A Method for Decentralized Formation Building for Unicycle-like Mobile Robots

Andrey V. Savkin[1], Chao Wang[1], Ahmad Baranzadeh[1], Zhiyu Xi[1] and Hung T. Nguyen[2]

Abstract— **The paper presents a method for decentralized flocking and global formation building for a network of unicycles described by the standard kinematics equations with hard constraints on the vehicles linear and angular velocities. We propose decentralized motion coordination control algorithms for the robots so that they collectively move in a desired geometric pattern from any initial position. There are no predefined leaders in the group and only local information is required for the control. The effectiveness of the proposed control algorithms is illustrated via computer simulations.**

I. Introduction

The study of decentralized control laws for groups of mobile autonomous robots has emerged as a challenging new research area in recent years (see, e.g., [1]–[8] and references therein). Broadly speaking, this problem falls within the domain of decentralized control, but the unique aspect of it is that groups of mobile robots are dynamically decoupled, meaning that the motion of one robot does not directly affect that of the others. Researchers in this new emerging area are finding much inspiration from biology, where the problem of animal aggregation is central in both ecological and evolutionary theory. Animal aggregations, such as schools of fish, flocks of birds, groups of bees, or swarms of social bacteria, are believed to use simple, local motion coordination rules at the individual level that result in remarkable and complex intelligent behaviour at the group level (see, e.g., [9]–[11]). Such intelligent behaviour is expected from very large scale robotic systems. The term 'very large scale robotic system' was introduced in [12] for a system consisting of autonomous robots numbering from hundreds to tens of thousands or even more. Because of decreasing costs of robots, interest in very-large-scale robotic systems is growing rapidly. In such systems, robots should exhibit some forms of cooperative behaviour.

In 1995, Vicsek et al. proposed a simple, but interesting discrete-time model of a system consisting of several autonomous agents, e.g., particles, moving in the plane [13]. The motion of each agent is updated using a local rule based on its own state and the state of its neighbours. This model can be viewed as a special case of a computer model proposed in [14] for the computer animation industry and mimicking animal aggregation. Simulation results in [13] show that in Vicseks model, all agents might eventually move in the same direction, despite the absence of centralized coordination. In [5], a modification of the Vicseks model is introduced and considered. Here, the heading of each robot is updated as the average headings of its neighbours. Compare this with [13], where the heading is given as the heading of the average velocity vector of its neighbours. The modification in [5] results in simpler mathematical analysis and allows linear tools to be applied. The first results on mathematical analysis of this model were given in [5]. The main results of [5] are sufficient conditions for coordination of the system of agents that are given in terms of a family of graphs characterizing all possible neighbour relationships among agents. Some further results have been obtained in other papers; see, e.g., [4] and [7]. There is a number of other recent results which show that for certain local rules the flocking problem will be solved, i.e. all vehicles will eventually move in the same direction despite the absence of centralized coordination.

A more difficult problem is to design a decentralized control law which guarantees that all vehicles will eventually move not only in the same direction but also in a desired geometric configuration. In this paper, we present a constructive and easily implementable decentralized control law for a group of unicycle robots under which the vehicles will eventually move in the same direction with the same speed and the shape of the group will converge to a given geometric pattern.F

It should be pointed out that many papers in this area consider simplest first- or second-order linear models for the motion of each robot; see, e.g., [15], [16] and [17]. Therefore, the obtained results are heavily based on tools and methods from linear system theory, such as stochastic matrices or graph Laplacians. It is known, that there are examples of unrealistic physically embodied behaviour that would be possible under such simplified models. In particular, the robot motion in such linear models does not satisfy the standard hard constraint on either robot speed or angular velocity. Furthermore, it can be shown that the models proposed in [7] and many other papers will result in arbitrarily large robot angular velocity and arbitrarily small robot turning radius, which is impossible on actual wheeled robots. This paper considers the much more difficult problem to design a similar decentralized control law for a multirobot system, in which the motion of each robot is described by a nonlinear

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model with the standard hard constraints on the vehicle angular acceleration and linear velocity. Such models can describe the kinematics of unmanned aerial vehicles (UAVs) or wheeled mobile robots; see, e.g., [18] and [19]. In this situation, linear system approaches of [5] and many other papers are not applicable. The considered multi-robot system is an example of a networked control system; see e.g. [20]–[23] and references therein. We present a constructive and easily implementable decentralized control law for a group of autonomous wheeled robots such that the vehicles will eventually move in the same direction with the same speed. Furthermore, we give a mathematically rigorous analysis of the proposed decentralized robot navigation scheme. In our problem, there are no leaders assigned a priori, and the robots have to coordinate with each other in the group relying on some global consensus in order to achieve and maintain a desired pattern. Furthermore, we consider the problem of formation building with anonymous robots. In this problem statement, each robot does not know a priori its position in the desired configuration, and the robots should reach a consensus on their positions. We propose a randomized navigation algorithm and prove its convergence with probability 1.

The effectiveness of the proposed control algorithms is verified via computer simulations.

Potential applications of our formation control of a group of robots are for sweep coverage [24] in operations like mine sweeping [25], boarder patrolling [26], environmental monitoring of disposal sites on the deep ocean floor [27], and sea floor surveying for hydrocarbon exploration [28].

The proofs of the main results will be given in the journal version of this paper

II. Multi-vehicle System

The system under consideration consists of n autonomous vehicles labelled 1 through n. All these vehicles moving in a plane in continuous time. Let $(x_i(t), y_i(t))$ be the Cartesian coordinates of the vehicle i. Also, let $\theta_i(t)$ be the orientation of this vehicle with respect to the $x-$axis, that is $\theta_i(t)$ is measured from the $x-$axis in the counterclockwise direction, it takes values in the interval $(-\pi, \pi]$ (so the $x-$axis corresponds to the orientation $\theta = 0$).

Furthermore, let $v_i(t)$ be the speed of the vehicle i, $\omega_i(t)$ be its angular acceleration. Then, the kinematic equations of the vehicle motion are given by

$$\begin{aligned} \dot{x}_i(t) = &\quad v_i(t)\cos(\theta_i(t)) \quad ; \\ \dot{y}_i(t) = &\quad v_i(t) sin(\theta_i(t)) \quad ; \\ \dot{\theta}_i(t) = &\quad \omega_i(t) \end{aligned} \tag{2.1}$$

for all $i = 1, 2, \ldots, n$. Here $\omega_i(t)$ and $v_i(t)$ are the control inputs, $\omega_i(t)$ is the angular velocity, and $v_i(t)$ is the linear velocity. The equations (2.1) can describe the kinematics of tactical missiles, UAVs or wheeled mobile robots; see e.g. [18], [19], [29]–[31]. The standard design specifications impose the following input constraints:

$$-\omega^{max} \le \omega_i(t) \le \omega^{max} \qquad \forall t \ge 0, \tag{2.2}$$

$$V^m \le v_i(t) \le V^M \qquad \forall t \ge 0 \tag{2.3}$$

for all $i = 1, 2, \ldots, n$. Here $\omega^{max} > 0$ and $0 < V^m < V^M$ are given constants.

Introduce two-dimensional vectors $z_i(t)$ of the vehicles' coordinates and vectors $V_i(t)$ of the vehicles' velocities by

$$z_i(t) := \begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix}, \qquad V_i(t) := \begin{pmatrix} v_i(t)\cos(\theta_i(t)) \\ v_i(t)\sin(\theta_i(t)) \end{pmatrix} \tag{2.4}$$

for all $i = 1, 2, \ldots, n$.

We assume that the vehicles communicate at discrete time instants $k = 0, 1, 2, \ldots$. The information available to the vehicles at time k is determined by a connected undirected graph $\mathcal{G}(k)$ with n vertices corresponding to the vehicles. Two vertices are connected by an edge in the graph $\mathcal{G}(k)$ if and only if the corresponding robots communicate at time k. Introduce the following notations.

Notation 2.1: For any vertex $i \in \{2, 3, \ldots, n\}$, $\mathcal{N}_i(k)$ denotes the set of all vertices $j \neq i$ of $\mathcal{G}$ that are connected to i by an edge. Furthermore, $n_i(k)$ denotes the number of vertices in the set $\mathcal{N}_i(k)$.

We will also need the following assumption.

Assumption 2.1: There exists an infinite sequence of contiguous, non-empty, bounded time-intervals $[k_j, k_{j+1})$, $j = 0, 1, 2, \ldots$, starting at $k_0 = 0$, such that across each $[k_j, k_{j+1})$, the union of the collection $\{G(k) : k \in [k_j, k_{j+1})\}$ is a connected graph.

For each robot i, we will utilize the consensus variables $\tilde{\theta}_i(k)$, $\tilde{x}_i(k)$, $\tilde{y}_i(k)$ and $\tilde{v}_i(k)$. The consensus variable $\tilde{\theta}_i(k)$ is used to achieve the common heading of the formation, the consensus variables $\tilde{x}_i(k)$ and $\tilde{y}_i(k)$ are used to achieve the common origin of coordinates of the formation, and the consensus variable $\tilde{v}_i(k)$ is used to achieve the common speed of the formation. In other words, at any time k, the robot i has the estimates $\tilde{\theta}_i(k)$, $\tilde{x}_i(k)$, $\tilde{y}_i(k)$ and $\tilde{v}_i(k)$ of the consensus formation parameters. The robots will start with different values of $\tilde{\theta}_i(0)$, $\tilde{x}_i(0)$, $\tilde{y}_i(0)$ and $\tilde{v}_i(0)$, and eventually converge to some consensus values which define a common formation orientation and speed for all the robots.

Assumption 2.2: The initial values of the consensus variables θ_i satisfy $\theta_i(0) \in [0, \pi)$ for all $i = 1, 2, \ldots, n$.

We assume that the information on other vehicles that is available to the vehicle i at time k is the coordinates $(x_j(k), y_j(k))$ and the consensus variables $\tilde{\theta}_j(k)$, $\tilde{x}_j(k)$, $\tilde{y}_j(k)$ and $\tilde{v}_j(k)$ for all $j \in \mathcal{N}_i(k)$. Also, the vehicle's own coordinates, orientation and speed are measured at any time $t \ge 0$.

III. Formation Building

We propose the following rules for updating the consensus variables $\tilde{\theta}_i(k)$, $\tilde{x}_i(k)$, $\tilde{y}_i(k)$ and $\tilde{v}_i(k)$:

$$\begin{aligned} \tilde{\theta}_i(k+1) &= \tfrac{\tilde{\theta}_i(k) + \sum_{j\in\mathcal{N}_i(k)} \tilde{\theta}_j(k)}{1+|\mathcal{N}_i(k)|}; \\ \tilde{x}_i(k+1) &= \tfrac{x_i(k)+\tilde{x}_i(k)+\sum_{j\in\mathcal{N}_i(k)}(x_j(k)+\tilde{x}_j(k))}{1+|\mathcal{N}_i(k)|} - x_i(k+1); \\ \tilde{y}_i(k+1) &= \tfrac{y_i(k)+\tilde{y}_i(k)+\sum_{j\in\mathcal{N}_i(k)}(y_j(k)+\tilde{y}_j(k))}{1+|\mathcal{N}_i(k)|} - y_i(k+1); \\ \tilde{v}_i(k+1) &= \tfrac{\tilde{v}_i(k) + \sum_{j\in\mathcal{N}_i(k)} \tilde{v}_j(k)}{1+|\mathcal{N}_i(k)|}. \end{aligned} \tag{3.5}$$

The algorithm (3.5) can be summarized as follows. The mobile robots use the consensus variables to achieve a consensus on the heading, speed and mass centre of the formation.

Lemma 3.1: Suppose that Assumptions 2.1 and 2.2 hold and the consensus variables are updated according to the decentralized control algorithm (3.5). Then there exist constants $\tilde{\theta}_0$, $\tilde{X}_0$, $\tilde{Y}_0$ and $\tilde{v}_0$ such that

$$\begin{aligned} \lim_{k\to\infty} \tilde{\theta}_i(k) &= \tilde{\theta}_0; \\ \lim_{k\to\infty} \tilde{v}_i(k) &= \tilde{v}_0; \\ \lim_{k\to\infty} (x_i(k) + \tilde{x}_i(k)) &= \tilde{X}_0; \\ \lim_{k\to\infty} (y_i(k) + \tilde{y}_i(k)) &= \tilde{Y}_0. \end{aligned} \tag{3.6}$$

for all $i = 1, 2, \ldots, n$. Moreover, the convergence in (3.6) is exponentially fast.

The statement of Lemma 3.1 immediately follows from the main result of [5]).

Assume now that $X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n$ are given numbers.

Definition 3.1: A navigation law is said to be globally stabilizing with initial conditions $(x_i(0), y_i(0), \theta_i(0))$, $i = 1, 2, \ldots, n$ and the configuration $\mathcal{C} = \{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n\}$, if there exist a Cartesian coordinate system and $\tilde{v}_0$ such that the solution of the closed-loop system (2.1) with these initial conditions and the proposed navigation law in this Cartesian coordinate system satisfies:

$$\begin{aligned} \lim_{t\to\infty} (x_i(t) - x_j(t)) &= X_i - X_j, \\ \lim_{t\to\infty} (y_i(t) - y_j(t)) &= Y_i - Y_j, \end{aligned} \tag{3.7}$$

$$\begin{aligned} \lim_{t\to\infty} \theta_i(t) &= 0, \\ \lim_{t\to\infty} v_i(t) &= \tilde{v}_0, \end{aligned} \tag{3.8}$$

for all $1 \le i \ne j \le n$.

In other words, Definition 3.1 means that the robots will eventually move in the same direction (along the $x-$axis in this Cartesian coordinate system), with the same speed and in the geometric configuration defined by $\mathcal{C} = \{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n\}$.

The objective of this section is to present a globally stabilizing control law for the multi-vehicle formation under consideration.

In our globally stabilizing law, we will use the consensus variables $\tilde{\theta}_i(k)$, $\tilde{x}_i(k)$, $\tilde{y}_i(k)$ and $\tilde{v}_i(k)$ updated according to (3.5). Introduce the corresponding piecewise constant continuous time variables as

$$\begin{aligned} \tilde{\theta}_i(t) &:= \tilde{\theta}_i(k) \quad \forall t \in (k, k+1); \\ \tilde{x}_i(t) &:= \tilde{x}_i(k) \quad \forall t \in (k, k+1); \\ \tilde{y}_i(t) &:= \tilde{y}_i(k) \quad \forall t \in (k, k+1); \\ \tilde{v}_i(t) &:= \tilde{v}_i(k) \quad \forall t \in (k, k+1). \end{aligned} \tag{3.9}$$

Let $c > 0$ be any constant such that

$$c > \frac{2V^M}{\omega^{max}}. \tag{3.10}$$

Furthermore, let h_1 and h_2 be non-zero two-dimensional vectors, and let α be the angle between the vectors h_1 and h_2 measured from h_1 in the counter-clockwise direction, $-\pi < \alpha \le \pi$, i.e. $\alpha = 0$ if $h_1 = h_2$. Now introduce the following function:

$$f(h_1, h_2) := sign(\alpha) \tag{3.11}$$

where $sign(\cdot)$ is defined by

$$sign(\alpha) := \begin{cases} -1 & if \quad \alpha < 0 \\ 0 & if \quad \alpha = 0 \\ 1 & if \quad \alpha > 0 \end{cases} \tag{3.12}$$

For any time t and any robot i, we consider a Cartesian coordinate system with the $x-$axis in the direction $\tilde{\theta}_i(t)$ (according to the definition (3.9), $\tilde{\theta}_i(t)$ is piecewise constant). In other words, in this coordinate system $\tilde{\theta}_i(t) = 0$, $x_i(t), y_i(t)$ are now coordinates of the robot i in this system.

Introduce the functions $h_i(t)$ as

$$h(t) := (x_i(t) + \tilde{x}_i(t)) + X_i + t\tilde{v}_i(t) \tag{3.13}$$

for all i. Also, for all $i = 2, \ldots, n$ introduce two-dimensional vectors $g_i(t)$ as

$$\begin{aligned} g_i^x(t) \quad &: \quad = \begin{cases} h_i(t) + c & if \ x_i(t) \le h_i(t) \\ x_i(t) + c & if \ x_i(t) > h_i(t) \end{cases} \\ g_i^y(t) \quad &: \quad = (y_i(t) + \tilde{y}_i(t)) + Y_i, \\ g_i(t) \quad &: \quad = \begin{pmatrix} g_i^x(t) \\ g_i^y(t) \end{pmatrix} \end{aligned} \tag{3.14}$$

and two-dimensional vectors $d_i(t)$ as

$$d_i(t) := g_i(t) - z_i(t)$$

where $z_i(t)$ is defined by (2.4).

Remark 3.1: We assume that the configuration $\mathcal{C} = \{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n\}$ and the constant c satisfying (3.10). are known to each vehicle i.

Now we introduce the following decentralized control law:

$$\begin{aligned} v_i(t) &= \begin{cases} V^M & if \quad x_i(t) \le h_i(t) \\ V^m & if \quad x_i(t) > h_i(t) \end{cases} \\ \omega_i(t) &= \omega^{max} f(V_i(t), d_i(t)), \end{aligned} \tag{3.15}$$

for all $i = 2, 3, \ldots, n$. Here $f(\cdot, \cdot), V_i(\cdot), d_i(\cdot)$ are defined by (2.4), (3.11).

We will need the following assumption.

Assumption 3.1: The initial robots' speeds satisfy

$$V^m < v_i(0) < V^M$$

for all $i = 1, 2, \ldots, n$.

Now we are in a position to present the main result of this section.

Theorem 3.1: Consider the autonomous vehicles described by the equations (2.1) and the constraints (2.2), (2.3). Let $\mathcal{C} = \{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n\}$ be a given configuration. Suppose that Assumptions 2.1, 2.2 and 3.1 hold, and c is a constant satisfying (3.10). Then, the decentralized control law (3.5), (3.15) is globally stabilizing with any initial conditions and the configuration $\mathcal{C}$.

The proof of Theorem 3.1 will be given in the journal version of this paper.

IV. Formation Building with Anonymous Robots

In this section, we consider the problem of formation building with anonymous robots. In other words, each robot does not know a priori its position in the configuration $\mathcal{C} = \{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n\}$, and the robots should reach a consensus on their positions.

Definition 4.1: A navigation law is said to be globally stabilizing with anonymous robots and the configuration $\mathcal{C} = \{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n\}$ if for any initial conditions $(x_i(0), y_i(0), \theta_i(0))$, there exists a permutation $r(i)$ of the index set $\{1, 2, \ldots, n\}$ such that for any $i = 1, 2, \ldots, n$, there exist a Cartesian coordinate system and $\tilde{v}_0$ such that the solution of the closed-loop system (2.1) with the proposed navigation law in this Cartesian coordinate system satisfies (3.8) and

$$\lim_{t\to\infty}(x_i(t) - x_j(t)) = X_{r(i)} - X_{r(j)},$$
$$\lim_{t\to\infty}(y_i(t) - y_j(t)) = Y_{r(i)} - Y_{r(j)}, \tag{4.16}$$

for all $1 \leq i \neq j \leq n$.

Let $R > 0$ be a given constant. We assume that each robot i has the capacity to detect all other robots inside the circle of radius R centred at the current position of robot i. Furthermore, let $0 < \epsilon < \frac{R}{2}$ be a given constant. For any configuration $\mathcal{C} = \{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n\}$ introduce a undirected graph $\mathcal{P}$ consisting of n vertices. Vertices i and j of the graph $\mathcal{P}$ are connected by an edge if and only if $\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2)} \leq R - 2\epsilon$. We will need the following assumption.

Assumption 4.1: The graph $\mathcal{P}$ is connected.

We present a randomized algorithm to build an index permutation function $r(i)$. Let $N \geq 1$ be a given integer. Let $r(0, i) \in \{1, 2, ..., n\}$ be any initial index values where $i = 1, 2, ..., n$.

As in the navigation law (3.5), (3.15), for any time t and any robot i, we consider a Cartesian coordinate system with the $x-$axis in the direction $\tilde{\theta}_i(t)$ (according to the definition (3.9), $\tilde{\theta}_i(t)$ is piecewise constant). In other words, in this coordinate system $\tilde{\theta}_i(t) = 0$, $x_i(t), y_i(t)$ are now coordinates of the robot i in this system. Furthermore, we say that a vertex j of the graph $\mathcal{P}$ is vacant at time kN for robot i if there is no any robot inside the circle of radius ϵ centred at the point

$$\begin{pmatrix} (x_i(kN) + \tilde{x}_i(kN)) + X_j + kN\tilde{v}_i(kN) \\ (y_i(kN) + \tilde{y}_i(kN)) + Y_j \end{pmatrix}$$

Let $\mathcal{S}(kN, i)$ denote the set of vertices of $\mathcal{P}$ consisting of $r(kN, i)$ and those of vertices of $\mathcal{P}$ that are connected to $r(kN, i)$ and vacant at time kN for robot i. Let $|\mathcal{S}(kN, i)|$ be the number of elements in $\mathcal{S}(kN, i)$. It is clear that $1 \leq |\mathcal{S}(kN, i)|$ because $r(kN, i) \in \hat{\mathcal{S}}(kN, i)$. Moreover, introduce the Boolean variable $b_i(kN)$ such that $b_i(kN) := 1$ if there exists another robot $j \neq i$ that is inside of the circle of radius ϵ centred at

$$\begin{pmatrix} (x_i(kN) + \tilde{x}_i(kN)) + X_i + kN\tilde{v}_i(kN) \\ (y_i(kN) + \tilde{y}_i(kN)) + Y_i \end{pmatrix}$$

at time kN, and $b_i(kN) := 0$ otherwise. We propose the following random algorithm:

$$\begin{gathered} r((k+1)N, i) = r(kN, i) \quad if \\ (b_i(kN) = 0 \; or \; (b_i(kN) = 1 \; and \; |\mathcal{S}(kN, i)| = 1)); \\ r((k+1)N, i) = j \\ with \;\; probability \;\; \tfrac{1}{|\mathcal{S}(kN,i)|} \quad \forall j \in \mathcal{S}(kN, i) \\ if \, (b_i(kN) = 1 \quad and \quad |\mathcal{S}(kN, i)| > 1). \end{gathered} \tag{4.17}$$

This random algorithm is close to the random algorithm proposed in [32].

Now we are in a position to present the main result of this section.

Theorem 4.1: Consider the autonomous vehicles described by the equations (2.1) and the constraints (2.2), (2.3). Let $\mathcal{C} = \{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n\}$ be a given configuration. Suppose that Assumptions 2.1, 2.2, 3.1 and 4.1 hold, and c is a constant satisfying (3.10). Then, initial conditions $(x_i(0), y_i(0), \theta_i(0))$, $i = 1, 2, \ldots, n$ there exists an integer $N_0 > 0$ such that for any $N \geq N_0$, the decentralized control law (3.5), (3.15), (4.17) with probability 1 is globally stabilizing with these initial conditions and the configuration $\mathcal{C}$.

The proof of Theorem 4.1 will be given in the journal version of this paper.

V. Computer Simulations

In this section, some simulation results are presented. Figure 3 and Figure 4 show simulation results with algorithms proposed in Section III applied. Here we consider five robots whose motions are governed by (2.1), (2.2) and (2.3) with decentralized law (3.5), (3.15) employed. The shape of undirected graph $\mathcal{G}(k)$ may change from time to time. It is shown that with any initial conditions and various configurations $\mathcal{C}$, global stability is achieved.

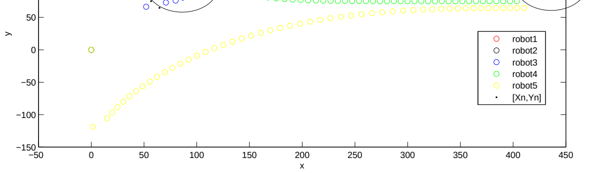

Fig. 1. Robots form an arch

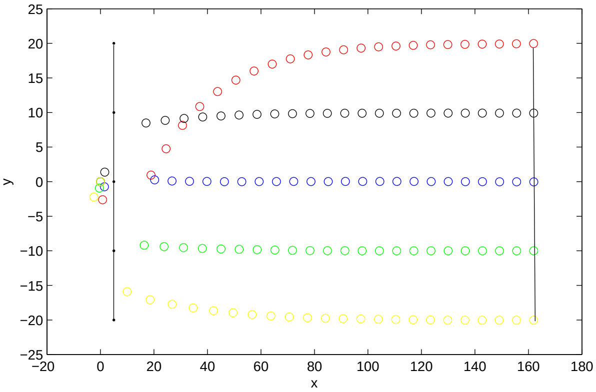

Fig. 2. Robots form a straight line

REFERENCES

[1] J. Feddema, C. Lewis, and D. Schoenwald, "Decentralized control of cooperative robotic vehicles: theory and application," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 5, pp. 852–864, Oct. 2002.

[2] J. Fredslund and M. Mataric, "A general algorithm for robot formations using local sensing and minimal communication," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 5, pp. 837–846, Oct 2002.

[3] I. Harmati and K. Skrzypezyk, "Robot team coordination for target tracking using fuzzy logic controller in game theoretic framework," *Robotics and Autonomous Systems*, vol. 57, no. 1, pp. 75–86, 2009.

[4] Y. G. Hong, H. P. Hu, and L. X. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, no. 7, pp. 1177–1182, 2006.

[5] A. Jadbabaie, J. Lin, and A. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, Jun. 2003.

[6] G. M. Mathews, H. Durrant-Whyte, and M. Prokopenko, "Decentralised decision making in heterogeneous teams using anonymous optimisation," *Robotics and Autonomous Systems*, vol. 57, no. 3, pp. 310–320, 2009.

[7] A. V. Savkin, "Coordinated collective motion of groups of autonomous mobile robots: analysis of Vicsek's model," *IEEE Transactions on Automatic Control*, vol. 49, no. 6, pp. 981–982, Jun. 2004.

[8] H. Yu and Y. Wang, "Coordinated collective motion of groups of autonomous mobile robots with directed interconnected topology," *Journal of Intelligent and Robotic Systems*, vol. 53, pp. 87–98.

[9] G. Flierl, D. Grunbaum, S. Levin, and D. Olson, "From individuals to aggregations: The interplay between behavior and physics," *Journal of Theoretical biology*, vol. 196, pp. 397–454, 1999.

[10] A. Okubo, "Dynamical aspects of animal grouping: swarms, schools, flocks, and herds," *Advances in Biophysics*, vol. 22, pp. 1–94, 1986.

[11] E. Shaw, "The schooling of fishes," *Scientific American*, vol. 206, pp. 128–138, 1962.

[12] J. H. Reif and H. Wang, "Social potential fields: A distributed behavioral control for autonomous robots," *Robotics and Autonomous Systems*, vol. 27, no. 3, pp. 171 – 194, 1999.

[13] T. Vicsek, A. Czirok, E. B. Jacob, I. Cohen, and O. Schochet, "Novel type of phase transitions in a system of self-driven particles," *Physical Review Letters*, vol. 75, pp. 1226–1229, 1995.

[14] C. W. Reynolds, "Flocks, birds, and schools: a distributed behavioral model," *Computer Graphics*, vol. 21, pp. 25–34, 1986.

[15] R. Olfati-Saber and R. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, Sept. 2004.

[16] H. Tanner, A. Jadbabaie, and G. Pappas, "Stable flocking of mobile agents, part i: fixed topology," in *Proceedings of the 42nd IEEE Conference on Decision and Control*, vol. 2, Dec. 2003, pp. 2010–2015.

[17] ——, "Stable flocking of mobile agents part ii: dynamic topology," in *Proceedings of the 42nd IEEE Conference on Decision and Control, 2003*, vol. 2, Dec. 2003, pp. 2016–2021.

[18] W. E. Dixon, D. M. Dawson, E. Zergeroglu, and A. Behal, *Nonlinear Control of Wheeled Mobile Robots*. London: Springer-Verlag, 2001.

[19] E. M. P. Low, I. R. Manchester, and A. V. Savkin, "A biologically inspired method for for vision-based docking of wheeled mobile robots," *Robotics and Autonomous Systems*, vol. 55, no. 10, pp. 769–784, 2007.

[20] A. S. Matveev and A. V. Savkin, "Estimation and control over communication networks," *Birkhauser*, 2009.

[21] A. V. Savkin, "Analysis and synthesis of networked control systems: topological entropy, observability, robustness, and optimal control." *Automatica*, vol. 42, no. 1, pp. 51–62, Jan. 2006.

[22] A. S. Matveev and A. V. Savkin, "An analogue of Shannon information theory for networked control systems. Stabilization via a noisy discrete channel," in *Proceedings of the 43rd IEEE Conference on Decision and Control*, vol. 4, Dec. 2004, pp. 4491–4496.

[23] ——, "Comments on "Control over noisy channels" and relevant negative results," *IEEE Transactions on Automatic Control*, vol. 50, no. 12, pp. 2105–2110, 2005.

[24] H. Choset, "Coverage for robotics - a survey of recent results," *Annals of Mathematics and Artificial Intelligence*, pp. 113–126, 2001.

[25] R. Cassinis, G. Bianco, A. Cavagnini, and P. Ransenigo, "Strategies for navigation of robot swarms to be used in landmines detection," in *Proceedings of the 3rd European Workshop on Advanced Mobile Robots*, 1999, pp. 211–218.

[26] S. Kumar, T. H. Lai, and A. Arora, "Barrier coverage with wireless sensors," in *Proceedings of the 11th Annual International Conference on Mobile Computing and Networking*, 2005, pp. 284–298.

[27] A. Jeremic and A. Nehorai, "Design of chemical sensor arrays for monitoring disposal sites on the ocean floor," *IEEE Journal of Oceanic Engineering*, vol. 23, no. 4, pp. 334–343, Oct. 1998.

[28] E. Borhaug, A. Pavlov, and K. Pettersen, "Straight line path following for formations of underactuated underwater vehicles," in *Proceedings of the 46th IEEE Conference on Decision and Control*, Dec. 2007, pp. 2905–2912.

[29] H. Teimoori and A. V. Savkin, "A biologically inspired method for robot navigation in a cluttered environment," *Robotica*, vol. 28, no. 5, pp. 637–648, 2010.

[30] ——, "Equiangular navigation and guidance of a wheeled mobile robot based on range-only measurements," *Robotics and Autonomous Systems*, vol. 58, no. 2, pp. 203–215, Feb. 2010.

[31] A. S. Matveev, H. Teimoori, and A. V. Savkin, "Method for tracking of environmental level sets by a unicycle-like vehicle," *Automatica*, vol. 48, no. 9, pp. 2252–2261, Sep. 2012.

[32] A. V. Savkin, F. Javed, and A. S. Matveev., "Optimal distributed blanket coverage self-deployment of mobile wireless sensor networks," *IEEE Communications Letters*, vol. 16, no. 6, pp. 949–951, 2012.