemma 2, let $F(t, x(t), x(t-\tau), v(t), v(t-\tau)) - 1_N \otimes f(t, x_0(t), x_0(t-\tau), v_0(t), v_0(t-\tau))$ be shorthand for $F - 1_N \otimes f$, then

$$\begin{aligned}
&\tilde{x}^T(t)\,[F - 1_N \otimes f] \\
&\leq m_1\tilde{x}^T(t)\tilde{x}(t) + m_2\|\tilde{x}(t)\|\|\tilde{v}(t)\| \\
&+ m_3\|\tilde{x}(t)\|\|\tilde{x}(t-\tau)\| + m_4\|\tilde{x}(t)\|\|\tilde{v}(t-\tau)\| \\
&\leq m_1\tilde{x}^T(t)\tilde{x}(t) + \frac{1}{2}m_2\tilde{x}^T(t)\tilde{x}(t) + \frac{1}{2}m_2\tilde{v}^T(t)\tilde{v}(t) \\
&+ \frac{1}{2}k\tilde{x}^T(t-\tau)[3(cL+D) - I_N]\tilde{x}(t-\tau) \\
&+ \frac{1}{2}\frac{1}{k}m_3^2\tilde{x}^T(t)[3(cL+D) - I_N]^{-1}\tilde{x}(t) \\
&+ \frac{1}{2}k\tilde{v}^T(t-\tau)I_N\tilde{v}(t-\tau) + \frac{1}{2}\frac{1}{k}m_4^2\tilde{x}^T(t)\tilde{x}(t),
\end{aligned}$$

and

$$\begin{aligned}
&2\tilde{v}^T(t)\,[F - 1_N \otimes f] \\
&\leq 2m_1\|\tilde{v}(t)\|\|\tilde{x}(t)\| + 2m_2\tilde{v}^T(t)\tilde{v}(t) \\
&+ 2m_3\|\tilde{v}(t)\|\|\tilde{v}(t-\tau)\| + 2m_4\|\tilde{v}(t)\|\|\tilde{v}(t-\tau)\| \\
&\leq m_1\tilde{v}^T(t)\tilde{v}(t) + m_1\tilde{x}^T(t)\tilde{x}(t) + 2m_2\tilde{v}^T(t)\tilde{v}(t) \\
&+ \frac{1}{2}k\tilde{x}^T(t-\tau)[3(cL+D) - I_N]\tilde{x}(t-\tau) \\
&+ \frac{1}{2}\frac{1}{k}4m_3^2\tilde{v}^T(t)[3(cL+D) - I_N]^{-1}\tilde{v}(t) \\
&+ \frac{1}{2}k\tilde{v}^T(t-\tau)I_N\tilde{v}(t-\tau) + \frac{1}{2}\frac{1}{k}4m_4^2\tilde{v}^T(t)\tilde{v}(t).
\end{aligned}$$

Thus,

$$\begin{aligned}
&\tilde{x}^T(t)\,[F - 1_N \otimes f] + 2\tilde{v}^T(t)\,[F - 1_N \otimes f] \\
&\leq \tilde{x}^T(t)\left[\left(2m_1 + \frac{m_2}{2} + \frac{m_4^2}{2k}\right)I_N + \frac{m_3^2}{2k}[3(cL+D) - I_N]^{-1}\right]\tilde{x}(t) \\
&+ \tilde{v}^T(t)\left[\left(m_1 + \frac{5m_2}{2} + \frac{2m_4^2}{k}\right)I_N + \frac{2m_3^2}{k}[3(cL+D) - I_N]^{-1}\right]\tilde{v}(t) \\
&+ k\tilde{x}^T(t-\tau)[3(cL+D) - I_N]\tilde{x}(t-\tau) + k\tilde{v}^T(t-\tau)I_N\tilde{v}(t-\tau).
\end{aligned}$$

Let

$$pI_N = \left(2m_1 + \frac{m_2}{2} + \frac{m_4^2}{2k}\right) I_N + \frac{m_3^2}{2k}[3(cL+D) - I_N]^{-1},$$

$$qI_N = \left(m_1 + \frac{5m_2}{2} + \frac{2m_4^2}{k}\right) I_N + \frac{2m_3^2}{k}[3(cL+D) - I_N]^{-1},$$

then,

$$\begin{aligned}
&\tilde{x}^T(t)[F - 1_N \otimes f] + 2\tilde{v}^T(t)[F - 1_N \otimes f] \\
&\le \tilde{x}^T(t)(pI_N)\tilde{x}(t) + \tilde{v}^T(t)(qI_N)\tilde{v}(t) \\
&+ k\tilde{x}^T(t-\tau)[3(cL+D) - I_N]\tilde{x}(t-\tau) + k\tilde{v}^T(t-\tau)I_N\tilde{v}(t-\tau) \\
&= \tilde{x}^T(t)(pI_N)\tilde{x}(t) + \tilde{v}^T(t)(qI_N)\tilde{v}(t) \\
&+ k\tilde{x}^T(t-\tau)[3(cL+D)]\tilde{x}(t-\tau) + 2k\tilde{v}^T(t-\tau)I_N\tilde{v}(t-\tau) \\
&- k\tilde{x}^T(t-\tau)I_N\tilde{x}(t-\tau) - k\tilde{v}^T(t-\tau)I_N\tilde{v}(t-\tau).
\end{aligned}$$

From Lemma 3,

$$\begin{aligned}
&- k\tilde{x}^T(t-\tau)I_N\tilde{x}(t-\tau) - k\tilde{v}^T(t-\tau)I_N\tilde{v}(t-\tau) \\
&\le 2k\tilde{x}^T(t-\tau)I_N\tilde{v}(t-\tau).
\end{aligned}$$

Consequently,

$$\begin{aligned}
&\tilde{x}^T(t)[F - 1_N \otimes f] + 2\tilde{v}^T(t)[F - 1_N \otimes f] \\
&\le \tilde{x}^T(t)(pI_N)\tilde{x}(t) + \tilde{v}^T(t)(qI_N)\tilde{v}(t) \\
&+ k\tilde{x}^T(t-\tau)[3(cL+D)]\tilde{x}(t-\tau) \\
&+ 2k\tilde{v}^T(t-\tau)I_N\tilde{v}(t-\tau) + 2k\tilde{x}^T(t-\tau)I_N\tilde{v}(t-\tau) \\
&\le \tilde{x}^T(t)(pI_N)\tilde{x}(t) + \tilde{v}^T(t)(qI_N)\tilde{v}(t) \\
&+ k\left(\tilde{x}^T(t-\tau), \tilde{v}^T(t-\tau)\right) \begin{bmatrix} 3(cL+D) & I_N \\ I_N & 2I_N \end{bmatrix} \begin{bmatrix} \tilde{x}(t-\tau) \\ \tilde{v}(t-\tau) \end{bmatrix} \\
&= \tilde{x}^T(t)(pI_N)\tilde{x}(t) + \tilde{v}^T(t)(qI_N)\tilde{v}(t) + kJ(t-\tau).
\end{aligned}$$

Take the derivative of $J(t)$ with respective to t along with Eq.(6) and the results are presented in the following.

When $mT \le t < mT + h (m = 0, 1, \cdots)$,

$$\begin{aligned}
&\dot{J}(t) = \left(\tilde{x}^T(t), \tilde{v}^T(t)\right) \begin{bmatrix} 3(cL+D) & I_N \\ I_N & 2I_N \end{bmatrix} \\
&\times \begin{bmatrix} \tilde{v}(t) \\ F - 1_N \otimes f - (cL+D)\tilde{x}(t) - (cL+D)\tilde{v}(t) \end{bmatrix} \\
&= \left(\tilde{x}^T(t)[3(cL+D)] + \tilde{v}^T(t)I_N,\ \tilde{x}^T(t)I_N + 2\tilde{v}^T(t)I_N\right) \\
&\times \begin{bmatrix} \tilde{v}(t) \\ F - 1_N \otimes f - (cL+D)\tilde{x}(t) - (cL+D)\tilde{v}(t) \end{bmatrix} \\
&= \tilde{x}^T(t)[3(cL+D)]\tilde{v}(t) + \tilde{v}^T(t)I_N\tilde{v}(t) \\
&+ \tilde{x}^T(t)[F - 1_N \otimes f] - \tilde{x}^T(t)(cL+D)\tilde{x}(t) \\
&- \tilde{x}^T(t)(cL+D)\tilde{v}(t) \\
&+ 2\tilde{v}^T(t)[F - 1_N \otimes f] - 2\tilde{v}^T(t)(cL+D)\tilde{x}(t) \\
&- 2\tilde{v}^T(t)(cL+D)\tilde{v}(t) \\
&= \left(\tilde{x}^T(t), \tilde{v}^T(t)\right) \begin{bmatrix} -(cL+D) + pI_N & 0 \\ 0 & -2(cL+D) + (q+1)I_N \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{v}(t) \end{bmatrix} \\
&+ kJ(t-\tau) \\
&= \left(\tilde{x}^T(t), \tilde{v}^T(t)\right) \\
&\times \begin{bmatrix} -(cL+D) + pI_N + \frac{3a_1}{2}(cL+D) & \frac{a_1}{2}I_N \\ \frac{a_1}{2}I_N & -2(cL+D) + (q+1+a_1)I_N \end{bmatrix} \\
&\times \begin{bmatrix} \tilde{x}(t) \\ \tilde{v}(t) \end{bmatrix} - a_1J(t) + kJ(t-\tau).
\end{aligned}$$

$$\begin{bmatrix} -(cL+D) + pI_N + \frac{3a_1}{2}(cL+D) & \frac{a_1}{2}I_N \\ \frac{a_1}{2}I_N & -2(cL+D) + (q+1+a_1)I_N \end{bmatrix} < 0, \tag{7}$$

i.e.,

$$\begin{bmatrix} \left(1 - \frac{3}{2}a_1\right)(cL+D) - pI_N & -\frac{a_1}{2}I_N \\ -\frac{a_1}{2}I_N & 2(cL+D) - (q+1+a_1)I_N \end{bmatrix} > 0. \tag{8}$$

Condition (i) indicates that $2(cL+D) - (q+1+a_1)I_N > 0$. Based on Lemma 1,

$$\left(1 - \frac{3}{2}a_1\right)(cL+D) - pI_N - \frac{a_1^2}{4}[2(cL+D) - (q+1+a_1)I_N]^{-1} > 0$$

is equivalent to (8). Thus,

$$\dot{J}(t) \le -a_1J(t) + kJ(t-\tau).$$

Based on condition (iv) $a_1 > k$, it follows from Lemma 4 that

$$J(t) \le \max_{mT-\tau \le \theta \le mT} J(\theta)\exp\{-\gamma(t - mT)\}. \tag{9}$$

When $mT + h \le t < (m+1)T (m = 0, 1, \cdots)$, we can obtain

$$\begin{aligned}
&\dot{J}(t) = \left(\tilde{x}^T(t), \tilde{v}^T(t)\right) \begin{bmatrix} 3(cL+D) & I_N \\ I_N & 2I_N \end{bmatrix} \\
&\times \begin{bmatrix} \tilde{v}(t) \\ F - 1_N \otimes f - (cL)\tilde{x}(t) - (cL)\tilde{v}(t) \end{bmatrix} \\
&= \left(\tilde{x}^T(t)[3(cL+D)] + \tilde{v}^T(t)I_N,\ \tilde{x}^T(t)I_N + 2\tilde{v}^T(t)I_N\right) \\
&\times \begin{bmatrix} \tilde{v}(t) \\ F - 1_N \otimes f - (cL)\tilde{x}(t) - (cL)\tilde{v}(t) \end{bmatrix} \\
&= \tilde{x}^T(t)[3(cL+D)]\tilde{v}(t) + \tilde{v}^T(t)I_N\tilde{v}(t) \\
&+ \tilde{x}^T(t)[F - 1_N \otimes f] - \tilde{x}^T(t)(cL)\tilde{x}(t) - \tilde{x}^T(t)(cL)\tilde{v}(t) \\
&+ 2\tilde{v}^T(t)[F - 1_N \otimes f] - 2\tilde{v}^T(t)(cL)\tilde{x}(t) - 2\tilde{v}^T(t)(cL)\tilde{v}(t) \\
&+ kJ(t-\tau) \\
&\le -\tilde{x}^T(t)(cL)\tilde{x}(t) + \tilde{x}^T(t)(pI_N)\tilde{x}(t) + 3\tilde{x}^T(t)D\tilde{v}(t) \\
&+ \tilde{v}^T(t)I_N\tilde{v}(t) + \tilde{v}^T(t)(qI_N)\tilde{v}(t) - 2\tilde{v}^T(t)(cL)\tilde{v}(t) \\
&+ kJ(t-\tau) \\
&= -\tilde{x}^T(t)(cL)\tilde{x}(t) + \tilde{x}^T(t)(pI_N)\tilde{x}(t) + 2 \times \frac{3}{2}(D\tilde{x}(t))^T\tilde{v}(t) \\
&+ \tilde{v}^T(t)I_N\tilde{v}(t) + \tilde{v}^T(t)(qI_N)\tilde{v}(t) - 2\tilde{v}^T(t)(cL)\tilde{v}(t) + kJ(t-\tau).
\end{aligned}$$

By Lemma 2, choosing the positive matrix $G = I_N$, we can have

$$\begin{aligned}
&\dot{J}(t) \le -\tilde{x}^T(t)(cL)\tilde{x}(t) + \tilde{x}^T(t)(pI_N)\tilde{x}(t) \\
&+ \frac{3}{2}\tilde{x}^T(t)D^TD\tilde{x}(t) + \frac{3}{2}\tilde{v}^T(t)\tilde{v}(t) \\
&+ \tilde{v}^T(t)I_N\tilde{v}(t) + \tilde{v}^T(t)(qI_N)\tilde{v}(t) - 2\tilde{v}^T(t)(cL)\tilde{v}(t) + kJ(t-\tau) \\
&= \left(\tilde{x}^T(t), \tilde{v}^T(t)\right) \begin{bmatrix} -cL + pI_N + \frac{3}{2}D^TD & 0 \\ 0 & -2cL + \left(q + \frac{5}{2}\right)I_N \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{v}(t) \end{bmatrix} \\
&+ kJ(t-\tau) \\
&= \left(\tilde{x}^T(t), \tilde{v}^T(t)\right) \\
&\times \begin{bmatrix} -cL + pI_N + \frac{3}{2}D^TD - \frac{a_2}{2}[3(cL+D)] & -\frac{a_2}{2}I_N \\ -\frac{a_2}{2}I_N & -2cL + \left(q + \frac{5}{2} - a_2\right)I_N \end{bmatrix} \\
&\times \begin{bmatrix} \tilde{x}(t) \\ \tilde{v}(t) \end{bmatrix} + a_2J(t) + kJ(t-\tau).
\end{aligned}$$

Condition (i) indicates that $-\big(-2cL+(q+\frac{5}{2}-a_2)I_N\big)>0$. According to Lemma 1, condition (ii) comes up to

$$\begin{bmatrix} cL-pI_N-\frac{3}{2}D^TD+\frac{a_2}{2}[3(cL+D)] & \frac{a_2}{2}I_N \\ \frac{a_2}{2}I_N & 2cL-(q+\frac{5}{2}-a_2)I_N \end{bmatrix}>0,$$

in sense that

$$\begin{bmatrix} -cL+pI_N+\frac{3}{2}D^TD-\frac{3a_2}{2}(cL+D) & -\frac{a_2}{2}I_N \\ -\frac{a_2}{2}I_N & -2cL+(q+\frac{5}{2}-a_2)I_N \end{bmatrix}<0,$$

then one has

$$\dot{J}(t)\leq a_2J(t)+kJ(t-\tau).$$

Lemma 5 demonstrates that

$$J(t)\leq \max_{mT+h-\tau\leq\theta\leq mT+h} J(\theta)\exp\{(a_2+k)(t-mT-h)\}. \tag{10}$$

Next, we can summary the solution of system (1) upon Eq.(9) and Eq.(10) as follows:

1: For $0\leq t<h$,

$$J(t)\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{-\gamma t\}.$$

For $h\leq t<T$,

$$J(t)\leq \sup_{h-\tau\leq\theta\leq h} J(\theta)\exp\{(a_2+k)(t-h)\}.$$

Since $\tau\leq h$, $[h-\tau,h]\subset[0,h]$, then

$$\sup_{h-\tau\leq\theta\leq h} J(\theta)\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{-\gamma t\}\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{-\gamma(h-\tau)\}.$$

So, for $h\leq t<T$,

$$J(t)\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{(a_2+k)(t-h)-\gamma(h-\tau)\}.$$

2: For $T\leq t<T+h$,

$$J(t)\leq \sup_{T-\tau\leq\theta\leq T} J(\theta)\exp\{-\gamma(t-T)\}.$$

Since $\tau\leq T-h$, $[T-h,T]\subset[h,T]$, then,

$$\begin{aligned} J(t)&\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{(a_2+k)(T-h)-\gamma(h-\tau)\}\exp\{-\gamma(t-T)\}\\ &=\sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{-\gamma(t-T)+(a_2+k)(T-h)-\gamma(h-\tau)\}. \end{aligned}$$

For $T+h\leq t<2T$,

$$\begin{aligned} J(t)&\leq \sup_{T+h-\tau\leq\theta\leq T+h} J(\theta)\exp\{(a_2+k)(t-T-h)\}\\ &=\sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{(a_2+k)(t-T-h)+(a_2+k)(T-h)-2\gamma(h-\tau)\}. \end{aligned}$$

3: For $2T\leq t<2T+h$,

$$\begin{aligned} J(t)&\leq \sup_{2T-\tau\leq\theta\leq2T} J(\theta)\exp\{-\gamma(t-2T)\}\\ &=\sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{-\gamma(t-2T)+2(a_2+k)(T-h)-2\gamma(h-\tau)\}. \end{aligned}$$

For $2T+h\leq t<3T$,

$$\begin{aligned} J(t)&\leq \sup_{2T+h-\tau\leq\theta\leq2T+h} J(\theta)\exp\{(a_2+k)(t-2T-h)\}\\ &=\sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{(a_2+k)(t-2T-h)+2(a_2+k)(T-h)\\ &\quad-3\gamma(h-\tau)\}. \end{aligned}$$

Note that $h=E_1T$ and $\tau=E_2T$, by induction, for $mT\leq t<mT+h$ $(m=0,1,\cdots)$,

$$\begin{aligned} J(t)&\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{-\gamma(t-mT)+m(a_2+k)(T-h)-m\gamma(h-\tau)\}\\ &\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{m(a_2+k)(1-E_1)T-m\gamma(E_1-E_2)T\}\\ &\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{(a_2+k)(1-E_1)t-\gamma(E_1-E_2)t+\gamma(E_1-E_2)h\}\\ &=\sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{-[\gamma(E_1-E_2)-(a_2+k)(1-E_1)]t+\gamma(E_1-E_2)h\}; \end{aligned}$$

for $mT+h\leq t<(m+1)T$ $(m=0,1,\cdots)$,

$$\begin{aligned} J(t)&\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{(a_2+k)(t-mT-h)\\ &\quad+m(a_2+k)(T-h)-(m+1)\gamma(h-\tau)\}\\ &=\sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{(a_2+k)t-(m+1)(a_2+k)h-(m+1)\gamma(h-\tau)\}\\ &=\sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{(a_2+k)t-(m+1)(a_2+k)E_1T\\ &\quad-(m+1)\gamma(E_1-E_2)T\}\\ &\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{(a_2+k)t-(a_2+k)E_1T-\gamma(E_1-E_2)t\}\\ &\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{-[\gamma(E_1-E_2)-(a_2+k)(1-E_1)]t\\ &\quad+\gamma(E_1-E_2)h\}. \end{aligned}$$

Thus, for any $t\geq0$, we can obtain

$$\begin{aligned} J(t)&\leq \sup_{-\tau\leq\theta\leq0} J(\theta)\exp\{-[\gamma(E_1-E_2)-(a_2+k)(1-E_1)]t\\ &\quad+\gamma(E_1-E_2)h\}. \end{aligned}$$

As condition (v), the result of Theorem 1 holds. ■

IV. Simulations

In this section, a numerical example is presented to show the effectiveness of the main results. The intrinsic dynamics is

$$\begin{aligned} &f(t,x_i(t)),x_i(t-\tau)),v_i(t)),v_i(t-\tau)))\\ &=\tanh\left(\frac{1}{45}x_i(t)+\frac{1}{15}x_i^{\frac{1}{6}}(t-\tau)+\frac{1}{20}v_i^{\frac{1}{3}}(t)+\frac{1}{30}v_i(t-\tau)\right). \end{aligned}$$

It is easy to get that

$$\begin{aligned} &\|F(t,x(t),x(t-\tau),v(t),v(t-\tau))\\ &-1_N\otimes f(t,x_0(t),x_0(t-\tau),v_0(t),v_0(t-\tau))\|\\ &\leq\frac{4}{45}\|x(t)-x_0(t)\|+\frac{1}{15}\|v(t)-v_0(t)\|\\ &+\frac{2}{45}\|x(t-\tau)-x_0(t-\tau)\|+\frac{2}{15}\|v(t-\tau)-v_0(t-\tau)\|. \end{aligned}$$

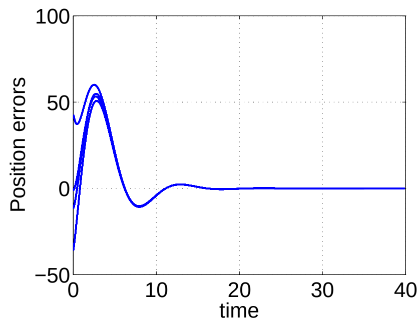

Fig. 2. Position convergence when $\tau = 0.02$.

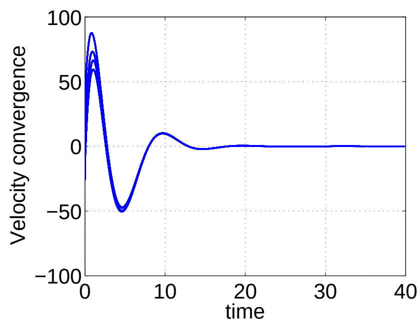

Fig. 3. Velocity convergence when $\tau = 0.02$.

The coupling configuration matrix is

$$A = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

Without loss of generality, we select the matrix D randomly as

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

in sense that only the second and the third nodes are informed by the virtual leader.

Selecting $c = 12, a_1 = \frac{1}{4}, a_2 = 8, T = 5, h = 4.9$, and we can verify easily that the conditions in Theorem 1 can be satisfied.

Figure 2 describes the position errors convergence of all agents when the time delay is $\tau = 0.02$. It is obvious in the figure that all the position errors tend to zero. Figure 3 depicts the convergence of the velocity errors and shows that the velocity of each agent converge to zero when time delay is $\tau = 0.02$. These two figures illustrate that all the positions and velocities of the multi-agent system can achieve those of the virtual leader respectively, even though only a small fraction of agents are informed by the virtual leader during part of time in each period.

V. Conclusion

In this paper, we have investigated second-order consensus of multi-agent system with time delay by adding periodically intermittent controllers to a fraction of agents. Furthermore, the nonlinear dynamics relates to the position and delayed position, the velocity and the delayed velocity, which is assumed to satisfy the Lipschita-like condition. Based on the Lyapunov stability theory and the mathematical induction method, we have derived the LMI criteria for the second-order consensus of the multi-agent systems. Finally, a simulation example is presented to illustrate the effectiveness of our theoretical results.

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