A Lyapunov Method Based Multiple-Model Adaptive Actuator Failure Compensation Scheme for Control of Near-Space Vehicles

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***Abstract*—In a recent paper [7], a multiple-model adaptive actuator failure compensation control scheme is proposed for the control of a near-space vehicle, using the gradient algorithm, to achieve fast and accurate actuator failures compensation. In this paper, a new multiple-model adaptive actuator failure compensation control scheme is developed for nonlinear systems motivated from a near-space vehicle control application. Such a design also employs multiple controllers based on multiple-model failure estimations and a control switching mechanism, based on finding the minimal performance cost index, to select the most appropriate controller. Different from [7], each estimator is designed based on the Lyapunov method, which ensures the system stability and desired tracking properties. Moreover, a smooth control are introduced to the multiple-model control system frame to avoid the discontinuity problem from the control switching, to widen the application of such design. Simulation results for a near-space vehicle dynamic model are presented to show the desired failure compensation performance.**

I. INTRODUCTION

In recent years, near-space vehicles (NSVs), as a kind of new aerospace vehicles, attract the attentions of researchers for its economics and homeland security reasons, many significance research results are developed [5], [6]. Moreover, to improve the reliability and safety of NSVs, more and more researchers work at failure compensation mechanisms, and make some contributions [3], [10]. Adaptive control, for its capability of accommodating system parametric and environmental uncertainties, is widely used for system failure compensation [9], [12]. However, actuator failures may cause the system parameters to change abruptly from one parameter region to another, which may bring a large transient tracking error from a large parameter error, so that an adaptive control law may take a relatively long time to compensate the uncertainties caused by the actuator failures if the control law is based on a single-model design. To quickly and accurately compensate the uncertainties caused by the failures, researchers employed the multiple-model approach to design the compensation control schemes [1], [2], which design algorithms to assure that the schemes switch to the controller corresponding to the model closet to the failed plant to achieve the desired system performance in the presence of the failures.

In this paper, we develop a new multiple-model adaptive actuator failure compensation control scheme for control of near-space vehicles based on the Lyapunov method. Different from the designs in [1] and [2] which use multiple-model adaptive reconfigurable control schemes for control of some linearized aircraft models with some specified types of effector failures, our design in this paper is aimed at control of a near-space vehicle whose dynamics are nonlinear and the considered failure model is described in a general form. Also different from our previous design in [7] whose each estimator is designed based on the gradient algorithm, the estimator designed in this paper is developed based on the Lyapunov method which ensures the system stability and the desired tracking properties. Moreover, we introduce a smooth control into the multiple-model switching control scheme to generate a continuous input signal to widen the application of such design. In summary, the new contributions of this paper are

- development of a new multiple-model adaptive actuator failure compensation control scheme for a class of nonlinear systems whose each estimator is designed based on the Lyapunov method to ensure the system stability and the desired tracking properties;
- introduction of a smooth control into the multiple-model adaptive scheme frame to widen the application of such design;
- design of a multiple-model adaptive actuator failure compensation scheme for a near-space vehicle with uncertain actuator failures.

This paper is organized as follows. In Section II, we formulate the adaptive control problem for a NSV with the uncertain actuator failures. In Section III, we design a multiple-model adaptive actuator failure compensation scheme, by calculating the controller parameters from adaptive estimates of the failure signal parameters, each estimator is designed based on the Lyapunov method, and setting up a switching algorithm to generate the current controller. In Section IV, we present simulation results for a NSV model to verify the desired adaptive failure compensation performance.

II. PROBLEM FORMULATION

The problem to be solved in this paper is to use a multiple-model approach to deal with the actuator failure compensation for control of a NSV. In this section, we first present the NSV attitude dynamics and the actuator failure model, and then address the control problem.

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A. NSV Attitude Dynamics and Failure Model

Consider the NSV attitude dynamics given by [10]:

$$\begin{aligned} \dot{\gamma} &= \Xi(\gamma)\omega \\ \dot{\omega} &= J^{-1}\Omega(\omega)J\omega + J^{-1}\Psi u, \end{aligned} \quad (1)$$

where $\gamma = [\mu, \beta, \alpha]^T$ is the attitude angle vector and $\omega = [p, q, r]^T$ is the angular rate vector, μ is the bank angle, β is the sideslip angle, α is the angle of attack, p is the roll rate, q is the pitch rate and r is the yaw rate. The control input $u(t) = [u_1(t), u_2(t), u_3(t), u_4(t), u_5(t), u_6(t)]^T$ represents six rudder servos: δ_e, δ_a, δ_r, δ_x, δ_y, δ_z, which are the elevator deflection, the aileron deflection, the rudder deflection, the equivalent control surface deflection of x-axis, y-axis and z-axis to the body frame, respectively. The skew symmetric matrix $\Omega(\omega)$, the symmetric positive definite moment of the inertia tensor J, and matrices $\Xi(r)$ and Ψ are

$$\Omega(\omega)=\begin{pmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{pmatrix}, \ \Xi(r)=\begin{pmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ \sin(\alpha) & 0 & -\cos(\alpha) \\ 0 & 1 & 0 \end{pmatrix}, \quad (2)$$

$$J = \begin{pmatrix} 554486 & 0 & -23002 \\ 0 & 1136949 & 0 \\ -23002 & 1 & 1376852 \end{pmatrix}, \quad (3)$$

$$\Psi = \begin{pmatrix} g_{p,\delta_e} & g_{p,\delta_a} & g_{p,\delta_r} & g_{p,\delta_x} & 0 & 0 \\ g_{q,\delta_e} & g_q, \delta_a & g_{q,\delta_r} & 0 & 0 & g_{q,\delta_z} \\ g_{r,\delta_e} & g_{r,\delta_a} & g_{r,\delta_r} & 0 & g_{r,\delta_y} & 0 \end{pmatrix}. \quad (4)$$

For the convenience of our study, we define $x_1 = \gamma$, $x_2 = \omega$ and $y = x_1$, and rewrite NSV dynamics (1) in the form:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1)x_2, \\ \dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)u, \end{aligned} \quad (5)$$

where $f_1 \in R^{(n-p)\times p}$, $f_2 \in R^{p\times 1}$ and $g_2 \in R^{p\times m}$ with $p < m$, are known. Our goal is to design a control law $u(t)$ to make the system achieve the desired performance. However, some actuators of NSV may fail during system operation, and some components $u_i(t)$ of $u(t) = [u_1(t), u_2(t), \ldots, u_m(t)]^T$ may not be influenced by the applied control signal $v_i(t)$, that is $u_i(t) \neq v_i(t)$, which causes the system performance to degrade inevitably.

To study the problem of actuator failure compensation, we consider a practical model of actuator failures [9]

$$u_j = \bar{u}_j(t) = \bar{u}_{j0} + \sum_{i=1}^{n_j} \bar{u}_{ji} f_{ji}(t) = \bar{\theta}_j^{*T}\omega_j(t), \ t \geq t_j, \quad (6)$$

for some unknown actuator j, unknown time instant t_j and unknown constants $\bar{u}_{j0}$ and $\bar{u}_{ji}$, and some known bounded functions $f_{ji}(t)$, $i = 1, 2, \ldots, n_j$, $j \in \{1, 2, \ldots, m\}$, where $\bar{\theta}_j^* = [\bar{u}_{j0}, \bar{u}_{j1}, \ldots, \bar{u}_{j\,n_j}]^T \in R^{n_j+1}$, and $\omega_j^T(t) = [1, f_{j1}(t), \ldots, f_{jn_j}(t)]^T \in R^{n_j+1}$. In the presence of actuator failures, the input signal $u(t)$ can be expressed as

$$u(t) = (I - \sigma(t))v(t) + \sigma(t)\bar{u}(t), \quad (7)$$

where $v(t) = [v_1(t), v_2(t), \ldots, v_m(t)]^T$ is the applied control input to be designed, $\bar{u}(t) = [\bar{u}_1(t), \bar{u}_2(t), \ldots, \bar{u}_m(t)]^T$ is the failure vector, and $\sigma(t) = \mathrm{diag}\{\sigma_1(t), \sigma_2(t), \ldots, \sigma_m(t)\}$ is the diagonal actuator failure pattern matrix, with $\sigma_i(t) = 1$ if the ith actuator fails and $\sigma_i(t) = 0$ otherwise.

B. Control Problem

The control objective in this paper is to use a multiple-model approach to find the applied feedback control signals $v(t) = [v_1(t), v_2(t), \ldots, v_m(t)]^T$ for the plant (5) such that the closed-loop signals are bounded and the system output $y(t) = x_1(t) \in R^p$ tracks a given reference output $y_m(t) \in R^p$, in the presence of up to $m-p$ uncertain actuator failures.

For the p-dimensional output $y(t) = x_1(t)$ to track a reference output $y_m(t)$, generically, only p input signals are needed. There are up to $m-p$ redundant actuators as determined by the control allocation matrix $g_2(x_1, x_2)$, which are needed for failure compensation. For $w \stackrel{\triangle}{=} g_2(x_1, x_2)u \in R^p$, there is a desired effective control signal $w_d(t)$ for w, which is generated from a nonlinear control design such as backstepping for the system (5) to meet the desired system performance. For control action implementation, the control signal equation

$$w_d = g_2(x_1, x_2)u, \quad (8)$$

needs to be met. For the system input $u(t) = (I-\sigma(t))v(t) + \sigma(t)\bar{u}(t)$, there has two cases: the no failure case with $\sigma(t) = 0$ and the failure case with $\sigma(t) \neq 0$. For the no failure case, $u(t) = v(t)$, (8) may be easy to satisfy, by making $u = (g_2 h(x_1, x_2))^{-1} w_d \in R^p$ with a chosen matrix $h(x_1, x_2) \in R^{m\times p}$. For the failure case, $u(t)$ consists of the designed control signal $v(t)$ and the unknown failure part $\bar{u}_j(t)$, (8) is not easily satisfied. In an adaptive control system, we can estimate the failure $\bar{u}_j(t)$ and make $g_{2a}v_a(t) + g_{2j}\bar{u}_j(t) = w_d$.

Basic multiple-model based control design procedure: The development of a multiple-model adaptive actuator failure compensation scheme for a NSV system includes three steps. We first select some possible failure patterns of interest for compensation to set up a possible failure pattern set Σ, then design a controller $v_{(i)}(t)$ corresponding to a failure pattern belonging to the failure pattern set, by calculating its parameters from the adaptive estimation, based on the Lyapunov method, of the failure parameters, to develop a bank of controllers, and finally design a switch mechanism by design of a performance cost index function based on the estimation errors to select the most appropriate controller, and introducing a smooth control to generate a continuous input signal $v(t)$ to control the system (5) with uncertain actuator failures.

III. Adaptive Actuator Failure Compensation

In this section, we develop the multiple-model adaptive actuator failure compensation scheme in detail.

A. Failure Pattern Set

The considered near-space vehicle model has six actuators, that is $m = 6$. From the analysis of the model, we can divide the six actuators into two groups: one consists of the first three actuators and the other consists of the rest actuators, according to their functions. In this paper, we consider the failure cases that one actuator in each group may fail. Since the adaptive failure compensation designs for choice of u_1 and u_6 and choice of u_2 and u_4 have the same technical complexity, to simpler notation for presentation, we choose the case of u_1 failure and u_6 failure to study the failure compensation design. So we consider the case when there are three failure patterns in the failure pattern set Σ of interest for compensation, which are represented as $\sigma_{(i)}(i=1,2,3)$. The three failure patterns can be described in detail as follows:

(I) the no failure case: $u_i(t)=v_i(t)$, $i=1,2,\ldots,6$, for some applied control signals $v_i(t)$ from some feedback control law;
(II) the u_1 failure case: $u_1(t)=\bar{u}_1(t)$, $t\geq t_1$, and $u_i(t)=v_i(t)$ for $i=2,3,\ldots,6$; and
(III) the u_6 failure case: $u_6(t)=\bar{u}_6(t)$, $t\geq t_6$, and $u_i=v_i(t)$ for $i=1,2,3,\ldots,5$.

The corresponding failure patterns $\sigma_{(i)}$ are $\sigma_{(1)} = \mathrm{diag}\{0,0,0,0,0,0\}$, $\sigma_{(2)}=\mathrm{diag}\{1,0,0,0,0,0\}$, and $\sigma_{(3)}=\mathrm{diag}\{0,0,0,0,0,1\}$.

With the failure pattern set Σ, we can design different control law for different failure pattern to compose a controller bank, as stated in the next subsection.

B. Bank of Control Laws

In this subsection we develop a bank of controllers, each of which is designed under a failure pattern pattern, and combined with a backstepping feedback control law. In the sequel, we first present the backstepping control design, then design a nominal controller for each failure pattern, and finally develop the adaptive controller for each failure pattern.

1) Backstepping Control Design: Backstepping is a nonlinear design method effective for control of nonlinear systems, it has been widely used in control system design [11]. A backstepping control design for the system (5) generates a desired effective control signal w_d for w, as shown next, based on which the controller $v(t)$ can be designed. Here we only give some main equations, due to limited space. Introducing $z_1=x_1-y_m$ and $z_2=x_2-\alpha_1$, and choosing a virtual control variable

$$\alpha_1=f_1^{-1}(x_1)(-c_1z_1+\dot{y}_m), \tag{9}$$

with a chosen constant $c_1>0$, we have

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1-\dot{y}_m=-c_1z_1+f_1(x_1)z_2,, &(10)\\ \dot{z}_2 &= \dot{x}_2-\dot{\alpha}_1=f_2(x_1,x_2)+g_2(x_1,x_2)u-\dot{\alpha}_1, &(11)\end{aligned}$$

and obtain the virtual control signal $w=g_2(x_1,x_2)u$ as

$$w=w_d=-f_2(x_1,x_2)-c_2z_2-f_1^T(x_1)z_1+\dot{\alpha}_1, \tag{12}$$

with a chosen constant $c_2>0$, which ensure the desired control objective is met.

2) Nominal Controller Design for Each Failure Pattern: In this study, we design three individual compensation controllers respectively for three failure patterns belonging to the failure pattern set Σ: (I) the no failure case with $\sigma_{(1)}$, (II) the u_1 failure case with $\sigma_{(2)}$, and (III) the u_6 failure case with $\sigma_{(3)}$.

Design for the no failure case. In this case, $u(t)=v(t)$ for all $t\geq 0$, and the control signal equation (8) is $g_2(x_1,x_2)v=w_d$. We design the signal $v(t)$ as

$$v(t)=v^*_{(1)}=h_{21}(x_1,x_2)v^*_{0(1)}(t), \tag{13}$$

for some chosen matrix function $h_{21}(x_1,x_2)\in R^{m\times(m-1)}$ and some signal $v^*_{0(1)}$ to be determined from

$$g_2(x_1,x_2)h_{21}(x_1,x_2)v^*_{0(1)}=w_d. \tag{14}$$

The explicit form of the solution $v^*_{0(1)}$ may be expressed as

$$v^*_{0(1)}=K_{21}(x_1,x_2)w_d, \tag{15}$$

for some non-unique matrix function $K_{21}(x_1,x_2)\in R^{(m-1)\times p}$. Note that this signal $v^*_{0(1)}$ is unrelated to any of the potential actuator failure signals $\bar{u}_j$, $j=1,2,\ldots,6$, and thus is a known signal with K_{21} known, and so is $v^*_{(1)}$ in (13) with h_{21} being chosen.

Design for the u_1 failure case. When $u_1(t)=\bar{u}_1(t)$ and $u_i(t)=v_i(t)$ for $i=2,\ldots,6$, with $g_2=[g_{21},g_{22},\ldots,g_{26}]=[g_{21},g_{2(2)}]\in R^{p\times m}$ for $g_{2(2)}=[g_{22},\ldots,g_{26}]\in R^{p\times(m-1)}$, $v(t)=[v_1(t),v_2(t),\ldots,v_6(t)]^T=[v_1(t),v^T_{a(2)}(t)]^T\in R^6$ for $v_{a(2)}(t)=[v_2(t),\ldots,v_6(t)]^T\in R^5$, the control signal equation $g_2(x_1,x_2)u=w_d$ becomes

$$g_{21}\bar{u}_1+g_{2(2)}v_{a(2)}=w_d. \tag{16}$$

In this case, the signal $v_1(t)$ is chosen to be $v_1(t)=0$ as $u_1(t)=\bar{u}_1(t)$ is failed. One may also choose a nonsingular matrix function $h_{22}(x_1,x_2)\in R^{(m-1)\times(m-1)}$ to set

$$\begin{aligned}v(t) &= [v_1(t),v^T_{a(2)}(t)]^T=v^*_{(2)}(t)=[0,v^{*T}_{a(2)}(t)]^T,\\ &\quad v^*_{a(2)}(t)=h_{22}(x_1,x_2)v^*_{0(2)}(t), &(17)\end{aligned}$$

for some signal $v^*_{0(2)}\in R^{m-1}$ to be determined from

$$g_{21}\bar{u}_1+g_{2(2)}h_{22}(x_1,x_2)v^*_{0(2)}=w_d. \tag{18}$$

The explicit form of the solution $v^*_{0(2)}$ may be expressed as

$$v^*_{0(2)}=K_{22}(x_1,x_2)w_d+K_{221}(x_1,x_2)\bar{u}_1, \tag{19}$$

for some non-unique matrix function $K_{22}(x_1,x_2)\in R^{(m-1)\times p}$ and vector $K_{221}(x_1,x_2)\in R^{(m-1)}$. Note that this signal $v^*_{0(2)}$ is related to the actuator failure signal $\bar{u}_1$, and so is $v^*_{(2)}$ in (17). However, the functions K_{22} and K_{221} in (19) are known by solving (18), and so is h_{22} in (17) by a pre-specified choice.

Design for the u_6 failure case. With a similar design process of the u_1 failure case, when $u_6(t)=\bar{u}_6(t)$ and $u_i(t)=v_i(t)$ for $i=1,2,\ldots,5$, we have the following equations:

$$\begin{aligned}& g_{2(1)}v_{a(3)}+g_{26}\bar{u}_6=w_d, &(20)\\ v(t) &= [v^T_{a(3)}(t),v_6(t)]^T=v^*_{(3)}(t)=[v^{*T}_{a(3)}(t),0]^T,\\ &\quad v^*_{a(3)}(t)=\ h_{23}(x_1,x_2)v^*_{0(3)}(t), &(21)\\ & g_{2(1)}h_{23}(x_1,x_2)v^*_{0(3)}+g_{26}\bar{u}_6=w_d, &(22)\\ v^*_{0(3)} &= K_{23}(x_1,x_2)w_d+K_{236}(x_1,x_2)\bar{u}_6, &(23)\end{aligned}$$

where all constants and variables have the similar meanings and mathematical expressions as those in the u_1 failure case.

Special designs: the unique solutions. The choice of $h_{2j}(x_1,x_2)$, $j=1,2,3$, should make the equations (14), (18) and (22) to have a solution $v^*_{0(j)}(t)$ respectively, which are possible and may not be unique but have some design freedom for certain optimality. We may choose h_{21} in (13) to be $m\times p$ matrix, h_{22} in (17) and h_{23} in (21) to be $(m-1)\times p$ matrices, and the corresponding $v^*_{0(j)}$ to be p-dimensional vectors to make the above individual solutions $v^*_{0(j)}$ be unique.

3) Adaptive Control Design for Each Failure Pattern: For the unknown actuator failures, the nominal control signals $v^*_{(2)}(t)$ in (17) and $v^*_{(3)}(t)$ in (21) are not available for actuator failure compensation control. We need to develop an adaptive scheme to adaptively estimate the unknown actuator failure parameters. As adaptive versions of (17) and (21), we use the control laws:

$$v_{(2)}(t) = [0, [h_{22}K_{22}w_d + h_{22}K_{221}\overline{\theta}_1^T\omega_1]^T]^T, \quad (24)$$
$$v_{(3)}(t) = [[h_{23}K_{23}w_d + h_{23}K_{236}\overline{\theta}_6^T\omega_6]^T, 0]^T, \quad (25)$$

where $\overline{\theta}_1$ and $\overline{\theta}_6$ are the estimates of the unknown parameters $\overline{\theta}_1^*$ and $\overline{\theta}_6^*$ respectively. For the system in the non-failure case, the system dynamics are known, so we use

$$v_{(1)}(t) = v^*_{(1)}(t) = h_{21}K_{21}w_d. \quad (26)$$

Next we design parameter estimators for the three failure patterns, each estimator generates an estimation error which is stable and convergent for the corresponding case but may not be convergent for other cases while the parameter boundedness can be ensured by parameter projection. To build parameter estimators, we express (10)–(12) as

$$\dot{z}_1 = -c_1z_1 + f_1(x_1)z_2, \quad (27)$$
$$\dot{z}_2 = -c_2z_2 - f_1^T(x_1)z_1 + g_2(x_1,x_2)u - w_d. \quad (28)$$

Parameter estimator for the no failure case. In this case, $g_2(x_1,x_2)u = g_2(x_1,x_2)v$ (this v could be any of $v_{(1)}$, $v_{(2)}$ and $v_{(3)}$ defined in (24)–(26), to be selected by a control switching algorithm), and we define the parameter estimator state variables $\hat{z}_{(1)1}$ and $\hat{z}_{(1)2}$ from

$$\dot{\hat{z}}_{(1)1} = -c_1\hat{z}_{(1)1} + f_1(x_1)\hat{z}_{(1)2}, \quad (29)$$
$$\dot{\hat{z}}_{(1)2} = -c_2\hat{z}_{(1)2} - f_1^T(x_1)\hat{z}_{(1)1} + g_2(x_1,x_2)v - w_d. \quad (30)$$

For the estimator state errors $e_{(1)1} = z_1 - \hat{z}_{(1)1}$ and $e_{(1)2} = z_2 - \hat{z}_{(1)2}$, from (27)–(30), we obtain

$$\dot{e}_{(1)1} = -c_1e_{(1)1} + f_1(x_1)e_{(1)2}, \quad (31)$$
$$\dot{e}_{(1)2} = -c_2e_{(1)2} - f_1^T(x_1)e_{(1)1} + g_2(x_1,x_2)(u-v). \quad (32)$$

Since the system $\dot{e}_{(1)1} = -c_1e_{(1)1} + f_1(x_1)e_{(1)2}$, $\dot{e}_{(1)2} = -c_2e_{(1)2} - f_1^T(x_1)e_{(1)1}$ is exponentially stable (which can be verified from $V_1 = \frac{1}{2}\left(e^T_{(1)1}e_{(1)1} + e^T_{(1)2}e_{(1)2}\right)$ and $\dot{V}_1 = -c_1e^T_{(1)1}e_{(1)1} - c_2e^T_{(1)2}e_{(1)2}$), the estimator state error model (31)–(32) has the desired convergent property for the no failure case when $u = v$. For the other two cases, it may not have the desired convergent property because $u \neq v$.

In this case, there is no explicit parameter estimation because the actuator failures are not involved, and the errors $e_{(1)1}$ and $e_{(1)2}$ are to be used for the control switching algorithm to be developed.

Parameter estimator for the u_1 failure case. In this case, $u_1 = \bar{u}_1$ so that $g_2(x_1,x_2)u = g_{21}(x_1,x_2)\bar{u}_1 + g_{2(2)}(x_1,x_2)v_{a(2)}$, for $g_2 = [g_{21}, g^T_{2(2)}]^T$ and $v = [v_1, v^T_{a(2)}]^T$ with $v_{a(2)} \in R^5$ being the sub-vector of the applied input $v(t)$, which could also be any of $v_{(1)}$, $v_{(2)}$ and $v_{(3)}$ defined in (24)–(26). In view of the parametrization $\bar{u}_1 = \bar{\theta}_1^{*T}\omega_1(t)$, letting $\bar{\theta}_1$ be the estimate of $\bar{\theta}_1^*$, we define the parameter estimator state variables $\hat{z}_{(2)1}$ and $\hat{z}_{(2)2}$ from

$$\dot{\hat{z}}_{(2)1} = -c_1\hat{z}_{(2)1} + f_1(x_1)\hat{z}_{(2)2}, \quad (33)$$
$$\dot{\hat{z}}_{(2)2} = -c_2\hat{z}_{(2)2} - f_1^T(x_1)\hat{z}_{(2)1} + g_{21}(x_1,x_2)\bar{\theta}_1^T\omega_1 + g_{2(2)}(x_1,x_2)v_{a(2)} - w_d. \quad (34)$$

For the estimator state errors $e_{(2)1} = z_1 - \hat{z}_{(2)1}$ and $e_{(2)2} = z_2 - \hat{z}_{(2)2}$, from (27), (28), (33) and (34) with $g_2(x_1,x_2)u = g_{21}(x_1,x_2)\bar{u}_1 + g_{2(2)}(x_1,x_2)v_{a(2)}$, we obtain

$$\dot{e}_{(2)1} = -c_1e_{(2)1} + f_1(x_1)e_{(2)2}, \quad (35)$$
$$\dot{e}_{(2)2} = -c_2e_{(2)2} - f_1^T(x_1)e_{(2)1} - g_{21}(x_1,x_2)(\bar{\theta}_1 - \bar{\theta}_1^*)^T\omega_1, \quad (36)$$

which, with the complete parametrization term $g_{21}(x_1,x_2)(\bar{\theta}_1 - \bar{\theta}_1^*)^T\omega_1$, indicates that this estimator state error model has the desired convergent property for this u_1 failure case but may not for other two cases when $g_2(x_1,x_2)u = g_{21}(x_1,x_2)\bar{\theta}_1^{*T}\omega_1(t) + g_{2(2)}(x_1,x_2)v_{a(2)}$ may not hold.

Considering the positive definite function

$$V_2 = \frac{1}{2}\left(e^T_{(2)1}e_{(2)1} + e^T_{(2)2}e_{(2)2} + (\bar{\theta}_1 - \bar{\theta}_1^*)^T\Gamma_1^{-1}(\bar{\theta}_1 - \bar{\theta}_1^*)\right), \quad (37)$$

where Γ_1 is a diagonal matrix with positive elements, and its time-derivative

$$\dot{V}_2 = -c_1e^T_{(2)1}e_{(2)1} - c_2e^T_{(2)2}e_{(2)2} - e^T_{(2)2}g_{21}(x_1,x_2)\tilde{\theta}_1^T\omega_1 + \tilde{\theta}_1^T\Gamma_1^{-1}\dot{\bar{\theta}}_1, \quad (38)$$

with $\tilde{\theta}_1 = \bar{\theta}_1 - \bar{\theta}_1^*$, we choose the adaptive law for $\bar{\theta}_1$ as

$$\dot{\bar{\theta}}_1 = \Gamma_1\omega_1e^T_{(2)2}g_{21}(x_1,x_2) + f_1, \quad (39)$$

where f_1 is the parameter projection signal which has the desired property: $(\bar{\theta}_1 - \bar{\theta}_1^*)^T\Gamma_1^{-1}f_1 \leq 0$ and the components of θ_1 are projected to some intervals. This adaptive law leads $\dot{V}_2$ to

$$\dot{V}_2 = -c_1e^T_{(2)1}e_{(2)1} - c_2e^T_{(2)2}e_{(2)2} + (\bar{\theta}_1 - \bar{\theta}_1^*)^T\Gamma_1^{-1}f_1, \quad (40)$$

which indicates that $e_{(2)1} \in L^\infty \cap L^2$ and $e_{(2)2} \in L^\infty \cap L^2$. Note that this desired property is for the u_1 failure case. For the no failure case, from (27) and (28) with $u = v$, this parameter estimator consisting of (33) and (34) makes the estimator state errors $e_{(2)1}$ and $e_{(2)2}$, satisfy $\dot{e}_{(2)1} = -c_1e_{(2)1} + f_1(x_1)e_{(2)2}$, $\dot{e}_{(2)2} = -c_2e_{(2)2} - f_1^T(x_1)e_{(2)1} + g_2(x_1,x_2)u - g_{21}(x_1,x_2)\bar{\theta}_1^T\omega_1 - g_{2(2)}(x_1,x_2)v_{a(2)}$. Further, with (39), it makes $\dot{V}_2$ have an additional term $e^T_{(2)2}\left(g_2(x_1,x_2)v - g_{21}(x_1,x_2)\bar{\theta}_1^{*T}\omega_1 - g_{2(2)}(x_1,x_2)v_{a(2)}\right)$, which shows that the parameter estimator designed for the u_1 failure case may not have the desired convergent property. Similarly, for the u_6 failure case, this estimator makes $\dot{V}_2$ have an additional term $e^T_{(2)2}(g_{2(1)}(x_1,x_2)v_{a_{(3)}} + g_{26}\bar{\theta}_6^{*T}\omega_6 - g_{21}(x_1,x_2)\bar{\theta}_1^{*T}\omega_1 - g_{2(2)}(x_1,x_2)\ v_{a(2)})$, which also suggests that the desired convergent property may not hold. However the use of parameter projection ensures that the parameter estimation $\bar{\theta}_1$ is bounded in both no failure and u_6 failure case.

Parameter estimator for the u_6 failure case. In this case, $u_6 = \bar{u}_6$ so that $g_2(x_1,x_2)u = g_{2(1)}(x_1,x_2)v_{a(3)} + g_6(x_1,x_2)\bar{u}_6$. Similarly, we define the parameter estimator state variables $\hat{z}_{(3)1}$ and $\hat{z}_{(3)2}$

$$\dot{\hat{z}}_{(3)1} = -c_1\hat{z}_{(3)1} + f_1(x_1)\hat{z}_{(3)2}, \quad (41)$$
$$\dot{\hat{z}}_{(3)2} = -c_2\hat{z}_{(3)2} - f_1^T(x_1)\hat{z}_{(3)1} + g_{2(1)}(x_1,x_2)v_{a(3)} + g_{26}(x_1,x_2)\bar{\theta}_6^T\omega_6 - w_d, \quad (42)$$

where $\bar{\theta}_6$ is the estimate of $\bar{\theta}_6^*$, and the estimator state errors $e_{(3)1} = z_1 - \hat{z}_{(3)1}$, $e_{(3)2} = z_2 - \hat{z}_{(3)2}$, obtain

$$\dot{e}_{(3)1} = -c_1 e_{(3)1} + f_1(x_1) e_{(3)2}, \tag{43}$$

$$\dot{e}_{(3)2} = -c_2 e_{(3)2} - f_1^T(x_1) e_{(3)1} - g_{26}(x_1, x_2)(\bar{\theta}_6 - \bar{\theta}_6^*)^T \omega_6. \tag{44}$$

and choose the adaptive law for $\bar{\theta}_6$ as

$$\dot{\bar{\theta}}_6 = \Gamma_6 \omega_6 e_{(3)2}^T g_{26}(x_1, x_2) + f_6, \tag{45}$$

where f_6 is the parameter projection signal. This adaptive law also guarantees $e_{(3)1} \in L^\infty \cap L^2$ and $e_{(3)2} \in L^\infty \cap L^2$, and this desired property is only for the u_6 failure case.

According to these above analysis, this adaptive scheme has the following properties:

Lemma 1: The adaptive law (39) guarantees the desired properties: (i) $\bar{\theta}_1(t) \in L^\infty$, $e_{(2)1} \in L^\infty \cap L^2$ and $e_{(2)2} \in L^\infty \cap L^2$, for the system (5) with $\sigma = \sigma_{(2)}$, and (ii) the adaptive law (45) guarantees: $\bar{\theta}_6(t) \in L^\infty$, $e_{(3)1} \in L^\infty \cap L^2$ and $e_{(3)2} \in L^\infty \cap L^2$, for the system (5) with $\sigma = \sigma_{(3)}$.

Controller bank. The adaptive controllers $v_{(1)}(t)$, $v_{(2)}(t)$ and $v_{(3)}(t)$ make up a controller bank, for which we need to design a switching scheme to select the most appropriate controller from the controller bank to control the system (5).

C. Control Switching Scheme

Designing a control switching scheme is an important step in a multiple-model control scheme. The switching scheme used in this paper is to first form cost functions for the designed estimation errors, which correspond to different failure patterns, then calculate and compare the costs, and finally select the controller with the minimum cost function as the current control signal. Further, to avoid the discontinuity problem from the control switching, we introduce an exponential function to smooth the input signal when switching occurs. To ensure system performance, we also introduce a nonzero waiting time $T_{min} > 0$ between every two control switchings.

1) Performance Indices: Referring the form of that from [4], we determine cost functions based on the estimation errors which are defined in (31), (32), (35), (36), (43), and (44). The cost functions are:

$$J_i(t) = a\left(e_{(i)1}^T(t) e_{(i)1}(t) + e_{(i)2}^T(t) e_{(i)2}(t)\right) + b\int_0^t e^{-\lambda(t-\tau)}\left(e_{(i)1}^T(\tau) e_{(i)1}(\tau) + e_{(i)2}^T(\tau) e_{(i)2}(\tau)\right) d\tau, \tag{46}$$

for $i = 1, 2, 3$, where a, b and λ are constant design parameters. Different a and b make different effect of instantaneous and long-term accuracy measures, and λ is the forgetting factor which determines the memory of the indices in rapidly switching environments and ensures boundedness of $J_i(t)$ for bounded $e_{(i)1}(t)$ and $e_{(i)2}(t)$.

The control switching scheme is implemented by calculating and comparing the cost functions for $i = 1, 2, 3$, and choosing the control law $v(t)$ as $v(t) = v_j(t)$, where j is from

$$j = \arg \min_{j=1,2,3} J_i(t). \tag{47}$$

2) Properties of the Overall Control System: The stability and tracking properties of the closed-loop control system are given below.

Theorem 1: All signals in the multiple-model adaptive actuator failure compensation control system consisting of the system (5), the failure model (6), the controller (24), (25) and (26), the adaptive laws (39) and (45), and the switching algorithm based on (46), are bounded, and the tracking error $e(t) = y(t) - y_m(t)$ satisfies

$$\lim_{t\to\infty} (y(t) - y_m(t)) = 0. \tag{48}$$

This theorem can be proved for two cases: (i) the desired stabilization and tracking control performance of the select control law matches the actual actuator failure pattern, and (ii) the desired multiple-model switching control mechanism which selects the control law corresponding to the minimum cost functions.

3) Smooth Control: The switching-based control law may inevitably bring the problem of discontinuous control inputs. However, NSV control system is a kind of aircraft motion flight control system, whose actuators may not move fast enough to response the discontinuous control inputs. In this paper, we introduce an exponential function to slow down the jumping of control input signal when control switching occurs. We assume the control switching occurs at t_1, when $t < t_1$, the controller $v_{(i)}$ has the minimum cost function, and when $t > t_1$, the controller $v_{(j)}$ has the minimum cost function, so we have the smooth control signal as

$$v(t) = v_{(i)}(t) + \delta(t)(v_{(j)}(t) - v_{(i)}(t)), \tag{49}$$

where

$$\delta(t) = \begin{cases} 0, & \text{if } t < t_1, \\ e^{T_{min}(t - t_1 - T_{min})}, & \text{if } t_1 \le t < t_1 + T_{min}, \\ 1, & \text{if } t \ge t_1 + T_{min}, \end{cases}$$

with T_{min} being a design parameter, which should be chosen as small as the actuator permits, to ensure the accurate selection of the controller.

IV. Simulation Study

In this section, we use the simulation results to verify the desired failure compensation performance of the designed multiple-model control system for control of the NSV attitude dynamic model in [10]. The attitude dynamic equations of a NSV at a Mach = 3.16 and an altitude of height = 97,167 ft are given in (1). Writing the NSV attitude model (1) as the form (5) with $f_1(x_1) = \Xi(\gamma)$, $f_2(x_1, x_2) = J^{-1}\Omega(\omega)J\omega$, and $g_2(x_1, x_2) = J^{-1}\Psi$.

We then simulate the multiple-model adaptive failure compensation control system stated in Section III for four cases: (i) no failure: $u_i(t) = v_i(t)$, $i = 1, 2, \ldots, 6$, for $t < 50s$; (ii) u_1 failure: $u_1(t) = 0.2$ rad, $u_i(t) = v_i(t)$, $i = 2, 3, \ldots, 6$, for $50s \le t < 100s$; (iii) u_1 becomes normal, no failure: $u_i(t) = v_i(t)$, $i = 1, 2, \ldots, 6$, for $100s \le t < 150s$; (iv) u_6 failure: $u_6(t) = 0.2 \sin t$ rad, $u_i(t) = v_i(t)$, $i = 1, 2, \ldots, 5$, for $t \ge 150s$.

In the simulation study, we choose the design and simulation parameters as follows: $c_1 = 0.8$, $c_2 = 2.2$, $\Gamma_1 = 100$,

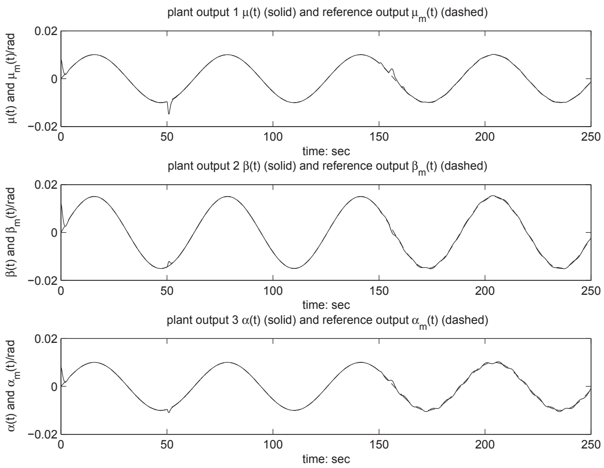

Fig. 1. System responses.

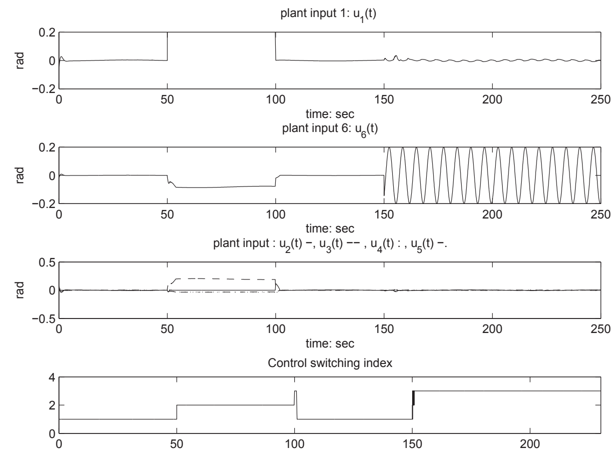

Fig. 2. Control input signal and control switching index.

$\Gamma_6 = 100$, $a = 1$, $b = 1$, $T_{min} = 0.01$, $y(0) = [\mu_0, \beta_0, \alpha_0]^T = [0.08, 0.012, 0.08]^T$, $\omega_1(t) = 1$, $\omega_6(t) = \sin(t)$, $\theta_1(0) = 0$, and $\theta_6(0) = 0$, (because the system operates under no failure case at the beginning). A given reference output is chosen as: $y_m(t) = [0.01\sin(0.1t), 0.015\sin(0.1t),\ 0.01\sin(0.1t)]^T rad$, to make a reasonable trajectory. The obtained simulation results are shown in Figures 1–2, respectively for the plant output $y(t)$, the reference output $y_m(t)$, the control input $u_i(t), i = 1, 2, \ldots, 6$, and the control switching index.

The system response results in Fig. 1 show the desired system performance: the closed-loop system is stable and the output $y(t)$ tracks $y_m(t)$ asymptotically. When an actuator failure occurs, there is a transient response in the tracking errors, and as time goes on, the output tracking error $e(t) = y(t) - y_m(t)$, starting from a transient value, becomes smaller, which verifies the effective failure compensation of the designed multiple-model adaptive control system. The simulation results in Fig. 2 show the improved control input signal and the quick and accurate control switching. The switching process settles down quickly at the most appropriate controller after a short initial period of rapid switching, when an actuator failure occurs or disappears.

V. Conclusion

This paper develops a new multiple-model adaptive actuator failure compensation control scheme for a class of nonlinear systems with some redundant actuators motivated from a near-space vehicle control application. Such design uses multiple controllers, each of which is designed with a parameter estimate and corresponds to a failure pattern of interest, to compensate the uncertainties caused by actuator failures. Each parameter estimator is designed by the Lyapunov method and leads to a performance cost function. The minimum of such cost functions is used as the indicator to select the most appropriate controller. A smooth control is introduced to the multiple-model control frame to avoid the discontinuity problem from the control switching. Such an adaptive failure compensation scheme has been evaluated on a near-space vehicle dynamic model, to show the desired failure compensation performance.

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References

[1] J. D. Boskovic and R. K. Mehra. Multiple-model adaptive ight control scheme for accommodation of actuator failures. *AIAA Journal of Guidance, Control, and Dynamics*, 25:712–724, 2002.

[2] Y. Y. Guo, B. Jiang, Y. M. Zhang. Actuator fault compensation via multiple model based adaptive control. *in Proceedings of 7th World Congress on Intelligent Control and Automationl*, Chongqing, China, 4246-4250, 2008.

[3] B. Jiang, Z. Gao, P. Shi, and Y. Xu. Adaptive fault-tolerant tracking control of Near-Space Vehicle using takagi-sugeno fuzzy models. *IEEE Transactions on fuzzy systems*, 18:1000–1007, 2010.

[4] K. S. Narendra and J. Balakrishnan. Adaptive control using multiple models. *IEEE Trans. on Automatic Control*, 42:171–187, 1997.

[5] J. T. Parker, A. Serrani, S. Yurkovich et al. Control-oriented modeling of an air-breathing hypersonic vehicle. *Journal of Guidance, Control, and Dynamics*, 30(3)856–868, 2007.

[6] D. O. Sigthorsson, P. Jankovsky, Serrani A, et al. Robust linear output feedback control of an airbreathing hypersonic vehicle. *Journal of Guidance, Control, and Dynamics*, 31(4)1052–1066, 2008.

[7] C. Tan, X. Yao, G. Tao, and R. Qi. A multiple-model based adaptive actuator failure compensation scheme for control of near-space vehicles. *the 8th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes (SAFEPROCESS)*, Mexico City, Mexico, August 29-31, 594–599, 2012.

[8] G. Tao, *Adaptive Control Design and Analysis*, John Wiley & Sons, Hoboken, NJ, 2003.

[9] G. Tao, S. Chen, X. Tang, and S. M. Joshi. Adaptive Control of Systems with Actuator Failures. *Springer- Verlag*, London, 2004.

[10] Y. Xu, B. Jiang, G. Tao and Z. Gao. Fault tolerant control for a class of nonlinear systems with application to near space vehicle. *Circuir Syst Signal Process*, 30:655–672, 2011.

[11] X. Yao, G. Tao, R. Qi, and B. Jiang. An adaptive actuator failure compensation scheme for an attitude dynamic model of near space vehicles. *2012 American Control Conference*, Montréal, Canada, June 27-June 29, 368–373, 2012.

[12] Y. W. Zhang and S. J. Qin. Adaptive actuator fault compensation for linear systems with matching and unmatching uncertainties. *Journal of Process Control*, 19:985–990, 2009.