Non-fragile Fuzzy Control Design for Nonlinear Time-Delay Systems

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***Abstract*— In this paper, a non-fragile fuzzy control design is proposed for a class of nonlinear systems with mixed discrete and distributed time delays. The Takagi and Sugeno (T-S) fuzzy set approach is applied to the modelling of the nonlinear dynamics, and a T-S fuzzy model is constructed, which can represent the nonlinear system. Then, based on the fuzzy linear model, a fuzzy linear controller is developed to stabilize the nonlinear system. The control law is obtained to ensure stochastically exponentially stability in the mean square. The sufficient conditions for the existence of such a control are proposed in terms of certain linear matrix inequalities.**

Keywords*— *Fuzzy control; LMI; delay; stochastic systems

I. Introduction

Most of the systems, which are encountered in control engineering, contain various nonlinearities and are affected by random disturbance signals. Nonlinear systems with time-delay constitute basic mathematical models of real phenomena, for instant in biology, mechanics and economics, see e.g. [8, 18]. Control of time-delay systems has been a subject of great practical importance, which has attracted a great deal of interest for several decades. On the other hand, it turns out that the delayed state is very often the cause for instability and poor performance of systems. Moreover, considerable attention has been given to both the problems of robust stabilization and robust control for linear systems with unavoidable time-varying parameter uncertainties in modeling dynamical systems and certain types of time-delays [14].

Since the introduction of fuzzy set theory by Zadeh in [29], many people have devoted a great deal of time and effort to both theoretical research and implementation technique for fuzzy logic controllers [15, 22]. With the development of fuzzy systems, it is known that the qualitative knowledge of a system can also be represented in nonlinear functional form. On the basis of this idea, some fuzzy models based control system design methods have appeared in the fuzzy control field [3, 22, 23]. These methods are conceptually simple and straightforward. Fuzzy controllers are usually characterized using Mamdani and T-S type. In general, Mamdani type fuzzy controllers are designed empirically. However, T-S controllers can be designed using the information of several local linearized models of a given system via the so-called parallel-distributed compensation scheme. Various stability conditions of fuzzy systems have been obtained by employing Lyapunov stability theory [4, 9, 10], passivity theory [20], and other methods [5, 12, 22]. Problem of control design based on the state feedback for T-S fuzzy systems using LMI approach has been studied in [28] and the delay-independent stability of T-S fuzzy model for a class of nonlinear time-delay systems investigated in [7]. Extension of the T-S fuzzy model approach to the stability analysis and control design for both continuous and discrete-time nonlinear systems with time-varying delay has been considered in [2] and also Lee et al. [11] presented design of an output feedback robust H_∞ controller based on T-S fuzzy model for uncertain fuzzy dynamic systems with time-varying delayed state. T-S modeling technique is an effective approach of nonlinear systems, which could approximate any smooth nonlinear function to any specified accuracy within any compact set (see for instance [9]). The T-S fuzzy models are represented by a set of linear models by fuzzy IF-THEN rules, so it is possible for the existing traditional linear systems results to be applied to analysis and synthesis of nonlinear systems based on the parallel-distributed compensation (PDC) scheme.

Recently, several criteria of input-to-bounded state (IBS) stabilization and bounded-input-bounded-output (BIBO) stabilization in mean square for nonlinear and quasi-linear stochastic control systems with time-varying uncertainties has been investigated in [6], also, another stability concepts in the mean-square sense such as mean-square stability (MSS) and the internal mean-square stability (IMSS) have been studied in [13]. The stabilization of stochastic systems with multiplicative noise has been studied since the late sixties, particularly in the context of linear quadratic optimal control,

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see e.g., [17, 24]. Also, a stochastic fuzzy control has been proposed by applying the stochastic control theory, instead of using a traditional fuzzy reasoning in [25] and a class of fuzzy stochastic control systems with random delays investigated in [19]. In this paper, the non-fragile fuzzy linear control problem for a class of stochastic nonlinear time-delay systems with mixed discrete and distributed time delays has been investigated and the attention was focused on the design of state feedback controller which ensures stochastically exponentially stable in the mean square.

Notation: The notations used throughout the paper are fairly standard. I and 0 represent identity matrix and zero matrix, respectively; the superscript $'T'$ stands for matrix transposition. $\|.\|$ refers to the Euclidean vector norm or the induced matrix 2-norm. $diag\{\cdots\}$ represents a block diagonal matrix and the operator $sym(A)$ represents $A+A^T$. The notation $P>0$ means that P is real symmetric and positive definite; the symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated.

II. PROBLEM FORMULATION

Consider a class of nonlinear continuous-time state delayed stochastic system described by

$$dx(t) = [A(x(t))\,x(t) + A_d(x(t))\,x(t-d) + A_\tau(x(t))\int_{t-\tau}^{t} x(\sigma)\,d\sigma + B(x(t))\,u(t)]\,dt + E_1\,dw(t) \tag{1}$$

$$x(t) = \varphi(t), \qquad t \in [-h, 0] \tag{2}$$

where $x(t) = [x_1(t), x_2(t), ..., x_n(t)]^T \in \Re^n$ is the state vector, $u(t) = [u_1(t), u_2(t), ..., u_m(t)]^T \in \Re^m$ is the control input and $h = \max\{d, \tau\}$, d is the discrete time delay and τ is the distributed time delay. $\varphi(t)$ is the continuous vector valued initial function and $w(t) = [w_1(t), w_2(t), ..., w_n(t)]^T \in \Re^n$ is a scalar Brownian motion defined on the probability space $(\Omega, F, \{F_t\}_{t\ge 0}, P)$.

A fuzzy dynamic model has been proposed by Takagi and Sugeno [21] to represent local linear input-output relations of nonlinear systems. This fuzzy linear model is described by fuzzy If-Then rules and will be employed here to deal with the control design problem of the nonlinear system (1-2). The ith rule of this fuzzy model for the nonlinear system (1-2) is of the following form [9, 21, 23]:

Plant Rule i:

$$\begin{aligned}&\textit{If } z_1(t) \textit{ is } F_{i1} \textit{ and} \ldots \textit{and } z_g(t) \textit{ is } F_{ig},\\ &\textit{Then } dx(t) = [A_i\,x(t) + A_{id}\,x(t-d) + A_{i\tau}\int_{t-\tau}^{t} x(\sigma)\,d\sigma + B_i\,u(t)]\,dt + E_1\,dw(t)\end{aligned} \tag{3}$$

for $i = 1, 2, \ldots, L$ where F_{ij} is the fuzzy set, $A_i \in \Re^{n\times n}$, $A_{id} \in \Re^{n\times n}$, $A_{i\tau} \in \Re^{n\times n}$, $B_i \in \Re^{n\times m}$ and L is the number of If-Then rules, and $z_1(t)$, $z_2(t)$,..., $z_g(t)$ are the premise variables.

The overall fuzzy system is inferred as follows:

$$dx(t) = \sum_{i=1}^{L} h_i(z(t))\,(A_i x(t) + A_{id}\,x(t-d) + A_{i\tau}\int_{t-\tau}^{t} x(\sigma)\,d\sigma + B_i u(t))\,dt + E_1 dw(t) \tag{4}$$

where

$$z(t) = [z_1(t), z_2(t), ..., z_g(t)] \tag{5}$$

$$\mu_i(z(t)) = \prod_{j=1}^{g} F_{ij}(z_j(t)) \tag{6}$$

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{L} \mu_i(z(t))} \tag{7}$$

and $F_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in F_{ij}.

Assumption 1. we assume $\mu_i(z(t)) \ge 0$ for $i = 1, 2, ..., L$ and $\sum_{i=1}^{L} \mu_i(z(t)) > 0$ for all t.

Therefore, we get

$$h_i(z(t)) \ge 0 \tag{8}$$

for $i = 1, 2, ..., L$ and

$$\sum_{i=1}^{L} h_i(z(t)) = 1. \tag{9}$$

Therefore, from (1) we get [4]

$$\begin{aligned} dx(t) &= [A(x(t))\,x(t) + A_d(x(t))\,x(t-d) + A_\tau(x(t))\int_{t-\tau}^{t} x(\sigma)\,d\sigma + B(x(t))\,u(t)]\,dt + E_1\,dw(t)\\ &= [\sum_{i=1}^{L} h_i(z(t))\,(A_i x(t) + A_{id}\,x(t-d) + A_{i\tau}\int_{t-\tau}^{t} x(\sigma)\,d\sigma + B_i u(t)) + \Delta A + \Delta A_d + \Delta A_\tau + \Delta B]\,dt + E_1 dw(t)\end{aligned} \tag{10}$$

where

$$\Delta A = (A(x(t)) - \sum_{i=1}^{L} h_i(z(t)) A_i)\,x(t)$$

$$\Delta A_d = (A_d(x(t)) - \sum_{i=1}^{L} h_i(z(t)) A_{id})\,x(t-h)$$

$$\Delta A_\tau = (A_\tau(x(t)) - \sum_{i=1}^{L} h_i(z(t))\,A_{i\tau})\int_{t-\tau}^{t} x(\sigma)\,d\sigma$$

$$\Delta B = \sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))\,(B(x(t)) - B_i)\,u(t).$$

denote the approximation errors between the nonlinear system (1) and the fuzzy model (4).

Assumption 2. There exist bounding matrices ΔA_i, ΔA_{id}, $\Delta A_{i\tau}$ and ΔB_i such that for all trajectory $x(t)$

$$\|\Delta A\| \le \left\|\sum_{i=1}^{L} h_i(z(t))\,\Delta A_i\, x(t)\right\| \tag{11}$$

$$\|\Delta A_d\| \le \left\|\sum_{i=1}^{L} h_i(z(t))\,\Delta A_{id}\, x(t-h)\right\| \tag{12}$$

$$\|\Delta A_\tau\| \le \left\|\sum_{i=1}^{L} h_i(z(t))\,\Delta A_{i\tau} \int_{t-\tau}^{t} x(\sigma)\, d\sigma\right\| \tag{13}$$

$$\|\Delta B\| \le \left\|\sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))\,\Delta B_i\, u(t)\right\| \tag{14}$$

and the bounding matrices ΔA_i, ΔA_{id}, $\Delta A_{i\tau}$ and ΔB_i can be described by

$$\begin{bmatrix} \Delta A_i \\ \Delta A_{id} \\ \Delta A_{i\tau} \\ \Delta B_i \end{bmatrix} = \begin{bmatrix} \delta_i A_p \\ \delta_{id} A_{pd} \\ \delta_{i\tau} A_{p\tau} \\ \eta_i B_p \end{bmatrix} \tag{15}$$

where $\|\delta_i\| \le 1$, $\|\delta_{id}\| \le 1$, $\|\delta_{i\tau}\| \le 1$ and $\|\eta_i\| \le 1$, for $i = 1,2,\ldots,L$ [1]. According to Assumption 2, we get

$$(\Delta A)^T(\Delta A) \le (A_p x(t))^T (A_p x(t)) \tag{16}$$

$$(\Delta A_d)^T(\Delta A_d) \le (A_{pd} x(t-h))^T (A_{pd} x(t-h)) \tag{17}$$

$$(\Delta A_\tau)^T(\Delta A_\tau) \le (A_{p\tau} \int_{t-\tau}^{t} x(\sigma)\, d\sigma)^T (A_{p\tau} \int_{t-\tau}^{t} x(\sigma)\, d\sigma) \tag{18}$$

$$(\Delta B)^T(\Delta B) \le (\sum_{j=1}^{L} h_j(z(t)) B_p u(t))^T (\sum_{j=1}^{L} h_j(z(t)) B_p u(t)) \tag{19}$$

i.e., the approximation error in the closed-loop nonlinear system is bounded by the specified structured bounding matrices A_p, A_{pd}, $A_{p\tau}$ and B_p.

Suppose the following fuzzy controller is employed to deal with the above control system design:

Control Rule j:

$$\begin{aligned} &\textit{If } z_1(t) \textit{ is } F_{j1} \textit{ and} \ldots \textit{and } z_g(t) \textit{ is } F_{jg}, \\ &\textit{Then} \quad u(t) = (K_j + \Delta K_j)\, x(t), \qquad j = 1,2,\ldots,L \end{aligned} \tag{20}$$

The matrices K_j are to be designed such that the closed-loop system is stable. ΔK_j is the gain variation of K_j and ΔK_j are assumed to be of the form

$$\Delta K_j = E_j F(t) H_j \tag{21}$$

where E_j and H_j are known real constant matrices and $F(t)$ is an unknown matrix function satisfying $F(t)^T F(t) \le I$.

Hence, the overall fuzzy controller is given by

$$u(t) = \sum_{j=1}^{L} h_j(z(t))\,(K_j + \Delta K_j) x(t) \tag{22}$$

where $h_j(z(t))$ is defined in (8) and (9) and K_j are the control parameters.

Substituting (22) into (10) yields the closed-loop nonlinear control system as follows:

$$\begin{aligned} &dx(t) = \\ &[\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t)) h_j(z(t)) (A_i + B_i(K_j + \Delta K_j)) x(t) + A_{id} x(t-d) \\ &+ A_{i\tau} \int_{t-\tau}^{t} x(\sigma)\, d\sigma\} + \Delta A + \Delta A_d + \Delta A_\tau + \Delta B] dt + E_1 dw(t) \end{aligned} \tag{23}$$

Next, observe the closed-loop system (23) and let $x(t;\zeta)$ denote the state trajectory from the initial data $x(\theta) = \zeta(\theta)$ on $-h \le \theta \le 0$ in $L^2_{F_0}([-h,0];\Re^{2n})$. Clearly, the system (23) admits a trivial solution $x(t;0) \equiv 0$ corresponding to the initial data $\zeta = 0$. We introduce the following stability and stabilizability concepts.

Definition 1. [27] For the system (23) and every $\zeta \in L^2_{F_0}([-h,0];\Re^{2n})$, the trivial solution is asymptotically stable in the mean square if

$$\underset{t\to\infty}{Lim}\, E|x(t;\zeta)|^2 = 0, \tag{24}$$

and is exponentially stable in the mean square if there exist constants $\alpha > 0$ and $\beta > 0$ such that

$$E|x(t;\zeta)|^2 \le \alpha e^{-\beta t} \underset{-h\le\theta\le 0}{Sup} E|\zeta(\theta)|^2. \tag{25}$$

Definition 2. [27] we say that the systems (1)-(2) is exponentially stabilizable in mean square if, for every $\zeta \in L^2_{F_0}([-h,0];\Re^{2n})$, there exists a fuzzy linear control law (22) such that the resulting closed-loop system is exponentially stable in mean square.

The objective of this paper is to design a fuzzy linear control for the stochastic nonlinear time-delay systems (1)-(2). More specifically, we are interested in seeking the control parameters K_j, for $j = 1,2,\ldots,L$, such that the closed-loop system (14) is exponentially stable in mean square. Moreover, for the existence of controller coefficients uncertainties, the proposed controller should be non-fragile.

III. MAIN RESULTS

We first give the following lemma, which will be used in the proof of our main results.

Lemma 1. [30] For any matrices X and Y with appropriate dimensions and for any constant $\eta > 0$, we have:

$$X^T Y + Y^T X \le \eta X^T X + \frac{1}{\eta} Y^T Y.$$

Lemma 2. [31] (*Jensen's Inequality*) Given a positive-definite matrix $P \in \Re^{n\times n}$ and two scalars $b > a \ge 0$ for any vector $x(t) \in \Re^n$, we have

$$\int_{t-b}^{t-a} x^T(\omega) P x(\omega)\, d\omega \ge \frac{1}{b-a}\Big(\int_{t-b}^{t-a} x(\omega)\, d\omega\Big)^T P\Big(\int_{t-b}^{t-a} x(\omega)\, d\omega\Big).$$

Lemma 3. The parameterized linear matrix inequalities,

$$\sum_{i=1}^{n}\sum_{j=1}^{n} h_i(z(t))h_j(z(t))M_{ij} < 0$$

is fulfilled, if the following condition holds:

$$M_{ii} < 0$$

$$\frac{1}{k-1}M_{ii} + \frac{1}{2}(M_{ij} + M_{ji}) < 0, \qquad 1 \le i \ne j \le k$$

3.1 Stochastic Stability Analysis

In this section, assuming that the fuzzy linear control is known and we will study the conditions under which the closed-loop system is stochastically exponentially stable in the mean square. The following theorem will play a key role in the stability analysis of closed-loop system and design of the expected fuzzy linear control.

Theorem 1. Let the control parameters K_j, for $j = 1,2,\ldots,L$, be given. If the fuzzy controller (22) is employed in the nonlinear system (1)-(2) and there exists positive scalars ε_1, ε_2, ε_3, ε_4, ε_5, η and a positive definite matrix P,S,R,U such that the following matrix inequalities

$$\Xi_{ii} < 0, \quad 1 \le i \le L \tag{26a}$$

$$\frac{1}{L-1}\Xi_{ii} + \frac{1}{2}(\Xi_{ij} + \Xi_{ji}) < 0, \quad 1 \le i \ne j \le L \tag{26b}$$

where

$$\Xi_{ij} := \begin{bmatrix} \Theta_{ij} & PA_{id} & PA_{i\tau} & \eta PB_iE_j & H_j^T \\ * & \varepsilon_2^{-1}A_{pd}^TA_{pd} - R & 0 & 0 & 0 \\ * & * & \varepsilon_3^{-1}A_{p\tau}^TA_{p\tau} - U & 0 & 0 \\ * & * & * & -\eta I & 0 \\ * & * & * & * & -\eta I \end{bmatrix}$$

with

$$\Theta_{ij} := sym(P(A_i + B_iK_j)) + S + hR + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5)P^2 + \varepsilon_1^{-1}A_p^TA_p + \varepsilon_4^{-1}(B_pK_j)^T(B_pK_i) + \varepsilon_5^{-1}\lambda_{\max}(E_j^TB_p^TB_pE_i)I$$

are satisfied for all $i, j = 1,2,\ldots,L$, then the closed-loop nonlinear system (23) is exponentially stable in the mean square.

Proof: Fix $\zeta \in L^2_{F_0}([-h,0];\Re^{2n})$ arbitrarily, and write $x(t;\zeta) = x(t)$. We define the lyapunov function candidate

$$V(x(t),t) = \sum_{i=1}^{3} V_i(x(t),t) \tag{27}$$

where $V_1(x(t),t) = x^T(t)Px(t)$ and

$$V_2(x,t) = \int_{t-d}^{t} x(\xi)^T S\, x(\xi)\, d\xi + \int_{t-d}^{t}\int_{\xi}^{t} x(s)^T R\, x(s)\, ds\, d\xi,$$

$$V_3(x,t) = \int_{t-\tau}^{t}\Big[\int_{s}^{t} x(\theta)^T d\theta\Big] U\Big[\int_{s}^{t} x(\theta)\, d\theta\Big] ds + \int_{0}^{\tau}\int_{t-s}^{t} (\theta - t + s)\, x(\theta)^T U\, x(\theta)\, d\theta\, ds$$

where P,S,R and U are positive definite matrices. Differentiating $V_1(x(t),t)$ in t we obtain

$$\begin{aligned} dV_1(x_t,t) &= 2x(t)^T P\big[\{\textstyle\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))(A_i + B_i(K_j + \Delta K_j))x(t) \\ &\quad + A_{id}x(t-d) + A_{i\tau}\textstyle\int_{t-\tau}^{t} x(\sigma)\, d\sigma\} + \Delta A + \Delta A_d + \Delta A_\tau + \Delta B\big]dt + 2x(t)^T PE_1 dw(t) \end{aligned} \tag{28}$$

Differentiating other Lyapunov terms in (27) and using Lemma 2 give

$$dV_2(x_t,t) = \Big[x(t)^T(S + hR)x(t) - x(t-d)^T Rx(t-d) - \int_{t-d}^{t} x(s)^T S\, x(s)\, ds\Big]dt \tag{29}$$

and

$$\begin{aligned} dV_3(x_t,t) &= \Big[-\Big[\int_{t-\tau}^{t} x(\theta)^T d\theta\Big]U\Big[\int_{t-\tau}^{t} x(\theta)\, d\theta\Big] + 2\int_{t-\tau}^{t}(\theta - t + \tau)x(t)^T Ux(\theta)\, d\theta \\ &\quad + \int_{0}^{\tau} s\, x(t)^T Ux(t)\, ds - \int_{0}^{\tau}\int_{t-s}^{t} x(\theta)^T Ux(\theta)\, d\theta\, ds\Big]dt \\ &\le \Big[h^2 x(t)^T Ux(t) - \Big[\int_{t-\tau}^{t} x(\theta)^T d\theta\Big]U\Big[\int_{t-\tau}^{t} x(\theta)\, d\theta\Big]\Big]dt \end{aligned} \tag{30}$$

The stochastic differential of $\Upsilon(x(t),t)$ along a given trajectory is obtained as

$$\begin{aligned} &dV(x(t),t) \\ &\le \{2x(t)^T P\big[\{\textstyle\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))(A_i + B_i(K_j + \Delta K_j))x(t) \\ &+ A_{id}x(t-d) + A_{i\tau}\textstyle\int_{t-\tau}^{t} x(\sigma)\, d\sigma\} + \Delta A + \Delta A_d + \Delta A_\tau + \Delta B\big] \\ &+ x(t)^T(S + hR)x(t) - x(t-d)^T Rx(t-d) - \int_{t-d}^{t} x(s)^T S\, x(s)\, ds \\ &+ h^2 x(t)^T Ux(t) - \Big[\int_{t-\tau}^{t} x(\theta)^T d\theta\Big]U\Big[\int_{t-\tau}^{t} x(\theta)\, d\theta\Big]\}dt + 2x^T(t)PE_1 dw(t) \end{aligned} \tag{31}$$

Now, by Lemma 1, it is trivial to show that for any positive scalars of ε_1, ε_2, ε_3, ε_4, ε_5 the following inequalities hold:

$$(\Delta A)^T Px(t) + x^T(t)P(\Delta A) \le x^T(t)(\varepsilon_1 P^2 + \varepsilon_1^{-1}A_p^TA_p)x(t) \tag{32}$$

$$\begin{aligned}(\Delta A_d)^T Px(t) + x^T(t)P(\Delta A_d) &\le \varepsilon_2 x^T(t)P^2x(t) \\ &+ \varepsilon_2^{-1}x(t-d)^T A_{pd}^TA_{pd}x(t-d)\end{aligned} \tag{33}$$

$$\begin{aligned}(\Delta A_\tau)^T Px(t) + x^T(t)P(\Delta A_\tau) &\le \varepsilon_3 x^T(t)P^2x(t) \\ &+ \varepsilon_3^{-1}\textstyle\int_{t-\tau}^{t} x(\sigma)^T d\sigma A_{p\tau}^TA_{p\tau}\int_{t-\tau}^{t} x(\sigma)\, d\sigma\end{aligned} \tag{34}$$

$$\begin{aligned}&(\Delta B)^T Px(t) + x^T(t)P(\Delta B) \\ &\le x^T(t)(\varepsilon_4 P^2 + \varepsilon_4^{-1}(\textstyle\sum_{j=1}^{L} h_j(z(t))B_pK_j)^T(\sum_{j=1}^{L} h_j(z(t))B_pK_j))x(t) \\ &\quad + x^T(t)(\varepsilon_5 P^2 + \varepsilon_5^{-1}(\textstyle\sum_{j=1}^{L} h_j(z(t))B_p\Delta K_j)^T(\sum_{j=1}^{L} h_j(z(t))B_p\Delta K_j))x(t)\end{aligned} \tag{35}$$

Then, noticing (27) and considering (28)-(35) results in

$$
\begin{aligned}
&dV(x(t),t) \le \\
&\{2x(t)^T P\Big[\textstyle\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))(A_i + B_i(K_j + \Delta K_j))x(t) \\
&+ A_{id}x(t-d) + A_{i\tau}\textstyle\int_{t-\tau}^{t} x(\sigma)\, d\sigma\Big] + x(t)^T (S + hR + (\varepsilon_1 + \varepsilon_2 \\
&+ \varepsilon_3 + \varepsilon_4)P^2 + \varepsilon_1^{-1} A_p^T A_p + \varepsilon_4^{-1}(\textstyle\sum_{j=1}^{L} h_j(z(t))B_p(K_j + \Delta K_j))^T \\
&\times (\textstyle\sum_{j=1}^{L} h_j(z(t))B_p(K_i + \Delta K_i)))x(t) + x(t-d)^T(\varepsilon_2^{-1} A_{pd}^T A_{pd} - R) \\
&\times x(t-d) + \varepsilon_3^{-1}\textstyle\int_{t-\tau}^{t} x(\sigma)^T\, d\sigma A_{p\tau}^T A_{p\tau}\int_{t-\tau}^{t} x(\sigma)\, d\sigma \\
&- \int_{t-d}^{t} x(s)^T S\, x(s)\ \ ds + h^2 x(t)^T U\, x(t) - [\int_{t-\tau}^{t} x(\theta)^T\ d\theta] U [\int_{t-\tau}^{t} x(\theta)\ \ d\theta]\}dt \\
&+ 2x^T(t)PE_1\, dw(t) \\
&\le \textstyle\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))\upsilon^T(t)(\Pi_{ij} + sym(\Phi F(t)\Psi))\upsilon(t)\, dt \\
&\quad + 2x^T(t)PE_1\, dw(t)
\end{aligned} \tag{36}
$$

where $\upsilon(t) = \begin{bmatrix} x^T(t) & x^T(t-d) & \int_{t-\tau}^{t} x(\theta)^T\ d\theta \end{bmatrix}^T$ and

$$
\Pi_{ij} = \begin{bmatrix} \Theta_1 & PA_{id} & PA_{i\tau} \\ * & \varepsilon_2^{-1}A_{pd}^T A_{pd} - R & 0 \\ * & * & \varepsilon_3^{-1}A_{p\tau}^T A_{p\tau} - U \end{bmatrix} < 0
$$

$$
\Phi = \begin{bmatrix} PB_iE_j \\ 0 \\ 0 \end{bmatrix}, \quad \Psi = \begin{bmatrix} H_j^T \\ 0 \\ 0 \end{bmatrix}^T
$$

By Lemma 1, a necessary and sufficient condition for (36) is that there exists a scalar $\eta > 0$ such that

$$
\begin{aligned}
&dV(x(t),t) \le \\
&\le \textstyle\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))\upsilon^T(t)(\Pi_{ij} + \eta\Phi\Phi^T + \eta^{-1}\Psi^T\Psi)\upsilon(t)\, dt \\
&\quad + 2x^T(t)PE_1\, dw(t)
\end{aligned} \tag{37}
$$

Then, according to the inequality above and Lemma 3, we find the inequalities (26). Consequently, the inequalities (26) mean that the nonlinear stochastic time-delay closed-loop system (23) is asymptotically stable by the fuzzy control law (22).

The expected exponential stability (in the mean square) of the closed-loop system (23) can be proved by making some standard manipulation on (37), see [16]. Let β_{ij} be the unique root of the equation

$$
\lambda_{\min}(-\Pi_{ij}) - \beta_{ij}\,\lambda_{\max}(P) - \beta_{ij}\, h\,\lambda_{\max}(S)e^{\beta_{ij}h} = 0 \tag{38}
$$

Then, by [26], we have

$$
\begin{aligned}
E\left|x(t)\right|^2 \le\ & \lambda_{\min}^{-1}(P)\ ([\lambda_{\max}(P) + h\,\lambda_{\max}(S)] \\
& + \beta_{ij}\,\lambda_{\max}(S)\,h^2\, e^{\beta_{ij}h})\ \underset{-h\le\theta\le 0}{Sup}\ E\left|\zeta(\theta)\right|^2 e^{-\beta_{ij}t}
\end{aligned} \tag{39}
$$

Notice that, according to (39), the definition of exponential stable in Definition 1 is satisfied and this complete the proof of Theorem 1.

The result of Theorem 1 may be conservative due to the use of inequalities (32)-(35). However, such conservativeness can be significantly reduced by appropriate choices of the parameters ε_1, ε_2, ε_3, ε_4, ε_5, η in a matrix norm sense.

3.2 Fuzzy Control Design

This subsection is devoted to the design of control parameters K_j, for $j = 1, 2, \ldots, L$, by using the result in Theorem 1. We will show that the design of control parameters problem can be solved via the resolution of matrix inequalities.

By performing a congruence transformation $diag\{\overline{P}, \overline{P}, \overline{P}, I, \cdots, I\}$, where $\overline{P} := P^{-1}$ to both sides of (24), applying Schur complements and considering $\overline{K}_j := K_j\overline{P}$ result in

$$
\hat{\Xi}_{ii} < 0, \quad 1 \le i \le L \tag{40a}
$$

$$
\frac{1}{L-1}\hat{\Xi}_{ii} + \frac{1}{2}(\hat{\Xi}_{ij} + \hat{\Xi}_{ji}) < 0, \quad 1 \le i \ne j \le L \tag{40b}
$$

where

$$
\hat{\Xi}_{ij} := \begin{bmatrix}
\hat{\Theta}_{ij} & A_{id} & A_{i\tau} & \eta B_i E_j & \overline{P}H_j^T & \overline{P}A_p^T & 0 & 0 & \overline{K}_j^T B_p^T & \lambda_{\max}^{0.5}(E_j^T B_p^T B_p E_i) \\
* & -\widetilde{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -\widetilde{U} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\eta I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\eta I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\varepsilon_1 I & \overline{P}A_{pd}^T & 0 & 0 & 0 \\
* & * & * & * & * & * & -\varepsilon_2 I & \overline{P}A_{p\tau}^T & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon_3 I & 0 & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon_4 I & 0 \\
* & * & * & * & * & * & * & * & * & -\varepsilon_5 I
\end{bmatrix}
$$

where

$\hat{\Theta}_{ij} := sym(A_i\overline{P} + B_i\overline{K}_j) + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5)I + \widetilde{S} + h\widetilde{R}$.

Theorem 2. The closed-loop fuzzy system (14) is exponentially stable in the mean square if there exist positive scalars ε_1, ε_2, ε_3, ε_4, ε_5, η, matrices $\overline{K}_j$ ($j = 1, 2, \ldots, L$) and positive definite matrices $\overline{P}, \widetilde{R}, \widetilde{S}, \widetilde{U}$, satisfying the LMIs (40). Then, the desired control gains in (20) are given by $K_j = \overline{K}_j\overline{P}^{-1}$.

IV. SIMULATION RESULTS

In this section, we will design a non-fragile fuzzy linear controller for the following stochastic nonlinear time-delay system

$$
dx(t) = [-0.1x(t)^3 + x(t-d) + \textstyle\int_{t-\tau}^{t} x(\sigma)\, d\sigma + u(t)]dt + dw(t)
$$

$$
x(t) = 1, \qquad t \in [-h, 0]
$$

Consider $d = 1, \tau = 0.5$ seconds with $h = 1$. We represent the system above by the following T-S fuzzy model

Plant Rule 1:

If $x(t)$ *is* F_{11},

Then $dx(t) = [-3x(t) + 0.2x(t-d) + 0.5\int_{t-\tau}^{t} x(\sigma)\, d\sigma + 2u(t)]dt + dw(t)$

Plant Rule 2:

If x *is* F_{21},

Then $dx(t) = [-2x(t) - 0.1x(t-d) + 0.1\int_{t-\tau}^{t} x(\sigma)\, d\sigma + u(t)]dt + dw(t)$

where $F_{21} = \dfrac{1}{1 + e^{-x^2}}$, $F_{11} = 1 - F_{12}$, $A_p = A_{pd} = A_{p\tau} = 0.5$ and $\mathbf{B}_p = 1$. Substituting the above parameters into Theorem 2, using the LMI toolbox in MATLAB, robust stability of the state of system in the presence of disturbance, i.e. Brownian motions has been depicted in Figure 1 and the overall fuzzy controller is shown in Figure 2.

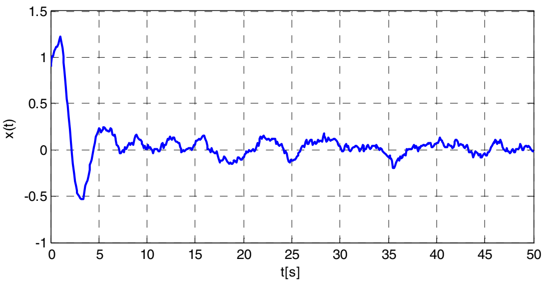

Fig. 1. Time behavior of the state of system

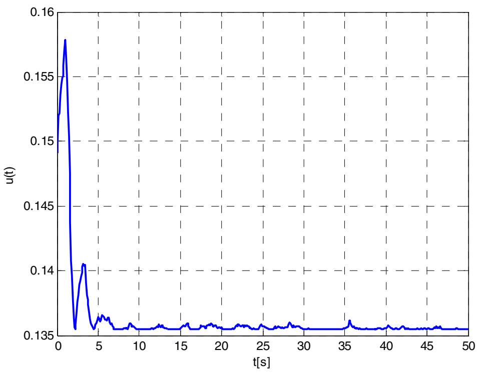

Fig. 2. Control input

V. CONCLUSIONS

The non-fragile fuzzy linear control design method for a class of stochastic nonlinear time-delay systems with state feedback and mixed discrete and distributed time delays was developed in this paper. First, the Takagi and Sugeno fuzzy linear model was employed to approximate a nonlinear system. Next, based on the fuzzy linear model, a fuzzy linear controller was developed to stabilize the non-linear system. The control law has been obtained such that ensures stochastically exponentially stable in the mean square and the sufficient conditions for the existence of such a control was proposed in terms of certain linear matrix inequality.

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