Robust Adaptive Attitude Synchronization of Rigid Body Networks with Unknown Inertias

Iman Fadakar
Dept. of Mechanical Engineering
University of Waterloo
Email: iman512003@gmail.com

Baris Fidan
Dept. of Mechanical Engineering
University of Waterloo
Email: fidan@uwaterloo.ca

Jan Huissoon
Dept. of Mechanical Engineering
University of Waterloo
Email: jph@uwaterloo.ca

***Abstract*—In this paper we consider the attitude synchronization task for rigid body networks. The configurations of such systems evolve on $SO(3)$, which is a nonlinear manifold and needs a special treatment for fulfilling this task. We propose two robust adaptive algorithms to handle the effects of unmodeled dynamics and unknown inertia matrices of agents. Our first approach is based on $\sigma - modification$ method which "trap" the trajectories of the system in to a small neighborhood around the origin. In the second algorithm, the effects of external disturbances are removed using a discontinuous function. We use Lyapunov methods to prove the main results of this paper. Simulation results show the effectiveness of our proposed methods.**

I. Introduction

Most of the previous works in the field of control and motion planning for robotic networks were based on centralized methods [1], Compared to distributed algorithms, centralized controllers usually need more time and resources to achieve a particular task. Hence, there has been a growing interest in cooperative and distributed control of robotic networks.

These algorithms have proven to be more flexible and stable and as stated before are more efficient in terms of costs and time [2]. In the past decade, many distributed algorithms have been proposed for multi agents systems to solve a wide range of problems such as, consensus [3], flocking [4], coverage [5], synchronization [6], formation control [7]. Among them consensus task has received a particular attention since it can encompass many other problems. For example, flocking can be seen as a consensus task between the agent bearings in 2D or 3D.

In this paper, we consider the attitude synchronization problem for multiple agents. The goal is to design a set of control laws, one for each agent, that steer them to a common attitude. This problem in its very nature, can be expressed as a consensus problem. The main difference between this problem and previous synchronization problems such as [3],[8],[9], is the fact that attitude synchronization problem is required to be solved on a general nonlinear manifold instead of a vector space. The latter causes many difficulties in attitude control tasks. For instance, the intrinsic nature of this nonlinear manifold would not permit us to construct a continuous feedback control law that globally stabilizes the attitude of a rigid body on it. In [10], the authors considered this problem in detail.

The same hindrance also exists in nonholonomic systems but from a different nature which is the inability of these systems to satisfy the Brocketts [11] necessity condition. In order to cope with these difficulties several methods have been proposed such as using quaternions as coordinate charts on $SO(3)$for representing the attitude of a rigid body [12] and geometric approach [13].

Both the quaternions and geometric approaches have their own capabilities and weaknesses. For example, quaternions double cover the rotation manifold of a rigid body and hence they cause ambiguities in representation of the attitude of a rigid body, but they are easy to work with. Geometric methods, on the other hand, give more insight about the nature of the system and enable us to follow a systematic approach for the purpose of control.

A particular geometric approach was discussed in [14] where by choosing an appropriate error function on the configuration manifold of the system, the error vectors and corresponding control laws would be constructed. Following this procedure, the control laws will be independent of any charts, and for the purpose of implementation one can use both Euler angles and quaternions.

Attitude synchronization task has many applications in astronomy and defence. For example, in [15] the problem of imaging using multiple synthetic aperture radars (SAR) has been considered. Another practical application of attitude synchronization problem is in satellite formation control for the on orbit assembly tasks or interferometery missions.

Attitude synchronization problem can be approached from several different directions. For instance, in [16], the authors used reduction techniques and potential shaping approach [17] to derive the control laws. They studied the equilibrium of the system by using energy-cassimir methods. [18] proposed a quaternion based method. [19] authors used a geometric approach to tackle this problem. In this paper we use a geometric framework for the purpose of designing control laws.

We propose two different strategies to handle the effects of external noise or unmodeled dynamics. We show that in the first method there is a trade off between the control effort and the efficiency of the synchronization task. Using the second approach, the effects of external disturbances are completely removed at the expense of using a discontinuous function in the input. We use Lyapunov methods to study the stability of our proposed control algorithms. The rest of this paper is organized as follows: in Section III, gives a quick review on rigid body dynamics and formally introduces the control problems. Section IV presents the adaptive control strategy to address the first problem. Section V presents our robust adaptive approach to handle the second problem. In Section VI, we study the artificial potential energy of the network and its relation to the total "disagreement" in the network. We

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present simulation results in Section VIII.

II. Preliminiaries

In this section we presen a quickreview of (a) mathematical concepts and definitions and (b) governing dynamic equations, which we used to prove the main results of this paper.

A. Mathematical Background

Graph theory is a powerful tool for representing the network topology of a group of rigid bodies. Here, we state the theorems and definitions in the abriged form and refer to [20] for full version of these theorems. A directed graph $G = (V, \varepsilon)$ of order n consists of a vertex set V which has n elements and another set which contains the edges of this directed graph, i.e. $\varepsilon = V \times V$. We indexed the elements of the vertex set with $\{1, ..., n\}$. A graph is called undirected if $(j, i) \in \varepsilon$ whenever $(i, j) \in \varepsilon$. A weighted graph is a triplet $D = (V, \varepsilon, A)$ where A stands for weighted adjacency matrix which has the following properties: for each i, j in the index set the entry $a_{ij} > 0$ if $(i, j) \in \varepsilon$ and $a_{ij} = 0$ otherwise. $SO(3)$ denotes the set of all orthogonal matrices in $\mathbb{R}^{3\times3}$ whose determinants are equal to 1. $so(3)$ denotes the Lie algebra of $SO(3)$ and it is defined as follows:

$$so(3) = \left\{O \in \mathbb{R}^{n\times n} | O^T = -O\right\}$$

$[\cdot]^\wedge : \mathbb{R}^3 \to so(3)$ is called *hat map* and it is defined as:

$$x = [x_1, x_2, x_3]^T \in \mathbb{R}^3, [x]^\wedge = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \quad (1)$$

Another useful map in this context is *vee map* $[\cdot]^\vee : so(3) \to \mathbb{R}^3$, which is the inverse of hat map. These maps have several important characteristics as follows [21]:

$$[a]^\wedge b = a \times b = -b \times a = -[b]^\wedge \times a \quad (2)$$

$$tr\left[Q[a]^\wedge\right] = \frac{1}{2} tr\left[[a]^\wedge\left(Q - Q^T\right)\right] = [a]^T\left(Q^T - Q\right)^\vee \quad (3)$$

for all $a, b \in \mathbb{R}^3$ and $Q \in \mathbb{R}^{3\times3}$. Computing the gradient of a function on $SO(3)$ has crucial importance in stability analysis since in the context of rigid body dynamics, the Lyapunov functions are usually defined on the general manifold of the system. It is shown in [22] that if $V(R) = trace\left(R^T A\right)$ where $A \in \mathbb{R}^{3\times3}$ and $R \in SO(3)$, then the gradient of V on $SO(3)$ is as follows:

$$grad(V)_{SO(3)} = \frac{1}{2} R\left[R^T A - A^T R\right]$$

B. Rigid Body Dynamics

The attitude of a general rigid body evolves on the nonlinear manifold $SO(3)$. This group represents the set of all rotations in $\mathbb{R}^{3\times3}$. Suppose that we have m rigid bodies whose rotation matrices with respect to an inertial frame are expressed by $R_k \in SO(3), k \in \{1, ..., m\}$. Then, the equations of motion of each agent can be expressed as follows:

$$J_k \dot{\Omega}_k = [J_k \Omega_k]^\wedge \Omega_k + u_k \quad (4)$$

$$\dot{R}_k = R_k [\Omega_k]^\wedge \quad (5)$$

where $J_k = diag(J_{k1}, J_{k2}, J_{k3})$ denotes the inertia matrix of $k{-}th$ agent, J_{k1}, J_{k2}, J_{k3} are principal moments of inertia, and without loss of generality we assume that $J_{k1} \leq J_{k2} \leq J_{k3}$. $u_k \in \mathbb{R}^3$ is the control input and $\Omega_k \in \mathbb{R}^3$ is the angular velocity of this agent. From (4) and (5) it is clear that these vectors are defined in body frame coordinate axis. The components of these vectors in inertial coordinate frame can be easily obtained by left multiplication of these vectors by the rotation matrix. The previous model has been widely used in control theory to represent the attitude dynamics of a fully actuated vehicle which can be modeled as a rigid body. A more accurate model can be obtained for the vehicles which operate in the presence of external field (such as wind, water, etc,...) by adding a damping term which is directly related to the angular velocity of the vehicle.

III. Problem Definition

In this section we formally introduce the problems which we want to tackle in this paper.

The main objectives is synchronization of the attitudes of multiple rigid bodies with unlnown inertia matrix and in the presence of bounded external noises and unmodeled dynamics. In this regard we introduce the following problems.

Problem.1 Consider a network of n rigid body agents in (Fig.1) whose dynamics are governed by (4) and (5) with unknown inertia matrix. Design a set of control laws that asymptotically synchronizes the attitudes of agents in the network, i.e. $\Omega_1 = \Omega_2 = \cdots = \Omega_n = \Omega_d$ and $R_1 = \cdots = R_2$, where Ω_d is the reference signal for angular velocity.

Problem.2 Consider Problem.1, but additionally, assume that there exist bounded disturbance and unmodeled dynamics that affects (4), i.e.

$$J_k \dot{\Omega}_k = [J_k \Omega_k]^\wedge \Omega_k + u_k + \delta_k$$
$$\dot{R}_k = R_k [\Omega_k]^\wedge$$

where the disturbance and uncertainties effects are modeled in the term δ_k. Design a method to attenuate or eliminate the effect of this term. For Problem.2, we can consider two

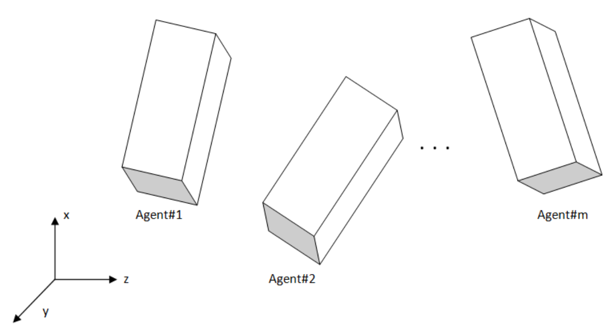

Fig. 1. A typical network of rigid bodies which consists of m agents.

different situations. In the first case, we assume that we know the upper bound of this unmodeled dynamics vector i.e. $\|\delta_k\| \leq \Delta_k,\ k \in \{1, \cdots, m\}$ where m is the number of agents in the network. In the second case, we assume that the upper bounds of each component of these vectors are known, i.e. $\|\delta_{k,j}\| \leq \Delta_{k,j},\ k \in \{1, \cdots, m\},\ j \in \{1, 2, 3\}$. Note that the latter can be regarded as a special case of the first one, using a conservative approximation.

IV. Adaptive Attitude Synchronization

The main goal of this section is to solve Problem.1 for a network of m agents. We will find a set of control inputs for the agents to synchronize their attitudes while having a synchronized velocity, Ω_d. In general, for stabilization problems, we do not need to have the value of the agent's inertia matrix but for tracking purpose we need to have this information to design the control laws. In this section, we propose an adaptive control law for this problem based on the results of [22] as follows:

$$u_k = -\left[\hat{J}_k\Omega_k\right]^{\wedge}\Omega_k + \hat{J}_k\dot{F}_k + \mu\left(F_k - \Omega_k + \Omega_d\right) \tag{6}$$

where, $F_k = \alpha\sum_{j=1}^{m} a_{jk}\left[R_k^T R_j - R_j^T R_k\right]^{\vee}, k = 1,\cdots,m, \alpha \geq 0$. μ is a positive constant and $\hat{J}_k$ is the estimate of inertia matrix of agent k and a_{jk} is the (j,k) element of the adjacency matrix. To design an adaptive law for obtaining $\hat{J}_k$. We consider the modified version of the Lyapunov function proposed in [22]:$(l > 0)$

$$V = \frac{-1}{2}\sum_{k=1}^{m}\sum_{j=1}^{m} a_{jk} trace\left(R_k^T R_j\right) + \sum_{k=1}^{m}\frac{1}{2d_k}\left\|J_k - \hat{J}_k\right\|_F^2 + \frac{l}{2}\sum_{k=1}^{m}\left(\Omega_k^r - F_k\right)^T J_k\left(\Omega_k^r - F_k\right), \tag{7}$$

where $l, d_k, k \in \{1,\cdots,m\}$, are positive constants and $\Omega_k^r = \Omega_k - \Omega_d$. $\|\cdot\|_F$ denotes the Frobenius norm. In fact, the values of d_k are acting as tuning gains of the adaptation laws and could be changed to enhance the performance of the adaptation process. Next, we compute the time derivative of this Lyapunov function as follows:

$$\dot{V} = \frac{-1}{\alpha}\sum_{k=1}^{m} trace\left(-\left[F_k\right]^{\wedge}\left[\Omega_k\right]^{\wedge}\right) + \sum_{k=1}^{m}\frac{1}{d_k} trace\left[\tilde{J}_k\dot{\tilde{J}}_k\right] + l\sum_{k=1}^{m}\left(\Omega_k^r - F_k\right)^T\left(J_k\dot{\Omega}_k^r - J_k\dot{F}_k\right) \tag{8}$$

where $\tilde{J}_k = J_k - \hat{J}_k$,by using the fact that $trace\left(-[a]^{\wedge}[b]^{\wedge}\right) = 2a^T b$ for $a, b \in \mathbb{R}^3$ and the kinematic equations (4),(5), we would have:

$$\dot{V} = \frac{-2}{\alpha}\sum_{k=1}^{m} F_k^T\Omega_k + \sum_{k=1}^{m}\frac{1}{d_k} trace\left[\tilde{J}_k\dot{\tilde{J}}_k\right] + l\sum_{k=1}^{m}\left(\Omega_k^r - F_k\right)^T\left(\left[J_k\Omega_k\right]^{\wedge}\Omega_k + u_k - J_k\dot{F}_k\right) \tag{9}$$

Next, we put (6) into (9):

$$\dot{V} = \frac{-2}{\alpha}\sum_{k=1}^{m} F_k^T\Omega_K + \sum_{k=1}^{m}\frac{1}{d_k} trace\left[\tilde{J}_k\dot{\tilde{J}}_k\right] + l\sum_{k=1}^{m}\left(\Omega_k^r - F_k\right)^T\left(\left[J_k\Omega_k\right]^{\wedge}\Omega_k - \left[\hat{J}_k\Omega_k\right]^{\wedge}\Omega_k + \hat{J}_k\dot{F}_k + \mu\left(F_k - \Omega_k + \Omega_d\right) - J_k\dot{F}_k\right) \tag{10}$$

Here we can see the main rationale for using the Frobenius norm in the structure of the Lyapunov function (7). This particular norm enables us to design adaptation laws for each agent by combining the second and third term in (10). We rearrange (10) as follows:

$$\dot{V} = l\sum_{k=1}^{m}\left(\Omega_k^r - F_k\right)^T\left(\left[\tilde{J}_k\Omega_k\right]^{\wedge}\Omega_k + \mu(F_k - \Omega_k + \Omega_d) - \tilde{J}_k\dot{F}_k\right) + \frac{-2}{\alpha}\sum_{k=1}^{m} F_k^T\Omega_k + \sum_{k=1}^{m}\frac{1}{d_k} trace\left[\tilde{J}_k\dot{\tilde{J}}_k\right] \tag{11}$$

Next, using the identities $a\cdot b = trace(ab^T)$ and $a\cdot(b\times c) = b\cdot(c\times a) = c\cdot(a\times b)$, for $a, b, c \in \mathbb{R}^3$, we have

$$\dot{V} = l\mu\sum_{k=1}^{m}(\Omega_k^r - F_k)^T(F_k - \Omega_k + \Omega_d) \\ \sum_{k=1}^{m} trace\left[\tilde{J}_k\left(\frac{1}{d_k}\dot{\tilde{J}}_k - \dot{F}_k(\Omega_k^r - F_k)^T - \Omega_k((\Omega_k^r - F_k)\times\Omega_k)^T\right)\right] \\ \frac{-2}{\alpha}\sum_{k=1}^{m} F_k^T\Omega_k \tag{12}$$

Choosing the adaptation laws as:

$$\dot{\tilde{J}}_k = d_k\left[\dot{F}_k(\Omega_k^r - F_k)^T + \Omega_k((\Omega_k^r - F_k)\times\Omega_k)^T\right] \tag{13}$$

and using (13) in (12), after simplification, (12) would become as follows:

$$\dot{V} = \sum_{k=1}^{m} -\left(-\mu l - \frac{1}{\alpha}\right)\left\|\Omega_k^r - F_k\right\|^2 - \frac{1}{\alpha}\left(\left\|\Omega_k^r\right\|^2 + \left\|F_k\right\|^2\right) - \frac{2}{\alpha}F_k^T\Omega_0 \tag{14}$$

The summation over the last term of (14) is equal to zero, by using this fact and choosing μ in a way that $\mu l \geq \frac{1}{\alpha}$, we could conclude that the time derivative of (7) is negative semi definite. From this we can deduce that $\Omega_k^r, F_k, \tilde{J}_k$ are belonging to L_∞. From the fact that the Lyapunov function (7) is lower bounded and its time derivative is nonincreasing, we can find that it has a finite time i.e. $\lim_{t\to\infty} V(t) = V_\infty$. From (4) and (5), we can conclude that $\dot{\Omega}_k^r, \dot{F}_k \in L_\infty$.Furthermore, from (14), we have:

$$\dot{V} \leq -\frac{1}{\alpha}\sum_{k=1}^{m}\left(\left\|\Omega_k^r\right\|^2 + \left\|F_k\right\|^2\right) \\ \Rightarrow -\alpha\int_0^\infty \dot{V}\ dt \geq \sum_{k=1}^{m}\int_0^\infty \left\|\Omega_k^r\right\|^2\ dt + \int_0^\infty \left\|F_k\right\|^2\ dt \\ \Rightarrow \alpha(V(0) - V_\infty) \geq \sum_{k=1}^{m}\left\|\Omega_k^r\right\|_2^2 + \left\|F_k\right\|_2^2 \tag{15}$$

So we can deduce that $\Omega_k^r, F_k \in L_2$. By applying Barbalat's lemma [23]we would have $\Omega_k^r \to 0, F_k \to 0$. For solving the previous problem, we did not assume any reference signal for the rotation matrix, and the consensus will be acheived on the general manifold of the system. In order to perform the previous task in the presence of such a reference signal, by considering a tree like topolgy for the network communication graph, it is possible to consider virtual leader with a fixed angular velocity $\Omega_d$and desired rotation matrix $R_d(t)$ and then by using the same procedure we could prove the stability results.

V. Robust Adaptive Control

In this section, we consider the Problem.2. In fact, we want to design a more robust control law for the previous task. In the past approach, we assumed that the system is perfect and there is no external noise in the input. However, for aeronautic applications a small noise caused by unmodeled dynamics or from the external sources can cause a catastrophic failure. In order to handle these types of noises we need to modify our previous approach. Agents dynamic models for this case were discussed in section III. As stated before, first we assume that the upper bound for these vectors are known i.e $\|\delta_k\| \leq \Delta_k$. We use Lyapunov function (7) to analyze this problem. The time derivative of this Lyapunov function in this case would have the following form:

$$\dot{V} = l\sum_{k=1}^{m}(\Omega_k^r - F_k)^T([J_k\Omega_k]^\wedge \Omega_k + u_k + \delta_k - J_k\dot{F}_k)$$
$$\frac{-2}{\alpha}\sum_{k=1}^{m}F_k^T\Omega_k + \sum_{k=1}^{m}\frac{1}{d_k}trace\left[\tilde{J}_k\dot{\hat{J}}_k\right] \tag{16}$$

we need to devise a method to suppress the effects of noises on the dynamics of each of agents for acheiving the attitude synchronization task. The latter can be done by adding an extra term to the control law (6) which is related to the upper bound on the noise vector. Our proposed control law for this case is as follows:

$$u_k = -\left[\hat{J}_k\Omega_k\right]^\wedge \Omega_k + \hat{J}_k\frac{d}{dt}F_k$$
$$+\mu(F_k - \Omega_k + \Omega_d) - \frac{\Delta_k^2(\Omega_k^r - F_k)}{\Delta_k\|\Omega_k^r - F_k\| + \epsilon_k} \tag{17}$$
$$\dot{\hat{J}}_k = d_k[\dot{F}_k(\Omega_k^r - F_k)^T$$
$$\Omega_k((\Omega_k^r - F_k)\times\Omega_k)^T - 2\sigma_k\hat{J}_k] \tag{18}$$

where σ_k and ϵ_k are design parameters. (17) and (18) have an extra term compared to equations (6) and (13). The main purpose of these terms is to confine the trajectories of the system to a small neighborhood of the origin by choosing sufficiently small values for σ_k and ϵ_k. By choosing such values for these parameters we would increase the control effort and hence there is a tradeoff between the increasing the efficiency of the synchronization task and the amount of control effort. After the insertion of (17) and (18) in (16) we would have:

$$\dot{V} = \sum_{k=1}^{m} -\left(\mu l - \frac{1}{\alpha}\right)\|\Omega_k^r - F_k\|^2 - \frac{1}{\alpha}\left(\|\Omega_k^r\|^2 + \|F_k\|^2\right)$$
$$+(\Omega_k^r - F_k)^T\left(\delta_k - \frac{\Delta_k^2(\Omega_k^r - F_k)}{\Delta_k\|\Omega_k^r - F_k\| + \epsilon_k}\right)$$
$$\sigma_k trace[\tilde{J}_k\hat{J}_k] \tag{19}$$

since $\|\delta_k\| \leq \Delta_k$, we can find that:

$$\dot{V} \leq \sum_{k=1}^{m} -\left(\mu l - \frac{1}{\alpha}\right)\|\Omega_k^r - F_k\|^2 - \frac{1}{\alpha}(\|\Omega_k^r\|^2 + \|F_k\|^2)$$
$$\sigma_k trace[\tilde{J}_k\hat{J}_k] + \epsilon_k \tag{20}$$

Next we need to find some bounds for the $\sum_{k=1}^{m}\sigma_k trace[\tilde{J}_k\hat{J}_k]$ term. Here we use the approach from [24]:

$$\sum_{k=1}^{m}\sigma_k trace[\tilde{J}_k\hat{J}_k] = \sum_{k=1}^{m}\sigma_k trace[\tilde{J}_k(J_k - \tilde{J}_k)]$$
$$= \sum_{k=1}^{m}\sigma_k(-\tilde{J}_{k,ii}^2 + \tilde{J}_{k,ij}J_{k,ji}) \leq \sum_{k=1}^{m}\sigma_k\left(-\frac{\tilde{J}_{k,ii}^2}{2} + \frac{J_{k,ij}^2}{2}\right)$$
$$= \sum_{k=1}^{m}\frac{-\sigma_k}{2}trace[\tilde{J}_k^2] + \frac{\sigma_k}{2}trace[J_k^2]$$
$$= \sum_{k=1}^{m}\frac{-\sigma_k}{2}\left\|\tilde{J}_k\right\|_F^2 + \frac{\sigma_k}{2}\|J_k\|_F^2 \tag{21}$$

where we used summation convention for brevity, we need to assume that the values of the maximum eigenvalues $\lambda_{k,M}$ of the inertia matrices of agents are known. This assumption is not a restrictive assumption since we can easily approximate these values. By using this assumption, (21) would result in:

$$\sum_{k=1}^{m}\sigma_k trace[\tilde{J}_kJ_k] \leq \sum_{k=1}^{m}\frac{-\sigma_k}{2}\left\|\tilde{J}_k\right\|_F^2 + \frac{3\sigma_k}{2}\lambda_{K,M}^2 \tag{22}$$

then we could write (20) as follows:

$$\dot{V} \leq \sum_{k=1}^{m} -\left(\mu l - \frac{1}{\alpha}\right)\|\Omega_k^r - F_k\|^2 - \frac{1}{\alpha}(\|\Omega_k^r\|^2 + \|F_k\|^2)$$
$$\frac{-\sigma_k}{2}\left\|\tilde{J}_k\right\|_F^2 + \frac{3\sigma_k}{2}\lambda_{K,M}^2 + \epsilon_k \tag{23}$$

we can conclude that if the following condition holds:

$$\sum_{k=1}^{m}\left(\mu l - \frac{1}{\alpha}\right)\|\Omega_k^r - F_k\|^2 + \frac{1}{\alpha}(\|\Omega_k^r\|^2 + \|F_k\|^2)$$
$$+\frac{\sigma_k}{2}\left\|\tilde{J}_k\right\|_F^2 \geq \sum_{k=1}^{m}\left(\frac{3\sigma_k}{2}\lambda_{K,M}^2 + \epsilon_k\right) \geq \beta \tag{24}$$

then $\dot{V} \leq 0$. Here we can see the fact that by choosing sufficiently small ϵ_k and σ_k for each agent in the network we could 'trap' the trajectories of the system in to a small neighborhood around the origin.

In the next scenario, we assume that the upper bound for each element of the δ_k is known, i.e. $\|\delta_{k,i}\| \leq \Delta_{k,i}$, $i \in \{1,2,3\}$. The modified input for this case would be as follows:

$$u_k = -\left[\hat{J}_k\Omega_k\right]^\wedge \Omega_k + \hat{J}_k\frac{d}{dt}F_k$$
$$\mu(F_k - \Omega_k + \Omega_0) - P_k \tag{25}$$
$$\dot{\hat{J}}_k = d_k[\dot{F}_k(\Omega_k^r - F_k)^T + \Omega_k((\Omega_k^r - F_k)\times\Omega_k)^T] \tag{26}$$

where the $i-th$ element of P_k is as follows:

$$P_{k,i} = -\Delta_{k,i}sign(\Omega_k^r - F_k)_i \tag{27}$$

then we could find that by using (25) and (26) the time derivative of the Lyapunov function (7) would result in:

$$\dot{V} \leq \sum_{k=1}^{m} -\left(\mu l - \frac{1}{\alpha}\right)\|\Omega_k^r - F_k\|^2 - \frac{1}{\alpha}(\|\Omega_k^r\|^2 + \|F_k\|^2) \tag{28}$$

by using Barbalat lemma, we can prove the stability of the synchronization task.

VI. Disagreement Function

In order to prove the main stability results for previous sections we used (7) as a Lyapunov function. An important part of this function is the following term:

$$V_p = \sum_{k=1}^{m}\sum_{j=1}^{m} a_{jk} trace(I_{3\times 3} - R_k^T R_j) \tag{29}$$

The structure of control laws (6) and (17) depends on the gradient of this function. In fact, this function serves as a artificial potential energy between the attitude of agents [22],[16],[19]. In another word, this function measures the total "disagreement" in the network. In order to better understand this function we use the Rodrigues Formula [13]:

$$R_k^T R_j = exp(\zeta_{kj}) = 1 + \frac{\sin\|\zeta_{kj}\|}{\|\zeta_{kj}\|}\hat{\zeta}_{kj} + \frac{1-\cos\|\zeta_{kj}\|}{\|\zeta_{kj}\|^2}\hat{\zeta}_{kj}^2$$

to express (29) in terms of $\zeta_{kj} \in \mathbb{R}^3$ as follows:

$$V_p = \sum_{k=1}^{m}\sum_{j=1}^{m} 1 - \cos\|\zeta_{kj}\|$$

If the agents in the network are in a synchronized state, then this function would be equal to zero. It is also possible to choose different types of potential functions to fulfill the synchronization task.

VII. Simulation Results

In this section we present the simulation results for the previous algorithms. First, we consider the simulation results of the control law (6) together with adaptation law (13) for a network of three agents with a complete communication graph. Fig.2 illustrates the results for the angular velocity of these agents.

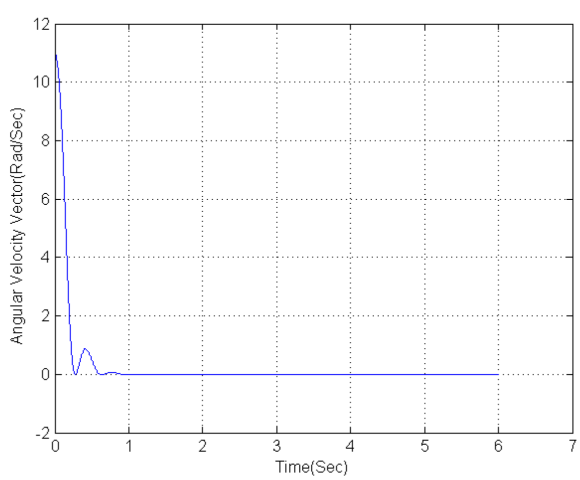

Fig. 2. Components of the angular velocity vector versus time

The desired angular velocity in this case is $[0,0,1]^T$. As it can be seen from Fig.2, the third component of the angular velocity vector of the agents in network converged to 1 after 2 second. Simulation parameters in this case are $\mu = 5$ and $\alpha = 1$. From Fig.3, we could deduce that the total disagreement in

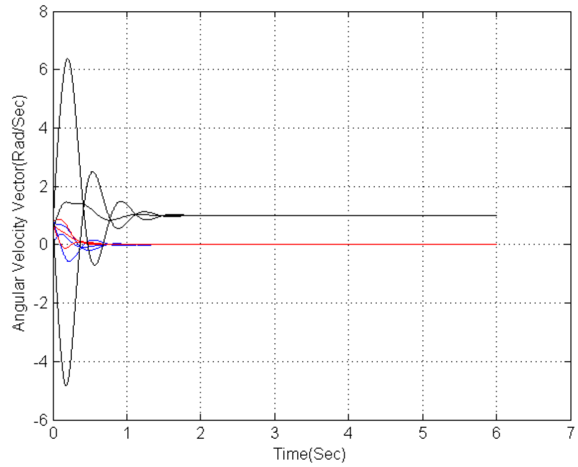

Fig. 3. Disagreement Function(dimensionless)

the network disappeared over time as a result of control law (6). This disagreement function of this network would have the following form:

$$\Psi = \sum_{i=1}^{3}\sum_{j=1}^{3}(I_{3\times 3} - R_i^T R_j) \to 0 \tag{30}$$

simulation results for the problem.2, are presented in Fig.4 and Fig.5. We used $\lambda_{K,M} = 1, \mu = 5, \alpha = 3$ for the control parameters. Network consists of three agents with complete communication graph. The robust control parameters are $\delta = \sqrt{300}$ and $\epsilon = 1$ for all agents. By using the results of the analysis in section V, we can deduce that the trajectories of the system eventually will reach to the following set:

$$\sum_{k=1}^{3}\left(5 - \frac{1}{3}\right)\|\Omega_k^r - F_k\|^2 + \frac{1}{3}(\|\Omega_k^r\|^2 + \|F_k\|^2) \tag{31}$$

$$\frac{\sigma_k}{2}\left\|\tilde{J}_k\right\|_F^2 \leq \sum_{k=1}^{m}\left(\frac{3\sigma_k}{2}\lambda_{K,M}^2 + \epsilon_k\right) \leq \frac{15}{2} \tag{32}$$

The norm of disturbance signal is less than $10\sqrt{3}$ and the desired angular velocity vector is $[0,0,1]^T$. As it can be seen from Fig.5, by choosing sufficiently small control parameters we can steer the system trajectoris near the desired value for the angular velocity vector. In this case the system states would be confined in the following set:

$$\sum_{k=1}^{m}\left(5 - \frac{1}{3}\right)\|\Omega_k^r - F_k\|^2 + \frac{1}{3}(\|\Omega_k^r\|^2 + \|F_k\|^2) + \frac{\sigma_k}{2}\left\|\tilde{J}_k\right\|_F^2$$

$$\leq \sum_{k=1}^{m}\left(\frac{3\sigma_k}{2}\lambda_{K,M}^2 + \epsilon_k\right) \leq \frac{0.15}{2} \tag{33}$$

The difference between the Fig.4 and Fig.5 stems from the fact that we choosed different values for the robust control parameters, In Fig.5 the robust control parameters, ϵ_k and δ_k, are very small and this will increase the efficiency of the network to fulfill the synchronization task.

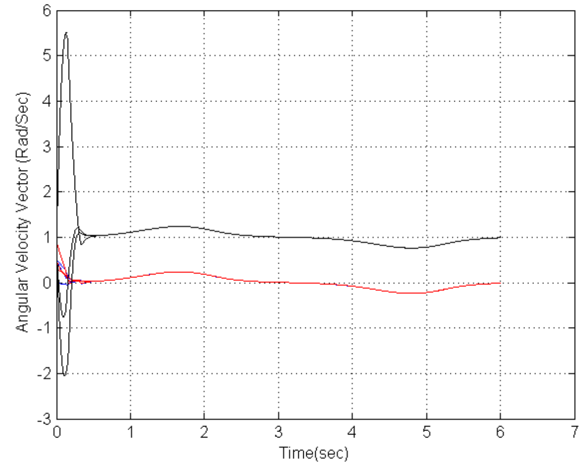

Fig. 4. Angular velocity vector component(robust adaptive approach with large values for control parameters) $\epsilon_k = 1, \delta_k = \sqrt{300}, \sigma_k = 1$

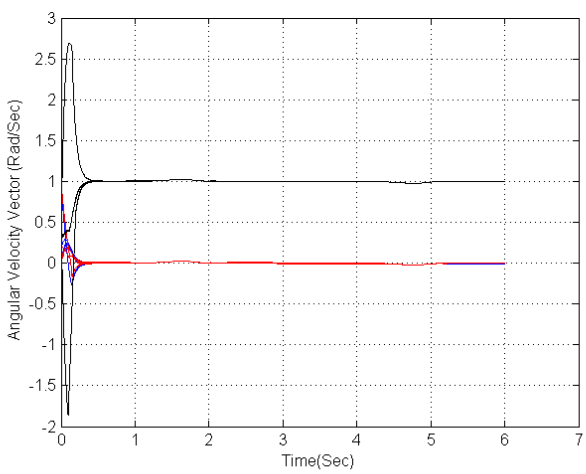

Fig. 5. Angular velocity vector component(robust adaptive approach with small values for control parameters) $\epsilon_k = 0.01, \delta_k = \sqrt{300}, \sigma_k = 0.01$

VIII. Conclusion

In this paper we considered the attitude synchronization task for a network of rigid bodies whose kinematics are evolving on the nonlinear manifold $SO(3)$. We proposed an adaptive control algorithm to achieve this task in the situations that inertia matrices of agents are unknown. Next, we proposed two robust adaptive methods to fulfill this task in the presence of unmodeled dynamics and external noises. A possible extension to this work is to modify the proposed control laws to achieve the attitude synchronization task for networks with switching topology graph.

IX. Acknowledgement

This work is supported by the Canadian NSERC Discovery Grant 116806 and 16922.

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