VB-AQKF-STF: A Novel Linear State Estimator for Stochastic Quantized Measurements Systems

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***Abstract*—Networked state estimation with adaptive bit quantization is studied for linear systems in this paper, for which sensor measurements are locally quantized and the taken quantized messages are sent to a processing center. Strong tracking filtering (STF) technology and variational Bayesian (VB) method are jointly adopted to deal with unknown variance of stochastic quantization error vector. A kind of novel quantized state estimator VB-AQKF-STF is proposed to effectively improve quantized estimate accuracy and performance to deal with sudden change of state. The variance of the quantization error is approximated by a known upper bound, and the STF with a time-variant fading factor is used to reduce influence of the approximation and achieve strong tracking performance for the inaccurate system model. The VB method is applied to dynamically evaluate the variance of the integrated message noise. In nature, this variance estimate essentially provides a basis for the quantized strong tracking filter. Two simulation examples are demonstrated to validate the proposed quantized estimators.**

I. Introduction

With fast development of modern information and network technologies, more and more applications about complex wireless data networks have been raised in recent decades [1,2]. Accordingly, distributed sensor networks, which is composed of many sensor nodes deployed dispersedly, has been extensively applied in many fields, for example, environment surveillance, target tracking, intelligent firefighting, and body health monitoring and so on. Normally, monitoring data taken by local sensors should be quantized before they are sent to a center for processing in order to meet requirements of digital transmission and limited bandwidth. For this reason, quantized filtering and fusion have been hot research topics in signal processing, communication, and control [3-7].

At present, adaptive bit quantization has been a popular way to get an approximative message of the original local sensor measurement because it can quantize measurement data according to realtime changing of available network bandwidth for sensor networks [8-10]. Thereby, there have been a lot of valuable research achievements and some of results have been applied in practical systems [4,7,11-17]. The result of stochastic bit quantization is to bring a quantization error vector with unknown accurate variance. The key idea to design quantized filters is how to deal with the stochastic quantization error. Usually, the quantization error is taken as a Gaussian white noise vector with zero mean. According to Kalman filtering theory, this variance should be exactly known and accurate in order to compute an optimal quantized state estimate. Unfortunately, it is difficult to get it accurately. An exact upper bound on this variance can be obtained and it relates to quantization interval [3,6,8,10]. Hence, this upper bound is usually used to substitute for the unknown real variance when designing quantized filters and fusion estimators [6,10]. However, there is a obvious problem that this upper bound approximation actually leads to uncertainty of system model-used. As a result, accuracy and stability of quantized state estimators must be influenced with different degrees. In order to improve performance of quantized Kalman estimators, strong tracking filtering (STF) technology has been used to overcome the unfavorable effect to some extent and help them get robustness for the sudden change of state [10]. Unfortunately, the STF cannot completely eliminate the influence induced by inaccurate system model and the variance of the quantization error can not also estimated in real time.

For most practical applications, it is necessary to quantificationally assess the influence of adaptive bit quantization and it is in favor of the assessing by evaluating the variance of the bit quantization error on line. Motivated from this, we study networked state estimation for linear dynamic systems with adaptive bit quantization by jointly using Variational Bayesian (VB) method and strong tracking filtering technology. The VB method can effectively evaluate the integrated variance of quantized message noise composed of original measurement noise and the quantization error [18]. The estimate of the integrated variance can be used in the strong tracking function. The strong tracking fading factor helps the quantized estimator to adapt the newest message and extract useful information from messages as much as possible. An upper bound is used to determine effectiveness of the variance estimate and an associated scheme is presented to decide an available variance. Accordingly, a novel variational Bayesian adaptive quantized Kalman filter based on the strong tracking filtering is proposed.

The rest of this paper is organized as follows. Section II is problem formulation including system description, adaptive bit quantization and motivation. Quantized Kalman filter based on the STF is introduced in section III. In section IV, a quantized Kalman filter using variational Bayesian method is presented. A novel variational Bayesian adaptive Kalman filter based on strong tracking filtering (VB-AQKF-STF) is proposed in section V. Simulation examples are demonstrated in section VI. Finally, we conclude this paper.

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II. Problem Formulation

A. System Equations

A kind of networked tracking system can be described by

$$\begin{cases} \mathbf{x}_k = \Phi_{k,k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k,k-1} \\ \mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k \end{cases} \tag{1}$$

where $\mathbf{x}_k \in \Re^{n\times 1}$ is system state vector and $\Phi_{k,k-1} \in \Re^{n\times n}$ is the corresponding system state transition matrix from $k-1$ to k. $\mathbf{w}_{k,k-1} \in \Re^{n\times 1}$ is a Gaussian white noise with zero mean and variance $\mathbf{Q}_{k,k-1}$. $\mathbf{z}_k \in \Re^{p\times 1}$ is sensor measurement and $\mathbf{H}_k \in \Re^{p\times n}$ is the associated measurement matrix. $\mathbf{v}_k \in \Re^{p\times 1}$ is also a Gaussian white noise with zero mean and variance $\mathbf{R}_k$ which is $diag\{(r_k^1)^2,\ (r_k^2)^2,\ \cdots\ (r_k^p)^2\}$.

It is assumed that initial state $\mathbf{x}_0$, which mean and covariance are $\hat{\mathbf{x}}_{0|0}$ and $\mathbf{P}_{0|0}$ respectively, is not related to $\mathbf{w}_{k,k-1}$ and $\mathbf{v}_k$ which are both uncorrelated.

B. Quantized System

For networked control or estimation systems with bandwidth L bits, local information, for example, original measurements, measurement innovations, and local estimates, should be sent to an estimation center via wireless/wire networks. In order to meet digital transmission and limited bandwidth, it is necessary to quantize local information. If data taken in local sensors is original measurements while the adaptive bit quantization strategy with given upper and lower bounds of quantization interval is used [6,8,10], then the quantized message taken by the center can be expressed as follows

$$\mathbf{z}_{\upsilon,k} = \mathbf{H}_k\mathbf{x}_k + \upsilon_k \tag{2}$$

where $\upsilon_k = \mathbf{v}_k + \mathbf{e}_k$ and it means that information received by the estimate center is not equal to the original one taken in local sensor. $\mathbf{z}_{\upsilon,k}$ is quantized massage and $\mathbf{e}_k$ is quantization error which variance can be approximatively expressed to

$$E\{\mathbf{e}_k\mathbf{e}_k^T\} = \mathbf{R}_{\mathbf{e},k} \preceq diag\{(\Delta_k^1)^2/4, \cdots, (\Delta_k^p)^2/4\} \tag{3}$$

where $\Delta_k^i (i = 1, 2, \cdots, p)$ is quantization interval for every component of measurement and can be computed according to [10]. Then, we have

$$\begin{aligned} \mathbf{R}_{\upsilon,k} &= E\{\upsilon_k\upsilon_k^T\} = \mathbf{R}_k + \mathbf{R}_{\mathbf{e},k} \\ &= diag\{(\sigma_k^1)^2,\ (\sigma_k^2)^2)\ \cdots,\ (\sigma_k^p)^2\} \\ &\preceq diag\{(r_k^1)^2 + (\Delta_k^1)^2/4, \cdots, (r_k^p)^2 + (\Delta_k^p)^2/4\} \end{aligned} \tag{4}$$

C. Motivation

For most of current work, Eq.(2) with the message noise variance $\mathbf{R}_{\upsilon,k}$, which includes $\mathbf{R}_{\mathbf{e},k}$, is directly used in traditional Kalman filter to obtain a quantized estimate. Actually, $\mathbf{R}_{\mathbf{e},k}$ is taken by using an upper bound on the variance of the quantization error $\mathbf{e}_k$ and mismatches to the real one. Clearly, it is a linear filtering problem with inaccurate noise variance. A strong tracking fading factor has been used to adjust predict error covariance in order to improve estimate performance of the linear filter. Accordingly, the estimate accuracy is improved while influence of approximative variance can be overcome to some extent. However, there are still two problems. The first is that the variance of the bit quantization error can not be estimated on line and secondly the estimate of quantized filter should be further improved.

In order to solve the two problems mentioned above, in this paper we introduce the variational Bayesian method to estimate the integrated quantized error's variance $\mathbf{R}_{\upsilon,k}$ in real time, and take a novel quantized state estimator for a kind of linear dynamic discrete system by combining the VB method and strong tracking filtering. Hence, a dynamical estimate on the variance of the stochastic quantization error can be obtained and the strong tracking performance can also be achieved at the same time for the novel quantized state estimator.

III. Quantized Kalman filter with Fading Factor

The strong tracking filter was firstly proposed to improve estimate performance of nonlinear extended Kalman filter (EKF) induced by linearization of nonlinear systems, which leads to that inaccuracy of system parameters taken after linearization are used in designing nonlinear filters [10,21,22]. Later, the abilities to handle inaccurate system model and to track sudden change of state were studied for linear and nonlinear systems. Its basic principle is to introduce a time-variant fading factor λ_k to automatically adjust one step state predict error covariance, namely

$$\mathbf{P}_{k|k-1} = \lambda_k\Phi_{k,k-1}\mathbf{P}_{k-1|k-1}\Phi_{k,k-1}^T + \mathbf{Q}_{k,k-1} \tag{5}$$

where λ_k can be taken by solve the following optimization problem, which is

$$\begin{aligned} &min\quad E\{[\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_{k|k}][\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_{k|k}]^T\} \\ &s.t.\quad E\{\gamma_{k+l}\gamma_k^T\} = \mathbf{0},\quad k = 1, 2, ...; l \geq 1 \\ &\qquad \gamma_k = \mathbf{z}_{\upsilon,k} - \hat{\mathbf{z}}_{k|k-1} \end{aligned} \tag{6}$$

where $\hat{\mathbf{x}}_{k|k}$ is quantized state estimate at time k and γ_k is innovation and $\hat{\mathbf{z}}_{k|k-1}$ is measurement or message prediction. Objective function in Eq.(6) is the optimal rule of state estimation, and the constraint indicates that innovations from different times should be orthogonal. The orthogonality means all of information provided by quantized messages is sufficiently and completely used in filtering process. Actually, nonorthogonality among innovations in different time is generally induced by mismatching between model-used and the real system. Then, a suboptimal solution of λ_k, which is popular in practical applications, can be evaluated in terms of the following formulas

$$\lambda_k = \begin{cases} c_k, c_k > 1 \\ 1,\ c_k \leq 1 \end{cases} \tag{7}$$

where

$$c_k = Tr(\mathbf{N}_k)/Tr(\mathbf{M}_k) \tag{8}$$

and

$$\begin{cases} \mathbf{N}_k = \mathbf{V}_{0,k} - \beta^*\mathbf{R}_{\upsilon,k} - \mathbf{H}_k\mathbf{Q}_{k,k-1}\mathbf{H}_k^T \\ \mathbf{M}_k = \mathbf{H}_k\Phi_{k,k-1}\mathbf{P}_{k-1|k-1}\Phi_{k,k-1}^T\mathbf{H}_k^T \end{cases} \tag{9}$$

$$\mathbf{V}_{0,k} = \begin{cases} \gamma_1\gamma_1^T,\ k = 1 \\ (\rho\mathbf{V}_{0,k-1} + \gamma_k\gamma_k^T)/(1+\rho),\ k > 1 \end{cases} \tag{10}$$

where $0 < \rho \leq 1$ and $\beta^* \geq 1$.

From the above formulas, we know that λ_k depends on the current innovation and it is able to reduce influence from history data and enhance effect of the current measurement. Accordingly, a quantized Kalman filter with strong tracking filtering (QKF-STF) can be taken [10].

IV. Adaptive Quantized Kalman Filter Using Variational Bayesian Method

A. Variational Bayesian Method

The variational Bayesian method is used to realize synchronous estimate of state and unknown variance of measurement noise [18]. As is known to all, under the assumption that dynamic model of system state and the variance parameters are independent, optimal Bayesian filtering is actually to compute the posterior distribution $p(\mathbf{x}_k, \mathbf{R}_{\upsilon,k}|\mathbf{z}_{\upsilon,k})$ for quantized state estimate systems. Although the VB method is used to estimate the interpreted variance of the message noise in designing quantized state estimator, it is also available by adding some operations when this variance is inaccurate in our work.

Generally, the recursive solution of the optimal Bayesian filtering includes prediction and update operations for quantized systems, which are

$$\begin{aligned} &p(\mathbf{x}_k, \mathbf{R}_{\upsilon,k}|\mathbf{z}_{\upsilon,1:k-1}) \\ &\quad = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) p(\mathbf{R}_{\upsilon,k}|\mathbf{R}_{\upsilon,k-1}) \\ &\quad \cdot p(\mathbf{x}_{k-1}, \mathbf{R}_{\upsilon,k-1}|\mathbf{z}_{\upsilon,1:k-1}) d\mathbf{x}_{k-1} d\mathbf{R}_{\upsilon,k-1} \end{aligned} \tag{11}$$

$$p(\mathbf{x}_k, \mathbf{R}_{\upsilon,k}|\mathbf{z}_{\upsilon,1:k}) \propto p(\mathbf{z}_{\upsilon,k}|\mathbf{x}_k, \mathbf{R}_{\upsilon,k}) p(\mathbf{x}_k, \mathbf{R}_{\upsilon,k}|\mathbf{z}_{\upsilon,1:k-1}) \tag{12}$$

where $\mathbf{z}_{\upsilon,1:k}$ is a set of $\mathbf{z}_{\upsilon,m}(m = 1, 2, \cdots, k)$. For the optimal Bayesian filtering, it is difficult to solve posterior probability density function variance because the variance of the integrated message noise is accurately unknown. The main idea of the variational Bayesian method uses many known distributions to approximate joint posterior distribution of state and message noise variance. Then, we can get [18]

$$\begin{aligned} &p(\mathbf{x}_k, \mathbf{R}_{\upsilon,k}|\mathbf{z}_{\upsilon,1:k}) \\ &\quad \approx \mathbf{N}(\mathbf{x}_k|\hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k}) \\ &\quad \times \prod_{i=1}^{p} Inv-Gamma((\sigma_k^i)^2|\alpha_{k|k-1}^i, \beta_{k|k-1}^i) \end{aligned} \tag{13}$$

Accordingly, a first-order approximation of prediction distribution of message noise's variance can be as follows

$$\alpha_{k|k-1}^i = \varrho^i \alpha_{k-1}^i, \beta_{k|k-1}^i = \varrho_k^i \beta_{k-1}^i \quad (i = 1, 2, \cdots, p) \tag{14}$$

where $i = 1, 2, \cdots, p$, and α_{k-1}^i and β_{k-1}^i are i^{th} component of parameters of Inv-Gamma distribution. $(\sigma_k^i)^2$ is i^{th} component of the unknown variance $\mathbf{R}_{\upsilon,k}$ and can be estimated by $(\hat{\sigma}_k^i)^2 = \beta_k^i / \alpha_k^i$. $\varrho_k^i \in (0, 1)$ is a weighted coefficient.

If we denote

$$\alpha_k = [\alpha_k^1 \ \ \alpha_k^2 \ \ \cdots \ \ \alpha_k^p]^T \tag{15}$$

$$\beta_k = [\beta_k^1 \ \ \beta_k^2 \ \ \cdots \ \ \beta_k^p]^T \tag{16}$$

$$\varrho_k = [\varrho_k^1 \ \ \varrho_k^2 \ \ \cdots \ \ \varrho_k^p]^T \tag{17}$$

then the estimate of $\mathbf{R}_{\upsilon,k}$ can be computed by

$$\hat{\mathbf{R}}_{\upsilon,k} = diag(\beta_k ./ \alpha_k) = diag\{(\hat{\sigma}_k^1)^2 \ \ \cdots \ \ (\hat{\sigma}_k^p)^2\} \tag{18}$$

where $'.'$ is the point operation in Matlab software.

In addition, different from the traditional variational Bayesian adaptive Kalman filter (VB-AKF) for which variance of measurement noise is absolutely unknown [18], we can get an approximation with upper bound and an inaccurate expression of the variance for quantized systems. Then, this inaccurate value can be used to distinguish effectiveness of the variance's estimate. Thereby, after $\hat{\mathbf{R}}_{\upsilon,k}$ is taken, a decision mechanism is available. Namely, if $(\hat{\sigma}_k^i)^2 > (r_k^i)^2 + (\triangle_k^i)^2/4 (i = 1, 2, \cdots, p)$, let $(\hat{\sigma}_k^i)^2 = (r_k^i)^2 + (\triangle_k^i)^2/4$; else use directly $(\hat{\sigma}_k^i)^2$ in the subsequent filtering steps. It is simple and easy to understand.

As a result, we can get a novel variational Bayesian adaptive quantized Kalman filter (VB-AQKF) similar to [18] for this kind of quantized linear system in this paper.

V. Variational Bayesian Adaptive Quantized Kalman Filter with Strong Tracking Filtering

As stated in the preceding, the strong tracking filter does not estimate the variance of the message noise and the variational Bayesian method cannot have strong tracking ability. In order to obtain both abilities, it is a good way to integrate them and a novel variational Bayesian adaptive quantized Kalman filter with strong tracking filtering (VB-AQKF-STF) can be taken for this kind of quantized system. By combining the QKF-STF in section III with the VB-AQKF in section IV, we can get the steps of VB-AQKF-STF as follows:

1) Evaluate one step state predict

$$\hat{\mathbf{x}}_{k|k-1} = \Phi_{k,k-1} \hat{\mathbf{x}}_{k-1|k-1} \tag{19}$$

2) Compute variance parameters prediction

$$\alpha_{k|k-1} = \varrho_k . \alpha_{k-1}, \ \beta_{k|k-1} = \varrho_k . \beta_{k-1} \tag{20}$$

3) Compute message prediction and innovation

$$\hat{\mathbf{z}}_{k|k-1} = \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}, \ \gamma_k = \mathbf{z}_{\upsilon,k} - \hat{\mathbf{z}}_{k|k-1} \tag{21}$$

4) Compute state prediction error covariance with λ_k

$$\mathbf{P}_{k|k-1} = \lambda_k \Phi_{k,k-1} \mathbf{P}_{k-1|k-1} \Phi_{k,k-1}^T + \mathbf{Q}_{k,k-1} \tag{22}$$

where the strong tracking fading factor λ_k can be recursively computed in terms of Eqs.(7)-(10).

5) Let $j = 0$ and initialization of iteratively estimating $\mathbf{R}_{\upsilon,k}$

$$\alpha_k = 1/2 + \alpha_{k|k-1}, \ \beta_{a,k}^0 = \beta_{k|k-1} \tag{23}$$

6) Evaluate iteratively $\hat{\mathbf{R}}_{\upsilon,k}^j (j = 0, 1, \cdots, N_1)$

$$\hat{\mathbf{R}}_{\upsilon,k}^j = diag(\beta_{a,k}^j ./ \alpha_k) = diag\{(\hat{\sigma}_k^{1,j})^2 \ \ \cdots \ \ (\hat{\sigma}_k^{p,j})^2\} \tag{24}$$

where $(\hat{\sigma}_k^{i,j})^2 (i = 1, 2, \cdots, p)$ indicates the variance of i^{th} component at j^{th} iteration.

7) If $(\hat{\sigma}_k^{i,j})^2 > (r_k^i)^2 + (\triangle_k^i)^2/4 (i = 1, 2, \cdots, p_l)$, let $(\hat{\sigma}_k^{i,j})^2 = (r_k^i)^2 + (\triangle_k^i)^2/4$; else think that $(\hat{\sigma}_k^{i,j})^2$ is directly available . Accordingly, an effective $\hat{\mathbf{R}}_{\upsilon,k}^j$ is completely taken.

8) Evaluate gain matrix

$$\mathbf{K}_k^{j+1} = \mathbf{P}_{k|k-1} \mathbf{H}_k^T [\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \hat{\mathbf{R}}_{\upsilon,k}^j]^{-1} \tag{25}$$

9) Compute iterative state estimate and the associated estimate error covariance

$$\begin{cases} \hat{\mathbf{x}}_{k|k}^{j+1} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k^{j+1}\ \gamma_k \\ \mathbf{P}_{k|k}^{j+1} = [\mathbf{I} - \mathbf{K}_k^{j+1}\mathbf{H}_k]\mathbf{P}_{k|k-1} \end{cases} \tag{26}$$

where $\mathbf{I}$ is unite matrix with $n \times n$ dimensions.

10) If $j < N_1$, where N_1 is iterative times, then update iteratively the estimate of parameter β_k^j according to

$$\begin{aligned} \beta_{a,k}^{j+1} = \beta_{k|k-1} + (\mathbf{z}_{\upsilon,k} - \mathbf{H}_k\hat{\mathbf{x}}_{k|k}^{j+1})^{.2}/2 \\ + diag(\mathbf{H}_k\mathbf{P}_{k|k}^{j+1}\mathbf{H}_k^T)/2 \end{aligned} \tag{27}$$

Afterwards, let $j = j+1$ and go to 6); else go to 11).

11) When $j = N_1$, we have

$$\begin{cases} \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k}^{N_1+1},\ \mathbf{P}_{k|k} = \mathbf{P}_{k|k}^{N_1+1} \\ \beta_k = \beta_{a,k}^{N_1},\ \hat{\mathbf{R}}_{\upsilon,k} = \hat{\mathbf{R}}_{\upsilon,k}^{N_1} \end{cases} \tag{28}$$

In terms of the running steps, we know that it does not strictly obey two obvious sections of the traditional Kalman filter, which are time and measurement (message) updates. This is because computation of λ_k, which is used to adjust prediction error covariance in time update, needs innovation γ_k which requires computation of measurement prediction in measurement update. Accordingly, it leads to that computation of $\mathbf{P}_{k|k-1}$ is actually done in message update process. As a result, time and message updates are mixed and it does not influence implementing of this quantized filter in nature.

VI. Simulations

In this section, two simulation examples are presented to show effectiveness of the proposed quantized state estimators for target tracking in this paper. All of simulations are average results of Monte-Carlo runs.

A. Simulation Setup

The following dynamic state equation is considered

$$\mathbf{x}_k = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \mathbf{x}_{k-1} + \mathbf{w}_{k,k-1} \tag{29}$$

where $\mathbf{x}_k = [\mathbf{x}_{1,k}\ \mathbf{x}_{2,k}]^T$ and $T = 0.5\ s$ is sample period. The measurement equations are

$$\mathbf{z}_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x}_k + \mathbf{v}_k \tag{30}$$

where the variances of $\mathbf{w}_{k,k-1}$ and $\mathbf{v}_k$ are respectively

$$\mathbf{Q}_{k,k-1} = \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix}, \mathbf{R}_k = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \tag{31}$$

The initial state estimate and its associated variance are $\hat{\mathbf{x}}_{0|0} = [0; 0.1]$ and $\mathbf{P}_{0|0} = [10\ 0;\ 0\ 1]$. Network bandwidth $L = 10\ bits$ and the upper and lower bounds of measurements are taken by adding and subtracting 20 on maximum and maximin of all of measurements respectively.

For the strong tracking filtering function, its parameters $\beta^* = 1.2$ and $\rho = 0.95$. In the variational Bayesian method

$$\varrho_0 = \begin{bmatrix} 1 - e^{10} \\ 1 - e^{10} \end{bmatrix}, \alpha_0 = \begin{bmatrix} 0.8 \\ 0.8 \end{bmatrix}, \beta_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{32}$$

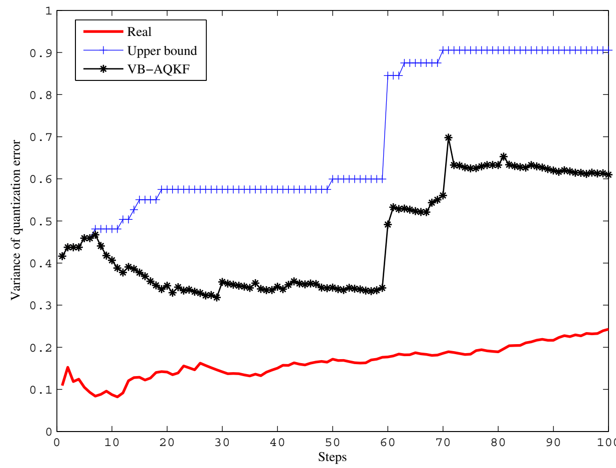

Fig. 1: Variance of quantization error

and the iterative times $N_1 = 3$.

Examples 1 and 2 both aim at a single sensor system. For simplicity, the simulation results of $\mathbf{x}_{2,k}$ (speed component of state) are only presented in the examples.

B. Example 1

This example demonstrates performances of the QKF-STF and the VB-AQKF when the state suddenly changes from time 60 to 70. The sudden change is made by $\mathbf{x}_k = \mathbf{x}_k + [16; 8]$ $(60 \leq k \leq 70)$. Simulation results see Fig.1 and Fig.2. Fig.1 shows the quantization error variance computed by $[(\hat{\sigma}_k^2)^2 - (r_k^2)^2]$ for second component $\mathbf{z}_{2,k}$ of measurement, and Fig.2 is the mean of absolute estimate error of $\mathbf{x}_{2,k}$. In Fig.1, bold red line indicates the real variance of the quantization error computed by $\sum_{k=1}^{k'}[(\mathbf{z}_k - \mathbf{z}_{\upsilon,k})(\mathbf{z}_k - \mathbf{z}_{\upsilon,k})']/(k'+1)$. Blue line with $'+'$ is the upper bound $(\Delta_k^2)^2/4$ and black line with $'*'$ is the estimate result of the VB-AQKF.

From Fig.1, we know that the variance estimate of the VB-AQKF becomes worse from time 60 when the sudden change of state appears and there is also an abrupt change like the state sudden change for variance estimate at time 60. This indicates that this function of variance estimate does not have good robustness for this case, although the result shows the taken variance estimate is better than the upper bound. Actually, it is possible to coincide with the upper bound when to encounter the state sudden change at worst. Fig.2 also indicates when state changes suddenly and the sudden change is over, the VB-AQKF can not track effectively the state but the QKF-STF can do. Clearly, the VB-AQKF is short of robustness for this kind of the state sudden change and in this case the STF can not still estimate the variance. However, it is necessary to achieve simultaneously the robustness of the state sudden change and the estimation of the variance of the quantization error. It is the motivation of this paper to combine the strong tracking filtering technology with the variational Bayesian method.

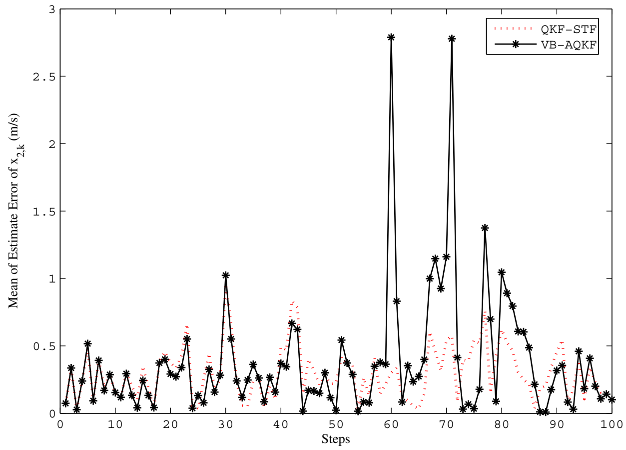

Fig. 2: Mean of absolute estimate error of $\mathbf{x}_{2,k}$

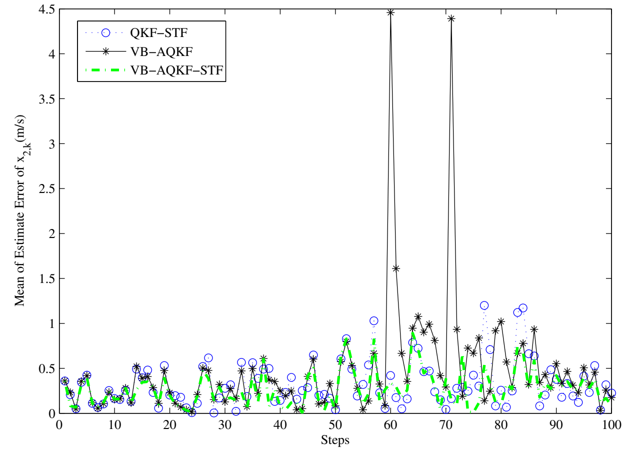

Fig. 4: Mean of absolute estimate error of $\mathbf{x}_{2,k}$

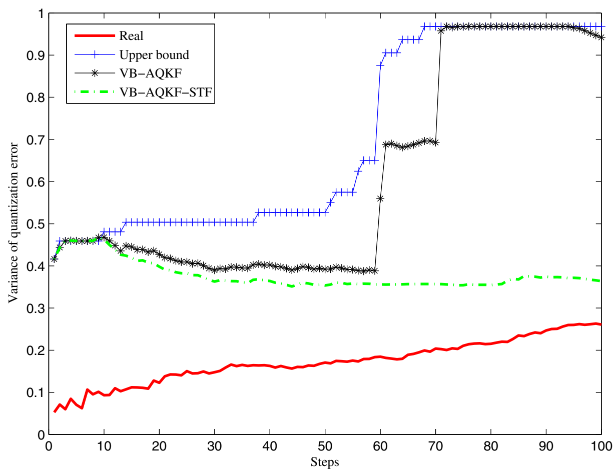

Fig. 3: Variance of quantization error

C. Example 2

The favorable performance of the VB-AQKF-STF is demonstrated in this example in the case with state sudden change. The state sudden change is to add $[20; 10]$ in the normal states from time 60 to 70. The simulation results are shown by Fig.3 and Fig.4. In Fig.3, when the sudden change happens at time 60, the variance estimate of the VB-AQKF also follows and soon afterwards it is equal to the upper bound, namely the worst case appears. However, the variance estimate for the VB-AQKF-STF is greatly stable and better than the VB-AQKF. Considering jointly Fig.3 and Fig.4, the VB-AQKF obtains the strong tracking ability of not only variance estimate but state estimate accuracy by introducing the fading factor.

VII. Conclusion

The networked state estimation is studied based on adaptive bits quantization for sensor networks with limited bandwidth in this paper. By perfectly combing the strong tracking filtering technology and the variational Bayesian method, a novel quantized Kalman estimator, the VB-AQKF-STF, is proposed to cope with universally quantized state estimation. This combination makes the proposed quantized filter integrate two advantages from the STF and the VB method respectively, which are the robustness to the sudden change of state and dynamical evaluation of the variance of the quantization error.The future work includes to extend the linear quantized filters and quantized fusion estimator to the cases with different correlated noises, multisensor systems, more networked constraints and nonlinear systems.

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