Consensus Control of Switching Directed Networks with General Linear Node Dynamics

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***Abstract*—Distributed consensus control for multi-agent systems with general linear node dynamics and switching balanced directed topologies is addressed in this paper. By using tools from algebraic graph theory and switching systems theory, it is proved that distributed consensus in the closed-loop linear multi-agent systems with switching balanced directed topologies can be achieved if, the feedback gain matrix of the protocol is appropriately designed and the coupling strength among neighboring agents is larger than a threshold value. The convergence rate for the achievement of consensus is further discussed. Interestingly, it is found that an arbitrarily given large convergence rate can be guaranteed if each agent is controllable. The proposed results are verified through numerical simulations.**

I. Introduction

Recently, theoretical study on distributed coordination of multi-agent systems has received much attention from various scientific communities ranging from mathematics to control engineering and to biology [1], [2]. The interest for this issue is motivated by benefits it offers such as the easy implementation, the large flexibility of design, and the robustness of operation. However, still much work need to be done before all the aforementioned advantages can be harvested. One critical issue to achieve coordination is to design a protocol based on the local relative information between neighboring agents to make the whole group reach an state agreement, which is known as the distributed consensus problem [3].

Much efforts have been made to investigate how to achieve distributed consensus in multi-agent systems under dynamically changing environments. As a result, a great deal of profound results have been reported in the literature [4]. By using tools from algebraic graph theory, it was shown in [3] that consensus in networks of agents with single-integrator dynamics can be achieved if and only if the time-varying directed network topology jointly contains a spanning tree as the systems evolve with time. Furthermore, consensus problem for multi-agent systems with second-order dynamics has been studied in [5]–[10]. Since many real coupled dynamical systems can be modeled as multi-agent systems with higher-order dynamics, such as the distributed unmanned air vehicles and the coupled manipulators, distributed consensus control for higher-order multi-agent systems were further addressed in [11]–[16]. Note that most of the above-mentioned works are mainly focused on solving consensus under a fixed communication topology. In reality, the underlying topology among the mobile agents may switch among some possible topologies due to, for instance, limited sensing radius, temporary sonar equipment failures or the presence of communication obstacles. Research along this line not only could yield some deep theoretical discoveries but also help researchers and engineers implement distributed coordination control strategies in real multi-agent systems.

In this paper, distributed consensus control for multi-agent systems with general higher-order dynamics and switching directed topologies is considered. Compared with the existing work in the field of distributed consensus control of multi-agent systems with higher-order dynamics, the underlying topology among the multiple agents is assumed to be switching in the present framework. More precisely, the topology switches over some given strongly connected and balanced directed graphs according to a piecewise constant switching signal as time evolves. It is further assumed that there is no leader in the multi-agent systems under consideration. Note that distributed consensus control for higher-order multi-agent systems with switching topologies in the presence of a leader has been recently addressed in [17] by using a multiple Lyapunov functions approach. It is also worth noting that the consensus error dynamical systems for multi-agent systems in [17] can be directly obtained, based on which some qualitative analysis can be further done. However, it is unclear how to derive the consensus error dynamics for multi-agent systems under switching directed topologies without a leader since the final consensus state is prior unknown. To solve such a challenging problem, a two-step design procedure is provided in the present work to construct the distributed protocol. Specifically, the first step deals with the agent dynamics and the feedback gain matrix of the distributed control protocol while the effect of the communication topologies on consensus is handled in the second step by designing the coupling strength. Then, by using a combination from the algebraic graph theory as well as switching systems theory,

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some sufficient conditions for achieving consensus in the closed-loop multi-agent systems are derived and analyzed. It is proved that consensus in a closed-loop multi-agent system with higher-order linear dynamics and switching topologies can be ensured if the protocol is appropriately designed. The interesting issue of how fast the distributed consensus in the closed-loop networks can be achieved is further addressed.

The rest of this paper is organized as follows. Some preliminaries and the problem formulation are given in Section II. Distributed consensus control for multi-agent systems with higher-order linear dynamics and switching directed topologies is addressed in Section III. In Section IV, numerical simulations are presented to verify the analytical results. Conclusions are finally drawn in Section V.

Throughout the paper, let $\mathbb{N}$ and $\mathbb{R}$ be the sets of natural and real numbers, respectively, and $\mathbb{R}^{n\times n}$ be the sets of $n\times n$ real matrices. Let I_n (O_n) be the $n\times n$ identity (zero) matrices, and $\mathbf{1}_n$ ($\mathbf{0}_n$) be the n-dimensional column vector with all entries equal to one (zero). Matrices, if not explicitly stated, are assumed to have compatible dimensions. The matrix inequality $A > B$ means that both A and B are symmetric matrices and that $A-B$ is positive-definite. $\mathrm{diag}\{a_1, a_2, \cdots, a_N\}$ represents a diagonal matrix with a_i, $i = 1, 2, \cdots, N$, being its diagonal elements. A column vector $x = (x_1, x_2, \cdots, x_N)^T \in \mathbb{R}^N$ is said to be positive if every entry $x_i > 0$ ($1 \le i \le N$). Notations $\otimes$ and $\|\cdot\|$ represent the Kronecker product and the Euclidian norm, respectively.

II. Preliminaries and problem formulation

In this section, some preliminaries on graph theory and the problem formulation are provided.

A. Preliminaries

Let $\mathcal{G}$ be a directed graph with the set of vertices $\mathcal{V} = \{1, 2, \cdots, N\}$, the set of directed edges $\mathcal{E} \subseteq \{(i, j),\, i, j \in \mathcal{V}\}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}]_{N\times N}$ with non-negative elements a_{ij}. The edge (i, j) in graph $\mathcal{G}$ originating at vertex j and ending at vertex i. A path on $\mathcal{G}$ from vertex i_1 to vertex i_s is a sequence of ordered edges of the form $(i_{k+1},\ i_k)$, $k = 1, 2, \cdots, s-1$. A directed graph is strongly connected if and only if there exists a path between every pair of distinct vertices. Furthermore, multiple edges and self-loops are forbidden in $\mathcal{G}$. The adjacency matrix $\mathcal{A} = [a_{ij}]_{N\times N}$ of a directed graph $\mathcal{G}$ is defined by $a_{ii} = 0$ for $i = 1, 2, \cdots, N$, and $a_{ij} > 0$ for $(i, j) \in \mathcal{E}$ but 0 otherwise. A directed graph is called balanced if $\sum_{j=1}^{N} a_{ij} = \sum_{j=1}^{N} a_{ji}$, for all $i = 1, 2, \cdots, N$. The Laplacian matrix $\mathcal{L} = [l_{ij}]_{N\times N}$ is defined as $l_{ij} = -a_{ij}$, $i \ne j$, and $l_{ii} = \sum_{k=1}^{N} a_{ik}$ for $i = 1, 2, \cdots, N$. The underlying topology among the multiple agents will be characterized by balanced directed graphs. And, the topology is assumed to be dynamically switching over graphs set $\widetilde{\mathcal{G}}$, where $\widetilde{\mathcal{G}} = \{\mathcal{G}^1, \mathcal{G}^2, \cdots, \mathcal{G}^p\}$, $p \ge 1$, denotes the set of all possible balanced directed topologies.

For a strongly connected and balanced directed graph $\mathcal{G}$, its Laplacian matrix $\mathcal{L}$ has the following properties (cf. [7], [18]).

Lemma 1: Suppose that directed graph $\mathcal{G}$ is strongly connected and balanced. Then, $\mathbf{1}_N^T\mathcal{L} = \mathbf{0}_N^T$. In addition, $\mathcal{L} + \mathcal{L}^T$ is positive semi-definite with zero being its simple eigenvalue.

B. Problem formulation

Consider a multi-agent system consists of N agents with general linear dynamics, described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state, $u_i(t) \in \mathbb{R}^m$ is the control input, A and B are constant real matrices with compatible dimensions. It is assumed that matrix pair (A, B) is stabilizable.

The communication topology among the N agents is switching among some balanced directed graphs. Suppose that there exists an infinite sequence of non-overlapping time intervals $[t_k,\ t_{k+1})$, $k \in \mathbb{N}$, with $t_1 = 0$, $t_{k+1} - t_k \ge \kappa_0$ and $\kappa_0 > 0$, across which the communication topology is fixed. Here, the positive constant κ_0 is called the dwell time. The time sequence $t_1, t_2, \cdots$, is called the switching sequence, at which the communication topology changes. For the convenience of analysis, introduce a switching signal: $\sigma(t)\ :\ [0, +\infty) \to \{1, 2, \cdots, p\}$. Then, one may let $\mathcal{G}^{\sigma(t)}$ be the communication topology of multi-agent system (1) at time t, where $t > 0$. It is easy to check that $\mathcal{G}^{\sigma(t)} \in \widetilde{\mathcal{G}}$.

To achieve consensus, the following control protocol based only on the relative information between agent i ($1 \le i \le N$) and its neighbors is proposed:

$$u_i(t) = \alpha K \sum_{j=1}^{N} a_{ij}^{\sigma(t)}[x_j(t) - x_i(t)], \tag{2}$$

where $\alpha > 0$ is the coupling strength, $K \in \mathbb{R}^{m\times n}$ is the feedback matrix to be designed, and $\mathcal{A}^{\sigma(t)} = [a_{ij}^{\sigma(t)}]_{N\times N}$ is the adjacency matrix of communication topology $\mathcal{G}^{\sigma(t)}$.

Then, it follows from (1) and (2) that

$$\dot{x}_i(t) = Ax_i(t) + \alpha BK \sum_{j=1}^{N} a_{ij}^{\sigma(t)}[x_j(t) - x_i(t)], \tag{3}$$

where $i = 1, 2, \cdots, N$.

Take $x(t) = \left[x_1^T(t), x_2^T(t), \cdots, x_N^T(t)\right]^T$, one gets

$$\dot{x}(t) = \left[(I_N \otimes A) - \alpha\left(\mathcal{L}^{\sigma(t)} \otimes BK\right)\right]x(t), \tag{4}$$

where $\mathcal{L}^{\sigma(t)}$ is the Laplacian matrix of communication topology $\mathcal{G}^{\sigma(t)}$.

Definition 1: The consensus problem of multi-agent system (1) is solved by protocol (2) if, for any initial conditions, the states of system (3) satisfy

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall\, i, j = 1, 2, \cdots, N. \tag{5}$$

III. Main results

In this section, the main results are provided and analyzed.

Before giving the main results, the following assumption is first introduced.

Assumption 1: For each $i \in \{1, 2, \cdots, p\}$, the communication topology $\mathcal{G}^i$ is strongly connected and balanced.

A. Consensus with switching topologies

Let $e(t) = (\Xi \otimes I_n)\, x(t)$, where $\Xi = \left[I_N - (1/N)\mathbf{1}_N\mathbf{1}_N^T\right] \in \mathbb{R}^{N\times N}$. Furthermore, taking the following similarity transformation $\Gamma = T^{-1}\Xi T$ with

$$T = \begin{pmatrix} 1 & \mathbf{0}_{N-1}^T \\ \mathbf{1}_{N-1} & I_{N-1} \end{pmatrix} \in \mathbb{R}^{N\times N}. \tag{6}$$

yields

$$\Gamma = \begin{pmatrix} 0 & \widetilde{\xi}^T \\ \mathbf{0}_{N-1} & I_{N-1} \end{pmatrix} \in \mathbb{R}^{N\times N}, \tag{7}$$

where $\widetilde{\xi} = (-1/N, -1/N, \cdots, -1/N)^T \in \mathbb{R}^{N-1}$. The above analysis indicates that 0 is a simple eigenvalue of Ξ. In addition, it is easy to check that $\mathbf{1}_N$ is a right eigenvector associated with the eigenvalue 0 of matrix Ξ. Thus, one can conclude that $e(t) = \mathbf{0}_{Nn}$ if and only if $x_1(t) = x_2(t) = \cdots = x_N(t)$, for $t \geq 0$.

By the definition of $e(t)$ and the fact that $\mathcal{G}^{\sigma(t)}$ is balanced, it thus follows from (4) that

$$\dot{e}(t) = \left[(I_N \otimes A) - \alpha\left(\mathcal{L}^{\sigma(t)} \otimes BK\right)\right] e(t). \tag{8}$$

Remark 1: It can be seen that the distributed consensus problem of multi-agent system (1) is solved by protocol (2) if the equilibrium point $\mathbf{0}_{Nn}$ for the switching linear systems (8) is globally asymptotically stable.

Before moving forward, the following algorithm is provided to select the feedback gain matrix K and coupling strength α of protocol (2) to achieve consensus in the switching multi-agent systems.

Algorithm 1: Suppose that (A, B) is stabilizable and Assumption 1 holds, the consensus protocol (2) can be designed as follows:

1) Solve the following linear matrix inequality (LMI):

$$AP + PA^T - BB^T < 0, \tag{9}$$

to get one feasible solution $P > 0$. Then, take $K = B^T P^{-1}$.

2) Choose the coupling strength $\alpha \geq \alpha_{\text{th}}$, where

$$\alpha_{\text{th}} = \frac{1}{\min_{i=1,2,\cdots,p}\{\lambda_2[\mathcal{L}^i + (\mathcal{L}^i)^T]\}},$$

with $\lambda_2[\mathcal{L}^i + (\mathcal{L}^i)^T]$ being the smallest nonzero eigenvalues of $\mathcal{L}^i + (\mathcal{L}^i)^T$.

Based on the above analysis, one can get the following theorem.

Theorem 1: Suppose that (A, B) is stabilizable and Assumption 1 holds. Then, consensus problem of multi-agent system (1) can be solved by protocol (2) constructed by Algorithm 1 for any given dwell time $\kappa_0 > 0$.

Proof: Consider the following common Lyapunov function for the switched systems (8):

$$V(t) = e(t)^T\left(I_N \otimes P^{-1}\right) e(t), \tag{10}$$

where P is a positive definite solution of LMI (9).

Taking the time derivative of $V(t)$ along the trajectories of systems (8) yields

$$\dot{V}(t) = e(t)^T \Big\{ I_N \otimes \left(A^T P^{-1} + P^{-1}A\right) - \alpha\Big[\left(\mathcal{L}^{\sigma(t)}\right)^T \otimes (K^T B^T P^{-1}) + \mathcal{L}^{\sigma(t)} \otimes (P^{-1}BK)\Big]\Big\} e(t). \tag{11}$$

Substituting $K = B^T P^{-1}$ into (11) gives

$$\dot{V}(t) = e(t)^T \Big\{ I_N \otimes \left(A^T P^{-1} + P^{-1}A\right) - \alpha\Big[\Big(\left(\mathcal{L}^{\sigma(t)}\right)^T + \mathcal{L}^{\sigma(t)}\Big) \otimes (P^{-1}BB^T P^{-1})\Big]\Big\} e(t). \tag{12}$$

Let $\varepsilon(t) = \left[\varepsilon_1(t)^T, \varepsilon_2(t)^T, \cdots, \varepsilon_N(t)^T\right]^T$, with $\varepsilon_i(t) = P^{-1}e_i(t),\ i = 1, 2, \cdots, N$. It is easy to check that $e(t) = (I_N \otimes P)\varepsilon(t)$. It then follows from (12) that

$$\dot{V}(t) = \varepsilon(t)^T \Big\{ I_N \otimes \left(PA^T + AP\right) - \alpha\Big[\Big(\left(\mathcal{L}^{\sigma(t)}\right)^T + \mathcal{L}^{\sigma(t)}\Big) \otimes (BB^T)\Big]\Big\} \varepsilon(t). \tag{13}$$

Based on Lemma 1, it follows from the Courant-Fischer minimum-maximum theory [19] and (13) that

$$\dot{V}(t) \leq \varepsilon(t)^T \left[I_N \otimes \left(PA^T + AP - \alpha\lambda_+ BB^T\right)\right]\varepsilon(t), \tag{14}$$

where $\lambda_+ = \min_{i=1,2,\cdots,p}\left\{\lambda_2[\mathcal{L}^i + \left(\mathcal{L}^i\right)^T]\right\}$. According to the fact $\alpha > 1/\lambda_+$ and LMI (9), one has that

$$\dot{V}(t) < 0, \tag{15}$$

for all $t \geq 0$. Thus, the consensus problem of multi-agent system (1) is indeed solved by protocol (2) constructed in Algorithm 1. ■

Remark 2: Note that LMI (9) in Algorithm 1 is feasible if and only if (A, B) is stabilizable. Furthermore, since a common Lyapunov function (10) is successfully constructed and employed for the error system (8), consensus in the closed-loop multi-agent system can be achieved for any given dwell time κ_0.

B. Discussions on convergence rate

From Theorem 1, one gets that consensus in multi-agent system (1) with protocol (2) constructed by Algorithm 1 can be achieved exponentially. However, the convergence rate can not be explicitly given in Algorithm 1, i.e., it is still unclear how fast consensus can be realized. In this subsection, a modified algorithm is proposed to design protocol (2) such that consensus in the closed-loop multi-agent system can be achieved with a given exponential convergence rate c_0.

Algorithm 2: Suppose that (A, B) is stabilizable and Assumption 1 holds, the consensus protocol (2) with a exponential convergence rate c_0 can be designed as follows:

1) Solve the following linear matrix inequality (LMI):

$$AP + PA^T - BB^T + 2c_0 P < 0, \tag{16}$$

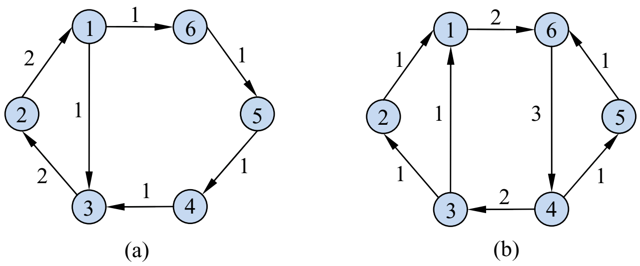

Fig. 1. Communication topologies $\mathcal{G}^1$ and $\mathcal{G}^2$.

to get one feasible solution $P > 0$. Then, take $K = B^T P^{-1}$.

2) Choose the coupling strength $\alpha \geq \alpha_{\rm th}$, where

$$\alpha_{\rm th} = \frac{1}{\min_{i=1,2,\cdots,p}\{\lambda_2[\mathcal{L}^i + (\mathcal{L}^i)^T]\}},$$

with $\lambda_2[\mathcal{L}^i + (\mathcal{L}^i)^T]$ being the smallest nonzero eigenvalues of $\mathcal{L}^i + (\mathcal{L}^i)^T$.

Theorem 2: Suppose that Assumption 1 holds and LMI (16) is feasible. Then, consensus in the closed-loop multi-agent system (1) with protocol (2) constructed by Algorithm 2 can be achieved with an exponential rate c_0 for any given dwell time $\kappa_0 > 0$.

Proof: Constructed the same Lyapunov function $V(t)$ as that used in the proof of Theorem 1, one has that

$$V(t) \leq V(0)exp(-2c_0 t). \tag{17}$$

It thus follows from (17) that

$$\|e(t)\| \leq exp(-c_0 t)\sqrt{\frac{V(0)}{\lambda_{\min}(P^{-1})}},$$

where $\lambda_{\min}(P^{-1})$ denotes the smallest eigenvalue of P^{-1}. ■

Remark 3: In the case of $(A,\, B)$ is controllable, there is a matrix K_1 and a positive definite matrix Q such that $(A - BK_1)^T Q + Q(A - BK_1) + 2c_0 Q < 0$ for any given $c_0 > 0$. By using the Finsler's lemma [21], it is not hard to get that LMI (16) is always feasible under the condition that $(A,\, B)$ is controllable, i.e., consensus in the closed-loop multi-agent system (1) with protocol (2) can be achieved with an arbitrarily given convergence rate c_0 by appropriately choosing the control parameters. Suppose that $(A,\, B)$ is stabilizable but not controllable, let $-\rho_0 < 0$ be the largest real part of the uncontrollable mode. Then, one gets that LMI (16) is feasible for any given $c_0 \in [0,\, \rho_0]$.

IV. Numerical simulations

In this section, some numerical simulations are performed to illustrate the theoretical analysis.

Consider a multi-agent system (1) of six agents with switching topologies $\mathcal{G}^1$ and $\mathcal{G}^2$ as shown in Figs. 1(a) and 1(b), respectively, where the weights are indicated on the edges. It can be seen that both $\mathcal{G}^1$ and $\mathcal{G}^2$ are strongly connected and balanced. In the simulations, the agents in systems (1) are the Caltech multi-vehicle wireless testbed vehicles [20], with $x_i(t) = [x_{i1}(t), x_{i2}(t), x_{i3}(t), \dot{x}_{i1}(t), \dot{x}_{i2}(t), \dot{x}_{i3}(t)]^T \in \mathbb{R}^6$,

$$A=\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & -0.2003 & -0.2003 & 0 & 0\\ 0 & 0 & 0.2003 & 0 & -0.2003 & 0\\ 0 & 0 & 0 & 0 & 0 & -1.6129 \end{pmatrix},$$

$$\text{and } B = \begin{pmatrix} 0 & 0\\ 0 & 0\\ 0 & 0\\ 0.9441 & 0.9441\\ 0.9441 & 0.9441\\ -28.7097 & 28.7097 \end{pmatrix},$$

where $x_{i1}(t)$ and $x_{i2}(t)$ are, respectively, the positions of the i-th $(1 \leq i \leq 6)$ vehicle along the x and y coordinates, $x_{i3}(t)$ is the orientation of the i-th $(1 \leq i \leq 6)$ vehicle.

It is easy to check that $(A,\, B)$ is controllable. Furthermore, some simple calculations give that $\min_{i\in\{1,2\}}\{\lambda_2[\mathcal{L}^i + (\mathcal{L}^i)^T]\} = 1.0746$. Set $c_0 = 1.0$ and $\alpha = 1.0$. Then, solving the LMI (16) gives that

$$K=\begin{bmatrix} 1.2379 & -0.1953 & -0.8927 & 2.6510 & -0.9694 & -0.4024\\ -0.1953 & 1.2379 & 0.8927 & -0.9694 & 2.6510 & 0.4024 \end{bmatrix}.$$

In the simulations, let the communication topology switch between $\mathcal{G}^1$ and $\mathcal{G}^2$ every 0.5s. The state trajectories of the closed-loop multi-agent systems are provided in Figs. 2-7, respectively. It can be seen that distributed consensus in the multi-agent system is indeed solved by the designed protocol. Furthermore, use $E(t) = \|e(t)\|$ to denote the consensus errors of the closed-loop multi-agent systems, where $e(t) = (\Xi \otimes I_n)\, x(t)$, $\Xi = \left[I_N - (1/N)\mathbf{1}_N\mathbf{1}_N^T\right]$ and $x(t) = [x_1(t)^T, x_2(t)^T, \cdots, x_6(t)^T]^T$. It can be seen from Fig. 8 that the larger the parameter c_0 is selected the faster the consensus rate will be yielded. However, it is also worth noting that the larger the parameter c_0 is selected the bigger the overshoot will be generated. Thus, a trade-off has to be made between the convergence rate and the overshoot of the closed-loop multi-agent systems.

V. Conclusion

By using tools from algebraic graph theory and switched systems theory, consensus for multi-agent systems with general linear higher-order dynamics and switching topologies has been addressed in this paper. With the assumption that each possible topology is strongly connected and balanced, a two-step design procedure is proposed to construct the consensus protocol. It is theoretically shown that consensus in the closed-loop multi-system can be ensured if the feedback gain matrix and the coupling strength of the network are appropriately designed, respectively. Some numerical simulations are finally given to verify the theoretical analysis. Future work will focus on solving an H_∞ consensus problem for multi-agent system with and general linear node dynamics and switching directed topologies without the balanced condition.

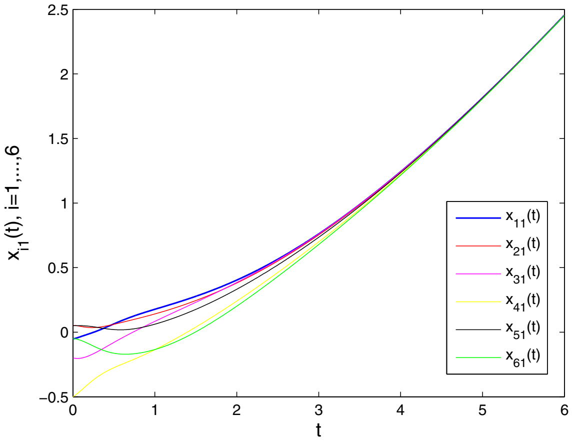

Fig. 2. State trajectories of $x_{i1}(t)$, $i = 1, 2, \cdots, 6$.

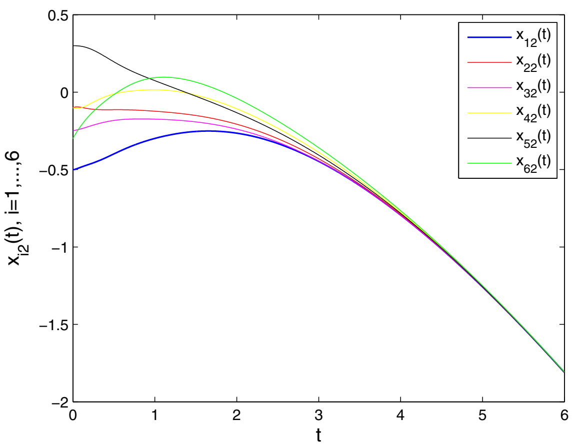

Fig. 3. State trajectories of $x_{i2}(t)$, $i = 1, 2, \cdots, 6$.

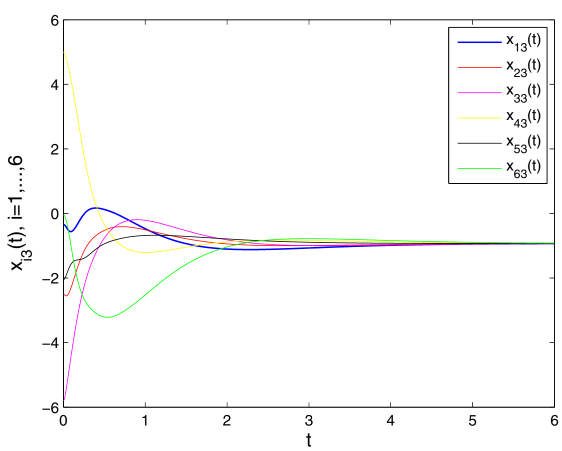

Fig. 4. State trajectories of $x_{i3}(t)$, $i = 1, 2, \cdots, 6$.

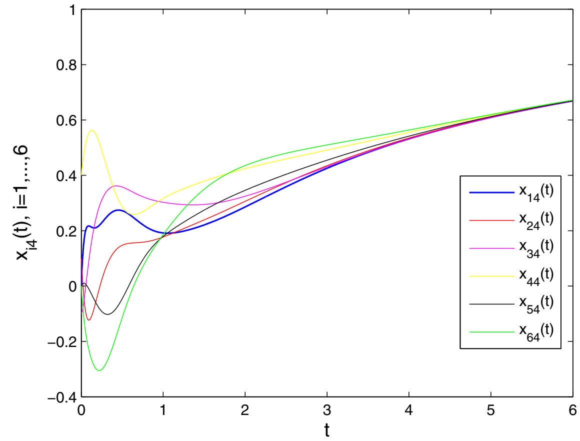

Fig. 5. State trajectories of $x_{i4}(t)$, $i = 1, 2, \cdots, 6$.

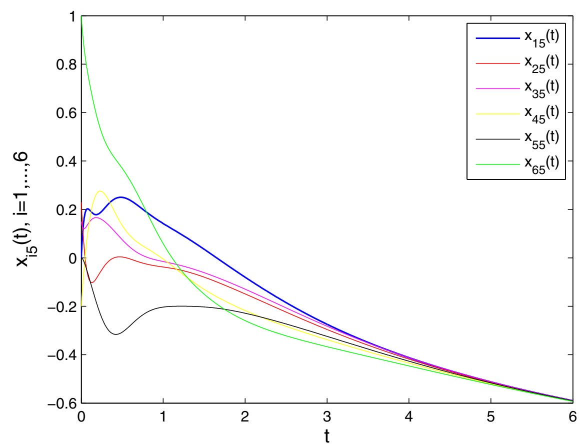

Fig. 6. State trajectories of $x_{i5}(t)$, $i = 1, 2, \cdots, 6$.

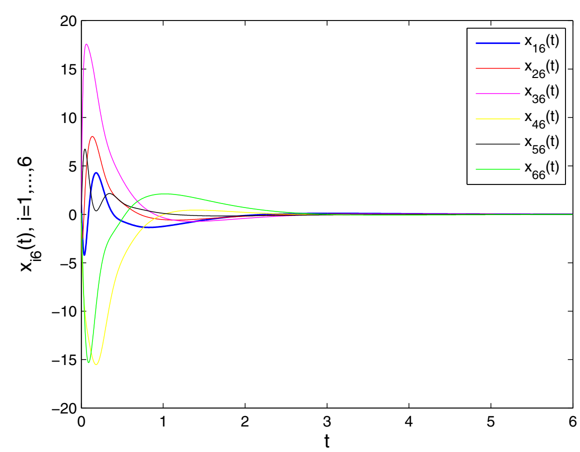

Fig. 7. State trajectories of $x_{i6}(t)$, $i = 1, 2, \cdots, 6$.

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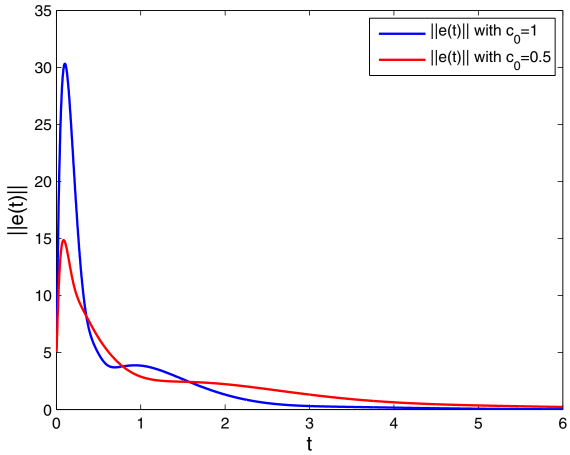

Fig. 8. State trajectories of $e(t)$.

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