Tracking Control for Piezo-Actuated Stage using Sliding Mode Controller with Observer-based Hysteresis Compensation

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***Abstract*—This paper presents a tracking control scheme for positioning of piezo-actuated stage under hysteresis effect. The hysteresis is estimated by an nonlinear observer and is compensated via feedforward control. Sliding mode controller is employed as a feedback control to track the reference. Experimental results showed effectiveness of the proposed scheme.**

Keywords – *Piezoelectric actuator, Hysteresis, Observer, Tracking Control*

I. INTRODUCTION

Micro and nano-manipulators are gaining importance in industries which see device miniaturization trend. Positioning stage is one of the devices employed in such industries. The stage applications include scanning microscopy, semiconductor testing, wafer positioning, etc. Some applications like scanning system or micromachining demand tracking of a certain reference. Therefore, a tracking controller for positioning stage is a necessity. Piezoelectric actuators (PA) has been extensively used as driver of positioning stage. However, due to ferroelectric nature of piezoelectric, the response of PA are nonlinear. The main component of this nonlinearity is hysteresis effect.

Hysteresis in PA has been studied extensively. Several hysteresis models have been reported in literature. They include the Preisach model [1], [2], Maxwell model [3], Duhem model [4], Dahl model [5], and Bouc-Wen model [6]. These models have been utilized for a model-based compensation scheme. Feedforward control using hysteresis model technique has been proposed in [7], [8] and [9]. The drawback of this model-based feedforward compensation is they are prone to modelling error and disturbance. To overcome these effects, feedback control for PA system has been employed.

Several feedback controllers has been studied for application into system with hysteresis. Li[10] designed an adaptive output feedback controller using RBF-Neural Network. Khan[11] used a discrete sliding mode control for piezo actuator. PID controller uses in disturbance rejection application has been described by Jayawardhana[12].

Most recent efforts [9] [13] use the combination of feedforward compensation and feedback control. This would utilize the knowledge about piezo actuators dynamics. To eliminate modelling errors and external disturbance, sliding mode controller is augmented as feedback controller. However, this approach introduced error because the input going into the inverse model is different than the real input of the system.

In the proposed scheme, instead of using an inverse model, hysteresis is compensated using estimated hysteresis. A hysteresis observer is employed to estimate hysteresis. Therefore, there is no error introduced by input difference, unlike observed in the existing scheme. Then a sliding mode control is used for feedback controller. Sliding mode controller is chosen due to its robustness to modelling error and disturbance. The performance of the proposed scheme then confirmed by experimental result. The scheme is able to compensate the effect of hysteresis. It also shows better tracking performance than existing methods.

II. PIEZO ACTUATED STAGE

The positioning mechanism of the stage is governed by PA systems. A PA system can be simply modelled as a mass-spring-damper-system with hysteresis effect:

$$m\ddot{q} + b\dot{q} + kq = k(d_e u - h) \tag{1}$$

where m is the mass of PA, b is damping coefficient, k is the stiffness factor and q is the displacement. Right hand side of (1) contains non-linear hysteresis term h, input voltage u.

One of the useful model to describe the hysteresis term is the Bouc-Wen model. The Bouc-Wen hysteresis dynamics is

$$\dot{h} = \alpha d_e \dot{q} - \beta|\dot{q}|h - \gamma\dot{q}|h| \tag{2}$$

with α, β, and γ are parameters which determine the shape of hysteresis curve.

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The system (1) and (2) can be rewritten as:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + \frac{k}{m}d_e Bu(t) - \frac{k}{m}Bh(t),\\ \dot{h}(t) &= \alpha d_e x_2(t) - f\big(x_2(t), h(t)\big)\end{aligned} \tag{3}$$

where $x_1(t) = q(t), x_2(t) = \dot{q}, A = \left[\begin{smallmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m}\end{smallmatrix}\right], B = \left[\begin{smallmatrix} 0 \\ 1\end{smallmatrix}\right]$, and

$$f\big(x_2(t), h(t)\big) = \beta|x_2|h + \gamma x_2|h|. \tag{4}$$

III. CONTROLLER DESIGN

The objective of the control law is to make the stage displacement to follow a given displacement trajectory x_d. A hysteresis observer is employed to estimate hysteresis. The estimated hysteresis and the steady state input then become the feedforward input. Additional error then eliminated with sliding mode feedback control. The proposed control scheme is shown by Fig. 1

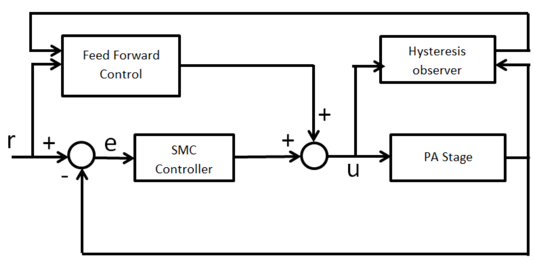

Fig. 1. Proposed Control Scheme

The subsequent development is based on the following preliminaries.

A. Preliminaries

The nonlinearity function (4) is assumed to be a globally Lipschitz function with respect to $h(t)$, i.e., there exist a positive constant θ such that

$$\|f\big(x_2(t), h_1(t)\big) - f\big(x_2(t), h_2(t)\big)\| < \theta\|h_1(t) - h_2(t)\|. \tag{5}$$

The following lemma will be used

Lemma1 [14] *Let $A(t)$ be a stable n x n matrix of bounded piecewise continuous functions. P(t) is a symmetric positive definite matrix of bounded continuous functions such that $\dot{P} + PA + A^T P$ is negative definite. Let B(t) be an n x m matrix of bounded piecewise continuous functions. Assume that there exist positive numbers T_0, ϵ_0, and δ_0 such as given $t_1 \geq 0$ and a unit vector $\omega \in \Re^m$, there is a $t_e \in [t_1, t_1 + T_0]$ such that*

$$\left|\int_{t_2}^{t_2+\delta_0} B^T(\tau)\omega d\tau\right| \geq \epsilon_0. \tag{6}$$

Then, the system

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix} A & -B^T \\ BP & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} \tag{7}$$

is uniformly asymptotically stable.

B. Hysteresis Observer

Hysteresis is the main factor of nonlinearity in piezoactuator. Here a hysteresis observer [15] is employed as follows:

$$\dot{\hat{x}}(t) = Ax(t) + \frac{k}{m}d_e Bu(t) - \frac{k}{m}B\hat{h}(t) + gE\big(\hat{x}(t) - x(t)\big), \tag{8}$$

$$\dot{\hat{h}}(t) = \alpha d_e x_2(t) - f(x_2, \hat{h}) + \frac{k}{m}g^2 B^T P\big(\hat{x}(t) - x(t)\big), \tag{9}$$

where $\hat{x}$ and $\hat{h}$ are estimated states and g is constant. E is a Hurwitz matrix to be designed and P is a positive definite solution such that

$$E^T P + PE = -Q \tag{10}$$

with Q a positive definite matrix.

Error states are defined by

$$\begin{aligned}&e_1(t) = \hat{x}(t) - x(t), \qquad e_2(t) = \hat{h}(t) - h(t)\\ &\tilde{f}\big(x_2(t), h(t), \hat{h}(t)\big) = f\big(x_2(t), \hat{h}(t)\big) - f\big(x_2(t), h(t)\big).\end{aligned}$$

Then, the error dynamics becomes

$$\begin{aligned}\dot{e}_1(t) &= gEe_1(t) - \frac{k}{m}Be_2(t)\\ \dot{e}_2(t) &= \frac{k}{m}g^2 B^T Pe_1(t) - \tilde{f}\big(x_2(t), h(t), \hat{h}(t)\big).\end{aligned} \tag{11}$$

Theorem1[15]The error dynamics (11) is globally asymptotically stable with a large enough g.

Proof Consider the error dynamics (11). Define a linear coordinate change

$$\xi(t) \equiv \begin{bmatrix} g^{-1}I & 0 \\ 0 & g^{-2}I\end{bmatrix} e(t), \tag{12}$$

where $e^T(t) = [e_1^T(t), e_2^T(t)]^T$ and I is an identity matrix. By coordinate change (12), the error dynamics (11) becomes

$$\begin{aligned}\dot{\xi}_1(t) &= gE\xi_1(t) - \frac{k}{m}B\xi_2(t)\\ \dot{\xi}_2(t) &= \frac{k}{m}gB^T P\xi_1(t) + g^{-2}\tilde{f}\big(x_2(t), h(t), \hat{h}(t)\big).\end{aligned} \tag{13}$$

Assume a temporary assumption as follow: *Temporary Assumption (t.a):* $\tilde{f}\big(x_2(t), h(t), \hat{h}(t)\big) = 0$. By the Temporary Assumption and change of variable $\tau = gt$, the error dynamics becomes

$$\begin{aligned}\frac{d}{d\tau}\xi_1 &= E\xi_1(t) - \frac{k}{m}B\xi_2\\ \frac{d}{d\tau}\xi_2 &= \frac{k}{m}B^T P\xi_1.\end{aligned} \tag{14}$$

Now the error dynamics (14) is in the form of (7).Since the constant matrix B obviously satisfy (6), by Lemma 1 the error dynamics is asymptotically stable. By converse theorem, there exist a Lyapunov function candidate $V(\xi, \tau)$ and constants $c_i, i = [1, 2, 3, 4]$ which satisfies

$$\begin{aligned}&c_1\|\xi(\tau)\| < V(\xi, \tau) < c_2\|\xi(\tau)\|,\\ &\dot{V}(\xi, \tau)|_{t.a} < -c_3\|\xi(\tau)\|^2,\\ &\left\|\frac{\partial}{\partial\xi}V(\xi, t)\right\| < c_4\|\xi(\tau)\|.\end{aligned} \tag{15}$$

To discard the temporary assumption, $V(\xi,\tau)$ is used as a Lyapunov candidate for system (13).

$$\begin{aligned}\frac{d}{d\tau}V(\xi,\tau) &= \frac{d}{d\tau}V(\xi)\bigg|_{t.a} + \frac{\partial V(\xi,\tau)}{\partial \xi} g^{-3}\tilde{f}(x_2,h,\hat{h}) \\ &\leq c3\|\xi(\tau)\|^2 + c_4\theta g^{-3}\|\xi(\tau)\|\|\hat{h}(t)-h(t)\| \\ &\leq c3\|\xi(\tau)\|^2 + c_4\theta g^{-1}\|\xi(\tau)\|^2 \\ &\leq \left(c3 + c_4\theta g^{-1}\right)\|\xi(\tau)\|^2\end{aligned} \tag{16}$$

$\frac{d}{d\tau}V(\xi,tau)$ is negative if we choose $g > c_4\theta/c_3$. This completes the proof.

C. Feed Forward Compensation

The feed forward compensation is constructed from the gain d_e and the estimated hysteresis from a hysteresis observer. Steady state input $u_s = x_d/d_e$ is added with estimated hysteresis $\hat{h}$. Then, the feed forward input u_{FF} is

$$u_{FF} = u_s + \frac{\hat{h}}{d_e} \tag{17}$$

D. Sliding Mode Feedback Controller

To take care of modeling errors, creep effect, and external disturbances, feedback controller is augmented in the control scheme. Sliding Mode Controller (SMC) is chosen as the feedback controller in the proposed scheme. When considering compensated hysteresis, parameters uncertainties and external disturbance d, Eq. (1) could be written as:

$$m\ddot{q}+b\dot{q}+kq = k\big(d_e u-(\hat{h}-h)\big)+\Delta m\ddot{q}+\Delta b\dot{q}+\Delta kq+d. \tag{18}$$

Equation (18) is rewritten in state-space canonical form as:

$$\begin{aligned}x_1 &\equiv q \\ x_2 &\equiv \dot{q} \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \frac{kd_e}{m}u + \frac{D(x,h)}{m},\end{aligned} \tag{19}$$

where D is the lumped disturbance term

$$D(x,h) \equiv k(h-\hat{h}) + \Delta m\ddot{x} + \Delta b\dot{x} + \Delta kx + d.$$

Equation (19) can be further generalized to be

$$\dot{x} = g(x,h) + Bu, \tag{20}$$

where B $\equiv [0 \quad \frac{kd_e}{m}]^T$.

The sliding surface s is defined as

$$s \equiv \begin{bmatrix}\epsilon & 1\end{bmatrix}\begin{bmatrix}e \\ \dot{e}\end{bmatrix} \equiv \begin{bmatrix}\epsilon & 1\end{bmatrix}\begin{bmatrix}x_1^d - x_1 \\ x_2^d - x_2\end{bmatrix} = Q(\mathbf{x^r} - \mathbf{x}) \tag{21}$$

where ϵ is a positive constant, $Q \equiv [\epsilon \quad 1]$, $\mathbf{x^r} \equiv [x_1^r \quad x_2^r]^T$, and $\mathbf{x} \equiv [x_1 \quad x_2]^T$.

Theorem2 Sliding function (21) is asymptotically stable.
Proof Lyapunov function candidate was chosen as

$$V(s) = \frac{1}{2}s^2. \tag{22}$$

The derivative of the Lyapunov function candidate is

$$\dot{V}(s) = s\dot{s}. \tag{23}$$

To make the states approach and stay at sliding surface s, then s should be asymptotically stable. That is, equations

$$\begin{aligned}&V(0) = 0, \\ &V(s) > 0, \quad s \neq 0 \\ &\dot{V}(s) < 0, \quad s \neq 0,\end{aligned} \tag{24}$$

must hold.

If the derivative of the sliding surface is designed as

$$\dot{s} = -\delta\ s, \tag{25}$$

where δ is a positive constant, then

$$\begin{aligned}&\dot{V}(s) = s\dot{s} = -\delta s^2 \\ &\dot{V}(s) < 0, \quad s \neq 0.\end{aligned} \tag{26}$$

By Eq. (26), the conditions in Eq. (24) are fulfilled.

Derivating Eq. (21) and substituting Eq. (20), the derivative of s is obtained as

$$\begin{aligned}\dot{s} &= Q(\dot{\mathbf{x}}^{\mathbf{r}} - \dot{\mathbf{x}}) = Q\dot{\mathbf{x}}^{\mathbf{r}} - Q\dot{\mathbf{x}} \\ &= Q\dot{\mathbf{x}}^{\mathbf{r}} - Q(g(\mathbf{x},h)Bu) \\ &= Q(\dot{\mathbf{x}}^{\mathbf{r}} - g(\mathbf{x},h)) - QBu \\ &= QB(u_{eq} - u),\end{aligned} \tag{27}$$

with $u_{eq} \equiv (QB)^{-1}Q(\dot{\mathbf{x}}^{\mathbf{r}} - g(\mathbf{x},h))$.

Equation (27) can be rewritten as

$$u_{eq} = u + (QB)^{-1}(\dot{s}). \tag{28}$$

By substituting Eq. (25), Eq. (28) becomes

$$u = u_{eq} + (QB)^{-1}(\delta s). \tag{29}$$

Equation (29) can be written for discrete system as

$$u(k) = u_{eq}(k) + (QB)^{-1}(\delta s(k)), \tag{30}$$

while Eq. (28) can be approximated as

$$u_{eq}(k) = u(k) + (QB)^{-1}\left(\tfrac{s(k+1)-s(k)}{T_s}\right). \tag{31}$$

T_s is the sampling time and k is the current time step. Since the term of $u_{eq}(k)$ is unknown, its value is approximated from its one-step past value, such as

$$\begin{aligned}\hat{u}_{eq}(k) &\equiv u_{eq}(k-1) \\ &= u(k-1) + (QB)^{-1}\left(\tfrac{s(k)-s(k-1)}{T_s}\right).\end{aligned} \tag{32}$$

By replacing $u_{eq}(k)$ in Eq. (30) with $\hat{e}u_{eq}$, the control input is obtained as

$$\begin{aligned}u(k) &= \hat{u}_{eq}(k) + (QB)^{-1}(\delta s(k)) \\ &= u(k-1) + (QB)^{-1}\left(\delta s(k) + \tfrac{s(k)-s(k-1)}{T_s}\right) \\ &= u(k-1) + \left(\tfrac{m}{kd_e}\right)\left(\delta s(k) + \tfrac{s(k)-s(k-1)}{T_s}\right)\end{aligned} \tag{33}$$

Thus control input (33) will make the sliding surface derivative as (27), thus stabilize the sliding function (21).

IV. EXPERIMENTAL RESULTS

A. Experimental Test Bed Description

The piezo actuated positioning device used in this experiments is the P-527.3CL from Physik Instrumente. The stage has 200 μm displacement range of each axis from 100 V range of input. The displacement axis is defined so that if the stage is centered x_A and x_B is equal to 100 μm The device is equipped with internal capacitive displacement sensors with under 1 nm resolution.

PC-based real-time control system is implemented in this research. The stage was interfaced to a PC by E-500 Modular Piezo Controller by Physik Instrumente. Communication between the piezo interface and PC was done through USB, utilizing USB driver from the manufacturer. This hardware configuration is able to operate at 20 Hz closed-loop sampling rate. For sampled-data systems, more than 20 times of the closed-loop system bandwidth is preferred for the sampling frequency of discrete control [16]. Therefore, the closed-loop control system is expected to have 1 Hz of bandwidth. The experiment setup is displayed in Fig. 2.

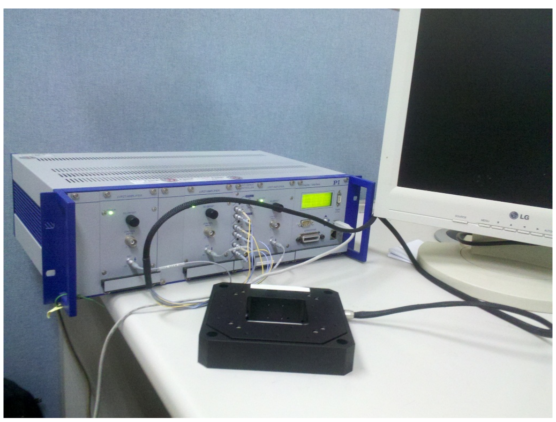

Fig. 2. Equipments used in experiment

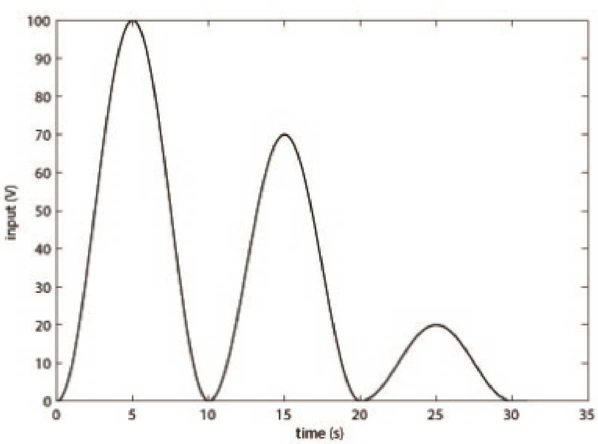

Fig. 3. Input signal for parameter identification

The plant parameters is identified using Particle Swarm Optimization [17] method. The identified parameters are listed in Table. I. Root mean square error (RMSE) by using the Bouc-Wen model (2) is 2.05. The input used is pictured in Fig. 3, similar to input used in [13] Parameter identification result is illustrated in hysteresis curve of Fig. 4

TABLE I
IDENTIFIED PLANT PARAMETERS

m	0.143	α	0.357
c	3.01	β	0.036
k	49.86	γ	0.027
d_e	2.55		

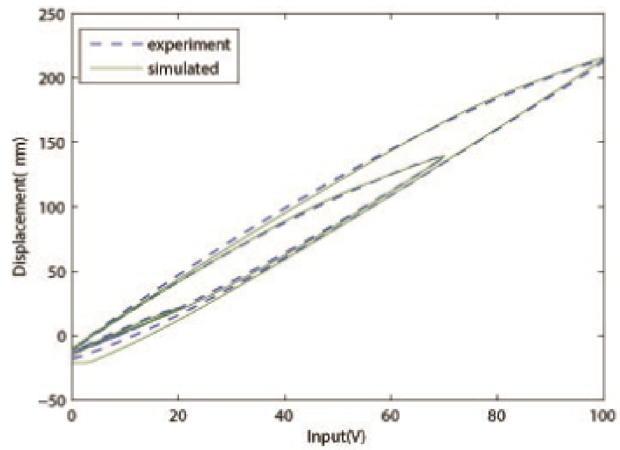

Fig. 4. Comparison of plant model and real plant output under same input

B. Simulation

Simulations are done to demonstrate the hysteresis observer capabilities. The simulated plant have parameters as listed in Table I. The observer parameter chosen are

$$E = \begin{bmatrix} 0 & 1 \\ -300 & -2000 \end{bmatrix}, P = \begin{bmatrix} 1.5572 & 0.0007 \\ 0.0007 & 0.0005 \end{bmatrix},$$

and the initial hysteresis value is chosen as $h(0) = -1$. Figure 5 shows that the reference is successfully tracked. This is because the hysteresis is correctly compensated as displayed by Fig. 6 and 7.

C. Experiment Performance Test

Experimental test was done by comparing with Lee's [9] method, which is a recent scheme employed a combination of feedback and feedforward controller. To confirm the tracking performance, sine reference with 0.5 Hz frequenct is used, as shown by Fig. 8.The tracking errors is displayed by Fig. 9. The figure shows that for same magnitude of overshoot, proposed method is superior to existing method. By proposed method he amplitude of error is halved from $1.5\mu m$ of existing method into $0.75\mu m$.

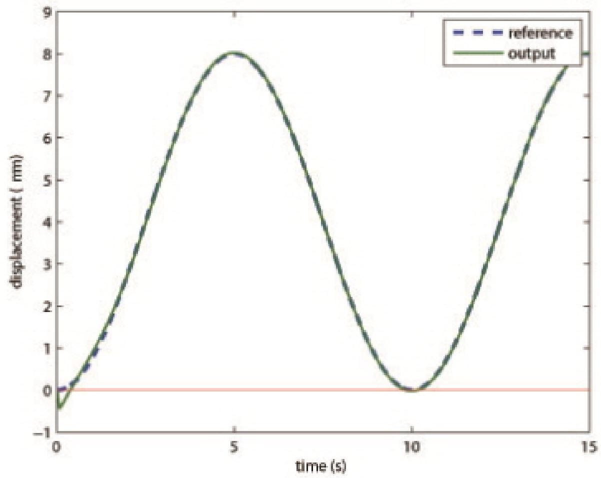

Fig. 5. Simulated Sinusoidal Tracking Output.

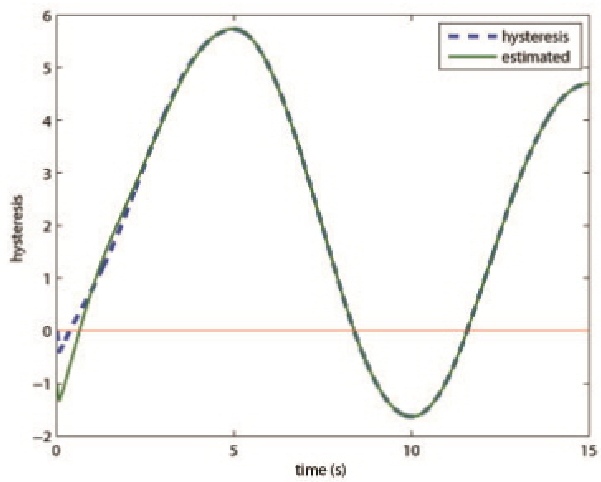

Fig. 6. Estimated hysteresis.

V. CONCLUSION

A tracking control scheme for a piezo-actuated positioning stage has been developed in this study. The control scheme brings the axis of movement of the stage to follow given reference. Hysteresis observer and feedforward controller are used to compensate the hysteresis effect. Sliding mode control is deployed to compensate unmodeled factor and eliminate errors. Experiment results show that proposed scheme demonstrated better performance compared to existing methods in the literature.

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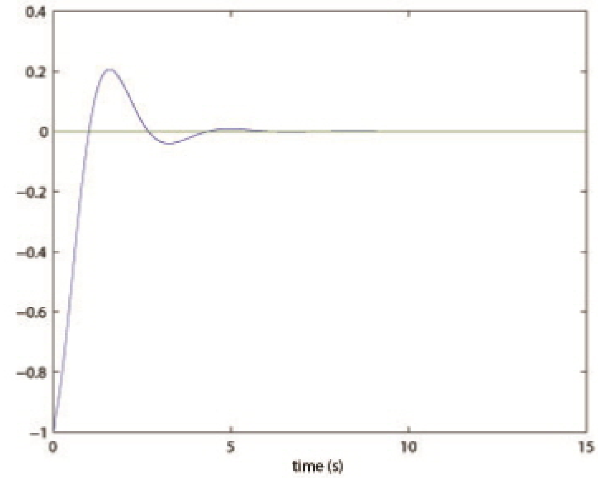

Fig. 7. Hysteresis estimation error.

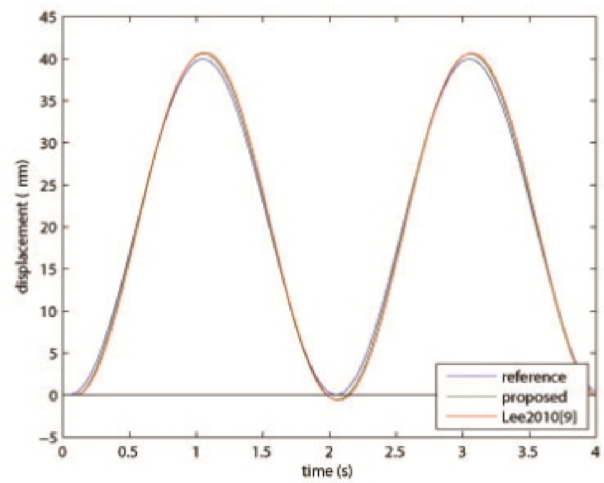

Fig. 8. Tracking ouput of sinusoidal reference.

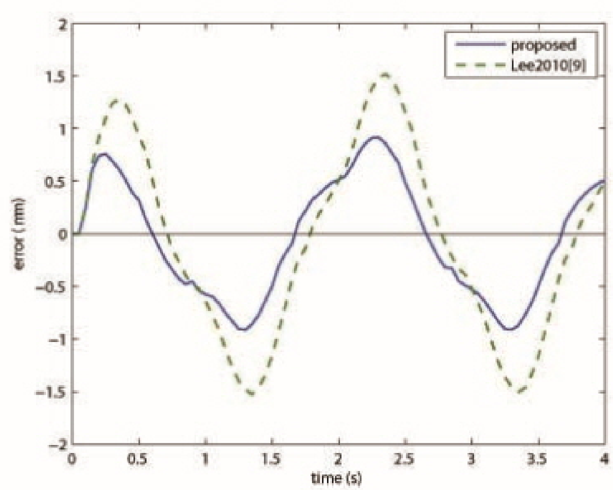

Fig. 9. Tracking errors of a sinusoidal reference.

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