Fuzzy Sampled Controller Design for Consensus of Multiagent Networks with Varying Connections

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***Abstract*—T-S fuzzy models are first presented to describe multiagent networks (MANs) with varying connections. The nodes of each T-S fuzzy model are rearranged so that the global fuzzy model is decomposed into independent and small-scale fuzzy models. It is shown that the consensus of the global fuzzy model is equivalent to that of its corresponding small-scale fuzzy models in which the sampled controller is applied. A sufficient condition is derived to ensure the consensus of the sampled controlling fuzzy models.**

I. Introduction

An important application area of multiagent networks is the consensus problem, since the pioneering work stemming from management science and statistics in 1960s (see [1] and the references therein). Consensus is a basic and fundamental research topic in decentralized control of networks of dynamic agents and has attracted great attention which is partly due to its broad applications in cooperative control of communication networks, design of sensor networks, swarm-based computing, etc. ([2]-[7]).

In the literature related to the consensus problem of multiagent networks, a connection between the nodes is assumed to either always exist or be always nonexistent. Obviously, network connections may be changed due to some influences such as time, external environment, and temperature. Over the past few decades, the Takagi-Sugeno (T-S) fuzzy model has been proven to be an effective model to describe many nonlinear and complex systems with unstructured uncertainties ([8]-[12]). Motivated by the characteristics of the fuzzy model, we shall try to regard the obscure influences caused by the varying network connections as a fuzzy set. That is, T-S fuzzy models will be applied to describe a network model with varying connections.

As we know, MANs may not reach a consensus when its connections are varying. As a result, effective control schemes have to be designed to force the complex network to achieve a consensus. Note that, some real-world applications can be modeled by continuous-time systems together with some discrete-time controllers such as impulsive responses, sampled data, and so on. Hence, the sampled controllers will be designed to achieve the consensus of multiagent networks with varying connections in this paper. The contribution of this paper is presented as follows:
1) T-S fuzzy systems will be first presented to address multiagent models with varying connections in this paper. For the proposed fuzzy models, a node-rearrangement method will be used to decompose the large-scale fuzzy models into independent and small-scale fuzzy models. Moreover, the consensus of every large-scale fuzzy model is equivalent to that of its corresponding small-scale fuzzy models.
2) Sampled controllers will be designed for the small-scale fuzzy models. With sampled controllers, the controlled fuzzy models are hybrid systems. The sampled controllers will be then considered as continuous delays with the a transformation in [15]. The hybrid fuzzy models are changed as continuous fuzzy systems with time-varying delays. A simple control method will be used to achieve a prescribed consensus.

The remainder of this paper is organized as follows. In Section 2, some definitions about directed graph are presented. The nodes rearrangement approach is addressed in Section 3. In Section 4, T-S fuzzy systems are first presented to address multiagent models with varying connections. A sampled controller is then designed to achieve the prescribed consensus. In Section 5, simulations are carried out to illustrate the effectiveness of the main results. Finally, conclusions are drawn in Section 6.

II. Preliminaries

Let $\mathcal{G}(\mathcal{V},\varepsilon,\mathcal{A})$ be a digraph of order n with the set of nodes $\mathcal{V}=\{v_1,v_2,\cdots,v_n\}$, the set of edges $\varepsilon\subseteq\mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $\mathcal{A}=(a_{ij})_{n\times n}$. An edge of $\mathcal{G}$ is denoted by $e_{ij}=(v_i,v_j)$, where $e_{ij}\in\varepsilon$ means that there is a directed connection from node v_j to node v_i. The entry $a_{ij}>0$ if $e_{ij}\in\varepsilon$, and $a_{ij}=0$ otherwise. Moreover, it is assumed that $a_{ii}=0$ for all $i\in\{1,2,\cdots,n\}$. The Laplacian of the directed graph is defined as $L=(l_{ij})_{n\times n}=\Delta-\mathcal{A}$, and

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$\Delta = (\Delta_{ij})_{n\times n}$ is a diagonal matrix with $\Delta_{ii} = \sum_{j=1}^{n} a_{ij}$. A digraph is a *spanning tree* if it has m vertices and $m-1$ edges and there exists a root vertex with directed paths to all other vertices.

Assume that a network system has n agents and each agent is regarded as a node in a directed graph $\mathcal{G}$. Let $\overline{x}_i(t) \in R$ denote the state of agent v_i, then $\mathcal{G}_{\overline{x}} = (\mathcal{G}, \overline{x}(t))$ with $\overline{x}(t) = (\overline{x}_1(t), \overline{x}_2(t), \cdots, \overline{x}_n(t))^T$ is a directed network. Agents v_i and v_j in the directed network are said to reach a (an) *consensus* (*agreement*) if and only if $||\overline{x}_i(t) - \overline{x}_j(t)|| \to 0$ as $t \to +\infty$, for any $i, j \in \{1, 2, \cdots, n\},\ i \neq j$. If the nodes are all in an agreement, the common value $\mathcal{X}(\overline{x})$ is called the *group decision value*.

The general multiagent network without missing connections has the following dynamics

$$\frac{d\overline{x}_i(t)}{dt} = \sum_{v_j \in N_i} a_{ij}(\overline{x}_j(t) - \overline{x}_i(t)), \tag{1}$$

where $N_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \varepsilon\}$ is the set of neighbors of node v_i, $i, j = 1, 2, \cdots, n$, and $\overline{x}_i(t)$ is the state of agent v_i. The dimension of $\overline{x}_i(t)$ could be arbitrary as long as it is the same for all agents. $A = (a_{ij})_{n\times n} \in R^{n\times n}$ is the weighted matrix. In this paper, for simplification, we only analyze the case when the dimension of $\overline{x}_i(t)$ is one. It is worth noticing that our analysis is valid for any dimension when the system models are rewritten with Kronecker products.

According to the definition of the Laplacian matrix L, (1) can be rearranged as

$$\frac{d\overline{x}(t)}{dt} = -L\overline{x}(t), \tag{2}$$

where $\overline{x}(t) = (\overline{x}_1(t), \overline{x}_2(t), \cdots, \overline{x}_n(t))^T$.

III. Nodes rearrangement approach

In this paper, we consider that the connections between nodes may change in the process of information transmission due to some affections such as time, external environment, temperature and so on. Here, the mentioned affections are obscure and difficult to be elaborated clearly, which can be regarded as a fuzzy set. A fuzzy dynamic model has been proposed by Takagi and Sugeno [8] to represent different linear/nonlinear systems of different rules. Based on this, we shall construct T-S fuzzy models to describe multiagent networks with varying connections.

Similar to [9], we consider a T-S fuzzy multiagent model, in which the ith rule is formulated in the following form:
Plant Rule i:
IF $\overline{x}_1(t)$ is η_{i1} and $\overline{x}_2(t)$ is η_{i2} and $\cdots$ and $\overline{x}_n(t)$ is η_{in},
THEN

$$\frac{d\overline{x}(t)}{dt} = -\overline{L}_i\overline{x}(t), \quad i = 1, 2, \cdots, r, \tag{3}$$

where $\overline{x}(t) = (\overline{x}_1(t), \overline{x}_2(t), \cdots, \overline{x}_n(t))^T$, η_{ij} is the Fuzzy set, and $\overline{L}_i$ is the ith Laplacian matrix corresponding to the ith rule.

The defuzzified output of the T-S fuzzy system (3) is represented as shown in the following

$$\frac{d\overline{x}(t)}{dt} = \frac{\sum\limits_{i=1}^{r} v_i(\overline{x}(t))[-\overline{L}_i\overline{x}(t)]}{\sum\limits_{i=1}^{r} v_i(\overline{x}(t))} = -\sum_{i=1}^{r} h_i(\overline{x}(t))\overline{L}_i\overline{x}(t), \tag{4}$$

where $i = 1, 2, \cdots, r,\ v_i(\overline{x}(t)) = \Pi_{j=1}^{n}\eta_{ij}(\overline{x}_j(t)),\ h_i(\overline{x}(t)) = \frac{v_i(\overline{x}(t))}{\sum_{i=1}^{r} v_i(\overline{x}(t))}$, and $\eta_{ij}(\overline{x}_j(t))$ is the membership function of $\overline{x}_j(t)$ in η_{ij}.

A basic property of $v_i(\overline{x}(t))$ is that

$$v_i(\overline{x}(t)) \geq 0, \quad i = 1, 2, \cdots, r, \quad \sum_{j=1}^{r} v_j(\overline{x}(t)) > 0, \tag{5}$$

and therefore,

$$h_i(\overline{x}(t)) \geq 0, \quad i = 1, 2, \cdots, r, \quad \sum_{j=1}^{r} h_j(\overline{x}(t)) = 1, \tag{6}$$

for $\forall\ t \in R$.

In this paper, we do not assume that graph $\mathcal{G}$ contains a spanning tree. Clearly, fuzzy network (4) may not reach a consensus when its connections are varying. As a result, some control schemes have to be designed to achieve a consensus of fuzzy network (4). As mentioned in [6], it is difficult to know which nodes are needed to be controlled for a large-scale network. Hence, we shall rearrange the node order of fuzzy network (4) in the following. In a graph, those root nodes are called the leaders and the other nodes are called the followers. For the original graph G_i, construct the rearranged graph $\widehat{G}_i$ as follows:

The rearrangement algorithm
1) Note that G_i may not be connected. For any $i \in \{1, 2, \cdots, r\}$, find out all Strongly Connected Components G_i^j $(j \in \{1, 2, \cdots, \Lambda\},\ \iota = \sum\limits_{j=1}^{\Lambda} i_j \leq n,\ i_j \geq 1$ is the number of nodes in G_i^j and $\Lambda \leq n$ is an integer) of graph G_i by using the algorithms in [13], [14]. Then, the nodes in G_i^j are all root nodes of graph G_i and the other nodes are all followers.
2) Rearrange the numerical orders of all nodes. Mark all root nodes as $1, 2, \cdots$ and number the followers behind the root nodes.
3) Graph G_i is rearranged as graph $\widehat{G}_i$. The Laplacian matrix L_i of graph $\widehat{G}_i$ can be written as

$$L_i = \begin{pmatrix} L_i^1 & & & 0 & \cdots & 0 \\ & \ddots & & \vdots & \cdots & \vdots \\ & & L_i^{\Lambda} & 0 & \cdots & 0 \\ L_i^{\iota+1,1} & \cdots & L_i^{\iota+1,\iota} & L_i^{\iota+1,\iota+1} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ L_i^{n,1} & \cdots & L_i^{n,\iota} & L_i^{n,\iota+1} & \cdots & L_i^{n,n} \end{pmatrix}.$$

Here, L_i^j $(j \in \{1, 2, \cdots, \Lambda\})$ are irreducible square matrices and in each line of the rest $n - \iota$ lines, there exists at least one entry satisfying $L_i^{\iota+i,j} \neq 0$ $(i = 1, 2, \cdots, n - \iota,\ j = 1, 2, \cdots, \iota + i - 1)$.

Based on the rearrangement algorithm, system (3) can be rewritten as
Plant Rule i:

IF $\overline{x}_1(t)$ is η_{i1} and $\overline{x}_2(t)$ is η_{i2} and $\cdots$ and $\overline{x}_n(t)$ is η_{in}, THEN

$$\frac{dx(t)}{dt} = -L_i x(t), \quad i = 1, 2, \cdots, r, \tag{7}$$

where $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ is a node-rearrangement of $\overline{x}(t)$. The defuzzified output of the T-S fuzzy system (7) is

$$\frac{dx(t)}{dt} = \frac{\sum_{i=1}^{r} v_i(\overline{x}(t))[-L_i x(t)]}{\sum_{i=1}^{r} v_i(\overline{x}(t))} = -\sum_{i=1}^{r} h_i L_i x(t), \tag{8}$$

where $h_i(\overline{x}(t))$ is simplified as h_i. We introduce a virtual leader such that system (3) want to realize the prescribed consensus value $s_0 \in R$ (let $\overline{x}_0 = (s_0, s_0, \cdots, s_0)^T \in R^n$). Correspondingly, the prescribed consensus vector of system (8) is also $x_0 = (s_0, s_0, \cdots, s_0)^T \in R^n$. We consider the sampled control law for the fuzzy system (8) as follows:
Controller Rule i:
IF $\overline{x}_1(t)$ is η_{i1} and $\overline{x}_2(t)$ is η_{i2} and $\cdots$ and $\overline{x}_n(t)$ is η_{in}, THEN

$$u_i(t) = u_i(t_l) = S_i(x(t_l) - x_0), \quad t_l \leq t < t_{l+1}, \tag{9}$$

where $i = 1, 2, \cdots, r, \quad S_i = diag(S_{i1}, S_{i2}, \cdots, S_{in})$ is a diagonal matrix. The discrete-time control signal is assumed to be generated by a zero-order hold function with a sequence of hold times $0 = t_0 < t_1 < \cdots < t_l < \cdots$. Here, $\forall\ l \geq 0,\ t_{l+1} - t_l \leq h$, $\lim_{t\to\infty} t_l = \infty$ and h is a positive number. Similar to the continuous control law, the overall sampled fuzzy controller can be given by $u(t_l) = \sum_{i=1}^{r} h_i S_i (x(t_l) - x_0)$. As a result, the T-S fuzzy discrete-time control system of model (8) is governed by

$$\frac{dx(t)}{dt} = -\sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p [L_i x(t) + S_p(x(t_l) - x_0)]. \tag{10}$$

IV. Main results

We first need the following assumption and lemma to be used in the proofs of our main results.

Lemma 1: For any vectors $x, y \in R^n$ and scalar $\varepsilon > 0$, the following inequality holds:

$$2x^T y \leq \varepsilon x^T x + \varepsilon^{-1} y^T y.$$

Let $y(t) = x(t) - x_0$, then System (10) can be respectively rewritten as

$$\frac{dy(t)}{dt} = -\sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p [L_i y(t) + S_p y(t_l)], \tag{11}$$

where $y(t_l) = x(t_l) - x_0$.

Following [15], we represent the digital control law in (11) as a delayed control as follows:

$$u_i(t_l) = u_i(t - \tau(t)), \quad \tau(t) = t - t_l, \quad t_l \leq t < t_{l+1}. \tag{12}$$

As a result, (11) can be rewritten as

$$\frac{dy(t)}{dt} = -\sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p [L_i y(t) + S_p y(t - \tau(t))]. \tag{13}$$

Theorem 1: Consider the fuzzy system (13). If there exist matrices $U_{ip}^k > 0$, Q^k and M^k with appropriate dimension such that

$$\Phi_1 = \begin{pmatrix} \Upsilon_1 & Q^k + M^k L_i^k + M^k S_p^k \\ * & hU_{ip}^k + M^k + (M^k)^T \end{pmatrix} < 0, \tag{14}$$

and

$$\Phi_2 = \begin{pmatrix} \Upsilon_1 & \Upsilon_2 & -h(Q^k)^T S_p^k \\ * & M^k + (M^k)^T & -hM^k S_p^k \\ * & * & -hU_{ip}^k \end{pmatrix} < 0, \tag{15}$$

where $\forall\ i, p = 1, 2, \cdots, r,\ \Upsilon_1 = (Q^k)^T L_i^k + (L_i^k)^T Q^k + (Q^k)^T S_p^k + S_p^k Q^k$ and $\Upsilon_2 = Q^k + M^k L_i^k + M^k S_p^k$, then system (13) can realize asymptotical stability, i.e., system (10) can achieve the prescribed consensus. Here, $k \in \{1, 2, \cdots, \Lambda\}$, $S_p = diag(S_p^1, S_p^2, \cdots, S_p^\Lambda, 0\cdots, 0)$ and $S_p^k \in R^{i_k \times i_k}$ ($i_k \geq 1$ is the number of nodes in G_i^j) are diagonal matrices.

Proof: System (13) can realize asymptotical stability if and only if the following systems can reach asymptotical stability

$$\frac{dy^k(t)}{dt} = -\sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p [L_i^k y^k(t) + S_p^k y(t - \tau(t))], \tag{16}$$

where $S_p = diag(S_p^1, S_p^2, \cdots, S_p^\Lambda, 0\cdots, 0)$ and $S_p^k \in R^{i_k \times i_k}$ ($k \in \{1, 2, \cdots, \Lambda\}$) are diagonal matrices. Construct the following Lyapunov function

$$V^k(t) = (h - \tau(t)) \int_{t-\tau(t)}^{t} (\dot{y}^k(s))^T U_{ip}^k \dot{y}^k(s) ds. \tag{17}$$

Note that $V^k(t)$ does not increase along the jumps $t_0, t_1, t_2, \cdots$ since $V^k(t) \geq 0$ and $V^k(t) = 0$ at the jumps $t_1, t_2, \cdots$. Thus, the condition $\lim_{t\to t_l^-} V^k(t) \geq V^k(t_l)$ holds.

Since $\frac{dy^k(t-\tau(t))}{dt} = (1 - \dot{\tau}(t))\dot{y}^k(t - \tau(t)) = 0$, one has

$$\begin{aligned} \frac{dV^k(t)}{dt} &= -\int_{t-\tau(t)}^{t} (\dot{y}^k(s))^T U_{ip}^k \dot{y}^k(s) ds \\ &\quad + (h - \tau(t))(\dot{y}^k(t))^T U_{ip}^k \dot{y}^k(t). \end{aligned} \tag{18}$$

Denoting $V_1 = \frac{1}{\tau(t)} \int_{t-\tau(t)}^{t} \dot{y}^k(s) ds$, one has that $\lim_{\tau(t)\to 0} V_1 = \dot{y}^k(t)$. From [15], one has that

$$\int_{t-\tau(t)}^{t} (\dot{y}^k(s))^T U_{ip}^k \dot{y}^k(s) ds \geq \tau(t)(V_1)^T U_{ip}^k V_1. \tag{19}$$

From (13) and $\sum_{i=1}^{r} h_i = 1$, one obtains that

$$\sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p [L_i^k y^k(t) + S_p^k y^k(t - \tau(t)) + \dot{y}^k(t)] = 0.$$

Then, one has

$$\sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p \cdot 2[(y^k(t))^T(Q^k)^T + (\dot{y}^k(t))^T M^k][L_i^k y^k(t) + S_p^k y^k(t) - S_p^k \int_{t-\tau(t)}^{t} \dot{y}^k(s)ds + \dot{y}^k(t)] = 0, \quad (20)$$

where matrices $Q^k,\ M^k \in R^{i_k \times i_k}$ $(k \in \{1, 2, \cdots, \Lambda\})$ are added into the left-hand side of (20).

Combining (18),(19) and (20), one has from $\sum_{i=1}^{r} h_i = 1$

$$\begin{aligned}\frac{dV^k(t)}{dt} &= \sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p\{-\int_{t-\tau(t)}^{t} (\dot{y}^k(s))^T U_{ip}^k \\ &\dot{y}^k(s)ds + (h-\tau(t))(\dot{y}^k(t))^T U_{ip}^k \dot{y}^k(t) + 2[(y^k(t))^T \\ &(Q^k)^T + (\dot{y}^k(t))^T M^k][L_i^k y^k(t) + S_p^k y^k(t) \\ &-S_p^k \int_{t-\tau(t)}^{t} \dot{y}^k(s)ds + \dot{y}^k(t)]\} \\ &\le \sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p[-\tau(t)(V_1)^T U_{ip}^k V_1 + (h-\tau(t))(\dot{y}^k(t))^T \\ &U_{ip}^k \dot{y}^k(t) + 2(y^k(t))^T (Q^k)^T L_i^k y^k(t) \\ &+2(y^k(t))^T (Q^k)^T S_p^k y^k(t) - 2(y^k(t))^T (Q^k)^T \\ &S_p^k \tau(t) V_1 + 2(y^k(t))^T (Q^k)^T \dot{y}^k(t) + 2((\dot{y}^k(t))^T \\ &M^k L_i^k y^k(t) + 2((\dot{y}^k(t))^T M^k S_p^k y^k(t) - 2\tau(t) \\ &(\dot{y}^k(t))^T M^k S_p^k V_1 + 2(\dot{y}^k(t))^T M^k \dot{y}^k(t)]. \end{aligned} \quad (21)$$

Setting $\eta_1(t) = col\{y^k(t), \dot{y}^k(t), V_1\}$, one obtains from (21)

$$\frac{dV^k(t)}{dt} \le \sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p (\eta_1(t))^T \Phi \eta_1(t), \quad (22)$$

where

$$\Phi = \begin{pmatrix} \Upsilon_1 & \Upsilon_2 & -\tau(t)(Q^k)^T S_p^k \\ * & \Upsilon_3 & -\tau(t) M^k S_p^k \\ * & * & -\tau(t) U_{ip}^k \end{pmatrix} < 0, \quad (23)$$

where $\Upsilon_1 = (Q^k)^T L_i^k + (L_i^k)^T Q^k + (Q^k)^T S_p^k + S_p^k Q^k$, $\Upsilon_2 = Q^k + M^k L_i^k + M^k S_p^k$, $\Upsilon_3 = (h-\tau(t))U_{ip}^k + M^k + (M^k)^T$. In (23), $\tau(t) \rightarrow 0$ and $\tau(t) \rightarrow h$ lead to the LMIs $\Phi_1 < 0$, $\Phi_2 < 0$, and Φ_1, Φ_2 are shown in (14) and (15). Let $\eta_0(t) = col\{y^k(t), \dot{y}^k(t)\}$, then (14) and (15) imply (23) since $(\eta_1(t))^T \Phi \eta_1(t) = \frac{h-\tau(t)}{h}\eta_0^T(t)\Phi_1\eta_0(t) + \frac{\tau(t)}{h}(\eta_1(t))^T \Phi_2 \eta_1(t) < 0$, $\forall\ \eta_1(t) \neq 0$.

One can conclude from (14) and (15) that the inequality (23) holds. That is, the fuzzy system (13) can realize asymptotical stability, i.e., system (10) can achieve the prescribed consensus. The proof is completed.

V. Illustrative Examples

In this section, a numerical example is presented to demonstrate the effectiveness of the developed results.

Consider a group of mobile agents with totally 8 agents where each agent has two sensors transmitting and receiving messages over the communication links. Here, the sensors monitor physical or environmental conditions, such as temperature, sound, pressure, etc, and cooperatively pass their data through the network. How to handle the different data to achieve the final asymptotic consensus state? As mentioned in Section 3, we will use the membership function in a fuzzy setting to describe the proportion of the data in different sensors and achieve the final data of the physical condition. For simplification, we only consider that one receives two different data for the same physical condition. Hence, a T-S fuzzy system with 8 nodes is proposed and its rule is as follows:

Plant Rule 1:
IF $\overline{x}_1(t)$ is $\eta_1(\overline{x}_1(t))$,
THEN

$$\frac{d\overline{x}(t)}{dt} = -\overline{L}_1 \overline{x}(t), \quad (24)$$

Plant Rule 2:
IF $\overline{x}_1(t)$ is $\eta_2(\overline{x}_1(t))$,
THEN

$$\frac{d\overline{x}(t)}{dt} = -\overline{L}_2 \overline{x}(t), \quad (25)$$

where $\overline{x}(t) = (\overline{x}_1(t), \overline{x}_2(t), \cdots, \overline{x}_8(t))^T$ and $\overline{x}_i(t)$ is the data state of the ith sensor. The weight communication matrices $\overline{L}_1$ and $\overline{L}_2$ among the 8 sensors are assumed to be

$$\overline{L}_1 = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 3 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 4 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix},$$

$$\overline{L}_2 = \begin{pmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & -1 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}.$$

The membership functions are assumed to be

$$h_1(\overline{x}_1(t)) = \frac{1-\sin^2(\overline{x}_1(t))}{2}, \quad h_2(\overline{x}_1(t)) = \frac{1+\sin^2(\overline{x}_1(t))}{2}.$$

Here, h_1 and h_2 can be seen as the proportion of different data in deciding the final data of the physical condition. The defuzzified output of the T-S fuzzy systems (24) and (25) is

$$\frac{d\overline{x}(t)}{dt} = -\sum_{i=1}^{2} h_i \overline{L}_i \overline{x}(t). \quad (26)$$

For any initial vector, we let $x_0 = (2, 2, \cdots, 2)^T \in R^8$ be the prescribed consensus vector. Fig. 1 shows the state responses for the uncontrolled fuzzy system, which apparently cannot reach a consensus. According to the rearrangment algorithm, $\overline{L}_1$ and $\overline{L}_2$ can be rearranged as

$$L_1 = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & -1 & -1 & -1 & -1 & 0 & 4 \end{pmatrix},$$

$$L_2 = \begin{pmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & -2 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -1 & -2 & -2 & 0 & 5 \end{pmatrix},$$

where $L_1^1 = L_2^2 = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$ and $L_1^2 = L_2^1 = \begin{pmatrix} 2 & 0 & -2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$. The discrete-time control matrices in (10) are designed as follows: $S_1^1 = S_1^2 = S_2^1 = S_2^2 = 4I_{3\times 3}$, and define $h = 0.01$. By using the MATLAB LMI toolbox, LMIs (14) and (15) can be solved with feasible solutions. According to Theorem 1, system (13) can realize asymptotical stability. That is, system (10) (here, $x(t) = x(t)$) can achieve the prescribed consensus. Fig. 2 shows the state responses for the discrete-time control fuzzy system (10), which apparently reach a consensus.

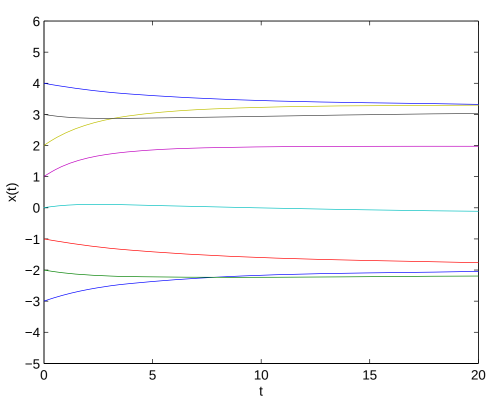

Fig. 1. The state trajectories of the uncontrolled fuzzy network (4).

VI. Conclusions

In this paper, we have discussed the consensus of a kind of multiagent networks with varying connections. T-S fuzzy models have been first addressed to describe multiagent networks with varying connections. For the proposed models, a node-rearrangement algorithm has been applied to decompose a large-scale fuzzy model into independent and small-scale fuzzy models. Moreover, the consensus of the large-scale fuzzy model is equivalent to that of its corresponding small-scale fuzzy models. Then, a sampled controller has been applied in the small-scale fuzzy models. Finally, numerical examples with the numerical simulations have been provided to illustrate the effectiveness of the obtained criteria.

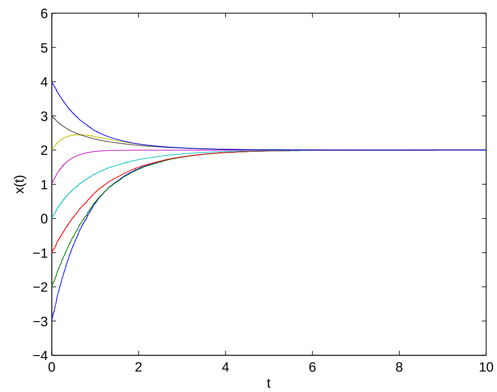

Fig. 2. The state trajectories of the sampled control fuzzy system (10).

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References

[1] M. DeGroot, “Reaching a consensus,” *Journal of the American Statistical Association*, pp. 118–121, 1974.

[2] A. Jadbabaie, J. Lin, and A. Morse, “Coordination of groups of mobile autonomous agents using nearest neighbor rules,” *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, 2003.

[3] L. Moreau, “Stability of multiagent systems with time-dependent communication links,” *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 169–182, 2005.

[4] R. Olfati-Saber and R. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[5] W. Ren and R. Beard, “Consensus seeking in multiagent systems under dynamically changing interaction topologies,” *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[6] W. Xiong, D. W. C. Ho, and Z. Wang, “Consensus analysis of multiagent networks via aggregated and pinning approaches,” *IEEE Transactions on Neural Networks*, vol. 22, no. 8, pp. 1231–1240, 2011.

[7] W. Yu, W. Zheng, G. Chen, W. Ren, and J. Cao, “Second-order consensus in multi-agent dynamical systems with sampled position data,” *Automatica*, vol. 47, no. 7, pp. 1946–1503, 2011.

[8] T. Takagi and M. Sugeno, “Fuzzy identification of system and its applications to modelling and control,” *IEEE Transactions on Systems Man and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.

[9] H. Dong, Z. Wang, and H. Gao, “h_∞ fuzzy control for systems with repeated scalar nonlinearities and random packet losses,” *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 2, pp. 440–450, 2009.

[10] Z. Wang, D. Ho, and X. Liu, "A note on the robust stability of uncertain stochastic fuzzy systems with time-delays," *IEEE Transactions on Systems, Man and Cybernetics, Part A*, vol. 34, no. 4, pp. 570–576, 2004.

[11] L. Wu and Z. Wang, "Fuzzy filtering of nonlinear fuzzy stochastic systems with time-varying delay," *Signal Processing*, vol. 89, no. 9, pp. 1739–1753, 2009.

[12] J. Liang, Z. Wang, and X. Liu, "On passivity and passification of stochastic fuzzy systems with delays: the discrete-time case," *IEEE Transactions on Systems, Man, and Cybernetics, Part B*, vol. 40, no. 3, pp. 964–969, 2010.

[13] E. Nuutila and E. Soisalon-Soininen, "On finding the strongly connected components in a directed graph," *Information Processing Letters*, vol. 49, no. 1, pp. 9–14, 1994.

[14] R. Bloem, H. Gabow, and F. Somenzi, "An algorithm for strongly connected component analysis in n log n symbolic steps," in *Formal Methods in Computer-Aided Design*, pp. 56–73, Springer, 2000.

[15] E. Fridman, "A refined input delay approach to sampled-data control," *Automatica*, vol. 46, no. 2, pp. 421–427, 2010.