

# A Frequency-based Method for Complete Identification of Some Types of Wiener-type Plants Based on Relay Feedback

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**Abstract**—This paper proposes a new closed loop frequency based method for a parametric identification of some types of Wiener-type nonlinear plants. It provides a parametric model of the linear part of the plant, and a point by point relationship between input and output of the nonlinear part of the plant. Two approaches are proposed for the identification of the nonlinear part. The first approach is slower, but it does not require any additional equipment expect the relay. The second approach is much faster, but it requires some additional equipment.

**Keywords**—*identification; closed-loop; nonlinear; Wiener-type; relay feedback*

## I. INTRODUCTION

The closed-loop experimental methods for the identification an unknown plant are popular, because they do not require the isolation of the plant from the control loop. Although a lot of such procedures are widely available, most of them assume that the plant is linear. As such assumption is usually false, these procedures may give completely unusable results.

So far, there is no any universally applicable method that works with an arbitrary nonlinear plant. However, sometimes the dynamic of the plant is mostly linear, and nonlinear effects are present only at the plant input or output. For example, any nonlinear actuator will turn a linear plant into a nonlinear plant with nonlinear effects that are present only at the plant input. Also, saturation effects that are always presented due to the limited amount of energy in any real system will effectively convert any linear plant into a nonlinear plant with nonlinear effects that are present only at the plant output. Such plants may be modeled using Hammerstein-type or Wiener-type models, which may be expressed as a cascade of an inertial linear block and a non-inertial nonlinear block. In the Hammerstein-type models, the nonlinear part precedes the linear part, but the opposite is true for the Wiener-type models.

Closed-loop methods are often frequency-based methods, where an appropriate system motion is generated automatically through a feedback supplied by a standard controller or some other tunable device. Many such methods use relays in the feedback. Some examples are simple relay method [1], relay-with-hysteresis method [2], Two Channel Relay (TCR) method [3] or Auto Tune Variation (ATV) method [4]. Such methods

often require extra equipment not usually included in the standard control loops. The method proposed in this paper is also based on relays, but it can be performed without any other equipment, although some extra equipment can considerably speed-up the procedure. Note that the behavior of relay-based feedback is well-known and described in [5] and [6], so the general results for such class of methods hold for the method described in this paper too.

Unfortunately, many closed-loop identification methods are based on the assumption that the plant is linear. Otherwise, they may produce useless results [7], especially if the method is based on obtaining the frequency response of the plant [8]. Therefore, the most of methods for the identification in presence of nonlinear effects are based on time-response approach. Although there are many such approaches that are developed for identification of Hammerstein-type plants, there are considerably smaller amount of approaches that are suitable for the identification of Wiener-type plants (actually, they are usually considered as much harder for the identification than Hammerstein-type plants). Some of them are described in [9], [10], [11], [12] and [13].

As the prevailing belief about the unsuitability of the frequency-based identification methods in presence of nonlinear effects is not entirely correct, some frequency-based methods for the identification of the Hammerstein type plants were also proposed in the past, e.g. methods described in [14], [15] and [16]. Unfortunately, they are open-loop methods that require special signal generators, and they are either too restrictive or too sensitive. A completely new approach for frequency-based closed-loop identification of Hammerstein-type plants is described in [17], [18], [19] and [20]. However, nothing similar is presented so far for the Wiener-type plants. This paper extends ideas presented in these papers to some types of Wiener-type plants.

The paper describes a simple frequency-based closed-loop identification method that may be applied for some types of Wiener-type plants. More concretely, the method is applicable to the Wiener-type plants whose dynamic may be described with an all-pole (poles-only) transfer function (i.e. transfer function without any zeros and without pure transport delay), and whose nonlinearity satisfies a simple regularity condition described later in the paper. It is based on a relay feedback, and

allows parametric identification of the linear part of the plant and obtaining a set of points on the input-output characteristic of the nonlinear part of the plant. The obtained set of points may also be used to obtain a parametric model of the nonlinearity.

The interesting point of the method is that it first identifies the linear part of the plant and later the knowledge of the linear part is used to identify the nonlinear part of the plant. This is in contrary to the most of other identification methods, which first try to identify somehow the nonlinear part of the plant, then use obtained knowledge for compensating the nonlinearity before identification the nonlinear part of the plant.

The method does not require any extra equipment (except the relay) for the identification of the linear part of the plant. The identification of the nonlinear part of the plant may be performed without extra equipment too, but in this case the procedure is somewhat lengthy. There is an alternative much faster approach, where the complete input-output characteristic of the nonlinear part is recorded in one experiment. However, it requires the  $x$ - $y$  registrator and the programmable linear block for creating the model of the identified linear part of the plant. For this purpose, a process computer may be used, which is usually present in any more complex process.

Section II of the paper shows the proposed procedure for the parametric identification of the linear part of the plant. Section III demonstrates the first (slower) approach for obtaining points on the input-output characteristic of the nonlinear part of the plant. Section IV demonstrates the second (faster) approach for identification of the nonlinear part of the plant. Finally, Section V illustrates the proposed theory through a simulation example.

## II. THE PARAMETRIC IDENTIFICATION OF THE LINEAR PART OF THE PLANT

The structure of the control loop extended with a relay in the feedback, as used for experiments, is shown in Fig. 1.

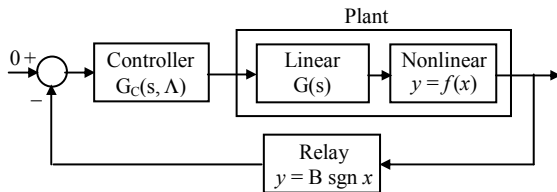


Fig. 1. Extended control loop with a relay in the feedback

The linear part of the plant has a transfer function  $G(s)$ , the nonlinear part of the plant is described by an input-output relation  $y = f(x)$ , and the relay  $y = B \operatorname{sgn} x$  may be as well described using the describing function  $N(a) = 4B/(\pi a)$ . Of course, both  $f(x)$ , and  $G(s)$  are unknown. However, for the identification purpose, it will be assumed that  $G(s)$  may be modeled as  $G_M(s, \Pi)$ , where  $G_M$  (model) is a function whose shape is principally known, but which depends of a vector  $\Pi = \{p_1, p_2, \dots, p_n\}$  of unknown parameters (that should be determined). In other words, for the identification purpose, it will be assumed that  $G(s) = G_M(s, \Pi)$ .  $G_C(s, \Lambda)$  is the known transfer function of the controller that depends of the vector of tunable parameters  $\Lambda$ . The relay parameter  $B$  is tunable too.

For the purpose of this paper, the following additional assumptions about the plant will be taken:

- It is known that the plant may be adequately represented as a Wiener-type non-linear model;
- The linear part of the plant has a transfer function  $G(s)$  which is an all-pole function (i.e. without finite zeros and without pure transport delay);
- The linear part of the plant has a low-pass frequency characteristic;
- The nonlinear part of the plant is described by an input-output relation  $y = f(x)$  where  $f(x)$  satisfies the regularity condition  $f(x) > 0$  for  $x > 0$  and  $f(x) < 0$  for  $x < 0$ ;
- The plant need not to be stable, but the used controller must be able to stabilize the plant.

Some of these assumptions may be relaxed. For example, if the frequency characteristic of the plant is not a low-pass characteristic, it is possible to insert an additional low-pass filter in the loop, whose transfer function may be included in the transfer function of the controller. Also, although the given regularity condition for  $f(x)$  is true for common nonlinearities, it is possible to extend the procedure for some other types of nonlinearities too. Moreover, there are some ideas how to relax the requirements about the all-pole nature of  $G(s)$ . Such extensions will be presented in follow-up papers.

Under the stated regularity assumption about  $f(x)$ , we have  $\operatorname{sgn} f(x) = \operatorname{sgn} x$ . Then, it follows that the cascade of the plant nonlinearity and the relay may be represented simply as a pure relay, so we have simply the linear object with the relay in the feedback. It is known that it is quite easy to establish stable periodical oscillations in such loops under very general conditions, as described in [5] and [6]. Assuming that the frequency of established oscillations is  $\omega^*$ , and that the oscillations arise under the controller setting  $\Lambda = \Lambda^*$  and the relay setting  $B = B^*$ , the principle of harmonic balance gives

$$1 + N(a^*) G_C(j\omega^*, \Lambda^*) G_M(j\omega^*, \Pi) = 0 \quad (1)$$

Here,  $a^*$  is the amplitude of established oscillations at the internal point between the linear and the nonlinear part of the plant. This point is inaccessible for measurements, and this fact is the main reason why Wiener-type plants are usually regarded as harder for the identification than Hammerstein-type plants. However, in the approach described in the paper, it is possible to eliminate  $N(a^*)$  completely, and consequently to remove dependence of  $a^*$ . Really, (1) may be rewritten as

$$G_C(j\omega^*, \Lambda^*) G_M(j\omega^*, \Pi) = -1 / N(a^*) \quad (2)$$

As  $N(a^*)$  is a real number, it follows that

$$\operatorname{Im} \{G_C(j\omega^*, \Lambda^*) G_M(j\omega^*, \Pi)\} = 0 \quad (3)$$

Also, under stated assumption about all-pole nature of  $G(s)$ , the model of the transfer function of the linear part of the plant may be expressed as  $G_M(s, \Pi) = 1/P_n(s, \Pi)$ , where  $P_n(s, \Pi)$  is a polynomial given by

$$P_n(s, \Pi) = p_n s^n + \dots + p_2 s^2 + p_1 s + 1 \quad (4)$$

The free term of  $P_n(s, \Pi)$  may be set to 1 without any loss of generality (in fact, it means that the static gain of the linear part of the plant is set to 1). Really, the eventual overall plant static gain different from 1 may be considered as the part of the nonlinear function  $y=f(x)$ . Now, (3) may be expressed as

$$\text{Im} \{G_C(j\omega^*, \Lambda^*) / P_n(j\omega^*, \Pi)\} = 0 \quad (5)$$

After simple transformations, (5) becomes

$$\begin{aligned} \text{Re} \{G_C(j\omega^*, \Lambda^*)\} (p_1 - p_3 \omega^2 + \dots) \omega = \\ = \text{Im} \{G_C(j\omega^*, \Lambda^*)\} (1 - p_2 \omega^2 + \dots) \end{aligned} \quad (6)$$

This equation is linear in all unknown coefficients from  $\Pi$ . So, to determine all of them uniquely, it is enough to collect  $n$  experimental pairs  $(\omega_k^*, \Lambda_k^*)$ ,  $k = 1..n$  with different frequencies adjusted by controller settings. It seems that the value of  $B^*$  is irrelevant, as (6) does not depend of it, but this is not quite true, because the obtained frequency  $\omega^*$  may depend of  $B^*$ . Note also that it is not necessary to measure  $a^*$  at all. Therefore, the linear part of the plant may be identified without any knowledge of the plant nonlinearity, and even without measuring the amplitude. However, the plant gain can not be determined yet, as it is amplitude-dependent and incorporated into  $f(x)$ .

Someone may object that the procedure given above is suspicious, due to an approximate nature of the principle of harmonic balance. However, it works quite well under the assumption that the product  $G_C(s, \Lambda) G_M(s, \Pi)$  is of enough high order so that its dynamic has strongly low-pass nature, which is usually true for industrial-type plants. Otherwise, it is always possible to insert an extra low-pass filter at the plant input to reduce the influence of higher harmonics. In such case, the (known) transfer function of the filter may be considered as the part of the controller transfer function.

Of course, it is possible to collect more than  $n$  experimental pairs. Then, the Mean Least Square (MLS) approach may be used to determine  $\Pi$  from such over-determined set of equations. In fact, such approach is not only possible, but strongly recommended, because it can reduce the influence of measurement errors and offer an indicator about the correctness of the assumed model of  $G(s)$ . Really, unacceptable high value of the mean square error (MSE) is a good sign that something is wrong with the model (for example, it will happen if the assumed order of  $G(s)$  is too small).

Note that from the theoretical aspect of view, the relay itself is not necessary at all to identify the linear part of the plant. Namely, the same experiment may at least in principle be performed without the relay, and the same equations will still hold. In such case,  $N(a)$  is simply the describing function of the unknown nonlinear part of the plant. However, it is much easier to establish stable oscillations in the closed loop with a relay. In addition, the presence of relay allows the regulation of the amplitude of established oscillations (by changing  $B$ ). Without the relay, the amplitude of generated oscillations in the loop is hardly controllable.

### III. THE IDENTIFICATION OF THE NONLINEAR PART OF THE PLANT – THE FIRST (SLOWER) APPROACH

After the linear part of the plant is identified, it is necessary to perform another set of experiments to identify the nonlinear part of the plant. This paper proposes two approaches to achieve this goal. In the first approach, no extra equipment is necessary, but it requires more experimental runs. To apply this approach, the relay from the feedback should be moved to the place between the controller and the plant, as shown in Fig. 2.

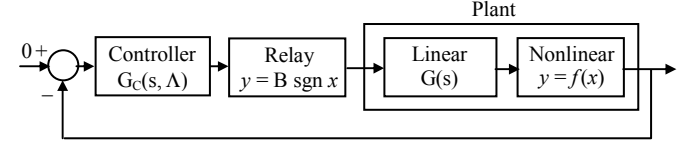


Fig. 2. Extended control loop for the identification of the nonlinear part

Note that the output of the linear part of the plant (i.e. the input of the nonlinear part of the plant) is an internal point of the plant that is not available for measurements. In fact, this is the main reason why Wiener-type systems are usually regarded as hard for the identification. However, it is obvious from Fig. 2 that the signal at the plant input (after initial transients vanishes) is a periodic square wave with the amplitude  $B^*$  and the fundamental frequency  $\omega^*$ , where  $B^*$  is the current setting of the relay. Such square wave may be represented using Fourier series, in which the first harmonic has the amplitude  $4B^*/\pi$ . Therefore, the signal at the output of the linear part of the plant is a periodic wave whose first harmonic has the amplitude  $4B^*|G(j\omega^*)|/\pi$ . But, under the stated assumption about low-pass nature of the plant, the amplitude of higher-order harmonics in the signal at the output of the linear part of the plant is negligible. So, it is possible to consider this signal as a pure sine wave with amplitude

$$A^* = 4B^*|G(j\omega^*)|/\pi \quad (7)$$

Strictly speaking,  $G(s)$  is not known, so  $A^*$  can not be calculated. But, after the identification of the linear part is performed, we have  $G(j\omega^*) \approx G_M(j\omega^*, \Pi)$ , so (7) becomes

$$A^* \approx 4B^*|G_M(j\omega^*, \Pi)|/\pi \quad (8)$$

This means that  $A^*$  may be easily calculated. But, if the signal at the input of the nonlinear part of the plant is really a sine wave with amplitude  $A^*$ , the signal at the output of the plant is a periodic wave with amplitude (more precise, the pick value)  $f(A^*)$ . As the plant output is available for the measurements, it is possible to measure  $f(A^*)$ . Therefore, the experiment determines uniquely one point  $(A^*, f(A^*))$  on the input-output characteristic of the nonlinear part of the plant. By performing more experiments with different settings of  $B$ , it is possible to obtain as many points as necessary. Note that it is not necessary to change the settings  $\Lambda^*$  of the controller.

The accuracy of the procedure depends of how close is  $A^*$  to the true amplitude of the signal at the output of the linear part of the plant, i. e. of how well this signal may be regarded as a pure sine wave. Therefore, a rough estimation of its

“purity” will be presented. It is known that the slope of the Bode plot of the linear part of the loop in the neighbor of the critical frequency  $\omega^*$  is about 40 db per decade. As the plant transfer function is all-pole function, the slope at the frequency  $3\omega^*$  can only be greater than this value. It follows that the ratio of the magnitudes of the loop transfer function at frequencies  $\omega^*$  and  $3\omega^*$  is at least  $3^2 = 9$  (it will be even greater if the plant has poles with absolute values between  $1/(3\omega^*)$  and  $1/\omega^*$ ). As the magnitude of the third harmonic of the square wave is one third of the magnitude of the principal harmonic, it follows that the magnitude of the third harmonic at the output of the linear part of the plant should be at least 27 times smaller than the magnitude of the principal harmonic.

Of course, some attenuation of the third harmonic (and other higher ones) is caused by the controller too. However, assuming that the controller is a PI controller, the attenuation of the higher harmonics caused by the controller is often not too significant. Namely, the transfer function of the PI controller is nearly constant on higher frequencies (i.e. the integral action of the controller on higher frequencies is relatively small). Therefore, it is possible to expect similar ratio of the magnitude of the third and the first harmonic (1/27 or less) in the signal measured on the plant output too. Similarly, the influence of higher harmonics is even much smaller, so it can be ignored. This means that it is possible to expect that the signal at the output of the linear part of the plant will really be nearly sinusoidal. More precisely, as  $1/27 \approx 0.037$ , the expected relative difference between  $A^*$  and the true value of the amplitude of the signal at the output of the linear part of the plant is smaller than 4%. Of course, after the linear part of the plant is estimated, it is possible to calculate the ratio  $3|G_M(j\omega^*, \Pi)|/|G_M(3j\omega^*, \Pi)|$  to check the validity of the previous intuitive reasoning.

After enough points on the input output relation  $f(x)$  are obtained, it is possible to form a parametric model of  $f(x)$  using some curve fitting. The polynomial model of  $f(x)$  may be quite adequate for this purpose.

#### IV. THE IDENTIFICATION OF THE NONLINEAR PART OF THE PLANT – THE SECOND (FASTER) APPROACH

The described approach works well, although it requires a lot of experimental runs to collect enough points to obtain good model of  $f(x)$ . It is possible to collect all information in just one experimental run, but it requires more equipment, as shown in Fig. 3.

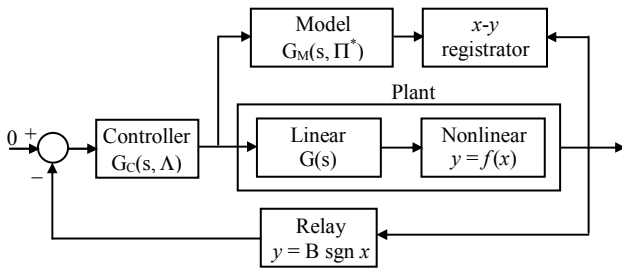


Fig. 3. Extended control loop for the identification of the nonlinear part

Really, after the linear part of the plant is identified, we have the model  $G_M(s, \Pi^*)$  of the linear part of the plant, where

$\Pi^*$  is obtained as the result of the identification. To identify the nonlinear part of the plant quickly, it is necessary to make the physical realization of the model  $G_M(s, \Pi^*)$ . The easiest way to do this is using a process computer, which is usually already presented in the more complex processes. After this, the output from the model and the output from the real plant should be connected on x-y registrator, as shown in Fig. 3.

In this approach, the main problem with the inaccessibility of the internal point in the Wiener-type plants that makes the identification of such models hard is completely avoided by forming a model of the linear part of the plant, which is already identified. The registration should be performed using such values of the controller parameters and the amplitude of the relay that the output value covers all values of  $y$  for which the input-output characteristic  $f(x)$  need to be recorded.

It is clear that if the model perfectly describes the exact transfer function of the linear part of the plant, then the x-y registrator will record the exact curve  $f(x)$ . However, in the practice, the recorded curve always has a hysteresis. Namely, if there is any difference between  $G(s)$  and  $G_M(s, \Pi^*)$ , the registrator will behave as if it is connected to the input and the output of the cascade of the linear block with the transfer function  $G(s)/G_M(s, \Pi^*)$ , which leads to the hysteresis, and the static nonlinearity  $f(x)$ . If the model is good enough, the hysteresis should be small. Therefore, significant hysteresis is a good indicator that the parametric identification of the linear part is not well done, or that the assumed model of the transfer function  $G_M(s, \Pi)$  does not describe adequately the dynamics of the plant. Therefore, the size of the hysteresis may be used to check whether the identification of the linear part of the plant is correctly performed or not.

#### V. THE SIMULATION EXAMPLE

The presented theory will be illustrated through a simulation example using SIMULINK. The linear part of the plant is the third order linear system with the transfer function

$$G(s) = 1 / (2s^3 + 3s^2 + 2s + 1) \quad (9)$$

For the identification purpose, it will be modeled as

$$G_M(s, \Pi) = 1 / (p_3 s^3 + p_2 s^2 + p_1 s + 1), \quad \Pi = \{p_1, p_2, p_3\} \quad (10)$$

The nonlinear part of the plant is a static nonlinearity described with the input-output characteristic  $f(x) = \arctan x$ . Such shape of nonlinearity is taken because it may be used as a simple first approximation of the saturation-type curves that arise in real objects. For example, all valve-based systems and systems where the output is limited due to any reason have similar shape of nonlinear characteristic. The controller is an ordinary PI controller with the transfer function

$$G_R(s, \Lambda) = \lambda_1 + \lambda_2 / s, \quad \Lambda = \{\lambda_1, \lambda_2\} \quad (11)$$

For the identification of the linear part of the plant, three experiments are performed. The obtained experimental results are summarized in Table I.

TABLE I. EXPERIMENTAL DATA FOR THE IDENTIFICATION OF THE LINEAR PART OF THE PLANT

Experiment	$\lambda_1^*$	$\lambda_2^*$	$B^*$	$\omega^*$
I	10	10	1.918	0.7088
II	10	0.1	1.918	1.0441
III	10	5	1.918	0.8185

Applying (6) and solving obtained set of equations gives  $p_3=1.883$ ,  $p_2=3.119$  and  $p_1=2.074$ . Obviously, the obtained values are quite close to the true values  $p_3=2$ ,  $p_2=3$  and  $p_1=2$ . The relative estimation error is smaller than 6%.

The next goal is to identify the nonlinear part of the plant. We will first demonstrate the slower approach (point-by-point recording). For this purpose, thirteen different experiments are performed, and the obtained experimental results are shown in Table II. In this table, values of  $\omega^*$  and  $f(A^*)$  are measured (from the output signal), and  $A^*$  is calculated using (8).

TABLE II. EXPERIMENTAL DATA FOR THE IDENTIFICATION OF THE LINEAR PART OF THE PLANT

Experiment	$\lambda_1^*$	$\lambda_2^*$	$B^*$	$\omega^*$	$A^*$	$f(A^*)$
I	35	0.001	0.1	0.9973	0.0603	0.0661
II	35	0.001	0.2	0.9973	0.1206	0.1282
III	35	0.001	0.5	0.9971	0.3016	0.3114
IV	35	0.001	0.7	0.9972	0.4222	0.4234
V	35	0.001	0.9	0.9972	0.5428	0.5250
VI	35	0.001	1.1	0.9972	0.6634	0.6161
VII	35	0.001	1.3	0.9971	0.7843	0.6967
VIII	35	0.001	1.5	0.9971	0.9049	0.7956
IX	35	0.4	2.0	0.9961	1.2099	0.9326
X	35	0.4	2.5	0.9958	1.5136	1.0323
XI	30	0.8	3.0	0.9951	1.8199	1.1070
XII	30	0.8	3.5	0.9943	2.1279	1.1644
XIII	30	0.8	4.0	0.9937	2.4360	1.2096

Fig. 4. shows the obtained points of the nonlinear input-output characteristic, together with the curve obtained using the linear interpolation (solid line), and the true input-output characteristic (dashed line).

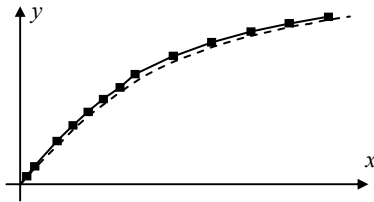


Fig. 4. True and identified input-output characteristic – the first approach

Obviously, the obtained points describe the true input-output characteristic quite well. Notice that all experiments in Table II are performed with nearly constant frequency. At the mean frequency  $\omega^*=0.9963$ , the expected attenuation of the third harmonic is

$$3 |G(j\omega^*)| / |G_M(3j\omega^*)| = 81.89 \quad (12)$$

based on the true transfer function, or

$$3 |G_M(j\omega^*, \Pi)| / |G_M(3j\omega^*, \Pi)| = 73.5 \quad (13)$$

based on the obtained model. Such strong attenuation is due to the presence of plant poles with absolute values that lay between  $1/(3\omega^*)$  and  $1/\omega^*$ .

Now, we will demonstrate the method for the quick identification of the nonlinear part of the plant from only one experimental run. To achieve this, a model of the linear part is formed and the procedure shown on Fig. 3 is applied. The obtained figure on the x-y registrator is shown on Fig. 5.

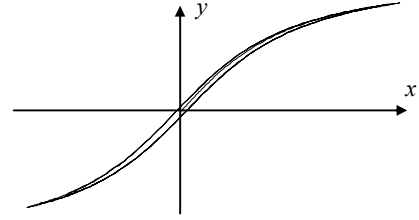


Fig. 5. The identified input-output characteristic – the second approach

As expected, a hysteresis is visible on the diagram. But, as the hysteresis is quite small, this is a good indication that the identification of the linear part of the plant is well performed. Also, the recorded curve is obviously very similar to the real shape of the nonlinearity  $y=f(x)$ .

To check the quality of the performed identification, two parametric models of  $f(x)$  are tried. In the first case,  $f(x)$  is modeled as a 3rd-order polynomial

$$f_{M1}(x) = a_3x^3 + a_2x^2 + a_1x + a_0 \quad (14)$$

The coefficients  $a_i, i=0..3$  are estimated using the MLS approach based on 800 samples of the recorded input-output characteristic. The obtained coefficients are  $a_3=-0.0923$ ,  $a_2=0.0072$ ,  $a_1=0.9098$  and  $a_0=-0.0019$ .

In the second case,  $f(x)$  is modeled as a 5th-order polynomial

$$f_{M2}(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 \quad (15)$$

The coefficients  $a_i, i=0..5$  are estimated using the same approach. The obtained values are  $a_5=0.0243$ ,  $a_4=-0.0034$ ,  $a_3=-0.2044$ ,  $a_2=0.0071$ ,  $a_1=1.0138$  and  $a_0=-0.0003$ . Both models  $f_{M1}(x)$  and  $f_{M2}(x)$  together with the appropriate data points are shown in Fig. 6.

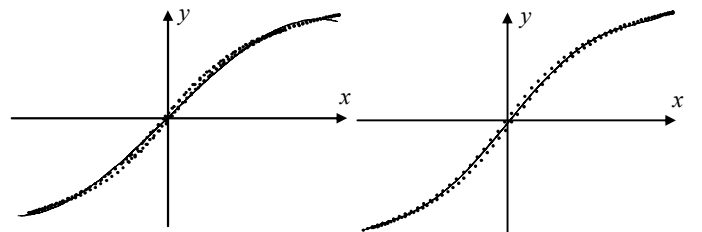


Fig. 6. 3<sup>rd</sup>-order (left) and 5<sup>th</sup>-order (right) model of the plant nonlinearity

Fig. 7 shows the true characteristic  $f(x)$  (solid line) and the characteristics based on the models  $f_{M1}(x)$  and  $f_{M2}(x)$  (dashed line) in the same coordinate system.

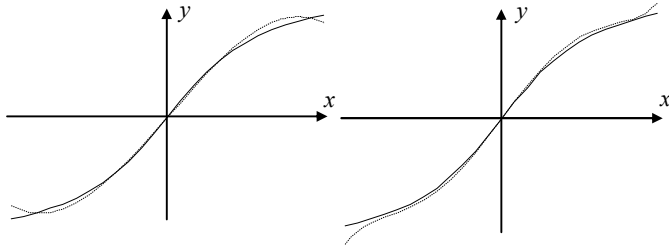


Fig. 7. Comparison between the true plant nonlinearity and the obtained 3<sup>rd</sup>-order (left) or 5<sup>th</sup>-order (right) model

These pictures show that both models approximate the real input-output characteristic of the nonlinear part of the plant quite well. The 5<sup>th</sup>-order model is slightly better and applicable for somewhat wider range of the input values.

Finally, to estimate the overall quality of the identification, the output of the real plant (in the complete loop used for the identification) is compared with the output of the identified model in Fig. 8. The dashed line is the output from the real plant, and the solid line is the output from the identified model.

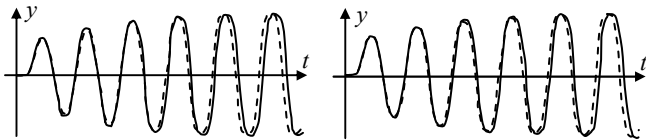


Fig. 8. Comparison of the output of the real and identified plant based on 3<sup>rd</sup>-order (left) and 5<sup>th</sup>-order nonlinear model

In both cases, the same model of the linear part of the plant is used. The similarity in behavior between the real plant and the identified model is quite satisfactory. The small difference between the period of the oscillations in the real and the identified plant is caused by errors in the identification of the linear part of the plant.

## VI. CONCLUSION

This paper proposes a novel frequency-based closed loop method for the identification of both the linear and the nonlinear part of some types of Wiener-type nonlinear plants. It takes into the consideration unknown non-inertial nonlinearities at the plant output, which usually cause severe errors in the most of the other frequency-based methods. The application of the method requires only the controller, usually PI or PID controller, and the relay. Optionally, for the quick identification of the nonlinear part of the plant, the  $x$ - $y$  registrator and the programmable linear block for creating the model of the identified linear part of the plant is necessary. Usually, the process computer is used for this purpose.

The procedure for the identification of the linear part of the plant is quite insensitive to a process noise, because the information is extracted from the frequency, which is not corrupted much even in a presence of noise. It is relatively fast and requires small amount of experimental runs. The same is true for the quick approach for recording the input-output

characteristic of the nonlinear part of the plant. However, the point-by-point recording of the input-output characteristic of the nonlinear part of the plant requires more experimental runs, and it is more sensitive to the measurement noise, because the amplitude should be measured too. The method probably cannot be used for online and real-time identifications.

This paper shows only how to identify a plant whose dynamic is low-pass and without finite zeros. How to overcome this limitation is the active topic of our current research, and eventual results will be presented in the follow up papers.

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