A Method for Performance Improvement of PID Control by Dual-Input Describing Function (DIDF) method

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***Abstract*— Though various techniques have been studied as a way of adjusting parameters of PID controllers, no perfect method of determining parameters is available to date. This paper proposes a new method for enhancing performance of PID controllers by using the characteristics of dual-input describing function (DIDF). In other words, if nonlinear elements with two inputs (DIDF) are connected in series to the plant, the critical point (-1+*j*0) for Nyquist stability theory can be moved to a position arbitrarily selected on the complex plane by determining necessary coefficients of the DIDF appropriately. This makes the application of the existing conventional PID parameter tuning methods a lot easier, and stability and robustness of the system are improved simultaneously due to the DIDF inserted.**

Keywords: PID controller tuning, Dual-Input Describing Functions.

I. INTRODUCTION

While various techniques have been studied as a way of adjusting PID parameters [1] ~ [6], no perfect method of determining parameters is available to date. In the case of the most widely-known Ziegler-Nichols method, it would not be permissible in a state where the system is in operation to increase proportional gains without securing safety measures and to bring the system next to the safety limit in actual circumstances. At the same time, lots of the methods suggested to improve conventional PID adjustment methods could be seen as an attempt to approximate the plant to 'first order delay + dead time[1], and determine PID parameters based on the dead time, time constants, and gains and so forth. As a result, such problems were identified that there is limitation on applicable plants, and stability of the close loop is not always guaranteed and so forth[2].

On the other hand, in case the level of non-linearity of nonlinear elements stands low (on-off, saturation, hysteresis and so forth, for example), describing functions are being used for the purpose of designing a certain specific nonlinear system. In this case, the describing functions, a sort of linearization technique of non-linearity, are used as mathematical linearization of non-linearity in case sine wave is permitted as an input [7]. Especially, in case a system having nonlinear elements is in a state of limit cycle, describing functions are used to measure the magnitude and frequency of the limit cycle, that is, sine wave for the input (or output) of non-linear elements.

West, et al. [8] has introduced a mathematical device which they call a dual-input describing function. It is an extension of the conventional describing function, which linearizes nonlinearity forced by two sinusoids. This method especially helps to understand input-output responses of the system which is in the state of limit cycle due to the presence of non-linear elements. If the application of DIDF is limited to the structure of feedback system tracking reference signals with non-linearity, the two inputs of the nonlinear element would be composed of two signals, that is, the error signal of steady-state and the sinusoid due to the limit cycle of the closed loop.

This paper proposes a new method for enhancing performance of PID controllers by using the characteristics of DIDF. In other words, if nonlinear elements with two inputs (DIDF) are connected in series to the plant, the critical point (-1+*j*0) for Nyquist stability theory can be moved to a position arbitrarily selected on the complex plane by determining necessary coefficients of the DIDF appropriately. This makes the application of the existing conventional PID parameter tuning methods a lot easier, and stability and robustness of the system are improved simultaneously due to the DIDF inserted. We propose a suitable DIDF structure and ways of determining its coefficients for this purpose, and verify its effectiveness by means of simulation.

II. DUAL-INPUT DESCRIBING FUNCTION

2.1 Dual-Input Describing Function

In case a signal having non-linearity of a high level such as a non-sinusoidal wave is input to a nonlinear element, it is difficult to analyze it with describing functions alone. It is thought that a signal having non-linearity of a high level is composed of the sum of two sine functions in most of the cases. Therefore, for the purpose of utilizing calculations of describing functions, an input to nonlinear element could be expressed as follows (Fig.1):

$$x(t) = x_1(t) + x_2(t) = h\sin(\omega_1 t + \theta_1) + l\sin(\omega_2 t + \theta_2) \qquad (1)$$

[1] Or the form 'integral + dead time' is also used.

[2] For overall evaluation including performance, stability and designing methods of PID controllers, refer to literature [6].

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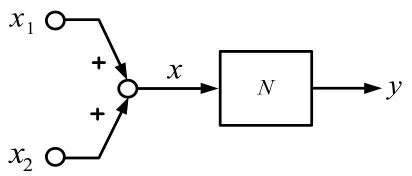


Fig. 1. Nonlinear element with two sinusoidal inputs

Since the amplitudes h, l and frequencies ω_1, ω_2 of two sine waves are determined by the state of the system and the external input, it seems reasonable that a harmonic relation is not thought to be existent between the frequencies of the two sine waves. As an example, we take the system which is in the state of limit cycle corresponding to a sinusoidal input. In this case, the frequencies of two inputs of nonlinear system are composed of ω_1 due to the limit cycle of the system (inner frequency) and ω_2 of an external sinusoidal input to nonlinear element.

Under the presupposition that the frequencies of the two inputs of sine waves to the nonlinear elements are not harmoniously connected, the describing function for the sine wave having the amplitude l could be expressed as follows[9]:

$$N_l = N_{pl} + jN_{ql} \tag{2.a}$$

$$N_{pl} = \frac{2}{l}\overline{y(0)\sin(\theta_2)},\ N_{ql} = \frac{2}{l}\overline{y(0)\cos(\theta_2)} \tag{2.b}$$

At the same time, the describing function for a signal having the amplitude h could be found by replacing θ_2 and l with θ_2 and h in Eq.(2).

On the other hand, let's think of a case where if signal x_1 in Eq.(1) 'varies slowly,' against the amplitude l of x_2, namely

$$T\left|\frac{dx_1(t)}{dt}\right| << l \tag{3}$$

is satisfied, where T is the period of $x_2(t)$. It could be that the servo system tracking step input stays in the state of limit cycle having amplitude l and frequency ω_2, to take an example. In this case, two inputs of the nonlinear elements are composed of two signals, that is, the steady-state error $e(t)=r(t)-y(t)$ and the sinusoid due to the limit cycle of the closed loop. If it is assumed that input and output of nonlinear elements are, for example

$$x(t) = B + A\sin(\omega t + \theta),\ y(t) = B' + A'\sin(\omega t + \theta) \tag{4}$$

then, describing functions representing nonlinear elements could be expressed as two DIDF's as follows, and as a result of this, it is confirmed that N, the nonlinear elements of Fig. 1, is composed of 'real part + imaginary part, ' that is, let $N=N_o+jN_s$, then (refer to Fig.2)

$$N_o = \frac{A'}{A}e^{-j\theta} \text{ : DIDF for limit cycle} \tag{5.a}$$

$$N_s = \frac{B'}{B} \text{ : DIDF for error signal} \tag{5.b}$$

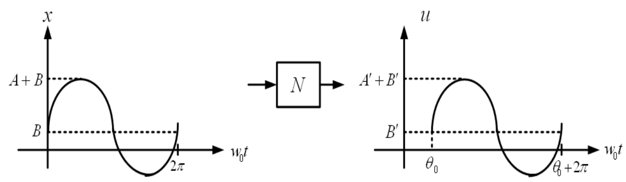


Fig.2. Linearization of nonlinearity in the presence of a sinusoid and DC term

2.2 Establishment of Proposition

This paper proposes to enhance the performance of PID controller by inserting DIDF compensator inside the loop, which is marked by dotted lines as shown in Fig. 3. In other words, the paper aims to enhance such performances as, for example, the tracking performance of output or disturbance suppression by changing dynamic characteristics of the closed loop, which is caused by a nonlinear compensator (namely, DIDF).

With the aid of the expressions of describing functions, the transfer function of nonlinear elements can be expressed as $N(A,\omega)$ as shown in Eq.(5), where A and ω represent the amplitude and frequency of the sinusoid, which is the external input of the nonlinear part. This way, intervention of nonlinear elements makes it possible to move Nyquist's critical point on a complex plane, thereby indicating that tuning of PID parameters is made simple and the whole performance could be enhanced.

The characteristic equation of Fig. 3 is given as follows:

$$1 + K_g(1 + N_d)G(s)C(s) = 0 \tag{6}$$

where K_g is a parameter used to adjust the gains of the DIDF compensator, $C(s)$ the transfer function of PID controller, is generally given in the form as follows:

$$C(s) = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) \tag{7}$$

where K_p, T_I and T_D refer to the PID controller's proportional gain, integral time and derivative time, respectively. In Fig.3, N_d represents a describing function of nonlinear element used for the purpose of performance improvement. As mentioned above, N_d has two inputs: limit cycle due to the nonlinear element and an extra external input.

The performance of the overall system of Fig.3 including stability is determined by the roots of Eq. (6) which contains the nonlinear part N_d. Accordingly, the establishment of the structure of N_d with two inputs and determination of its coefficients play an important role to improve its performance, that is, to change its existing characteristics.

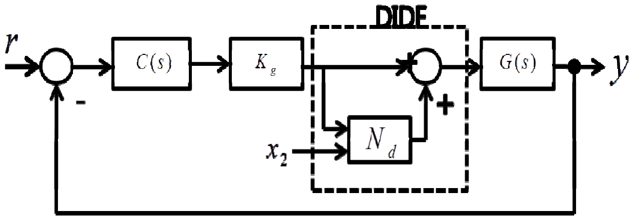


Fig.3. PID control with DIDF compensator

3 DIDF COEFFICIENTS ESTABLISHING METHOD FOR DESIGNING PID CONTROLLERS MATH

3.1 Complex DIDF and its composition

Here is how N_d, the describing function of nonlinear elements intended to enhance performance of a PID control system, is composed [10].

When Eq.(4) has been referred to, the two inputs that N_d has in Fig. 3 can be expressed as follows:

$$x_1(t) = h\sin(\omega_1 t)\ ,\ x_2(t) = l\sin(\omega_2 t + \theta) \tag{8}$$

where $x_1(t)$ refers to a signal in the steady state of the system, namely, 'B' of Eq.(5.a), and $x_2(t)$ could be regarded as $A\bullet\sin(\omega t+\theta)$ of Eq.(5.b). When such has been taken into consideration, the best way of expressing N_d, which includes nonlinear elements, is by using complex DIDF given as a function of l, the amplitude of external input $x_2(t)$ [11]. This method features the simple realizability of the imaginary part by applying the phase shift concept.

As a way of realizing the complex DIDF given as a function of l, the equation

$$N_d(l) = N_p(l) + jN_q(l), (l > 0) \tag{9.a}$$

$$N_p(l) = k_p l^2,\ N_q(l) = k_q l^2 \tag{9.b}$$

is used [11], where k_p and k_q are constants. Here it is known that, the rate of the frequencies of two inputs of N_d, that is, $\gamma=\omega_2/\omega_1$ being an irrational number has led to Eq.(9).

- Realization of N_d

Under the assumptions that ($\omega_2>>\omega_1$) is met and γ is an irrational number, the method set forth in literature [11] is used as a way of constructing N_d. Since it is assumable that x_1 stays almost constant, compared with x_2 under the assumption of the relation with Eq.(9) and the condition ($\omega_2>>\omega_1$), the real part, N_p, becomes the function of l. By using such a fact, it is easily derived that the input and output relationships (of the real part) of nonlinear $N_d(l)$ having two inputs are given as follows:

$$y(x_1, x_2) = 2k_p x_1 x_2^2,\ x_1 \in (-h, h), x_2 \in (-l, l) \tag{10}$$

By using similar techniques of the real part, the same type of equation of Eq.(10)

$$y(x_1, x_2) = 2k_q x_1 x_2^2 \tag{11}$$

is acquired with regard to the imaginary part of N_d. A '90°' phase shift, however, is required for the realization of '*j*', the imaginary number. A block diagram is shown in Fig. 4, designed to realize N_d, a nonlinear element, as a complex DIDF, based on the foregoing description.

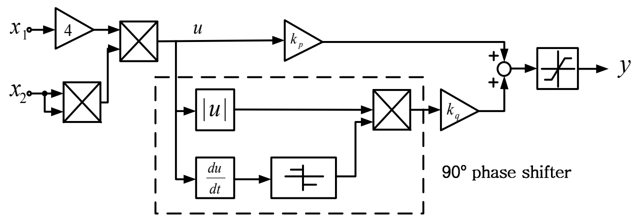


Fig. 4. The Construction of the complex DIDF

3.2 Coefficients determination for complex DIDF

This paper postulates a system in the form as shown in Fig. 3, and suggests a method for determining coefficients of DIDF inserted in series to plant G(s) which causes changes of characteristics of the closed loop system. As a result, we can apply the existing PID parameter tuning methods with ease to this structure, and obtain good control performances.

The problem here is how to set up k_p and k_q, the DIDF variables, which facilitates the setup of the parameters tuning for PID controller. The paper proposes a way of securing stability of the closed loop system composed of plant G(s) and the nonlinear part N_d of Eq.(9) by properly choosing coefficients N_d of Eq.(9) . In other words, the study uses, and is based on, the fact that stability of Fig. 3 (where $C(s)$=1), namely, the roots of the characteristic equation of the closed loop.

$$1 + K_g(1 + N_d)G(s) = 0 \tag{12}$$

are determined by $G(s)$-locus and $\{-1/K_g(1+N_d), j0\}$, a point on the complex plane.

The final goal of the paper is to find out proper parameters of PID controller Eq.(7). Therefore, if the features of a plant can be adjusted by adding DIDF so that the conventional PID tuning methods are easily applicable, it would be helpful to improve the performance of the entire system. Under this premise, we propose two methods for determining two variables k_p and k_q, of DIDF shown in Fig. 4.

(1) The frequency response of the closed-loop system:

In case of raising proportional gains of the closed loop composed of proportional action alone, normally the response (namely, output) to a step input grows more and more oscillatory, going beyond the stability limit and ultimately turning into the state of oscillation(that is in the stability limit). The method is to find the size of the proportional gain and the frequency of oscillation when the output is oscillating constantly, and then determine parameters of the PID based on such figures.

However, such a method is not applicable to all ordinary plants across the board. For example, cases are found where the plant has poles on the right half of the plane or, conversely, where increases and decreases in proportional gains alone cannot turn the output into oscillating state. In such cases, if the complex DIDF is inserted into the loop, the transfer function of the closed loop becomes as follows:

$$1+K_g\left[1+N_d(l)\right]G(j\omega)=0 \rightarrow G(j\omega)=-\frac{1}{K_g\left[1+N_d(l)\right]} \quad (13)$$

Then stability of the close loop comes to be dependent on the location of $\{-1/K_g(1+N_d), j0\}$, on the complex plane. As indicated in Eq.(9), the amplitude and phase of N_d are given as a function of l , the amplitude of external input $x_2(t)$, thereby making it possible to change the value of the right side of Eq.(13). When the characteristics of the close loop are adjusted by using the foregoing relations, and as a result, Ziegler-Nichols method is applied to the results with ease, a PID controller with desirable output responses could be constructed.

(2) The system's open-loop response:

If a step response of the plant is found by allowing a unit-step signal to the plant without using feedback, step responses of most processes show themselves as a diamond-shaped curve[3]. It has been known that there are several ways of setting up parameters of PID controller based on these curves [1].

However, when applying this method, approximation of the plant in the form of 'first order delay + dead time' is primarily presupposed. Therefore it is not generally so simple to obtain a diamond-shaped processor response curve under open loop situation. If the gain and phase of plant are appropriately transformed to the form of 'DIDF+plant', it is confirmed that the desirable step response could be obtained[4]. Then it is possible to acquire quite accurate parameters of the PID by means of conventional tuning methods.

Both methods suggested above proposes to induce changes in characteristics of existing systems by inserting DIDF, a sort of linearized representation of nonlinear elements.

- Effects of l, the amplitude of $x_2(t)$

As mentioned earlier, the characteristic equation of the close-loop of Fig. 3 is $1+[1+N_d(l)]G(s)=0$, where the variables k_p and k_q of N_d are determined so that the critical point $(-1+j0)$ may be moved to the designated point regardless of the value of l. While the relations of Eq.(9) show that the phase of complex DIDF is fixed according to k_p and k_q, it also shows that the size of the amplitude is depending on l, the amplitude of x_2. As a rule, the roots of the characteristic equation approach the imaginary axis as the variable l increases. Accordingly, in case it is difficult to find the two variables k_p and k_q that can bring the critical point $(-1+j0)$ to the selected point on the complex plane, it would be possible to accomplish it by changing l. In other words, to enhance the output response of the closed loop which has the two variables k_p and k_q in the loop, l can be used as an extra parameter.

- Stability of the closed loop

When the stable (or stabilized) system is in a state of limit cycle by inserting nonlinear elements, only in case loop gains in particular have decreased so much that it is difficult to continue the limit cycle state, it is known that the overall system loses its stability [9]. Since this paper assumes the case where the whole system is brought to the state of limit cycle by inserting non-linearity into a stable system, it is always possible to maintain stability of the whole system regardless of the insertion of non-linearity.

Accordingly, since stability of the closed loop in the structure of Fig. 3 remains unchanged, the fact that the output $y(t)$ will follow the reference input $r(t)$ also remains unchanged. However, though insertion of the complex DIDF turns the system to the limit cycle state centering around the equilibrium point, thereby causing minute oscillations to appear on the output, it is generally too tiny to pose any problem.

- Robustness of the closed loop

Sensitivity to modeling errors, namely, robustness is given as the maximum value of the sensitivity function:

$$M_s=\underset{\omega}{\mathrm{Max}}\left|\frac{1}{1+C(j\omega)(1+N_d)G(j\omega)}\right| \quad (14)$$

Here the size of M_s is given as the reciprocal number of the shortest distance between the critical point $\{-1/K_g(1+N_d), j0\}$,

[3] The slope of the tangent at the point of the sharpest slope in the curve is expressed as R (reaction speed), point of time when this tangent crosses X axis, L (dead time), and the final value of the volume to control (namely, output), K (normal gains), are used.

[4] When DIDF variables k_p, k_q and K_g are adjusted, a reaction curve is readily found.

on the complex plane and the Nyquist locus of the plant. It is known that the less value M_s has, the more robust (namely, strong against model errors) it is in the frequency given. Due to the reason mentioned above, M_s can be adjusted to some degrees.

4 EXAMPLES

(1) Comparison with the existing methods

To conduct comparisons with the results of the existing well-arranged PID controller parameter tuning methods, we adopt examples from two papers [1] and [5].

[A]. Plant from [1]:

First, since the point where Nyquist diagram of $G_3(s)$ intersects the real axis is -0.986, we know the stability margin is very low. Here, inserting the complex DIDF with k_p=0.7, k_q=0.9, the critical point was moved to -0.4595+j0.2432. Following values were found from the open-loop step responses of the system with and without DIDF.

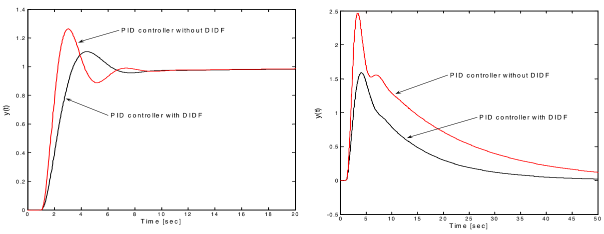
From the foregoing results two kinds of PID parameters were obtained.

(A.1): R=2.667, K=32.45, L=0.7692, T=K/R=12.1672 (without DIDF)
(A.2): R=2.066, K=16.08, L=0.7692, T=K/R=7.7832 (with DIDF)

With these values, PID parameters are determined by CHR method [1].

(A.1): K_P=0.4631, T_I=16.5474, T_D=0.3615 (without DIDF)
(A.2): K_P=0.5978, T_I=10.5852, T_D=0.3615 (with DIDF)

The results of simulations with these parameters are shown in Fig.5. Though the simulation results of all nine systems are not shown due to lack of space, we confirmed that the performances of PID controllers are greatly improved due to the DIDF compensators.



(a). Output response (b) Disturbance response

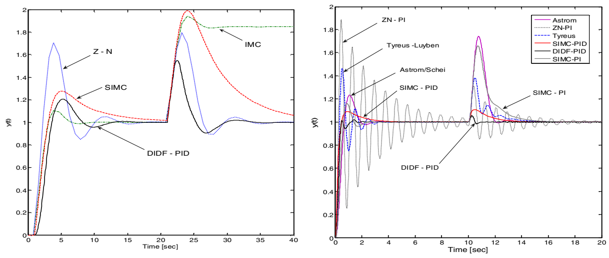
Fig. 5.

[B]. Plants from [5]

In [5], analytic rules for PID controller tuning are presented. In order, however, to apply these rules, model reduction is required. We borrow two examples from [5], that is:

(1) Integrating process $g(s)=e^{-s}/s$, (2) fourth-order process $g(s)=1/(s+1)(0.2s+1)(0.04s+1)(0.008s+1)$

Simulation results are shown in Fig.6, where we used the same parameters given in [5].



(a) Responses of $g(s)=e^{-s}/s$ (b) of $g(s)=e^{-s}/s$:

Fig.6.

(2) Example of unstable plants

DIDF effects for unstable systems are shown here, citing two examples used in literature [12]. Here are two plants and their controllers.

$$\text{(a)}\ \ G_1(s)=\frac{e^{-0.2s}}{s-1},\ \ C_1(s)=4.2148+\frac{4.1667}{s}+0.0527s$$

$$\text{(b)}\ \ G_2(s)=\frac{27e^{-0.5s}}{(s-1)(s+2.8)},\ \ C_2(s)=0.5668+\frac{0.1048}{s}+0.4071s$$

The results of simulation are shown in Fig. 7, where the coefficients of DIDF used are set as

(a) K_p=0.7, k_q=0.6, l=0.9, (b) K_p=0.2, k_q=0.3, l=0.6

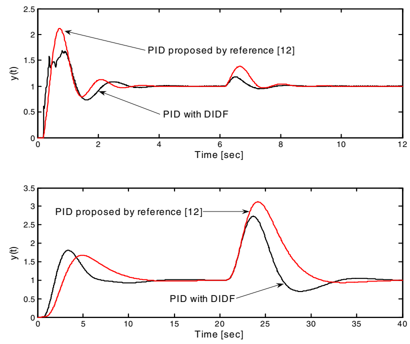


Fig. 7. Simulation Results

(3) Example of a non-minimum phase plant.

$$G_2(s)=\frac{(s-1)^2}{(s^2+1)(s+2)(s^2+s+1)} \tag{15}$$

is considered. The coefficients of DIDF based on the method mentioned in Chapter 3 are k_p=0.53, k_q=0.08 and l=1.0. The simulation results for two kinds of parameters used in literature [13] and DIDF are shown in Fig. 8.

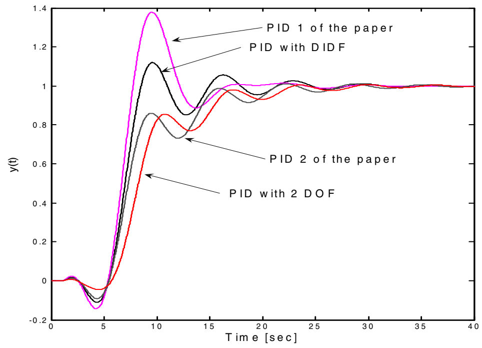


Fig.8 Results of non-minimum phase plant

(4) Comparison with the results of literature

Among the 8 models presented in literature [4], we use two examples:

$$\text{(a)}\ G_2(s)=\frac{e^{-0.5s}}{(s+1)^3}\ ,\ \text{(b)}\ G_n(s)=\frac{1}{(s+1)^n},(\mathrm{n}=4\sim 7)$$

From Fig.9, we confirmed that, in the case of $G_2(s)$, the output response by using DIDF did not change noticeably in spite of a long dead time. M_s=1.4, or M_s=2.0 in the figure represent the parameter type of the PID controller which was obtained with M_s as a standard index.

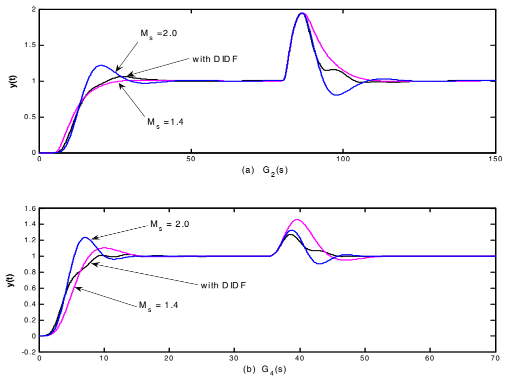


Fig.9. Comparison with the results of literature [4]

5 CONCLUSION

This paper proposed a method whereby to enhance performance of PID type controllers. While many of the existing tuning methods worked well for particular plants, in other words, plants suitable for the its derivative process for parameter tuning, it was hardly applicable to all the plants across the board. With this in mind, the paper aimed to transform the plant into an ideal form out of existing tuning methods by inserting DIDF in front of a plant. Because of this, the critical point (-1+j0) of Nyquist theory could be moved to an appropriate location, that is, we can modify the characteristics of the plant into a suitable form by which conventional PID tuning methods are easily applicable. Two kinds of techniques of determining proper coefficients k_p, k_q and l for DIDF are suggested. To sum up:

(1) It is possible to set up parameters of PID controllers through simple calculations by applying properly designed DIDF, thereby improving input and disturbance responses.

(2) It is possible to enhance robustness of the closed loop due to nonlinear element inserted

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