The Effect of Measurement for Time Synchronization Error in the Tightly Coupled GPS/INS Integration

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***Abstract*—The performance of tightly coupled GPS/INS integration becomes worse if the time synchronization error occurs between the GPS receiver and INS(Inertial Navigation System). In this paper the effect of measurement due to time synchronization error is investigated for tightly coupled GPS/INS integration. Two kinds of measurement is used; one is pseudorange only, and the other is both pseudorange and pseudorange rate.**

Keywords—GPS; INS; tightly coupled GPS/INS integration; time synchronization error

I. INTRODUCTION

GPS(Global Positioning System) and INS(Inertial Navigation System) are widely used as standalone navigation systems, respectively. The integration of GPS and INS not only leads to high accuracy of vehicle's position, velocity, and attitude, but also provides position error bound. Recently there are more useful applications rather than pure navigation [1,2].

To maximize the performance of GPS/INS integrated systems, time synchronization between GPS and INS is an important issue [4,5,6,7]. Various methods have been developed to reduce the effect of the time synchronization error in fusion of multiple sensors [8,9,10]. It was shown that time delay may cause the acceleration bias error relatively large [3]. Skog et.al. [5] shows that large time synchronization error in a loosely coupled GPS/INS integration may seriously degrade the performance of the system such as acceleration bias error, the gyro bias error, and the position error.

There has been some research for tightly coupled GPS/INS systems. For tightly coupled GPS/INS integration, we are supposed to analyze the effect of time delay for two cases, one using only pseudorange information as Kalman filter measurement, the other using both pseudorange and pseudorange information as Kalman filter measurement.

To compare the performance of the two measurements for the tightly coupled integration systems, it is analytically studied how the time delay between GPS and INS has effect on the Kalman filter innovation. Then based on the analysis results, computer simulations are performed to check how each measurement has effect on the navigation performance such as position, velocity, and attitude of the vehicle. The tightly coupled GPS/INS integration method is reviewed briefly in section 2, and the effects of time delay are analyzed in section 3. Computer simulations are performed in section 4 and conclusions are given in section 5.

II. THE TIGHTLY COUPLED GPS/INS NAVIGATION

The block diagram of the tightly coupled GPS/INS integration is shown in Fig. 1. The GPS pseudorange and pseudorange rate are used directly in the measurement of Kalman filter and the clock error of the GPS receiver should be included in the state variable to be estimated. Thus the Kalman filter error model contains position error, velocity error, attitude error, accelerometer bias, gyroscope bias, clock bias and clock drift of GPS receiver, resulting in 17 state variables.

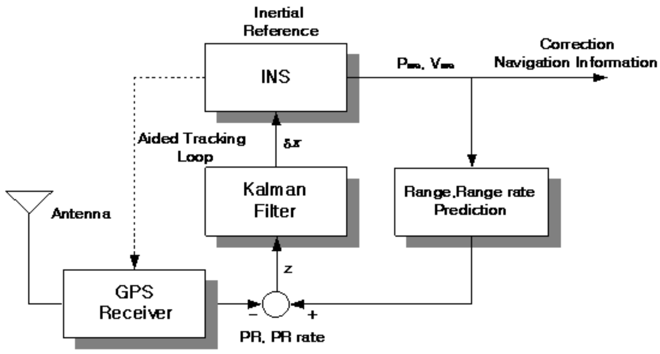


Fig. 1. Tightly coupled GPS/INS integration

The system can be described as in (1)

$$\begin{bmatrix} \dot{x}_{INS} \\ \dot{x}_{clock} \end{bmatrix} = \begin{bmatrix} F_{INS} & 0_{15\times 2} \\ 0_{2\times 15} & F_{clock} \end{bmatrix} \begin{bmatrix} x_{INS} \\ x_{clock} \end{bmatrix} + w_T \ , \ w_T \sim N(0, Q_T) \quad (1)$$

$$x_{INS} = [\delta L \ \delta l \ \delta h \ \delta v^n \ \Phi^n B_{accel} B_{gyro}]^{\mathrm{T}}, x_{clock} = \left[\delta c_{bias} \ \delta c_{drift} \right]^{\mathrm{T}} \quad (2)$$

where $F_{clock} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, F_{INS} are described in [11].

The state variable contains the position error with latitude, longitude, and height $(\delta L, \delta l, \delta h)$, and velocity error (δv^n), attitude error $(\delta\Phi^n)$ expressed in the navigation frame, and bias errors (B_{accel}, B_{gyro}) of accelerometer and gyroscopes(refer to [11] for detail error model). Variables δc_{bias} and δc_{drift} denote for clock bias and clock drift of GPS receiver.

The measurement of Kalman filter for tightly coupled GPS/INS systems is the pseudorange and the pseudorange rate. In this paper we are going to investigate the performance of two measurements: pseudorange only, and both pseudorange and pseudorange rate.

(1) Type 1 (T1) : the case of using only pseudorange as measurement

$$z_{T1} = \rho_{INS} - \rho_{GPS} = \mathrm{H}_{T1}\left[x_{INS}\ x_{clock}\right]^{\mathrm{T}} + v_{T1}\ ,\ v_{T1} \sim N(0, R_{T1}) \quad (3)$$

(2) Type 2 (T2) : the case of using both the pseudorange and the pseudorange rate as measurement

$$z_{T2} = \begin{bmatrix} \rho_{INS} \\ \dot{\rho}_{INS} \end{bmatrix} - \begin{bmatrix} \rho_{GPS} \\ \dot{\rho}_{GPS} \end{bmatrix} = \mathrm{H}_{T2}\left[x_{INS}\ x_{clock}\right]^{\mathrm{T}} + v_{T2}\ ,\ v_{T2} \sim N(0, R_{T2}) \quad (4)$$

Where ρ and $\dot{\rho}$ are the pseudorange and the pseudorange rate, respectively. The v_{T1} and v_{T2} are white noises, and the measurement matrix H_{T1} and H_{T2} are described in [11].

III. EFFECTS OF THE TIME SYNCHRONIZATION ERROR

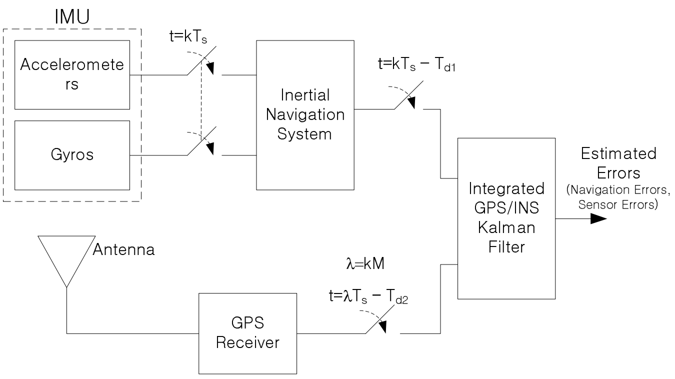


Fig. 2. Structure of synchronization error Td(=Td2 -Td1)in the integrated GPS/INS navigation system

For the GPS/INS integration there exists time delay between GPS receiver and INS since not only sampling time but also the signal processing time is different as in Fig. 2. For simplified analysis, the GPS receiver sampling period is assumed to be a multiple M of the IMU sampling period Ts. Td1 and Td2 are processing time delays of the inertial navigation system(INS) and the GPS receiver, respectively and Td =Td2 -Td1, i.e., the difference of the INS and GPS processing time.

A. EKF algorithm

Consider an error model of INS and EKF equation to analyze the effect of time delay in measurement. The error model can be described as follows:

$$x_{k+1} = \Psi_k x_k + G_k e_k \quad (5)$$

$$z_k = \mathrm{H}_k x_k + w_k \quad (6)$$

Here, Ψ_k is the error state transition matrix, e_k is the process noise, G_k is the process noise gain, H_k is the measurement matrix, and w_k is the measurement noise. The subscript k implies k-th sampling sequence. The process noise e_k and the measurement noise w_k are considered as white noise, uncorrelated with each other and the covariance matrices are $R = \mathrm{E}\{w_k w_k^{\mathrm{T}}\}$ and $Q = \mathrm{E}\{e_k e_k^{\mathrm{T}}\}$, where, $\mathrm{E}\{\ \}$ denotes the expectation operator.

The EKF equations for the tightly coupled GPS/INS integration are given in Table 1.

Table 1. EKF Algorithm for Integration between the GPS and the INS

Kalman gain update	$\mathrm{K}_k = \mathrm{P}_k - \mathrm{H}_k^{\mathrm{T}}(\mathrm{H}_k\mathrm{P}_k - \mathrm{H}_k^{\mathrm{T}} + R)^{-1}$
Difference of the two measurements	$z_k = \hat{p}_{k,INS} - \hat{p}_{k.GPS}$
Estimation of the state errors	$\hat{x}_k = \begin{bmatrix} \delta\hat{r}_k \\ \delta\hat{\xi}_k \end{bmatrix} = \mathrm{K}_k z_k$
Error correction	$\begin{bmatrix} \hat{r}_k \\ \hat{\xi}_k \end{bmatrix} = \begin{bmatrix} \delta\hat{r}_k^- \\ \delta\hat{\xi}_k^- \end{bmatrix} + \hat{x}_k$
Covariance update	$\mathrm{P}_k = (\mathrm{I} - \mathrm{K}_k\mathrm{H}_k)\mathrm{P}_k^-$
Sensor error compensation	$\hat{u}_k = g(\tilde{u}_k, \hat{\xi}_k)$
Navigation equation update	$\hat{r}_{k+1}^- = f(\hat{r}_k, \hat{u}_k)$
Sensor error update	$\hat{\xi}_{k+1}^- = \hat{\xi}_k$
Covariance update	$\mathrm{P}_{k+1}^- = \Psi_k \mathrm{P}_k^- \Psi_k^{\mathrm{T}} + G_k Q G_k^{\mathrm{T}}$

B. Effects of time synchronization error

Variables $\hat{r}_k$, $\hat{\xi}_k$ and $\hat{u}_k$ in Table 1 denote the estimated navigation state, the estimated sensor errors, and the IMU measurements, respectively. Effects of time synchronization error between GPS receiver and INS can be analyzed as follows. Let the misalignment of sampling measurement between GPS receiver and INS be T_d and suppose that INS sampling has time synchronization error less than 1 sec.

Suppose that the pseudorange, which is the measurement of tightly coupled integration, has time synchronization error less than 1sec, then the pseudorange estimated in INS can be described as (7) by Taylor series, where $\rho(t)$ is true pseudorange.

$$\hat{\rho}_{k,INS} = \rho(k\mathrm{T}_S - \mathrm{T}_d) \cong \rho_k - \dot{\rho}_k \mathrm{T}_d + 0.5\ddot{\rho}_k \mathrm{T}_d^2 + w_{1k} \qquad (7)$$

where $|T_d| \leq 1$, the pseudorange rate is $\dot{\rho}_k = (v_{k,GPS} - v_k) l_k$, pseudorange acceleration is $\ddot{\rho}_{k,INS} = (\alpha_{k,GPS} - a_k) l_k$, l_k is LOS(line of sight) vector between satellite and vehicle, and $v_{k,GPS}$ and $\alpha_{k,GPS}$ are velocity and acceleration of satellite, respectively. And v_k is the velocity of the vehicle and a_k is the acceleration.

Similarly suppose that the pseudorange rate has time synchronization error less than 1sec, then the pseudorange rate estimated in INS can be described as (8) by Taylor series, where $\dot{\rho}(t)$ is true pseudorange rate.

$$\hat{\dot{\rho}}_{k,INS} = \dot{\rho}(k\mathrm{T}_S - \mathrm{T}_d) \cong \dot{\rho}_k - \ddot{\rho}_k \mathrm{T}_d + 0.5\dddot{\rho}_k \mathrm{T}_d^2 + w_{2k} \qquad (8)$$

where the pseudorange jerk is $\dddot{\rho}_{k,INS} = (\dot{\alpha}_{k,GPS} - \dot{a}_k) l_k$, $\dot{\alpha}_{k,GPS}$ is jerk of satellite and $\dot{a}_k$ is jerk of the vehicle.

Thus the true observation z_k^a of two integration systems is obtained as in (9).

For the tightly coupled GPS/INS system of the type T1

$$z_k^a = \hat{\rho}_{k,INS} - \hat{\rho}_{k,GPS} \qquad (9a)$$
$$\cong \rho_k - \dot{\rho}_k \mathrm{T}_d + \ddot{\rho}_k \frac{\mathrm{T}_d^2}{2} + w_k - \hat{\rho}_{k,GPS}$$

For the tightly coupled GPS/INS system of the type T2

$$z_k^a = \begin{bmatrix} \hat{\rho}_{k,INS} \\ \hat{\dot{\rho}}_{k,INS} \end{bmatrix} - \begin{bmatrix} \hat{\rho}_{k,GPS} \\ \hat{\dot{\rho}}_{k,GPS} \end{bmatrix} \qquad (9b)$$
$$\cong \begin{bmatrix} \rho_k \\ \dot{\rho}_k \end{bmatrix} - \begin{bmatrix} \hat{\rho}_{k,GPS} \\ \hat{\dot{\rho}}_{k,GPS} \end{bmatrix} + \begin{bmatrix} -\dot{\rho}_k \mathrm{T}_d + 0.5\ddot{\rho}_k \mathrm{T}_d^2 \\ -\ddot{\rho}_k \mathrm{T}_d + 0.5\dddot{\rho}_k \mathrm{T}_d^2 \end{bmatrix} + \begin{bmatrix} w_{1k} \\ w_{2k} \end{bmatrix}$$

Let $\mathrm{H}_k x_k = \rho_k - \hat{\rho}_{k,GPS}$ or $\mathrm{H}_k x_k = \begin{bmatrix} \rho_k \\ \dot{\rho}_k \end{bmatrix} - \begin{bmatrix} \hat{\rho}_{k,GPS} \\ \hat{\dot{\rho}}_{k,GPS} \end{bmatrix}$, then (9) becomes (10).

$$z_k^a = \mathrm{H}_k x_k + w_k + d_k \qquad (10)$$

In (10), the pseudorange error bias vector or the pseudorange and pseudorange rate error bias vector is introduced and defined as

$$d_k = -\dot{\rho}_k \mathrm{T}_d + \ddot{\rho}_k \frac{\mathrm{T}_d^2}{2} \quad \text{or} \quad d_k = \begin{bmatrix} -\dot{\rho}_k \mathrm{T}_d + 0.5\ddot{\rho}_k \mathrm{T}_d^2 \\ -\ddot{\rho}_k \mathrm{T}_d + 0.5\dddot{\rho}_k \mathrm{T}_d^2 \end{bmatrix} \qquad (11)$$

In [5], the MSE(mean square error) of the navigation solution may be expressed as

$$\begin{aligned} \mathrm{P}_{k+1} &= \mathrm{E}\{x_{k+1}(x_{k+1})^\mathrm{T}\} \qquad (12) \\ &= \Phi_k \mathrm{P}_k \Phi_k^\mathrm{T} + \mathrm{K}_{p,k} R \mathrm{K}_{p,k}^\mathrm{T} + G_k Q G_k^\mathrm{T} \\ &\quad + \mathrm{K}_{p,k} d_k d_k^\mathrm{T} \mathrm{K}_{p,k}^\mathrm{T} - \Phi_k \bar{x}_k d_k^\mathrm{T} \mathrm{K}_{p,k}^\mathrm{T} - \mathrm{K}_{p,k} d_k \bar{x}_k^\mathrm{T} \Phi_k^\mathrm{T} \end{aligned}$$

where $\mathrm{K}_{p,k} = \Psi_k \mathrm{K}_k$, $\Phi_k = \Psi_k - \mathrm{K}_{p,k} \mathrm{H}_k$ and the mean error $\bar{x}_k = \mathrm{E}\{x_k\}$ can be calculated as $\bar{x}_{k+1} = \Phi_k x_k - \mathrm{K}_{p,k} d_k$.

Here, the speed, $v_{k,GPS}$, of the satellite is large about 3.9km/sec, and the acceleration, $\alpha_{k,GPS}$, is about 0.05 m/sec^2, and the jerk, $\dot{\alpha}_{k,GPS}$, is small about 10^{-5}m/sec^3. Thus the speed and acceleration are not large, (7) and (8) can be expressed approximately as (13).

$$\hat{\rho}_{k,INS} \simeq \rho_k - (v_{k,GPS} l_k) \mathrm{T}_d + w_{1k} \qquad (13a)$$

$$\hat{\dot{\rho}}_{k,INS} \simeq \dot{\rho}_k + w_{2k} \qquad (13b)$$

Equation (13) means that when a vehicle moves slowly the effect of time delay on the pseudorange measurement comes from mainly satellite speed, while the effect of time delay on the pseudorange rate measurement is almost zero. That is, the bias error vector can be described as follows.

For the tightly coupled integration of the type 1

$$d_k \approx -(v_{k,GPS} l_k) \mathrm{T}_d \qquad (14a)$$

For the tightly coupled integration of the type 2

$$d_k \approx [-(v_{k,GPS} l_k)\mathrm{T}_d \quad 0]^T \tag{14b}$$

Thus we can notice that time delay in the tightly coupled integration of the type 2 has more positive effect on the navigation performance than in the tightly coupled integration of the type 1 when velocity and acceleration of vehicle are not large.

IV. SIMULAITONS

In this section computer simulation is performed to verify the analysis result on the effects of time delay for the two measurements in the case of 1 sec delay. Table 2 shows the specification of INS and GPS used in the simulation.

The vehicle trajectory is assumed as circle with constant speed and it takes 200sec for every circulation. Two vehicle speeds are used such as 30km/hr and 150km/hr to investigate the relation between the vehicle speed and time delay. The vehicle runs the circle nine times each.

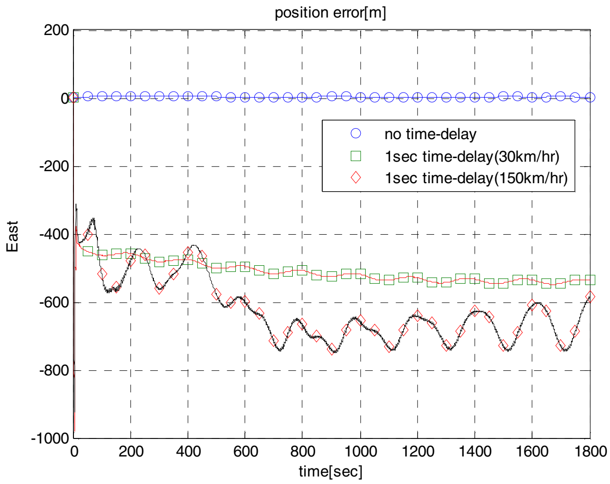
Table 2. Specification of error sources of the INS and the GPS

	Error Sources	1-σ value
INS	initial position error	10m
	initial velocity error	1m/sec
	initial horizontal attitude error	0.03 deg
	initial vertical attitude error	5 deg
	accelerometer bias	500 μg
	gyro bias	3 deg/hr
GPS	clock bias	10m
	clock drift	1m/sec

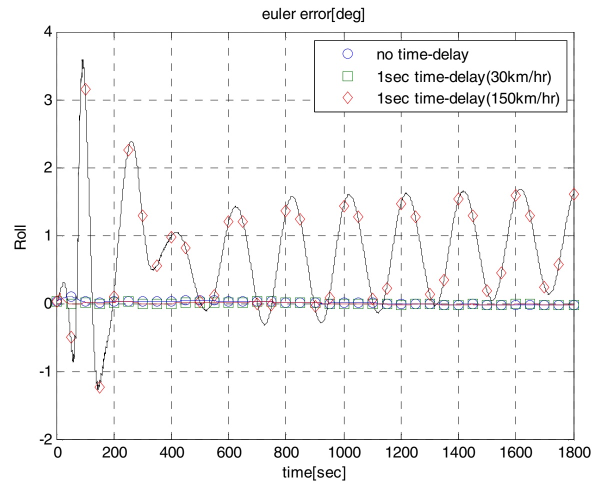
Figure 3 shows the east position errors and roll angle errors in the tightly coupled GPS/INS according to the two vehicle speeds when pseudorange is the only measurement. It shows that time delay has large effect on the performance regardless of the size of vehicle speed. This is because the satellite speed has more effect than the vehicle speed on the pseudorange bias error vector d_k. We can notice the oscillation in the position and attitude error for the vehicle speed of 150km/hr. It is due to the bias error term of vehicle speed, $-v_k l_k T_d$.

Figure 4 shows the east position errors and roll angle errors in the tightly coupled GPS/INS according to the two vehicle speeds when both pseudorange and pseudorange rate are used as measurement. The simulation shows that the position error performance is similar as in Figure 3(a) meaning that the position error increases as the position bias error vector increases due to the satellite speed. In case of vehicle speed of 150km/hr, the position error becomes smaller than that in Fig.3(a). It is because the pseudorange rate has little effect from time delay and thus the Kalman filter has better degree of observability. Notice the scale of Fig. 3(a) and Fig. 4(a). The attitude error in Fig.4(b) is much smaller than that in Fig.3(b).

Figure 3 and Figure 4 show that when there exists time delay between GPS receiver and INS for the tightly coupled GPS/INS integration, it is better to use both pseudorange and pseudorange rate rather than to use only pseudorange as Kalman measurement in order to obtain better estimation performance.

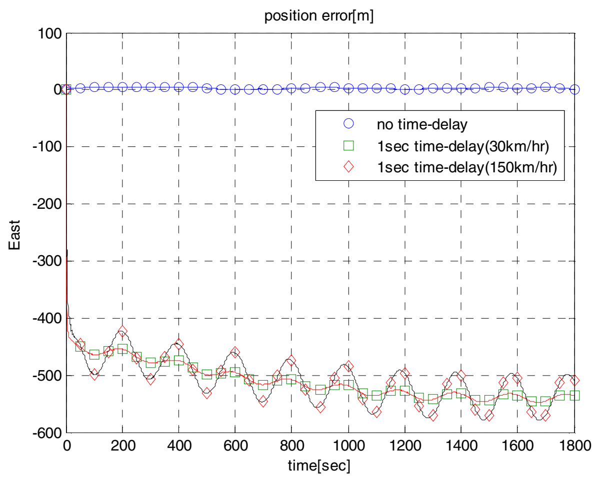


(a) east position error

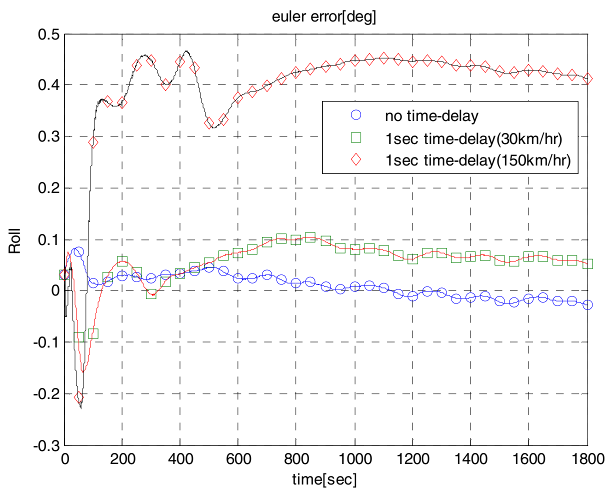


(b) the roll angle error

Fig. 3. The position error and the attitude error of the tightly coupled GPS/INS system using only pseudorange measurements



(a) the east position error



(b) the roll angle error

Fig. 4. The position error and the attitude error of the tightly coupled GPS/INS system using pseudorange measurements and pseudorange rate measurements

V. CONCLUSIONS

In this paper the effect of measurement for time synchronization error is investigated for the tightly coupled GPS/INS system. The measurement for Kalman filter is considered as two cases. One is the pseudorange only, and the other is both pseudorange and pseudorange rate. The estimation performance is compared for the two measurement cases.

For the analysis Kalman filter measurement is expressed in terms of time delay, and velocity, acceleration, and jerk of the speed of vehicle and satellite. With the theoretically obtained terms, simulations are performed to verify the analysis result for the two measurements. The simulation shows that time synchronization error has effect more on the tightly coupled integration than the loosely coupled integration [5].

For the tightly coupled GPS/INS system, if there exists time delay between the GPS receiver and INS, the Kalman filter measurement contains a big bias term due to the GPS satellite speed regardless of the vehicle speed, resulting in the worse Kalman filter performance

If the vehicle speed is not large, it is better to use both pseudorange and pseudorange rate rather than to use only pseudorange as Kalman measurement in order not to receive much effect from the time delay.

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