A New Technique in Multi-Model Adaptive Control: Sequential Parameter Discrimination and Hybrid Parameter Vector

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***Abstract*—We propose a new methodology in order to provide faster convergence in adaptive control of a class of nonlinear plants. Currently, each model in a multi-model adaptive system is evaluated as a whole, using a cost function derived from estimation errors. Therefore the number of fixed models required for improvement in transient response becomes quite large, for the plants having several unknown parameters. The proposed scheme removes this difficulty by considering each parameter sequentially and individually; and provides better convergence as compared to classical multi-model adaptive systems by using an assumption that a decrease in any element of the parameter error vector results in decrease in the state estimation error and vice-versa.**

***Keywords*—Adaptive control, transient performance, multiple models**

I. Introduction

The past of multi-model adaptive control (MMAC) for improvement of transient response exceeds nearly two decades [1][2]. The theory was first proposed for linear systems and matured with switching and tuning in the subsequent years. The success of inclusion of multiple models in adaptive control of linear systems drew interest to nonlinear systems shortly thereafter. In 2000s, several works began to appear in the literature that promise better transient responses for some classes of nonlinear systems. The first attempts used adaptive backstepping [3]. Later, MMAC systems using indirect [4] and direct [5] adaptive schemes came for a larger class of nonlinear systems. The combination of both schemes was also proposed for a sub-class of these systems [6]. Some recent works extend the application of the theory to non-affine nonlinear systems [7].

While the achievement of the MMAC in improvement of the transient response of both linear and nonlinear plants is evident, the focus turned to forming the control input in a more sophisticated manner in order to gain further improvement in the transient response and increase the robustness of the adaptive system [8]–[11]. In [8] and [9], adaptive mixing control scheme providing smooth control signal is proposed for both transient improvement and stability robustness. On the other hand, second level adaptation instead of switching is described in [10] and [11] for linear plants so that any convex combination of the estimates is also an estimate of the actual parameter vector. All these works linearly combine either the controller signals or the parameter estimates, that removes the discontinuities in the applied control input. A much earlier application of the similar idea can be found in [12], where averaging of the parameter estimates instead of switching was proposed for the control of a linear dc motor.

In this work, we propose an alternative approach for the fulfillment of the above objectives. The number of the identification models and locations of the parameter estimates in the parameter space are important in determining the performance of the MMAC system. The higher the number of models, the larger part of the parameter space is covered and hence the faster adaptation is expected, which consequently provides better transient performance. However, when the number of the unknown plant parameters is high, the MMAC system requires exponentially high number of fixed models to be able to cover all preset combinations. Eventually, such an increase in the estimation models may cause prohibitive complexity in realization of the adaptive system. The proposed methodology provides better coverage of the parameter space by evaluating the parameters sequentially and discriminatively, and building a hybrid parameter estimate vector, without creating such complexity in the overall system.

II. Problem Definition

The single-input single-output plant considered in this study is of the form

$$\left.\begin{array}{rcl} \dot{x} &=& f(x,\theta)+g(x,\theta)u \\ y &=& h(x) \end{array}\right\} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $f \in \mathbb{R}^n$, $g \in \mathbb{R}^n$ and $h \in \mathbb{R}$ are sufficiently smooth vector fields,

$$\theta = \begin{bmatrix} \vartheta_1 \\ \vdots \\ \vartheta_p \end{bmatrix} \in \mathcal{S} \tag{2}$$

is the unknown parameter vector with $\mathcal{S} \subset \mathbb{R}^p$ being a compact set, $u \in \mathbb{R}$ is the input and $y \in \mathbb{R}$ is the output.

We assume that the full state vector is available for feedback, the system has constant relative degree γ and the zero-dynamics of the system is exponentially stable. We also assume that the dynamics defined in (1) is linear in its parameters. If the parameter vector $\theta \in \mathcal{S} \subset \mathbb{R}^p$ includes p unknown parameters, then the vector fields f and g can be expressed as,

$$f(x,\theta) = \sum_{k=1}^{p} \vartheta_k f_k(x) \tag{3}$$

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$$g(x,\theta)=\sum_{k=1}^{p}\vartheta_k g_k(x). \tag{4}$$

Without loss of generality, the elements ϑ_k of the parameter vector θ are the same for both vector fields f and g, as defined in (3) and (4).

Large and abrupt parameter changes may occur in some or all parameters of the actual plant. Control objective is to track a bounded, pre-specified trajectory $y_r(t)$, which has bounded derivatives $\dot{y}_r(t),\ldots,y_r^{(\gamma)}(t)$, with as small transient errors as possible under such changes in the parameters. In a typical MMAC system, one incorporates N fixed models and a switching mechanism, such that whenever one of the fixed models minimizes a cost function derived from identification errors, the parameter estimates are reset to those of the minimizing fixed model. Then, the adaptation continues using update dynamics, but consequent switchings may happen thereafter. The overall objective is to construct an adaptive control system for the nonlinear plant in (1) using N fixed models and a switching logic, which would lead to asymptotic tracking convergence with improved transient response under large parameter variations.

III. Review of Multi-Model Adaptive Control

In this section we will briefly review the typical multi-model adaptive control system. The MMAC requires a system identification part in order to determine the proximity of the parameter estimates to the actual parameter vector in the parameter space. Another reason for the separate system identification part is due to the fact that the adaptive control system is based on the indirect scheme. Generally the state estimation error is used as the identification error. Let $\tilde{x}\in\mathbb{R}^n$ be the vector of state estimation error, which is defined as

$$\tilde{x}\overset{def}{=}\hat{x}-x \tag{5}$$

where x is the state vector which is measurable, and $\hat{x}$ is estimate for the state vector. To obtain the state estimate $\hat{x}$, the system dynamics in (1) is first put into the regressor form as

$$\dot{x}=w^T(x,u)\theta \tag{6}$$

where w is the regressor matrix and θ is the parameter vector. The observer-based estimation model for the system is

$$\left.\begin{array}{rcl}\dot{\hat{x}} &=& A(\hat{x}-x)+w^T(x,u)\hat{\theta}\\ \dot{\hat{\theta}} &=& -w(x,u)P(\hat{x}-x)\end{array}\right\} \tag{7}$$

where $\hat{x}$ and $\hat{\theta}$ are estimates of x and θ respectively, $A\in\mathbb{R}^{n\times n}$ is a Hurwitz matrix, and a symmetric, positive definite matrix $P\in\mathbb{R}^{n\times n}$ is the solution of the Lyapunov equation

$$A^TP+PA=-Q \tag{8}$$

with Q being a symmetric, positive definite matrix. Similar to the state estimation error $\tilde{x}$ as given in (5), parameter error is defined as $\tilde{\theta}\overset{def}{=}\hat{\theta}-\theta$. Differentiating $\tilde{x}$, the error system

$$\left.\begin{array}{rcl}\dot{\tilde{x}} &=& A\tilde{x}+w^T(x,u)\tilde{\theta}\\ \dot{\tilde{\theta}} &=& -w(x,u)P\tilde{x}\end{array}\right\} \tag{9}$$

is obtained. Note that if the nonlinear system is bounded-input bounded-state (BIBS) stable, then $\lim_{t\to\infty}\tilde{x}=0$. Moreover, if the regressor matrix w is sufficiently rich, then $\lim_{t\to\infty}\tilde{\theta}=0$ [13].

The state estimation errors for the fixed models are computed similarly, as

$$\tilde{x}_j=\hat{x}_j-x \quad , \qquad j=1,\ldots,N. \tag{10}$$

The state estimates for the fixed models are obtained from the dynamics

$$\dot{\hat{x}}_j=A(\hat{x}_j-x)+w^T(x,u)\hat{\theta}_j \quad , \qquad j=1,\ldots,N \tag{11}$$

where $\hat{x}_j$ and $\hat{\theta}_j$ are the state and parameter estimate vectors for the j^{th} fixed model, respectively, and A is the same Hurwitz matrix as in (7). Typically, the state estimation errors are used in the switching mechanism of the multiple models scheme. Therefore, denoting the state estimation error of the adaptive model given in (5) by $\tilde{x}_0$, (5) and (10) can be combined in a single equation as,

$$\tilde{x}_j=\hat{x}_j-x \quad , \qquad j=0,\ldots,N \tag{12}$$

Then, we can consider a cost function in quadratic form

$$J_j(t)\overset{def}{=}\tilde{x}_j^T(t)G\tilde{x}_j(t) \quad , \qquad j=0,\ldots,N \tag{13}$$

where $G\in\mathbb{R}^{n\times n}$ is a positive-definite weight matrix. In (13), the index *zero* in J (i.e. J_0) represents the cost function calculated for the adaptive model, and the indices 1 through N correspond to the cost functions calculated for N fixed models.

Definition 1: A finite sequence $T_i\in\mathbb{R}_+$ is defined as a switching sequence if $T_0=0$ and $T_i<T_{i+1}$ for all i. Additionally, if there is a number $T_{min}>0$ such that $T_{i+1}-T_i\geq T_{min}$ for all i, then the sequence is called a permissible switching scheme.

This definition basically states that infinite switching is not allowed in finite time, in a permissible switching scheme. A cost function based switching logic can also be defined as follows:

Definition 2: With N fixed identification models defined in (10), (11), one adaptive model with estimation dynamics in (9), and a permissible switching scheme as defined in Definition 1, the switching logic which is based on the cost function in (13) is defined as,

$$j^\star(t)=\arg\max\{(J_0(t)-J_j(t))\geq\kappa>0\}, \qquad j=0,\ldots,N \quad , \quad t\in[T_i,T_{i+1}) \tag{14}$$

where $j^\star$ gives the index for the chosen identification model.

In the typical MMAC system, N fixed identification models are represented by N fixed parameter estimates, $\hat{\theta}_j$, with $j=1,\ldots,N$. According to the above definition, if the maximizing cost function has the index zero, the adaptive model is already the best model and no switching occurs at that evaluation instant. If the maximizing cost function has the index which is greater than zero and that cost function is less than the cost function of the adaptive model $J_0(t)$ at least by the amount of a positive hysteresis denoted by κ, then the parameter estimate vector $\hat{\theta}(t)$ of the adaptive model is reset to the parameter vector $\hat{\theta}_{j^\star}$ of the selected fixed model. The adaptive model continues adaptation re-initialized with this fixed model.

IV. Sequential Parameter Discrimination

In this section we describe a new methodology that improves transient response of MMAC system for a nonlinear plant with reasonable number of fixed models. The parameter vector θ has p elements as given in (2). As we have N fixed models for the identification, we have a total of Np scalar parameters. However, the total number all possible parameter vectors that could be used as the fixed models would be N^p. For better switching and hence faster convergence, we need large number of fixed models, but on the other hand, the possibilities grow exponentially as the number of the fixed models and the number of the unknown plant parameters increase [10][11].

For example, we have a plant with two unknown parameter and we have three supposefixed identification models for this plant. That is, $N = 3$ and $p = 2$. Let the models be given as $\theta_1 = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$, $\theta_2 = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$ and $\theta_3 = \begin{bmatrix} a_3 \\ b_3 \end{bmatrix}$. Obviously, there is no reason why a hybrid model derived from the above models would not give better result than any of these three models. If we consider each element of the parameter vectors independently, we obtain $3^2 = 9$ parameter vectors as $\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$, $\begin{bmatrix} a_1 \\ b_2 \end{bmatrix}$, $\begin{bmatrix} a_1 \\ b_3 \end{bmatrix}$, $\begin{bmatrix} a_2 \\ b_1 \end{bmatrix}$, $\begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$, $\begin{bmatrix} a_2 \\ b_3 \end{bmatrix}$, $\begin{bmatrix} a_3 \\ b_1 \end{bmatrix}$, $\begin{bmatrix} a_3 \\ b_2 \end{bmatrix}$, $\begin{bmatrix} a_3 \\ b_3 \end{bmatrix}$. Similarly, for a plant having five unknown parameters and five possible preset values for each parameter, there would be $5^5 = 3125$ possible models. It is evident that such a large number of identification models is not feasible for a real-word adaptive control system implementation. Below we present a methodology that needs only $5 \times 5 = 25$ sets for the case of five unknown parameters with five preset values each, which is of course much lower than 3125.

The performance index given in (13) is a quadratic form of the state estimation vector $\tilde{x}_j$, which is derived from the identification error dynamics in (9). For the j-th model, we use $\hat{\theta}_j$ in (9) and hence $\tilde{x}_j$ is obtained. Note that full $\hat{\theta}_j$ is incorporated in these calculations.

Using $p \times 1$ parameter estimate vector $\hat{\theta}$ and N fixed identification models, we define $p \times (N+1)$ matrices as

$$\Theta_d(t) \stackrel{def}{=} \begin{bmatrix} \hat{\vartheta}_1 & \hat{\vartheta}_1 & \hat{\vartheta}_1 & \cdots & \hat{\vartheta}_1 \\ \hat{\vartheta}_2 & \hat{\vartheta}_2 & \hat{\vartheta}_2 & \cdots & \hat{\vartheta}_2 \\ \vdots & \vdots & \vdots & & \vdots \\ \hat{\vartheta}_d & \hat{\vartheta}_{d1} & \hat{\vartheta}_{d2} & \ddots & \hat{\vartheta}_{dN} \\ \vdots & \vdots & \vdots & & \vdots \\ \hat{\vartheta}_p & \hat{\vartheta}_p & \hat{\vartheta}_p & \cdots & \hat{\vartheta}_p \end{bmatrix}, \; d = 1, \ldots, p \quad (15)$$

where the first column is identical to $\hat{\theta}$ and each of the remaining columns is the adaptive parameter vector with its d-th element replaced by that of the corrsponding fixed model. For instance, the second column of Θ_d replaces the d-th element of the $\hat{\theta}$ by the d-th element of the second fixed model $\hat{\theta}_2$.

We will now modify the state observer dynamics given in (11) as

$$\dot{\hat{x}}_{dj} = A(\hat{x}_{dj} - x) + w^T(x, u)\hat{\theta}_{dj}, \quad d = 1, \ldots, p \quad j = 0, \ldots, N \quad (16)$$

where $\hat{\theta}_{dj}$ is the corresponding column of Θ_d given in (15). Then we re-define the state estimation error accordingly, as $\tilde{x}_{dj} = \hat{x}_{dj} - x$, and the cost function as

$$J_{dj}(t) = \tilde{x}_{dj}^T(t) G \tilde{x}_{dj}(t) \quad (17)$$

where the subscript d varies from 1 to p sequentially. Using the cost function in (13), the switching logic given in Definition 2 assigns a j, which is scalar and denotes the selected parameter vector. However, with the use of (17), we obtain a j value for every d. Eventually this results in a $p \times 1$ selection vector.

Definition 3: With the discriminative matrices Θ_d as given in (15), the identification dynamics in (16), and a permissible switching scheme as defined in Definition 1, the switching logic which is based on the cost function in (17) is defined as,

$$j^\star(d, t) = \underset{j=0,\ldots,N;\, t\in[T_i, T_{i+1})}{\arg\min} \{J_{dj}(t)\} \quad (18)$$

for each value of $d = 1, \ldots p$.

The result of the above switching scheme is a *hybrid parameter vector*

$$\hat{\theta}_h(t) = \begin{bmatrix} \hat{\vartheta}_{1j^\star(1,t)} \\ \hat{\vartheta}_{2j^\star(2,t)} \\ \vdots \\ \hat{\vartheta}_{pj^\star(p,t)} \end{bmatrix}.$$

It is important to note that whenever the minimizing argument of (17) is zero for any d, then no resetting occurs for that element of the parameter estimate vector $\hat{\theta}$. If (17) is minimized with all zero arguments for all $d = 1, \ldots, p$, then $\hat{\theta}_h(t)$ is simply identical to $\hat{\theta}(t)$.

The proposed methodology basically suggests the use of the hybrid parameter vector and describes a way of achieving this. We can summarize the necessary steps as follows:

1) Begin with $d = 1$,
2) Construct Θ_d matrix discriminatively, i.e. by taking only the d-th elements of the fixed parameter vectors,
3) Evaluate the cost function in (17) for each column vector of Θ_d,
4) Obtain the d-th element of the hybrid parameter vector $\hat{\theta}_h$ using the switching logic in (18),
5) Set $d + 1 \to d$ and repeat the steps (2) through (5) sequentially until (and including) $d = p$; hence obtain the full $\hat{\theta}_h(t)$,
6) Reset $\hat{\theta}(t)$ to $\hat{\theta}_h(t)$.

The transient performance improvement of the proposed methodology as compared to the typical MMAC system stems from the assumption that the decrease in any element of the parameter error vector results in decrease in the state estimation error. Conversely, decrease in the state estimation error implies decrease in at least one of the components of the parameter error vector. That is,

$$\|\tilde{\theta}_{dj}\| \le \|\tilde{\theta}\| \Longleftrightarrow \|\tilde{x}_{dj}\| \le \|\tilde{x}\| \quad (19)$$

for any d and j.

V. Indirect Adaptive Control with Multiple Fixed Models

In this section we derive an indirect adaptive system using multiple models based on the indirect adaptive control scheme described in [13]. Since our approach is based on the indirect scheme, we use the separate estimator and the switching logic derived in the preceding section to identify the unknown system parameters. The control input u is then computed using these estimates via the certainty equivalence principle. To generate the control signal based on the estimated system parameters, the nonlinear system dynamics in (1) is written in the normal form. Let $\gamma \leq n$ be the relative degree of the nonlinear system. Then we can write (1) as,

$$\left.\begin{array}{rcl} \dot{\xi}_1 & = & \xi_2 \\ & \vdots & \\ \dot{\xi}_\gamma & = & L_f^\gamma h(x)(x,\theta) + L_g L_f^{\gamma-1} h(x)(x,\theta) \cdot u \\ \dot{\eta} & = & q(\xi,\eta) \\ y & = & \xi_1 \end{array}\right\} \quad (20)$$

where $\xi_1, \ldots, \xi_\gamma$ are the linearized states and $\dot{\eta} = q(\xi,\eta)$ defines the internal dynamics with $\dot{\eta} = q(0,\eta)$ being exponentially stable zero-dynamics. Note that $L_f^\gamma h(x)(x,\theta)$ and $L_g L_f^{\gamma-1} h(x)(x,\theta)$ in (20) can now be expanded in terms of multi-linear parameter elements as

$$\begin{array}{l} L_f^\gamma h(x)(x,\theta) \\ = \sum_{i_1}^p \sum_{i_2}^p \cdots \sum_{i_\gamma}^p \vartheta_{i_1} \vartheta_{i_2} \cdots \vartheta_{i_\gamma} \frac{\partial}{\partial x} \\ \times \left(\frac{\partial}{\partial x} (\cdots (\frac{\partial h}{\partial x} f_{i_1}(x)) \cdots) f_{i_{\gamma-1}}(x) \right) f_{i_\gamma}(x) \end{array} \quad (21)$$

and

$$\begin{array}{l} L_g L_f^{\gamma-1} h(x)(x,\theta) \\ = \sum_{j}^p \sum_{i_1}^p \cdots \sum_{i_{\gamma-1}}^p \vartheta_j \vartheta_{i_1} \cdots \vartheta_{i_{\gamma-1}} \frac{\partial}{\partial x} \\ \times \left(\frac{\partial}{\partial x} (\cdots (\frac{\partial h}{\partial x} f_{i_1}(x)) \cdots) f_{i_{\gamma-1}}(x) \right) g_j(x). \end{array} \quad (22)$$

The above representation is based on the definitions of f and g given in equations (3) and (4), respectively. For the brevity of the analysis to follow, equations (21) and (22) can be written in vector notation as

$$L_f^\gamma h(x)(x,\theta) \overset{def}{=} \mathcal{P}^T \mathcal{F}(x) \quad (23)$$

and

$$L_g L_f^{\gamma-1} h(x)(x,\theta) \overset{def}{=} \mathcal{P}^T \mathcal{G}(x) \quad (24)$$

where $\mathcal{P}$ represents the multi-linear parameter vector. Using this notation, the $\dot{\xi}_\gamma$ equation in the normal form given in (20) can be written as

$$\dot{\xi}_\gamma = \mathcal{P}^T \left(\mathcal{F}(x) + \mathcal{G}(x) u \right). \quad (25)$$

Based on the parameter estimates and the switching logic defined in Definition 3, we can now generate the control signal u using the certainty equivalence principle. First, we will define a notation to clearly show that the parameter estimates are held constant throughout the permissible switching period. Let us define a function $z(t) : \mathbb{R}_+ \mapsto 0, \ldots, N$ such that if $t \in [T_i, T_{i+1})$ for some $i < \infty$, then $z(t) = z(T_i)$. Here, T_i is the permissible switching sequence defined in Definition 1. Using this notation, under the switching logic the parameter estimate vector $\hat{\theta}$ and the multi-linear parameter estimate vector $\hat{\mathcal{P}}$ will take the form, respectively, $\hat{\theta}_{z(t)}$ and $\hat{\mathcal{P}}_{z(t)}$. For brevity of notation, we will simply use z to mean $z(t)$, and we will use $\hat{\theta}_z$ and $\hat{\mathcal{P}}_z$ in the rest of this work.

Using the switched parameter estimates, the control signal can be generated as,

$$u = \frac{1}{\hat{\mathcal{P}}_z^T \mathcal{G}(x)} (-\hat{\mathcal{P}}_z^T \mathcal{F}(x) + v) \quad (26)$$

where $\hat{\mathcal{P}}$ is the estimate of the multi-linear parameter vector and v is the tracking control signal to be designed using the given desired trajectory information, as

$$v = y_r^{(\gamma)} + \lambda_1 (y_r^{(\gamma-1)} - y^{(\gamma-1)}) + \cdots + \lambda_\gamma (y_r - y) \quad (27)$$

where $\lambda_1, \ldots, \lambda_\gamma$ are chosen positive constants, such that

$$s^\gamma + \lambda_1 s^{\gamma-1} + \cdots + \lambda_\gamma \quad (28)$$

represents a Hurwitz polynomial. Note that in (27), $y_r, \dot{y}_r, \ldots, y_r^{(\gamma)}$ are already available. On the other hand, $\dot{y}, \ldots, y^{(\gamma-1)}$ need to be calculated from $L_f h, L_f^2 h, \ldots, L_f^{\gamma-1} h$, respectively. However, since these Lie derivatives are not free of the unknown parameters, we apply the certainty equivalence principle once more, and we get

$$\hat{y}_z^{(k)} = \sum_{j_1}^p \cdots \sum_{j_k}^p \hat{\vartheta}_{z_{j_1}} \cdots \hat{\vartheta}_{z_{j_k}} \frac{\partial}{\partial x} (\cdots (\frac{\partial h}{\partial x} f_{j_1}) \cdots) f_{j_k} \quad (29)$$

where $1 \leq k \leq \gamma - 1$. Then using (29), we obtain the certainty equivalence based tracking control estimate $\hat{v}$ as,

$$\hat{v} = y_r^{(\gamma)} + \lambda_1 (y_r^{(\gamma-1)} - \hat{y}_z^{(\gamma-1)}) + \cdots + \lambda_{\gamma-1} (\dot{y}_r - \dot{\hat{y}}_z) + \lambda_\gamma (y_r - y). \quad (30)$$

The control input can now be fully constructed using only the known signals in (30) as,

$$u = \frac{1}{\hat{\mathcal{P}}_z^T \mathcal{G}(x)} (-\hat{\mathcal{P}}_z^T \mathcal{F}(x) + \hat{v}). \quad (31)$$

Plugging the control signal u in the system dynamics given in (25), one can now obtain the closed loop error dynamics. To do this, first we rewrite the $\dot{\xi}_\gamma$ term in (25) as

$$\begin{array}{r} \dot{\xi}_\gamma = \mathcal{P}^T \mathcal{F}(x) + \mathcal{P}^T \mathcal{G}(x) u - [\hat{\mathcal{P}}_z^T \mathcal{F}(x) + \hat{\mathcal{P}}_z^T \mathcal{G}(x) u] \\ + [\hat{\mathcal{P}}_z^T \mathcal{F}(x) + \hat{\mathcal{P}}_z^T \mathcal{G}(x) u]. \end{array} \quad (32)$$

Then defining the multi-linear parameter error vector as,

$$\tilde{\mathcal{P}}_z \overset{def}{=} \hat{\mathcal{P}}_z - \mathcal{P} \quad (33)$$

we obtain

$$\dot{\xi}_\gamma = \hat{\mathcal{P}}_z^T \mathcal{F}(x) + \hat{\mathcal{P}}_z^T \mathcal{G}(x) u + \tilde{\mathcal{P}}_z^T \varphi_1(x,u). \quad (34)$$

If we now plug in the certainty equivalence control u into (34), we get,

$$\dot{\xi}_\gamma = \hat{v} + \tilde{\mathcal{P}}_z^T \varphi_1(x,u). \quad (35)$$

Note that $\hat{v}$ can be written as

$$\begin{array}{r} \hat{v} = y_r^{(\gamma)} + \lambda_1 (y_r^{(\gamma-1)} - y^{(\gamma-1)}) + \lambda_1 (y^{(\gamma-1)} - \hat{y}_z^{(\gamma-1)}) \\ + \cdots + \lambda_\gamma (y_r - y) + \lambda_\gamma (y - \hat{y}_z) \end{array} \quad (36)$$

which is equivalent to the exact tracking control v plus an offset term which is a function of parameter errors,

$$\hat{v} = v + \tilde{\mathcal{P}}^T \varphi_2(x,u). \quad (37)$$

Then, equation (35) can now be written as

$$\dot{\xi}_\gamma = v + \tilde{\mathcal{P}}_z^T(\varphi_1(x,u) + \varphi_2(x,u)) \tag{38}$$

Let the dimensions of the multi-linear parameter vector $\mathcal{P}$ be $m \times 1$. Then, defining an $m \times n$ matrix

$$\varphi \overset{def}{=} [\ 0 \quad | \quad \cdots \quad | \quad 0 \quad | \quad \varphi_1 + \varphi_2\ ] \tag{39}$$

which includes all the offset terms due to parameter errors, and defining an error term as,

$$e_i = \xi_i - y_r^{(i-1)} \quad , \qquad i = 1, \ldots, \gamma \tag{40}$$

the closed loop error dynamics takes the following form:

$$\left.\begin{array}{rcl} \xi & = & e + r \\ \dot{e} & = & Ae + \varphi^T(x,u)\tilde{\mathcal{P}}_z \\ \dot{\eta} & = & q(\xi,\eta) \end{array}\right\} \tag{41}$$

where $r = \left[\begin{array}{cccc} y_r & \dot{y}_r & \cdots & y_r^{(\gamma-1)} \end{array}\right]^T$, A is a Hurwitz matrix, with parameters defined in (28), and $\varphi(x,u)$ represents the regressor matrix defined in (39).

Theorem 1: Consider the nonlinear system defined in (1) with the exponentially stable internal dynamics $q(\xi,\eta)$ which is globally Lipschitz in ξ, η, and a bounded tracking signal y_r, with bounded derivatives as $y_r, \dot{y}_r, \ldots, y_r^{(\gamma-1)}$. With the assumption in (19), if the regressor matrix φ is bounded for bounded ξ, η, u, such that $\|\varphi^T(\xi,\eta,u)\| \leq b_\varphi(\|\xi\| + \|\eta\|)$ holds, and if the input is persistently exciting such that $\tilde{\mathcal{P}} \to 0$ as $t \to \infty$, then the control laws given by (30) and (31) along with N fixed identification models incorporated in a switching logic as defined in Definition 3 result in an asymptotically stable closed loop error dynamics such that $y(t) \to y_r(t)$ as $t \to \infty$, with enhanced transient performance.

The proof of this theorem can be done as the one given in [4], using the similar arguments and incorporating the assumption in (19).

VI. Conclusions

A new methodology for the adaptive control of a class of single-input single-output nonlinear systems using multiple identification models is proposed in this study. In the proposed scheme, each parameter is evaluated in a discriminative manner and a hybrid vector of parameter estimates is obtained. In doing this, we invoke the assumption so that a decrease in even one element of the parameter error vector necessarily implies decrease in the state estimation error and vice-versa.

Using the described technique, one can distribute the fixed models evenly in the parameter space, without trying to create successful combinations of the parameters. The discriminative evaluation of each parameter makes it possible to obtain the best combination of the available preset values of the parameters, namely the hybrid parameter vector. Since the hybrid vector points to a location which is closer to the actual parameter vector, the transient response of the adaptive system is improved.

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