Passivity-based Finite-time Attitude Control Problem

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***Abstract*—In this paper the passivity-based finite-time attitude control problem of a rigid spacecraft is addressed. Firstly, for a certain class of nonlinear passive system we derive different control laws according to different choices of storage functions. Based on this result, combining the sliding mode control method, we propose a passivity-based finite-time controller for a rigid spacecraft. Performances of the proposed controllers are illustrated by simulation.**

Keywords—attitude control; passivity; finite-time control

I. Introduction

In recent years, the attitude control of a rigid spacecraft has been extensively studied. It is a very interesting problem due to its many different types of applications, such as pointing and slewing of spacecrafts, helicopters, satellites, underwater vehicles and robot manipulation [1], [2].

The attitude stabilization of a spacecraft using attitude and angular velocity in the feedback control law has been investigated by many researchers and a wide class of controllers has been proposed. Among the exiting control laws, most are asymptotically stable control laws [2]-[6]. Asymptotic stability implies that the system trajectories converge to the equilibrium as time goes to infinity. In [5], the attitude tracking control problem of a rigid spacecraft with external disturbances and an uncertain inertia matrix is addressed using the adaptive control method. The proposed quaternion-based hybrid feedback law in [6] solves the global attitude tracking problem in three scenarios: full state measurements, only measurements of attitude, and measurements of attitude with angular velocity measurements corrupted by a constant bias.

However, there is little result about finite-time attitude control for a rigid spacecraft. Obviously, finite-time stabilization of a dynamical system gives rise to a high-precision performance and better disturbance rejection properties [7] [8] [9]. In [8], the standard terminal sliding mode control technique was employed. In [9], a finite-time attitude tracking feedback control law using both attitude and angular velocity has been designed for a single spacecraft and a distributed finite-time attitude synchronization algorithm has also been developed for a group of spacecrafts.

The attitude control of a spacecraft with full state measurements has been directed towards removing the requirement of the angular velocity measurement due to the lack of tachometer of manipulators [3], [6], [11], [12], [13] and [14]. The passivity property was the main idea behind the design of controller without angular velocity measurement. In [3], the authors used the passivity-based adaptive control approach for robotic manipulators to derive the adaptive attitude control scheme without velocity measurement. The stabilization controller proposed in [6] used a nonlinear filter of quaternion to replace the angular velocity feedback. The authors in [12] showed that linear asymptotically stabilizing controllers without angular velocity measurements followed naturally from the passivity properties established for attitude motion of a rigid body. In [13], the velocity-free unit quaternion-based tracking controller guaranteeing almost global asymptotic stability was derived by using an auxiliary unit-quaternion dynamical system.

The existing finite-time attitude control laws mainly depend on the methods which have been explored in designing general nonlinear systems. It is possible to use the inherent properties of the system to benefit the designing process. Owing to the importance of the passivity properties of the system, we focus on finding the possibility of designing a finite-time attitude control law based on the passivity properties of the system. Firstly, by combining the finite-time stability theory with passivity approach, we derive a finite-time control law for a certain class of nonlinear passive system. Secondly, by combining the sliding mode control method, we derive a finite-time control law for the whole dynamic system. The performance of the proposed controller is illustrated by simulation. To the best of authors' knowledge, this is a new result about finite-time attitude control problem based on the passivity properties of the dynamical system.

The remainder of this paper is organized as follows. In section 2, some preliminaries and problem statement are given. For the set point control problem for a rigid spacecraft, the finite-time attitude control law is proposed in section 3. The numerical examples are given in section 4 to illustrate the results. Finally, we make concluding remarks in section 5.

II. Preliminaries and problem statement

A. Rotation motion a rigid spacecraft

Fundamental to any problem in this area is the necessity to identify and adopt a parameterization for attitude. Modified Rodrigues parameters (MRPs) are an attractive set of three-dimensional coordinates for attitude motions [15]. We adopt the MRPs to represent the attitude motion of a spacecraft.

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Let $\sigma = \eta \tan(\theta/4) \in \mathbb{R}^3$,$-2\pi < \theta < 2\pi$ represent the MRPs for a spacecraft, where η is the Euler axis and θ is the Euler angle. Given a vector $\nu = [\nu_1, \nu_2, \nu_3]^T$,the symbol $s(\cdot)$ denotes a 3×3 skew-symmetric matrix, that is $s(\nu) = [0, \nu_3, -\nu_2; -\nu_3, 0, \nu_1; \nu_2, -\nu_1, 0]$. It also performs the vector cross product between any two vectors, i.e., $s(a)b$=$-a \times b$, $a, b \in \mathbb{R}^3$.

Let $u(t) \in \mathbb{R}^3$ be an external torque vector acting on the spacecraft whose mass moment of inertia is given by the matrix $J \in \mathbb{R}^{3\times3}$. Usually we can transform J to a diagonal matrix. $\omega(t) \in \mathbb{R}^3$ is the angular velocity of the spacecraft with respect to the inertial frame expressed in the body frame and J is the symmetric inertia matrix. The dynamic motion the attitude of the spacecraft are

$$\begin{cases} \dot{\sigma} = G(\sigma)\omega & \text{(1a)} \\ J\dot{\omega} = s(\omega)J\omega + u & \text{(1b)} \end{cases}$$

where the matrix $G(\sigma)$ is given by

$$G(\sigma) = \frac{1}{2}\left[\frac{1-\sigma^T\sigma}{2} I_{3\times3} - s(\sigma) + \sigma\sigma^T\right] \quad (2)$$

with I_3 being the 3×3 identity matrix. For the matrix, the following properties are known [14]:

$$\sigma^T G(\sigma)\omega = \frac{1+\sigma^T\sigma}{4}\sigma^T\omega, G(\sigma)G(\sigma)^T = \frac{1+\sigma^T\sigma}{4} I_{3\times3} \quad (3)$$

B. Passivity

Definition 1[16]: For a dynamical system represented by the state model

$$\begin{cases} \dot{x} = f(x, u) & \text{(4a)} \\ y = h(x, u) & \text{(4b)} \end{cases}$$

where$f : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is locally Lipschitz, $h : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous, $f(0, 0) = 0$, and $h(0, 0) = 0$.

The system is said to be passive if there exists a continuously differentiable positive semi definite function (called the storage function) such that [16]:

$$u^T y \geq \dot{V} = \frac{\partial V}{\partial x} f(x, u), \forall (x, u) \in \mathbb{R}^n \times \mathbb{R}^n.$$

Moreover, it is said to be

1) Lossless if $u^T y = \dot{V}$.
2) Output feedback passive if $u^T y \geq \dot{V} + y^T\rho(y)$ for some function ρ.
3) Output strictly passive if $u^T y \geq \dot{V} + y^T\rho(y)$ and $y^T\rho(y) > 0$, $\forall y \neq 0$.

In all cases, the inequality should hold for all(x, u).

Definition 2[16]: The system (4a)-(4b) is said to be zero-state observable if no solution of $\dot{x} = f(x, 0)$ can stay identically in $S = \{x \in \mathbb{R}^n | h(x, 0) = 0\}$, other than the trivial solution $x(t) \equiv 0$.

Lemma 1[16]: Consider the system (4a)-(4b), the origin of $\dot{x} = f(x, 0)$ is asymptotically stable if the system is strictly passive or output strictly passive and zero-state observable. Furthermore, if the storage function is radially unbounded, the origin will be globally asymptotically stable.

C. Finite-time stability

Lemma 2[17]: Consider the system $\dot{x} = f(x)$, where$f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function. Suppose there exists a continuous, positive definite function $V(x) : U \to \mathbb{R}$ defined on an open neighborhood U of the origin such that$\dot{V}(x) + c(V(x))^\alpha \leq 0$ on U for some $c > 0$ and $\alpha \in (0, 1)$, then the origin is a finite-time stable equilibrium of system $\dot{x} = f(x)$ and the finite settling time T satisfies $T \leq (V(x(0)))^{1-\alpha}/c(1-\alpha)$. If $U = \mathbb{R}^n$ and V is radially unbounded, the origin is a globally finite-time stable equilibrium.

Lemma 3[18]: For any $x_i \in \mathbb{R}, i = 1, \cdots, n$, and a real number $p \in (0, 1]$,

$$(|x_1|+\cdots+|x_n|)^p \leq |x_1|^p+\cdots+|x_n|^p \leq n^{1-p}(|x_1|+\cdots+|x_n|)^p$$

D. Problem Formulation

Given the dynamical system described by (1a) and (1b), our control objective is to design a finite-time attitude controller for a rigid spacecraft. Under this control law, the desired steady state target attitude can be tracked in finite time.

III. Finite-time attitude control for a rigid spacecraft

A. Finite-time feedback control for a class of nonlinear passive system

Passivity provides us with a useful tool for the analysis of nonlinear systems, which relates nicely to Lyapunov stability. Based on the properties of a passive system, one can obtain the control law to globally stabilize the origin of a nonlinear system.

Proposition 1[16]:If the system $(4a) - (4b)$ is

1) Passive with a radially unbounded positive definite storage function and
2) Zero-state observable,

then the origin $x = 0$ can be globally stabilized by $u = -\varphi(y)$, where φ is any locally Lipschitz function such that $\varphi(0) = 0$ and $y^T\varphi(y) > 0$ for all $y \neq 0$.

The existing linear asymptotically stabilizing control laws for the attitude motion of a rigid body using minimal three-dimensional parameterizations are intimately related to the passivity properties of the corresponding kinematic systems. So it is natural to explore the possibility of designing the finite-time control law for certain passive systems.

Theorem 1: For the n-input-n-output system

$$\begin{cases} \dot{x} = f(x, u) & \text{(5a)} \\ y = x & \text{(5b)} \end{cases}$$

where $f : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is locally Lipschitz, $f(0, 0) = 0$, for some real numbers $c > 0$ and $\alpha \in (0, 1)$

1) If the system is passive with the storage function $V = 2ln(1+x^T x)$, choosing V as the candidate Lyapunov function, the origin of the system is finite-time stable with control $u = -c \cdot 2^\alpha \cdot x^{2\alpha-1}$.

2) If the system is passive with the storage function $V = \frac{1}{2}x^T Jx$, where J is a diagonal positive definite matrix, choosing V as the candidate Lyapunov function, then the origin of the system is finite-time stable with control $u = -c \cdot (\frac{1}{2})^\alpha \cdot J^\alpha \cdot x^{2\alpha-1}$.

Proof:

1) If we choose control

$$u_i = -c \cdot 2^\alpha \cdot x_i^{2\alpha-1}$$

then

$$u^T y = u^T x = -c \cdot 2^\alpha \cdot \sum_{i=1}^{n} x_i^{2\alpha}$$

since

$$(x^T x)^\alpha = (x_1^2 + \cdots + x_n^2)^\alpha \leq \sum_{i=1}^{n} x_i^{2\alpha}$$

$$= (x_1, \cdots, x_n) \cdot \begin{pmatrix} x_1^{2\alpha-1} \\ \vdots \\ x_n^{2\alpha-1} \end{pmatrix}$$

we get

$$u^T y + c \cdot 2^\alpha \cdot \sum_{i=1}^{n} x_i^{2\alpha} \leq 0$$

and also the system is passive:

$$u^T y \geq \dot{V}$$

we get

$$\dot{V} + c \cdot 2^\alpha \cdot \sum_{i=1}^{n} (x^T x)^\alpha \leq 0$$

obviously

$$ln(1 + x^T x) \leq x^T x$$

so we get

$$\dot{V} + c \cdot V^\alpha \leq 0$$

Choosing V as the Lyapunov candidate, then the origin of the passive system is finite-time stable.

The proof for 2) are similar to 1), thus omitted.

Proposition 2[14]:

1) System (1a) with input ω and output σ is passive
2) System (1b) with input u and output ω is passive.

Proof:

1) Taking the time derivative of the function $V_1(\sigma) = 2ln(1 + \sigma^T \sigma)$ along the trajectories of (1a) yields that $\dot{V}_1(\sigma) = \sigma^T \omega$. This shows the system is passive (lossless).
2) Taking the time derivative of the function $V_2(\omega) = \frac{1}{2}\omega^T J\omega$ along the trajectories of (1b) yields that $\dot{V}_2(\omega) = \omega^T u$. This shows the system is passive (lossless).

Proposition 3: If we can find real numbers satisfy $c > 0$ and $\alpha \in (0, 1)$, then the origin of system (1a) is finite-time stable with control

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = -c \cdot 2^\alpha \cdot \begin{pmatrix} \sigma_1^{2\alpha-1} \\ \sigma_2^{2\alpha-1} \\ \sigma_3^{2\alpha-1} \end{pmatrix} \quad (6a)$$

the origin of system (1b) is finite-time stable with control

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = -c \cdot (\frac{1}{2})^\alpha \cdot J^\alpha \begin{pmatrix} \omega_1^{2\alpha-1} \\ \omega_2^{2\alpha-1} \\ \omega_3^{2\alpha-1} \end{pmatrix} \quad (6b)$$

B. Finite-time set point control for a rigid spacecraft

Let σ_d, ω_d denote the desired attitude and the desired angular velocity, respectively. Define $e = [e_1, e_2, e_3]^T \in \mathbb{R}^3$ as the relative attitude error between the actual attitude and the desired attitude, where

$$e = \sigma \otimes \sigma_d^{-1} = \frac{\sigma_d(\sigma^T\sigma - 1) + \sigma(1 - \sigma_d^T\sigma_d) + 2s(\sigma_d)\sigma}{1 + \sigma_d^T\sigma_d\sigma^T\sigma + 2\sigma_d^T\sigma}.$$

Define $\nu = [\nu_1, \nu_2, \nu_3]^T = \omega - R_d^b\omega_d \in \mathbb{R}^3$ as the relative angular velocity error, where R_d^b is the rotation matrix from the desired reference frame to the body reference frame. The rotation matrix R_d^b is a proper orthogonal matrix and is given by $R_d^b = R(e)$, where

$$R(e) = I_3 + 4((1 - e^T e)/(1 + e^T e)^2)s(e) + 8s^2(e)/(1 + e^T e)^2,$$

then the relative kinematic and dynamic equations are given as in [9]

$$\dot{e} = G(e)\nu$$

$$J\dot{\nu} = s(\omega)J\omega + u - JR_d^b\dot{\omega}_d - Js(\nu)R_d^b\omega_d$$

In this paper, we consider the set point control problem of driving the attitude of a rigid spacecraft to a steady state target attitude. So $\omega_d = 0, \dot{\omega}_d = 0$ and the equation is given by

$$\begin{cases} \dot{e} = G(e)\omega & (7a) \\ J\dot{\omega} = s(\omega)J\omega + u & (7b) \end{cases}$$

Therefore, to solve the finite-time set point control problem, we need to design a control law such that $e \to 0$ in finite time.

The relative attitude kinematics and dynamics rotation equations (7a)-(7b) represent a system in cascade form. The control input drives the angular velocity equation and the angular velocity drives the kinematic equation .There is no direct connection between the kinematics subsystem and the torque input. The kinematic equation can be accessed and manipulated only through the angular velocity vector. For systems in cascade connection there is an intuitive way to achieve closed-loop stability. The methodology involves a two-step procedure. One can concentrate first on the stabilization of the second driven subsystem(the kinematic equations in our case) treating the driving state as a control-like variable (the angular velocity vector in our case) and then proceed to the stabilization of the complete system.

Our goal is to design a suitable controller which drives the attitude of a spacecraft to a given desired point in finite time. The technique based on sliding mode control approach was used in designing a finite-time convergent controller in [7]. The basic idea behind the technique is to choose a proper sliding surface such that on it the objective of control is achieved in finite time, and to design a control law which can drive the motion of the system on the sliding surface in finite time.

Theorem 2: Consider the system (7a) and (7b), if the control torque u is chosen as

$$\begin{aligned} u = -s(\omega)J\omega - k_v \cdot (\frac{1}{2})^{p_2} \cdot J^{p_2} \cdot (\omega - \omega^*)^{2p_2-1} \\ -k_p \cdot (2p_1 - 1) \cdot 2^{p_1} \cdot (J \cdot e^{2p_1-2}) \circ (G(e) \cdot \omega) \end{aligned} \tag{8}$$

where $\omega^* = -p_1 \cdot 2^{p_1} \cdot e^{2p_1-1}$, $k_v > 0, k_p > 0$, $0 < p_1, p_2 < 1$, symbol $\circ$ denotes the Hadamard product. Then e converges to zero in finite time.

Proof:

The proof procedure can be divided into two steps. First, ω is taken as a virtual input for (7a) and is designed such that e reaches zero in finite time. We design the sliding manifold $z = e - \psi(e)$ such that, when motion is restricted to the manifold, the reduced-order model $\dot{e} = G(e)\psi(e)$ has a finite-time stable equilibrium point at the origin. Then, the control law u is designed such that sliding surface $z = e - \psi(e)$ goes to zero in finite time and maintain it there for all future time.

Step 1: Virtual input ω design

Select a candidate Lyapunov function as

$$V_0 = 2ln(1 + e^T e)$$

along the trajectory of system (7a) and using (3) we have

$$\dot{V}_0 = \frac{4}{1 + e^T e} e^T \dot{e} = \frac{4}{1 + e^T e} e^T G(e)\omega = e^T \omega$$

using the Proposition 3, we can get

$$\omega = \psi(e) = -k_p \cdot 2^{p_1} \cdot J \cdot \sigma^{2p_1-1}$$

so we obtain

$$\dot{V}_0 = -k_p \cdot 2^{p_1} \sum_{i=1}^{3} \sigma_i^{2p_1} \le -k_p \cdot (2 \sum_{i=1}^{3} \sigma_i^2)^{p_1}$$

since

$$V_0 = 2ln(1 + \sigma^T \sigma) \le 2\sigma^T \sigma = 2 \sum_{i=1}^{3} \sigma_i^2$$

and

$$\dot{V}_0 + k_p V_0^{p_1} \le 0$$

by Lemma 2, we obtain $V_0(t)$ reaches zero in finite time and e reaches zero in finite time.

Step 2: Control law u design

The rotation equation of a rigid spacecraft can be transformed into the form:

$$\begin{cases} \dot{e} = G(e)\psi(e) + G(e)(\omega - \psi(e)) \\ \dot{\omega} = J^{-1}s(\omega)J\omega + J^{-1}u. \end{cases}$$

Design the sliding manifold $z = \omega - \psi(e) = 0$ and let $J^{-1}\nu = J^{-1}u - \dot{\psi}$, we obtain

$$\begin{cases} \dot{e} = G(e)\psi(e) + G(e)z \\ \dot{z} = J^{-1}(s(\omega)J\omega + \nu). \end{cases}$$

Taking $\nu = -s(\omega)J\omega + \varphi(z)$, the work reduces to design $\varphi(z)$ to bring z to zero in finite time and maintain it there for all future time. Choosing a candidate Lyapunov function as $V_1 = \frac{1}{2} z^T J z$, along the system (7b), we have $\dot{V}_1 = z^T \varphi(z)$. Taking

$$\varphi(z) = -k_v \cdot (\frac{1}{2})^{p_2} \cdot J^{p_2} \cdot z^{2p_2-1}$$

and by lemma 4, one obtains

$$\dot{V}_1 \le -k_v \cdot (\frac{1}{2})^{p_2} \cdot J^{p_2} \cdot (\sum_{i=1}^{3} z_i^2)^{p_2} = -k_v \cdot V_1^{p_2}$$

Thus we obtain z goes to zero in finite time and maintain it there for all future time.

Combining step 1 and step 2, we get the result proposed in Theorem 2.

IV. Numerical Simulations

We now demonstrate the previous theoretical results by means of numerical simulations. We first show the effectiveness of the results of Proposition 3. Consider system (1a), we suppose the initial MRPs vector is $\sigma_0 = [0.3, 0.5, 0.8]^T$. The values for the gains are chosen as $c = 1, 4, 6, 10, \alpha = 0.8$.

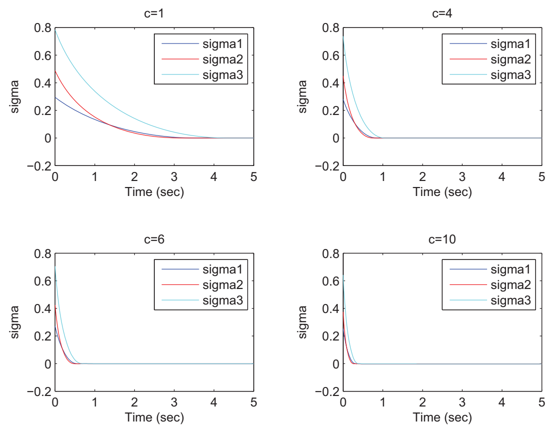

Fig. 1: Stabilization for system (1a)

The results are shown in Figure 1. Under the control law (6a), the origin of the system (1a) can be finite-time stabilized. The finite time T will decrease as we increase the value of parameter c which has a maximum value c_{max} satisfying$\dot{V}(x) + c(V(x))^{\alpha} \le 0$.

For system (1b), we suppose the initial angular velocity $\omega_0 = [0.3, 0.5, 0.8]^T$ and the inertia matrix is represented by $J = diag(1, 0.63, 0.85)$. The values for the gains are chosen as $c = 1, 4, 6, 10, \alpha = 0.8$. Figure 2 shows that under the control law (6b), the origin of the system (1b) can be finite-time stabilized. The finite time T will decrease as we increase the value of parameter c which has a maximum value c_{max} satisfying$\dot{V}(x) + c(V(x))^{\alpha} \le 0$.

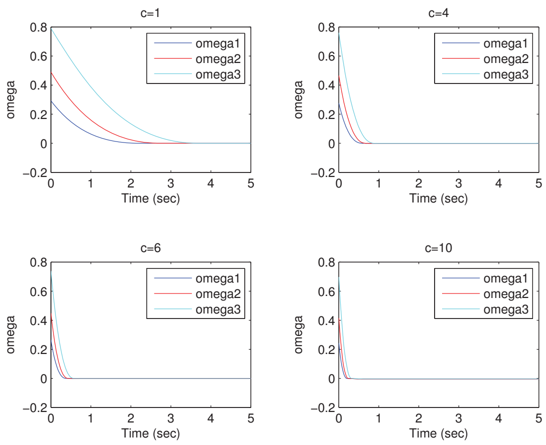

Fig. 2: Stabilization for system (1b)

In order to illustrate the results presented in Section 3, a simple example considered in [2] is addressed here. A body with the inertia matrix (expressed in the body frame) $J = diag(1, 0.63, 0.85)$ is considered.

The initial orientation corresponds to the identity matrix $I_{3\times 3}$ and the initial angular velocity is $[\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}]$. In terms of the equivalent axis and angle representation, the desired orientation corresponds to an eigenaxis/angle representation given by

$$\eta = [0.4896, 0.2032, 0.8480]^T, \theta = 2.4648rad.$$

The values for the gains are selected to be $k_p = 8$ and $k_v = 8$. In order to satisfy the finite-time stability theorem, we select $p_1 = 0.9$ and $p_2 = 0.9$. These values are chosen by trial and error in order to achieve good attitude control performance. Based on the control law (8), we get the following results: Figure 3 depicts the behavior of the Modified Rodrigues parameter vector, while Figure 4 shows the time history of the associated control effort.

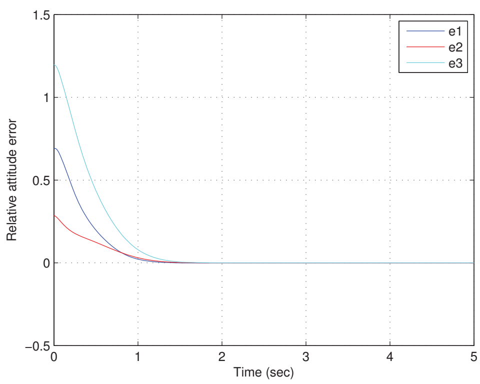

Fig. 3: Stabilization for system (7a)-(7b).

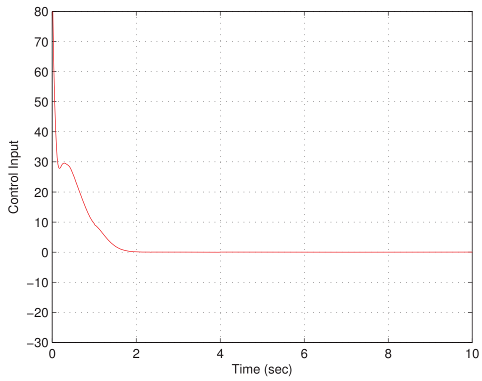

Fig. 4: Control input for system (7a)-(7b).

V. Conclusion

In this paper, a passivity-based finite-time attitude control law for a rigid spacecraft is proposed. A finite-time control law for a class of nonlinear passive system is derived. The corresponding storage functions include a quadratic term in the angular velocities and a logarithmic term in the attitude parameters. Combining the sliding mode control method, we derive the finite-time nonlinear control law for the whole dynamic system.

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