Mathematical Model of Group Robots

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***Abstract*—We have so far developed continuum mechanical simulation methods of group robots. Taking Hamiltonian with specified exponential function of time has allowed us to deal with motion of robots with friction also in canonical equation framework. We examine in this article results calculated by our continuum mechanical methods. We especially note the presence of collision among robots and dependence of calculation on number of robots. Also we try to make an object follow a required path.**

***Index Terms*—group robotics, Brownian motion, distribution function, energy disspation, collision among robots**

I. Introduction

Some researchers[1], [2] have been inspired by a fact that a huge number of liquid atoms can move pollen floated on liquid, that have contributed to develop a new motor device in nano region. According to Brownian motion, it worked under temperature gradients. We can also notice that we do not need to equip the atoms with any sensing devices. To apply the idea of "Brownian motors" to robot systems with extent $nm \sim m$ we have proposed[3] a transportation system of objects by group of robots under potential field controlling motion of robots. Robots corresponded to liquid atoms while objects that were transported by robots to floating pollen. We assumed $10^2 \sim 10^4$ constituent robots that move according to classical mechanics. Under external potential field, our robots moved aimlessly and only collided with each other. They had a chance to collide with an object. Repetitive collision of robots with the object **indirectly** carried the object. As applications in macroscopic world, we thought transporting garbage of various physical properties and removing obstacles at disaster spots. It was obvious that calculating Newton equations of each robot becomes difficult when a number N of robots increases. However, in designing systems of group robots, we needed mathematical model that appropriately describes trends of the systems in time. Brownian motion can be described by fluctuation terms in equations of motion. In contrast, since physical properties of the systems as "liquids" are not made preparations in advance, we need dynamical model for systems with extent $nm \sim m$ In addition to this, temperature distribution cannot move robots when they are heavier than liquid atoms. For full capabilities of the systems, however, it is essential that a mathematical model to predict trends of the systems be available. When a number of robots increases, difficulties in the predictions cannot be avoided. It was better to calculate an average of dynamical state of robots than to directly follow locations and momenta of each robot and of the object in time. Based on the idea, a main dynamical variable in our formulation was a number density of the robots. Liouville equation has been derived by Hamiltonian equations for the systems. For macroscopic systems, we have to take a friction into account. When frictional force is present, energy of systems dissipates. A well-known extension[4] of canonical formalism enabled us to take continuum mechanical picture[5], [6] also for group robots under friction. In this article we examine results calculated by our continuum mechanical model of group robots with energy dissipation. We especially note the presence of collision among robots. Results obtained are valid both for nano and for macro systems as long as the systems behave according to classical mechanics.

First in Section **II** we show the calculation formulae obtained in preceding articles[3], [5], [6]. Section **III** gives numerical simulation. Continuum mechanical calculation is compared with molecular dynamical one in the presence of collision among robots. Results feature how force acts on the object. Summary and discussion are given in Section **IV**.

II. Distribution Function[3], [5], [6]

Our system consists of N_0 robots and one object in rectangular plane region. Both the robots and the object are modeled as disc. We need following parameters to physically describe the system:

1) number N_0 of robots
2) mass M(the object) and m(each robot)
3) radius R_B(the object) and diameter a_R(each robot)
4) proportionality constant δ(the object) in the form of frictional force $-\delta\vec{V}$ and γ(each robot) in the form of frictional force $-\gamma\vec{v}$
5) wall reaction parameter c_B, n_{cB}(the object) and c_R, n_{cR}(each robot)
6) collision parameter σ_s, n_s between the object at $\vec{X}$ and each robot at $\vec{x}$ in the form "soft core" of potential energy $V_B(|\vec{X}-\vec{x}|) = \sigma_s(R_B/|\vec{X}-\vec{x}|)^{n_s}$ and that σ_v, n_v between the robots at $\vec{x}_i$ and $\vec{x}_j$ in the form of $V_{col}(|\vec{x}_i-\vec{x}_j|) = \sigma_v(a_R/|\vec{x}_i-\vec{x}_j|)^{n_v}$

We count the number N_0 of the robots as an initial energy of group robots. Only the collision among the robots is not

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taken into consideration in our continuum mechanical model of group robots.

Force of robots under equilibrium distribution acting on the object is calculated as follows. First, we note that the object velocity $\vec{V}$ increase(or decrease) by an increment $d\vec{V}^{'}$ after collision with each one of robots. Let the robot has relative velocity $\vec{v}_r$ with its absolute value v_r and an angle ϕ. The angle ϕ is measured from a vector $\vec{\theta}_{x_1}$ that indicates a direction from the center of the object to a collision point. This vector $\vec{\theta}_{x_1}$ itself is in a direction θ in our coordinate frame. When e denotes a coefficient of restitution absolute value of the increment $d\vec{V}^{'}$ is given as

$$dV^{'} = \frac{(1+e)v_r \cos\phi}{1+\frac{M}{m}} \tag{1}$$

Second, we calculate how many robots collide with the object. Let $\vec{p}$ and $\vec{x}$ denote momentum and coordinate of a robot, respectively. We introduce a distribution function $f_p(\vec{p},\vec{x};\vec{X};t)$ that is parametrized as

$$f_p(\vec{p},\vec{x};\vec{X};t) = C_p(t)e^{-\beta(t)H(\vec{p},\vec{x};\vec{X};t)} \tag{2}$$

under existence of Hamiltonian $H(\vec{p},\vec{x};\vec{X};t)$. Use of the distribution function f_p makes us possible to state that

$$d^2N = N_0 \times f_p(\vec{p},\vec{x};\vec{X};t) \times d^2\vec{v}_r \tag{3}$$

robots with relative velocity $\vec{v}_r \sim \vec{v}_r + d\vec{v}_r$ are in the unit area. A relation $\vec{p} = m(\vec{v}_r + \vec{V})$ is trivial. During time interval dt, with line element $R_B d\theta$ of the object only robots in the area

$$dS = R_B d\theta(-v_r \cos\phi)dt \tag{4}$$

can collide the object. Each robot in the number $d^2N \times dS$ gives impulse $Md\vec{V}^{'}$ to the object. When we integrate the impulse, net force acting on the object by robots is given as

$$\vec{F}(\vec{X};t) = \frac{1}{dt}\int_{\theta=0}^{2\pi}\int_{v_r=0}^{\infty}\int_{\phi=\frac{\pi}{2}}^{\frac{3\pi}{2}} Md\vec{V}^{'}d^2NdS \tag{5}$$

When frictional force is proportional to velocity, $-\gamma\vec{v}$, canonical formulation is possible[4]. We can start with a Lagrangian

$$L(\vec{x},\dot{\vec{x}};t) = e^{\frac{\gamma}{m}t}\left(\frac{m}{2}\dot{\vec{x}}^2 - V(\vec{x};\vec{X};t)\right) \tag{6}$$

The exponential factor $e^{\frac{\gamma}{m}t}$ in the Lagrangian, (6), allows us to calculate Hamiltonian $H = \vec{x}\cdot\vec{p} - L$. It is the only role of this time function $e^{\frac{\gamma}{m}t}$ that makes us possible to treat robot motion in canonical formulation. This exponential factor has no physical meaning. The potential energy $V(\vec{x};\vec{X};t)$ in (6) except wall reaction consists of external field $V_{cnt}(\vec{x};t)$ to control motion of robots and collision energy $V_B(|\vec{X}-\vec{x}|)$ between the object and each robot. Use of Hamiltonian-like function

$$\tilde{H}(\vec{v},\vec{x};\vec{X};t) \equiv \frac{m}{2}\vec{v}^2 + V(\vec{x};\vec{X};t) \tag{7}$$

and the "temperature" $\tilde{\beta}$ allows us to parametrize distribution function as

$$f_v(\vec{v},\vec{x};\vec{X};t) = C_v(t)e^{-\tilde{\beta}(t)\tilde{H}(\vec{v},\vec{x};\vec{X};t)} \tag{8}$$

under assumptions that variations of $V(\vec{x};\vec{X};t)$ and exponential factors $e^{\pm\frac{\gamma}{m}t}$ are slow enough compared with motion of robots[7]. In (7) the potential energy $V(\vec{x};\vec{X};t)$ except wall reaction consists of external field $V_{cnt}(\vec{x};t)$ to control motion of robots and collision energy $V_B(|\vec{X}-\vec{x}|)$ between the object and each robot. Tilde on the $\tilde{H}$ and $\tilde{\beta}$ are to distinguish from those without frictional force, $\gamma = 0$. We calculate these two parameters $C_v(t)$ and $\tilde{\beta}(t)$ by following two conditions. First, we have N_0 robots in the system.

$$N_0 = \int d^2\vec{v}\int d^2\vec{x} f_v(\vec{v},\vec{x};\vec{X};t) \tag{9}$$

Second,energy dissipation gives us an ordinary differential equation for $\tilde{\beta}(t)$ in time

$$\dot{\tilde{\beta}} = \frac{-\frac{2\gamma}{m}\frac{1}{\tilde{\beta}} - \frac{\delta\vec{V}^2}{N_0} + \overline{\frac{\partial V_0(\vec{x};t)}{\partial t}} - \frac{1}{N_0}\frac{dE_{Obj}}{dt} - \frac{\partial f}{\partial t}}{\frac{\partial f}{\partial\tilde{\beta}}} \tag{10}$$

where V_0 in the 3rd term of the numerator on the r.h.s. is a sum of V_{cnt}, control energy, and wall reaction energy to each robot and E_{obj} in the 4th term is a sum of kinetic energy $M/2\cdot\vec{V}^2$ of the object and wall reaction energy to the object. At each stage of calculation, $C_v(t)$ is given by the condition (9) using $\tilde{\beta}(t)$ at that stage of time.

III. Simulation

In actual calculation, we applied Taylor expansion regarding $\vec{V}$, velocity of the object. We set a linear function as a specific form for a potential

$$V_{cnt}(\vec{x}) = \alpha_1 x_1 + \alpha_2 x_2 \tag{11}$$

Parameters in MKS units are:

1) walls are modeled as $[-S_1,S_1]\times[-S_2,S_2] = [-1,1]\times[-1,1]$,
2) a number of robots is $N_0 = 200$ (in calculation given in Fig.2 we take also $N_0 = 50$ and 100 for comparison),
3) mass and radius of robots are set as $m = 0.01$ and $a_R = 0.01$, respectively,
4) for the object we set its radius $R_B = 0.1$ and mass $M = 0.5$, respectively,
5) proportionality constans are $\gamma = 0.1$ and $\delta = 0.5$ (in Fig.3 we also take $\gamma = 0.2$, $\delta = 0.2$ for comparison),
6) we set $e = 1$ as coefficient of restitution
7) for wall reaction and collision energy, we set $c_R = 3\times10^{-5}$, $n_{cR} = 4$, $c_B = 3\times10^{-7}$, $n_{cB} = 4$, $\sigma_v = 10$, $\sigma_s = 10$ and $n_s = 4$,
8) in (11) we set as $\alpha_1 = \alpha_2 = 0.1$ to make robots move from **upper right** →**lower left** .

Let us note that we take collision among robots into account by setting nonzero σ_v value. In an initial state at $t = 0.0$, the object is set at the origin, while robots are randomly laid out. Both are put at rest. Energy conditions are calculated to give

the initial value of $\tilde{\beta}(0)$. In the calculation we take up to $n = 3$ degree in the Taylor expansion of (5). In numerical integration in $\vec{x}$, $\int d^2\vec{x}e^{-\tilde{\beta}(t)}V(\vec{x};\vec{X};t)$, we apply 20 space-division. A forward difference scheme is applied with time division $dt = 5 \times 10^{-6}$. We compare trends by continuum mechanical methods in Fig.**1** with that obtained by molecular dynamical calculations[8], [9]. The **negative** gradient of V_{cnt} forces the

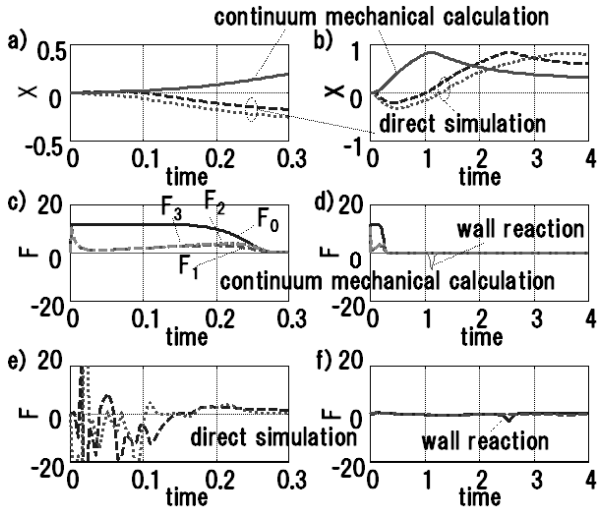

Fig. 1. Our continuum mechanical simulation results of the object coordinate $X_1(t)$ and $X_2(t)$(thick solid lines) are compared in a)(in detail near the initial time) and b) with Newton mechanical calculations. We show force $F_i(t)$(thick solid lines), calculated up to $\vec{V}^i$ degree in Taylor expansion, acted on the object in c)(in detail near the initial time) and d). Also wall reaction is shown in d). In figures a), b), c) and d), due to geometrical symmetry ($\alpha_1 = \alpha_2$), we have the same continuum mechanical trends $X_1(t) = X_2(t)$. Regarding the force we also have the same values of X_1 and X_2 components. We also show results of force calculated by direct simulation in d) and e). In e) only wall reaction is shown.

robots to move towards the area where we have small potential values. The robots collide more frequently than those in the area with large potential values. As shown in Fig.1 c) during $0 \sim time \sim 0.3$, this results in the **positive** sign for the force on the object. Near the initial $time \sim 0$, we see in c) that the 0-th component of the force, calculated by (1), dominates, since $\vec{V}$ still takes small values. Direct simulation as shown in d) indicates that only near initial time we have dominant force acting on the object. Peaky form of negative value in the force during $time \sim 1.1$ in d) expresses the reaction of the wall. The trends(thick solid line(s) in a) and b) of the object by continuum method do not have "dead time" characteristics $\tau \sim 1 \sim 1.2$ appearing in the trends $X_1(t)$(dashed line) and $X_2(t)$(dotted line) by direct simulation. Regarding the number of robots N_0, simply the more robots we prepare, the faster transportation speed as shown in Fig.**2**d) is obtained. Dead time characteristics as given in e) are not appropriately calculated, however. Friction coefficients take $\gamma = 0.1$ and $\delta = 0.5$ in Figs.1 and 2. As friction acts on robots proportional to velocity, we take two values of proportionality constant $\gamma = 0.1$ and 0.2. Also for frictional force of object, we apply $\delta = 0.2$ and 0.5. We give results of simulation in Fig.**3**. As

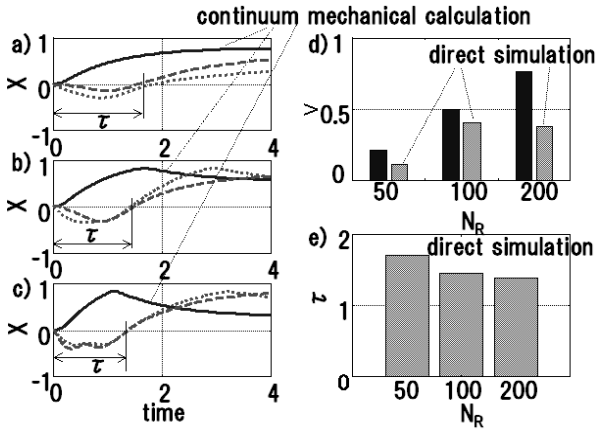

Fig. 2. a),b) and c) give trends by continuum mechanics(solid lines) and $X_1(t)$(dashed line) and $X_2(t)$(dotted line) by direct simulation, respectively. In a), b) and c) we take number of robots as $N_0 = 50$, 100 and 200, respectively. In d) we give mean velocity of the object for each N_0 value. Thick and thin colored bar represent the result by continuum mechanics and direct simulation, respectively. We see, in e), dead times τ predicted by direct simulation. These τ's are depicted in a), b) and c), respectively.

we set $\alpha_1 = \alpha_2$ in (11), $X_1(t) = X_2(t)$ and $V_1(t) = V_2(t)$ by symmetry between directions in x_1 and x_2. Trends in

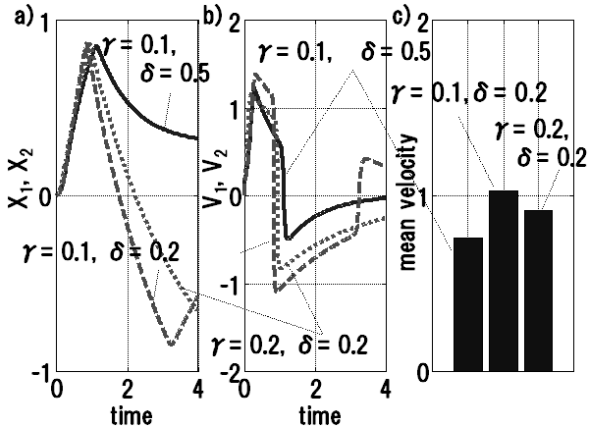

Fig. 3. For three combinations of proportionality constants γ and δ, we show in a) trends of the coordinate $X_1(t)$ and in b) the velocity $V_1(t)$ of the object. Among these three trends, we compare transportation capabilities in c) as values of mean velocity $\overline{V_1(t)}$.

the figure show that small/large friction force corresponds good/bad characteristics of transporting object. We calculate each value of mean velocity in Fig.3c) simply as

$$\overline{V_1(t)} \equiv \frac{1}{T}\int_0^T dt\, V_1(t) \tag{12}$$

In (12) T is a time when $X_1(t)$ takes a maximum value: $T = 1.12$ for a combination ($\gamma = 0.1$, $\delta = 0.5$), 0.84 for (0.1, 0.2) and 0.94 for (0.2, 0.2). For a fixed $\gamma = 0.1$, we have better(faster) $\overline{V_1(t)} = 1.03$ for $\delta = 0.2$ than 0.76 for $\delta = 0.5$. Meanwhile, the result $\overline{V_1(t)} = 1.03$ for $\gamma = 0.1$ is better than 0.92 for $\gamma = 0.2$, when we set $\delta = 0.2$. In trends of the object simulated by Newton mechanics, we have those that take large

deviation from the continuum mechanical prediction. Number of robots is not sufficient to completely describe the systems only in statistical methods. We must examine dependence of trends of the object on initial layout of robots. Such additional information can help our method.

When we make the object follows a path $\vec{X}^*(t)$ in the rectangular region, two parameters α_1 and α_2 in (11) are set as specified values. Newton equation

$$M\ddot{\vec{X}}^* = \vec{F}(\vec{X}^*; t) \tag{13}$$

where $\vec{F}$ is given by (5), holds. Parameters $\alpha_1^*(t)$ and $\alpha_2^*(t)$ are calculated as time functions that satisfy (13). We show below an example of calculation for frictionless systems with $\gamma = \delta = 0$. For the object path with the symmetry $X_1^* = X_2^*$ as depicted in Fig.4a), the calculated results of the parameters $\alpha_1^* = \alpha_2^*$ are shown in c). Residual error $|M\ddot{\vec{X}}^* - \vec{F}|^2$ of (13) is given in b). Discrepancies between the object path $\vec{X}(t)$

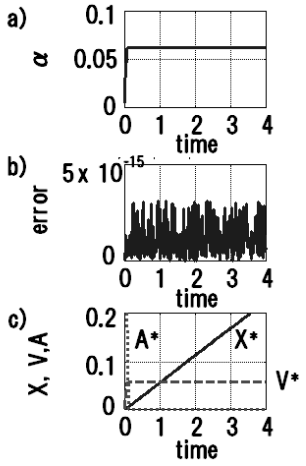

Fig. 4. Calculation of α_1^* and α_2^* that satisfy (13) under given required values of $\vec{X}^*$.

under the control potential V_{cnt}, (11), with α_1^* and α_2^*, and the required one $\vec{X}^*(t)$ are compensated by feedback action.

IV. Summary and discussion

We examined in this article results calculated by our continuum mechanical methods. We noted collision among robots that is essentially not incorporated in our continuum mechanical picture of group robots. Simulation studies showed that even in the presence of collision among robots the continuum method is physically acceptable. Also we tried to make the object follow a required path $\vec{X}^*$. Control potential $V_{cnt} = \alpha_1 x_1 + \alpha_2 x_2$ was set by giving specified parameter values $\alpha_1 = \alpha_1^*$ and $\alpha_2 = \alpha_2^*$ corresponding to the required path. We must take dead time characteristics into consideration in our continuum model. We also must examine our scheme of transportation by group of robots, especially without sensors for mutual information, in experiments.

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