Input-to-state stability for switched nonlinear time-delay systems

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***Abstract*—This paper investigates the problem of the input-to-state stability (ISS) for a class of switched nonlinear time-delay systems, in which time delays are involved in both the state and the switching signal of the controller. Because of the presence of the switching delay, the switching information available to the controller is a delayed information of the system, and then the closed-loop system will have two asynchronous switching signals. To study these two asynchronous switching signals in a unified framework, we adopt the technique of the merging switching signal. Based on a piecewise Lyapunov-Krasovskii functional method, some sufficient conditions are explicitly given to guarantee ISS of the switched nonlinear time-delay system under average dwell time scheme.**

I. Introduction

Switched nonlinear systems involving a coupling between nonlinear continuous dynamics and discrete switching events have been drawing considerable attention due to the significance both in theory development and practical applications [1]-[4]. For stability analysis and control synthesis of switched nonlinear systems, there are already a large number of achievements, see the references [5]-[8]. On the other side, in many practical systems, time delays are widespread and unavoidable. It has been well recognized that the existence of time delay affects not only the performance of the control, but also can even undermine the stability of the system [9]-[10]. Therefore, the influence of time delay for system performance can't be ignored. It is also known that switched time-delay systems are also often met in practical systems such as networked control system [11], drilling systems [12], mechanical rotational cutting process [13], etc. Compared with the switched system without time delay, the dynamical behaviors of the switched time-delay systems are very complex, and thus the study for switched time-delay systems is more important and challenging. In recent years, some results for switched time-delay systems have been derived, see for example [14]-[17].

In addition, it is interesting and significant to quantify the external inputs when a control system is influenced by them. In 1989, a precise definition of input-to-state stability was introduced in [18], and then a few basic results were obtained, see for instance [19]-[20]. As stated in [18], the definition of input-to-state stability is intend to capture the idea of "bounded input bounded output" behavior together with decay of states under small inputs. As a very powerful analysis tool, ISS has got the attention of a large number of scholars and the past two decades have witnessed the development of ISS [21]-[26].

In this paper, we are interested in the ISS for switched nonlinear time-delay systems for both the synchronous and the asynchronous case. In the synchronous case, it is expected that the controller has instant access to the switching information of the system. However, this expectation may be found unrealistic in practical applications because it is necessary to take some time to distinguish the active subsystem and then use the corresponding controller [14]-[17]. Hence, the closed-loop system will feature two asynchronous switching signal. The work [14] and [16] addressed the problem of asynchronous switching for switched linear systems. In [25], the state trajectory method was employed to discuss the problem of asynchronous switching for switched nonlinear systems under a class of switching signal satisfying dwell time, in which state delays are not involved. Like the work [16], [25], here, we do not consider the controller synthesis. Despite this, considering switched nonlinear systems with time delays involved in the switching signal is significant for the preceding reason. To the best of our knowledge, no results on this problem have been reported, which motivates the present study.

This paper investigates the problem of input-to-state stability (ISS) for a class of switched nonlinear time-delay systems, in which time delays are involved in both the state and the switching signal of the controller. Because the presence of the switching delay, the switching information available to the controller is a delayed version of that at the system, and then the closed-loop system can have two asynchronous switching signals. To study these two asynchronous switching signals in a unified framework, the merging switching signal is adopted. Based on a piecewise Lyapunov-Krasovskii functional method, some sufficient conditions are explicitly given to guarantee ISS of the switched nonlinear time-delay system under average dwell time scheme.

The paper is organized as follows. The problem formulation is stated in Section II, followed by the main results in Section III. The paper is concluded in section IV.

Notations. In this paper, the symbol $|\cdot|$ stands for the Euclidean norm of a real vector or induced matrix norm. The interval $[0, +\infty)$ in the space of real numbers R is denoted by R^+. R^n denotes the n-dimensional vector space. For a

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measurable and essentially bounded function $u : R^+ \to R^m$, we define its infinity norm $||u||_\infty = ess\sup_{t\geq 0}|u(t)|$. If we have $||u||_\infty < \infty$, then we write $u \in L^m_\infty$. $C([-\tau, 0]; R^n)$ denotes the set of the continuous functions mapping from $[-\tau, 0]$ to R^n, equipped with norm $||\phi||_\tau := \sup_{-\tau\leq s\leq 0}|\phi(s)|$. A function $\gamma : R^+ \to R^+$ is a K function if it is continuous, zero at zero, and strictly increasing. It is a K_∞ function if it is of class K and unbounded. A function $\beta : R^+ \times R^+ \to R^+$ is a KL function if $\beta(\cdot, t)$ is of class K for each fixed $t \geq 0$ and $\beta(s, \cdot)$ is decreasing to zero for each fixed $s \geq 0$. For a function $w : R^+ \to R^+$, we write $w \in P_0$ if it is continuous and satisfies $w(0) = 0$. If $w(s) > 0$ holds for all $s > 0$ additionally, we write $w \in P$.

II. Problem formulation

In this paper, we consider the following switched nonlinear time-delay system:

$$\dot{x}(t) = f_{\sigma(t)}(x_t, u(t)), t \geq 0 \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the input function, for $t \geq 0$, $x_t : [-\tau, 0] \to R^n$ is given by $x_t(s) = x(t+s)$, $\tau \geq 0$ is the maximum involved delay. $\xi_0 \in C([-\tau, 0]; R^n)$ is the initial state. $\sigma(t) : [0, \infty) \to M = \{1, 2, ..., m\}$ is the switching signal. Corresponding to the switching signal $\sigma(t)$, we have the switching sequence $\Sigma = \{\xi_0 : (i_0, t_0), ..., (i_k, t_k), ...|i_k \in M, k \in N\}$, which means that the i_kth subsystem, called a mode, is activated when $t \in [t_k, t_{k+1})$; $f_{i_k} : C([-\tau, 0]; R^n) \times R^m \to R^n$ is completely continuous and locally Lipschitz. Throughout the work, we assume that $f_{i_k}(0, 0) = 0$. The switching sequence Σ may or may not be infinite. In the first case, we may take $t_{k+1} = \infty$, with all further definitions and results will valid. We assume that no jump occurs in the state at a switching time and that only finitely many switchings can occur in any finite interval. In ideal cases, we assume that $u(t) = g_{\sigma(t)}(x_t, w)$ with w be the reference input such that the corresponding closed-loop system is

$$\begin{aligned}\dot{x}(t) &= f_{\sigma(t)}(x_t, g_{\sigma(t)}(x_t, w)), t \geq 0\\ &=: \bar{f}_{\sigma(t)}(x_t, w)\end{aligned} \tag{2}$$

The goal of this paper is to establish ISS property of the switched nonlinear time-delay system (1). To this end, we first recall the following definition:

Definition 1[26] System (1) is said to be ISS if there exist a function $\beta \in KL$ and a function $\sigma \in K$ such that for all $\xi_0 \in C([-\tau, 0]; R^n)$ and all $w \in L_\infty$, we have

$$|x(t)| \leq \beta(||\xi_0||_\tau, t) + \sigma(||w_{[0,t]}||_\infty), \forall t \geq 0. \tag{3}$$

Definition 2[1] For any $\tau_2 > \tau_1 \geq 0$, let $N_\sigma(\tau_1, \tau_2)$ denote the number of switching of $\sigma(t)$ over (τ_1, τ_2). If $N_\sigma(\tau_1, \tau_2) \leq N_0 + \frac{\tau_2-\tau_1}{\tau_a}$ holds for $\tau_a > 0, N_0 \geq 0$, then τ_a is called average dwell time and N_0 is called a chatter bound. Denote by $S_{ave}[\tau_a, N_0]$ the class of switching signals with average dwell time τ_a and chatter bound N_0.

To give the stability property of system (1), we consider the following piecewise Lyapunov-Krasovskii functional $V(\phi) = V_{\sigma(t)}(\phi)$, where $V_{i_k}(\phi)$ is continuously differentiable functional and $V_{i_k}(\phi) = V^1_{i_k}(\phi(0)) + V^2_{i_k}(\phi)$. For each $i_k \in M$, define the upper right-hand derivative of $V_{i_k}(\phi)$ with respect to i_kth mode of system (1) as follows:

$$D^+V_{i_k}(\phi, u) = \lim_{h\to 0^+}\sup\frac{1}{h}[V_{i_k}(\phi^*_h) - V_{i_k}(\phi)], \tag{4}$$

where $\phi^*_h \in C([-\tau, 0]; R^n)$ is given by

$$\phi^*_h(s) = \begin{cases}\phi(s+h), & s \in [-\tau, -h];\\ \phi(0) + f(\phi, u)(h+s), & s \in [-h, 0].\end{cases} \tag{5}$$

III. Main results

In this section, we will give some conditions to guarantee the ISS for the switched nonlinear time-delay system. Both synchronous switching and asynchronous switching are considered. We start with the case that the synchronous case.

A. Synchronous switching

Theorem 1 Consider the switched nonlinear time-delay system (2). Suppose that there exist a piecewise Lyapunov-Krasovskii functional $V(\phi) = V_{\sigma(t)}(\phi)$, where $V_{\sigma(t)}(\phi) = V^1_{\sigma(t)}(\phi(0)) + V^2_{\sigma(t)}(\phi)$, and functions $\alpha_i \in K_\infty (i = 1, 2, 3)$, $\rho \in K_\infty$, constants $\mu \geq 1$, $\lambda_s > 0$, such that, for all i_k, $i_j \in M$, $k, j \in N, i_k \neq i_j$,
(i) $\alpha_1(|\phi(0)|) \leq V^1_{i_k}(\phi(0)) \leq \alpha_2(|\phi(0)|)$,
$0 \leq V^2_{i_k}(\phi) \leq \alpha_3(||\phi||_\tau)$;
(ii) $||\phi||_\tau \geq \rho(|w|) \Rightarrow D^+V_{i_k}(\phi) \leq -\lambda_s V_{i_k}(\phi)$;
(iii) $V^1_{i_k}(\phi(0)) \leq \mu V^1_{i_j}(\phi(0)), V^2_{i_k}(\phi) \leq \mu V^2_{i_j}(\phi)$.
If the switching signal satisfy average dwell time $\tau_a > \tau^*_a = \frac{\ln\mu}{\lambda_s}$, then the system (2) is ISS.

Proof: The lines of a part of the proof of the main theorem in [23] is followed here. For $t \geq 0$, let $\vartheta(t) := \rho(||w_{[0,t]}||_\infty)$ and $\xi(t) = \alpha_1^{-1}(\mu^{N_0}\alpha(\vartheta(t)))$, where $\alpha = \alpha_2 + \alpha_3$. Furthermore, introduce the set $B_\vartheta(t) := \{x(t) : ||x(t)||_\tau \leq \vartheta(t)\}$. If $||x(t)||_\tau \geq \vartheta(t) \geq \rho(|w(t)|)$ during some time interval $t \in [t', t'']$, then the following is true

$$\begin{aligned}|x(t)| &\leq \alpha_1^{-1}(\mu^{N_0}e^{(-\lambda_s+\frac{\ln\mu}{\tau_a})(t-t')}\alpha(||x(t')||_\tau))\\ &:= \bar{\beta}(||x(t')||_\tau, t-t').\end{aligned} \tag{6}$$

Next, we give the proof of (6). First, consider the piecewise Lyapunov-Krasovskii functional $V_{\sigma(t)}(x_t)$. On any interval $[t_k, t_{k+1}) \cap [t', t'']$, condition (ii) yields $D^+V_{i_k}(x_t) \leq -\lambda_s V_{i_k}(x_t)$, then it can be derived that $V_{i_k}(x_t) \leq e^{-\lambda_s(t-t_k)}V_{i_k}(x_{t_k})$. In view of condition (iii), we can derive $V_{i_{k+1}}(x_{t_{k+1}}) \leq \mu e^{-\lambda_s(t_{k+1}-t_k)}V_{i_k}(x_{t_k})$ and thus, for any $t \in [t', t'']$, by the proof of theorem 1, it holds $V_{\sigma(t)}(x_t) \leq \mu^{N_\sigma(t',t)}e^{-\lambda_s(t-t')}V_{\sigma(t')}(x_{t'})$. Therefore, (6) follows.

Then, let $\check{t}_1 := \inf\{t \geq 0 : ||x(t)||_\tau \leq \vartheta(t)\}$. For $0 \leq t \leq \check{t}_1$, it is clear $|x(t)| \leq \bar{\beta}(||\xi_0||_\tau, t)$. If $\check{t}_1 = \infty$, which only can happen if $\vartheta(t) \equiv 0$ since $\bar{\beta} \in KL$. Then (3) is established and thus the switched nonlinear time-delay system (1) is ISS. Hence in the following we only consider the case where $\check{t}_1 < \infty$.

Let $\hat{t}_1 := \inf\{t > \check{t}_1 : ||x(t)||_\tau > \vartheta(t)\}$. If this is an empty set, let $\hat{t}_1 := \infty$. For all $t \in [\check{t}_1, \hat{t}_1)$, it is easy to obtain that $||x(t)||_\tau \leq \vartheta(t) \leq \xi(t)$.

For the case that $\hat{t}_1 < \infty$, by the continuity of $||x(\cdot)||_\tau$ along with the monotonicity of $\vartheta(\cdot)$, it gives $||x(\hat{t}_1)||_\tau = \vartheta(\hat{t}_1)$.

Furthermore, for all $\tau > \hat{t}_1$, if $||x(\tau)||_\tau > \vartheta(\tau)$ define $\hat{t} := \sup\{t < \tau : ||x(t)||_\tau \leq \vartheta(t)\}$. Again, follow the same line, it can be obtained that $||x(\hat{t})||_\tau = \vartheta(\hat{t})$. Then, according to (6), it holds that

$$\begin{aligned} |x(\tau)| &\leq \bar{\beta}(||x(\hat{t})||_\tau, \tau - \hat{t}) \\ &= \bar{\beta}(\vartheta(\hat{t}), \tau - \hat{t}) \\ &= \alpha_1^{-1}(\mu^{N_0} e^{(-\lambda_s + \frac{\ln \mu}{\tau_a})(\tau - \hat{t})} \alpha(\vartheta(\hat{t}))) \\ &= \xi(\hat{t}) \leq \xi(\tau). \end{aligned} \quad (7)$$

To sum up, for all $t \geq \check{t}_1$, it holds that

$$\begin{aligned} |x(t)| &\leq \xi(t) \\ &= \alpha_1^{-1}(\mu^{N_0} \alpha(\vartheta(t))) \\ &= \alpha_1^{-1}(\mu^{N_0} \alpha(\rho(||w_{[0,t]}||_\infty))) \\ &= \sigma(||w_{[t_0,t]}||_\infty). \end{aligned} \quad (8)$$

Then, (6) along with (8) leads to

$$|x(t)| \leq \bar{\beta}(||\xi_0||_\tau, t) + \sigma(||w_{[t_0,t]}||_\infty) \quad (9)$$

for all $t \geq 0$. It gives that system (1) is ISS.

B. Asynchronous switching

Due to the existence of switching delay τ_s, we consider the following input: $u(t) = g_{\sigma(t-\tau_s(t))}(x_t, w(t))$, where $\tau_s(t)$ is the uncertain switching delay, satisfying $0 \leq \tau_s(t) \leq \tau_s$. Here assume that the maximal switching delay τ_s is known without loss of generality. And we have the following switching sequence $\{x_{t_0} : (i_0, t_0 + \tau_s(t_0)), .., (i_k, t_k + \tau_s(t_k)), ..., |i_k \in M, k \in N\}$, which means that the i_kth controller is active when $t \in [t_k + \tau_s(t_k), t_{k+1} + \tau_s(t_{k+1}))$, $k \in N$.

The merging signal technique in [26] will be adopted to cope with the asynchronous switching signal. Similarly, first create a virtual switching signal $\sigma'(t) : [0, \infty) \to \bar{M} = M \times M$ as follows: $\sigma' = (\sigma_1(t), \sigma_2(t))$. The merging action is denoted by $\oplus$ such that $\sigma' = \sigma_1 \oplus \sigma_2$, which implies that the set of switching times of σ' is the union of the sets of switching times of σ_1 and of σ_2.

Lemma 1:[28] Given $\sigma_1(t) \in S_{ave}[\tau_a, N_0]$, and $\sigma_2(t) = \sigma_1(t - \tau_s(t))$, it has $\sigma_2 \in S_{ave}[\tau_a, N_0 + \frac{\tau_s}{\tau_a}]$, $\sigma' \in S_{ave}[\bar{\tau}_a, \bar{N}_0]$, where $\bar{\tau}_a = \frac{\tau_a}{2}$, $\bar{N}_0 = 2N_0 + \frac{\tau_s}{\tau_a}$.

Lemma 2:[16] Let $\sigma_1(t) \in S_{ave}[\tau_a, N_0]$, and $\sigma_2(t) = \sigma_1(t - \tau_s(t))$. Suppose that $0 \leq \tau_s(t) \leq \tau_s$ for all t, and $\tau_s < t_{k+1} - t_k, k \in N$. For an interval (t_0, t), let $m_{(t_0,t)}$ be the total time for which $\sigma_1(t) = \sigma_2(t)$, and let $\bar{m}_{(t_0,t)} = t - t_0 - m_{(t_0,t)}$. If $\tau_s(\lambda_s + \lambda_u) \leq (\lambda_s - \lambda)\tau_a$ for some positive constants λ_s, λ_u, and $\lambda \in [0, \lambda_s]$, then

$$-\lambda_s m_{(t_0,t)} + \lambda_u \bar{m}_{(t_0,t)} \leq c_T - \lambda(t - t_0), \forall t \geq t_0 \quad (10)$$

where $c_T = (\lambda_s + \lambda_u)(N_0 + 1)\tau_s$.

Now, rewriting the system and integrating the switching sequence of the system with the switching sequence of the controller, we can derive

$$\dot{x}(t) = \bar{f}_{\sigma'(t)}(x_t, w(t)). \quad (11)$$

Theorem 2 For system (11), suppose that there exists a piecewise Lyapunov-Krasovskii functional $V(\phi) = V_{\sigma'(t)}(\phi)$, where $V_{\sigma'(t)}(\phi) = V^1_{\sigma'(t)}(\phi(0)) + V^2_{\sigma'(t)}(\phi)$, and functions $\alpha_i \in K_\infty (i = 1, 2, 3)$, $\rho \in K_\infty$, constants $\mu \geq 1$, $\lambda_s > 0$, such that, for all i_k, $i_j \in M$, $k, j \in N, i_k \neq i_j$,

(i) $\alpha_1(|\phi(0)|) \leq V_i^1(\phi(0)) \leq \alpha_2(|\phi(0)|)$, $0 \leq V_i^2(\phi) \leq \alpha_3(||\phi||_\tau), i \in \bar{M}$;

(ii) $||\phi||_\tau \geq \rho(|w|) \Rightarrow \begin{cases} D^+ V_{i_k i_k}(x_t) \leq -\lambda_s V_{i_k i_k}(x_t), \\ D^+ V_{i_k i_j}(x_t) \leq \lambda_u V_{i_k i_j}(x_t), \end{cases}$

(iii) $V^1_{i_k i_k}(\phi(0)) \leq \mu V^1_{i_k i_j}(\phi(0)), V^2_{i_k i_k}(\phi) \leq \mu V^2_{i_k i_j}(\phi)$, $V^1_{i_k i_j}(\phi(0)) \leq \mu V^1_{i_j i_j}(\phi(0)), V^2_{i_k i_j}(\phi) \leq \mu V^2_{i_j i_j}(\phi)$.

If the switching signals satisfy average dwell time

$$\tau_a > \tau_a^* = \frac{2 \ln \mu + (\lambda_s + \lambda_u)\tau_s}{\lambda_s}, \quad (12)$$

then system (11) is ISS.

Proof: The proof is similar to that of Theorem 1. For $t \geq 0$, let $\vartheta(t) := \rho(||w_{[0,t]}||_\infty)$ and $\xi(t) = \alpha_1^{-1}(g_0 \alpha(\vartheta(t)))$, where $\alpha = \alpha_2 + \alpha_3$, and $g_0 = \mu^{\bar{N}_0} e^{(\lambda_s + \lambda_u)(N_0+1)\tau_s}$. Furthermore, introduce the set $B_\vartheta(t) := \{x(t) : ||x(t)||_\tau \leq \vartheta(t)\}$. If $||x(t)||_\tau \geq \vartheta(t) \geq \rho(|w|)$ during some time interval $t \in [t', t'']$, then the following is true

$$\begin{aligned} |x(t)| &\leq \alpha_1^{-1}(g_0 e^{-(\lambda - \frac{\ln \mu}{\bar{\tau}_a})(t - t')} \alpha(||x(t')||_\tau)) \\ &:= \bar{\beta}(||x(t')||_\tau, t - t'). \end{aligned} \quad (13)$$

Next, we give the proof of (13). First, consider the piecewise Lyapunov-Krasovskii functional $V(x_t) = V_{\sigma'(t)}(x_t)$. In any interval $[t', t'']$, let $\tau_1, ..., \tau_{N_{\sigma'}(t',t'')}$ denote the switching times of σ' in (t', t''), $\tau_0 = t'$, $\tau_{N_{\sigma'}(t',t'')+1} = t''$. For any $t \in [t', t'']$, condition (ii) along with condition (iii) gives

$$V(x_t) \leq \mu^{N_{\sigma'(t)}(t',t)} e^{-\lambda_s m_{(t',t)} + \lambda_u \bar{m}_{(t',t)}} V(x_{t'}). \quad (14)$$

The condition (12) implies the existence of λ such that

$$\frac{2 \ln \mu}{\tau_a} < \lambda < \lambda_s - \frac{(\lambda_s + \lambda_u)\tau_s}{\tau_a}, \quad (15)$$

which can be rewritten as

$$\begin{aligned} (\lambda_s + \lambda_u)\tau_s &< (\lambda_s - \lambda)\tau_a, \quad (16) \\ \lambda &> \frac{\ln \mu}{\bar{\tau}_a}. \quad (17) \end{aligned}$$

Then, by using Lemma 2, it yields

$$\begin{aligned} V(x_t) &\leq \mu^{N_{\sigma'(t)}(t',t)} e^{C_T - \lambda(t - t')} V(x_{t'}) \\ &\leq g_0 e^{-(\lambda - \frac{\ln \mu}{\bar{\tau}_a})(t - t')} V(x_{t'}). \end{aligned} \quad (18)$$

Therefore, (13) follows.

Then, let $\check{t}_1 := \inf\{t \geq 0 : ||x(t)||_\tau \leq \vartheta(t)\}$. For $0 \leq t \leq \check{t}_1$, it is clear

$$|x(t)| \leq \bar{\beta}(||\xi_0||_\tau, t). \quad (19)$$

If $\check{t}_1 = \infty$, which can only happen if $\vartheta(t) \equiv 0$, then (3) is established and thus system (1) is ISS. Hence in the following we only consider the case where $\check{t}_1 < \infty$.

Let $\hat{t}_1 := \inf\{t > \check{t}_1 : ||x(t)||_\tau > \vartheta(t)\}$. If this is an empty set, let $\hat{t}_1 := \infty$. Clearly, for all $t \in [\check{t}_1, \hat{t}_1)$, it holds that $||x(t)||_\tau \leq \vartheta(t) \leq \xi(t)$.

For the case of $\hat{t}_1 < \infty$, by the continuity of $||x(\cdot)||_\tau$ along with the monotonicity of $\vartheta(\cdot)$, it has $||x(\hat{t}_1)||_\tau = \vartheta(\hat{t}_1)$. Furthermore, for all $\tau > \hat{t}_1$, if $||x(\tau)||_\tau > \vartheta(\tau)$ define $\hat{t} := \sup\{t < \tau : ||x(t)||_\tau \leq \vartheta(t)\}$. Again, follow the same line, it can be obtained that $||x(\hat{t})||_\tau = \vartheta(\hat{t})$. Then, according to (13), it holds that $|x(\tau)| \leq \xi(\tau)$.

To sum up, for all $t \geq \check{t}_1$, it holds that

$$\begin{aligned} |x(t)| &\leq \xi(t) \\ &= \alpha_1^{-1}(g_0\alpha(\vartheta(t))) \\ &= \alpha_1^{-1}(g_0\alpha(\rho(||w_{[0,t]}||_\infty))) \\ &= \sigma(||w_{[t_0,t]}||_\infty). \end{aligned} \quad (20)$$

Furthermore, combining (19) with (20) leads to

$$|x(t)| \leq \bar{\beta}(||\xi_0||_\tau, t) + \sigma(||w_{[t_0,t]}||_\infty) \quad (21)$$

for all $t \geq 0$. It gives that system (1) is ISS.

Remark 1: The piecewise Lyapunov-Krasovskii functional method and merging switching signal technique are employed to deal with the problem of asynchronous switching for a class of switched nonlinear time-delay system. If the state delay is removed from the system (1) in our paper, the system exactly degenerates to the one studied in [25]. In [25], the state trajectory method was employed to discuss the problem of asynchronous switching for switched nonlinear systems under a class of switching signal satisfying dwell time. Therefore, even for such a special system, our results are still expected to be less conservative.

IV. CONCLUSION

This paper has investigated the issues of the input-to-state stability for switched nonlinear time-delay systems via piecewise Lyapunov-Krasovskii functional method. Both the synchronous and asynchronous switching have been considered. Some sufficient conditions have been provided to guarantee the input-to-state stability of the switched nonlinear time-delay system with average dwell time scheme.

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