Distributed State Estimation for Lur'e Systems in Sensor Networks with Impulsive Effects and Intermittent Measurements*

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Abstract— **This paper is concerned with the problem of distributed state estimation for Lur'e systems in sensor networks with impulsive effects and intermittent measurements. Based on a Lyapunov function method, sufficient conditions are presented which ensure all node estimators converging asymptotically. The gains of the estimators are obtained by solving certain linear matrix inequalities with some algebra constraints. Numerical examples are given to illustrate the effectiveness of the proposed estimation method.**

I. INTRODUCTION

Over the past few decades, sensor networks have been gaining increasing research attention because of their military and civilian applications such as battle-filed surveillance, target tracking, guidance for autonomous vehicles, infrastructure security, monitoring of manufacturing processes, etc. [1], [2]. In particular, the problem of distributed state estimation (or distributed filtering) has recently been an area of active research for sensor networks [3]–[8].

It is noted that impulses may occur in the process of the communications between sensor i and its neighboring sensors, which are similar to the cases in synchronization of complex dynamical networks [9], [10] and consensus of multi-agent systems [11], [12]. Moreover, sometimes impulses can play a negative role [13]. In addition, some communication constraints may occur when sensor i shares with its neighbors, which are similar to the cases in consensus of multi-agent systems [14].

This paper considers distributed state estimation for Lur'e systems in sensor networks with impulsive effects and intermittent measurements, where the target is a Lur'e model. In the proposed distributed estimators, the topology information of the sensor network is included. We shall present sufficient conditions for convergence of the distributed state estimators under consideration.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times n$ real matrices. $P > 0 (< 0, \leq 0, \geq 0)$ stands for a symmetrical positive(negative, semi-negative, semi-positive) define matrix P. X^T and X^{-1} denote, respectively, the transpose and the inverse of any square matrix X. diag$\{\cdots\}$ stands for a block-diagonal matrix. In symmetric block matrices, $*$ is used as an ellipsis for terms induced by symmetry. The notation sym$\{A\}$ is defined as sym$\{A\} = A^T + A$. $\lambda_{\max}(Q)(\lambda_{\min}(Q))$ stand for the maximum (minimum) eigenvalue of the matrix Q, respectively. I_n is the $n \times n$ identity matrix. The notation $\otimes$ denotes the Kronecker product. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

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II. PROBLEM FORMULATION AND PRELIMINARIES

Let the target be the following Lur'e model:

$$\dot{x}(t) = Ax(t) + B\varphi(Cx(t)) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}$ and $C = [C_1^T, C_2^T, \ldots, C_m^T]^T \in \mathbb{R}^{m\times n}$ are known constant matrices. $\varphi(Cx(t)) = [\varphi_1(C_1x(t)), \varphi_2(C_2x(t)), \ldots, \varphi_m(C_mx(t))]^T : \mathbb{R}^m \to \mathbb{R}^m$ is a memoryless nonlinear vector-valued function which is globally Lipschitz, $\varphi(0) = 0$. Assume that $\varphi_j(C_jx(t))$ belongs to a sector $[0, \delta_j]$, i.e., $\forall x(t) \in \mathbb{R}^n$,

$$\varphi_j(C_jx(t))(\varphi_j(C_jx(t)) - \delta_jC_jx(t)) \leq 0 \tag{2}$$

$(j = 1, 2, \ldots, m)$.

Consider a sensor network of size N. Suppose that the measurement of sensor i on state $x(t)$ is given by

$$y_i(t) = H_ix(t) \tag{3}$$

where $H_i \in \mathbb{R}^{q\times n}$ is a known constant matrix. Same as [9], the sensor network topology is represented by a directed graph $\mathcal{G}_a = (\mathcal{V}, \mathcal{E}_a, \mathcal{A}_a)$ of order N with the set of nodes $\mathcal{V} = \{1, \cdots, N\}$, the set of edges $\mathcal{E}_a = \{(i, j) : i, j \in \mathcal{V}\} \subset \mathcal{V} \times \mathcal{V}$, and an adjacency matrix $\mathcal{A}_a = [a_{ij}]_{N\times N}$ with adjacency elements $a_{ij} > 0$. In addition, it is assumed that $a_{ii} > 0$ for all $i \in \{1, 2, \cdots, N\} \triangleq \underline{N}$. The set of neighbors of node i including the node itself is denoted by $\mathcal{N}_{ai} = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}_a\}$.

Note that in some real situations, sensor nodes communicate with their neighbors during some time intervals and don't work during the other intervals in order to shorten communication time and/or minimize communication cost. Motivated by this observation and [15], the following state

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estimator on the sensor node $i(i \in \underline{N})$ is considered:

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + B\varphi(C\hat{x}_i(t)), t \in [l\omega, l\omega+\tau], \\ \begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + B\varphi(C\hat{x}_i(t)) \\ \qquad +G_i \sum\limits_{j\in\mathcal{N}_{ai}} a_{ij}(H_j\hat{x}_j(t) - y_j(t)), t \in (t_{l,k-1}, t_{l,k}], \\ \triangle\hat{x}_i(t) = K_i \sum\limits_{j\in\mathcal{N}_{bi}} b_{ij}(H_j\hat{x}_j(t) - y_j(t)), t = t_{l,k}, \end{cases} \\ \qquad\qquad\qquad t \in [l\omega+\tau, (l+1)\omega], \end{cases} \tag{4}$$

where $0 < \tau < \omega < \infty, l = 1, 2, \cdots, k = 1, 2, \cdots, r_l, r_l$ is a positive integer such that $l\omega + \tau = t_{l,0} < t_{l,1} < t_{l,2} < \cdots < t_{l,r_l} \le (l+1)\omega$, $\triangle\hat{x}_i(t_{l,k}) = \hat{x}_i(t^+_{l,k}) - \hat{x}_i(t^-_{l,k})$ with $\hat{x}_i(t^+_{l,k}) = \lim\limits_{h\to 0^+} x_i(t_{l,k}+h), \hat{x}_i(t^-_{l,k}) = \lim\limits_{h\to 0^+} x_i(t_{l,k}-h)$, G_i and K_i are parameters of the estimators to be determined. Without loss of generality, it is assumed that $\hat{x}_i(t^-_{l,k}) = \hat{x}_i(t_{l,k})$. Note that the sensor network topology at impulsive instants maybe changes, and it is represented by a directed graph $\mathcal{G}_b = (\mathcal{V}, \mathcal{E}_b, \mathcal{A}_b)$ with $\mathcal{E}_b = \{(i,j) : i,j \in \mathcal{V}\} \subset \mathcal{V}\times\mathcal{V}$, an adjacency matrix $\mathcal{A}_b = [b_{ij}]_{N\times N}$ with adjacency elements $b_{ij} > 0$. In addition, it is assumed that $b_{ii} > 0$ for all $i \in \underline{N}$. The set of neighbors of node i including the node itself is denoted by $\mathcal{N}_{bi} = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}_b\}$.

Denoting $e_i(t) = x(t) - \hat{x}_i(t)$, then the estimation error dynamics for the node i is given by

$$\begin{cases} \dot{e}_i(t) = Ae_i(t) + B\tilde{\Phi}_i(Ce_i(t), x(t)), t \in [l\omega, l\omega+\tau], \\ \begin{cases} \dot{e}_i(t) = Ae_i(t) + B\tilde{\Phi}_i(Ce_i(t), x(t)), \\ \qquad +G_i \sum\limits_{j\in\mathcal{N}_{ai}} a_{ij}H_je_j(t),\ t \in (t_{l,k-1}, t_{l,k}], \\ \triangle e_i(t) = K_i \sum\limits_{j\in\mathcal{N}_{bi}} b_{ij}H_je_j(t),\ t = t_{l,k}, \end{cases} \\ \qquad\qquad\qquad t \in [l\omega+\tau, (l+1)\omega], \end{cases} \tag{5}$$

where

$$\begin{aligned} \tilde{\Phi}_i(Ce_i(t), x(t)) &= \begin{bmatrix} \tilde{\Phi}_{i1}(C_1e_i(t), x(t)) \\ \vdots \\ \tilde{\Phi}_{im}(C_me_i(t), x(t)) \end{bmatrix} \\ &= \varphi(Cx(t)) - \varphi(Cx(t) - Ce_i(t)),\ i \in \underline{N}. \end{aligned}$$

It is assumed that $\tilde{\Phi}_{ij}(C_je_i(t), x(t))$ belongs to a sector $[0, \delta_j]$([16]), i.e., $\forall x(t), e_i(t) \in \mathbb{R}^n$,

$$\tilde{\Phi}_{ij}(C_je_i(t), x(t))(\tilde{\Phi}_{ij}(C_je_i(t), x(t)) - \delta_jC_je_i(t)) \le 0 \tag{6}$$

$(i \in \underline{N}, j = 1, 2, \ldots, m)$.

Define $e(t) = [e_1^T(t), \ldots, e_N^T(t)]^T$, then system (5) can be rewritten as

$$\begin{cases} \dot{e}(t) = (I_N\otimes A)e(t) + (I_N\otimes B)\tilde{\Phi}(\tilde{C}e(t), x(t)), \\ \qquad\qquad\qquad t \in [l\omega, l\omega+\tau], \\ \begin{cases} \dot{e}(t) = (I_N\otimes A + \bar{S})e(t) \\ \qquad +(I_N\otimes B)\tilde{\Phi}(\tilde{C}e(t), x(t)),\ t \in (t_{l,k-1}, t_{l,k}], \\ \triangle e(t) = \bar{W}e(t),\ t = t_{l,k}, \end{cases} \\ \qquad\qquad\qquad t \in [l\omega+\tau, (l+1)\omega], \end{cases} \tag{7}$$

where $\tilde{C} = I_N \otimes C$,

$$\bar{S} = (S_{ij})_{N\times N}, S_{ij} = \begin{cases} a_{ij}G_iH_j, \text{if } j \in \mathcal{N}_{ai}, \\ 0, \text{others}, \end{cases}$$
$$\bar{W} = (W_{ij})_{N\times N}, W_{ij} = \begin{cases} b_{ij}K_iH_j, \text{if } j \in \mathcal{N}_{bi}, \\ 0, \text{others}, \end{cases}$$
$$\tilde{\Phi}(\tilde{C}e(t), x(t)) = [\tilde{\Phi}_1(Ce_1(t), x(t))^T, \ldots, \tilde{\Phi}_N(Ce_N(t), x(t))^T]^T.$$

Let $\Delta = \mathrm{diag}\{\delta_1, \ldots, \delta_m\}$, $\Lambda = I_N \otimes \Delta$, then it follows from (6) that $\tilde{\Phi}(Ce(t), x(t))$ belongs to a sector $[0, \Lambda]$, i.e., $\forall e(t), x(t)$,

$$\tilde{\Phi}(\tilde{C}e(t), x(t))^T(\tilde{\Phi}(\tilde{C}e(t), x(t)) - \Lambda\tilde{C}e(t)) \le 0. \tag{8}$$

The purpose of this paper is to design parameters G_i and K_i in (4) such that the error system (7) is globally asymptotically stable.

Lemma 1 [17] The Kronecker product has the following properties:

(1) $(\kappa A)\otimes B = A\otimes(\kappa B)$, where κ is a scalar;
(2) $(A+B)\otimes C = A\otimes C + B\otimes C$;
(3) $(A\otimes B)(C\otimes D) = (AC)\otimes(BD)$;
(4) $(A\otimes B)^T = A^T\otimes B^T$.

III. MAIN RESULT

This section provides a method to construct the parameters G_i, K_i in (4). The sufficient condition presented below in Theorem 1 allows one to achieve this by solving a set of linear matrix inequalities with a algebra constraint.

Theorem 1 The error system (7) is globally asymptotically stable with

$$G_i = P_i^{-1}X_i, K_i = P_i^{-1}Y_i \tag{9}$$

if there exist matrices $\bar{P} = \mathrm{diag}\{P_1, \ldots, P_N\} > 0$, $X_i, Y_i (i \in \underline{N})$, $\Gamma_{si} = \mathrm{diag}\{\gamma_{si1}, \ldots, \gamma_{sim}\} > 0 (s = 1, 2, i \in \underline{N})$ and scalars $\varepsilon_1 > 0, \varepsilon_2 > 0, \beta > 0, 0 < \alpha < 1$ such that the following conditions hold.

$$\begin{bmatrix} \Sigma_{11} & \bar{P}(I_N\otimes B) + \tilde{C}^T\Lambda\bar{\Gamma}_1 \\ * & -2\bar{\Gamma}_1 \end{bmatrix} < 0 \tag{10}$$

$$\begin{bmatrix} \Xi_{11} & \bar{P}(I_N\otimes B) + \tilde{C}^T\Lambda\bar{\Gamma}_2 \\ * & -2\bar{\Gamma}_2 \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} -\beta\bar{P} & (I_N\otimes I_n)\bar{P} + \bar{R}^T \\ * & -\bar{P} \end{bmatrix} < 0 \tag{12}$$

$$\ln\beta - \varepsilon_2\triangle_{\max} + \varepsilon_1\tau = \ln\alpha \le 0, \tag{13}$$

where $\triangle_{\max} = \max\limits_{l,k}\{t_{l,k} - t_{l,k-1}\}, l = 1, 2, \cdots, k = 1, 2, \cdots, r_l$, $\bar{\Gamma}_s = \mathrm{diag}\{\Gamma_{s1}, \ldots, \Gamma_{sN}\}, s = 1, 2$,

$$\Sigma_{11} = \mathrm{sym}\{\bar{P}(I_N\otimes A)\} - \varepsilon_1\bar{P},$$
$$\Xi_{11} = \mathrm{sym}\{\bar{P}(I_N\otimes A) + \bar{U}\} + \varepsilon_2\bar{P},$$
$$\bar{U} = (U_{ij})_{N\times N}, U_{ij} = \begin{cases} a_{ij}X_iH_j, \text{if } j \in \mathcal{N}_{ai}, \\ 0, \text{others}, \end{cases}$$
$$\bar{R} = (R_{ij})_{N\times N}, R_{ij} = \begin{cases} b_{ij}Y_iH_j, \text{if } j \in \mathcal{N}_{bi}, \\ 0, \text{others}. \end{cases}$$

Proof. From (8) it follows that for any $\bar{\Gamma}_s = \text{diag}\{\Gamma_{s1}, \ldots, \Gamma_{sN}\}$ with $\Gamma_{si} = \text{diag}\{\gamma_{si1}, \ldots, \gamma_{sim}\} > 0 (s = 1, 2, i \in \underline{N})$

$$-2\tilde{\Phi}(\tilde{C}e(t), x(t))^T\bar{\Gamma}_s(\tilde{\Phi}(\tilde{C}e(t), x(t)) - \Lambda\tilde{C}e(t)) \geq 0. \quad (14)$$

Construct the following Lyapunov function:

$$V(e(t)) = e(t)^T\bar{P}e(t) \quad (15)$$

where $\bar{P} = \text{diag}\{P_1, \ldots, P_N\} > 0$.

For $t \in [l\omega, l\omega + \tau]$, the derivative of $V(e(t))$ with respective to (7) is

$$\begin{aligned} \dot{V}(e(t))|_{(7)} &= e(t)^T\text{sym}\{\bar{P}(I_N \otimes A)\}e(t) \\ &\quad + 2e(t)^T\bar{P}(I_N \otimes B)\tilde{\Phi}(\tilde{C}e(t), x(t)) \\ &\leq e(t)^T\text{sym}\{\bar{P}(I_N \otimes A)\}e(t) \\ &\quad + 2e(t)^T\bar{P}(I_N \otimes B)\tilde{\Phi}(\tilde{C}e(t), x(t)) \\ &\quad - 2\tilde{\Phi}(\tilde{C}e(t), x(t))^T\bar{\Gamma}_1(\tilde{\Phi}(\tilde{C}e(t), x(t)) - \Lambda\tilde{C}e(t)) \\ &= \xi(t)^T\Sigma\xi(t) + \varepsilon_1 V(e(t)) \end{aligned} \quad (16)$$

where $\xi(t) = [e(t)^T, \tilde{\Phi}(\tilde{C}e(t), x(t))^T]^T$,

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \bar{P}(I_N \otimes B) + \tilde{C}^T\Lambda\bar{\Gamma}_1 \\ * & -2\bar{\Gamma}_1 \end{bmatrix}.$$

From (28) it follows that for $t \in [l\omega, l\omega + \tau]$

$$\dot{V}(e(t))|_{(7)} \leq \varepsilon_1 V(e(t)) \quad (17)$$

and

$$V(e(l\omega + \tau)) \leq V(e(l\omega))e^{\varepsilon_1\tau}.$$

For $t \in (t_{l,k-1}, t_{l,k}] \subseteq [l\omega + \tau, (l+1)\omega]$,

$$\begin{aligned} &\dot{V}(e(t))|_{(7)} \leq e(t)^T\text{sym}\{\bar{P}(I_N \otimes A + \bar{S})\}e(t) \\ &+ 2e(t)^T\bar{P}(I_N \otimes B)\tilde{\Phi}(\tilde{C}e(t), x(t)) \\ &- 2\tilde{\Phi}(\tilde{C}e(t), x(t))^T\bar{\Gamma}_2(\tilde{\Phi}(\tilde{C}e(t), x(t)) - \Lambda\tilde{C}e(t)) \\ &= \xi(t)^T\Omega\xi(t) - \varepsilon_2 V(e(t)) \end{aligned} \quad (18)$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & \bar{P}(I_N \otimes B) + \tilde{C}^T\Lambda\bar{\Gamma}_2 \\ * & -2\bar{\Gamma}_2 \end{bmatrix},$$
$$\Omega_{11} = \text{sym}\{\bar{P}(I_N \otimes A + \bar{S})\} + \varepsilon_2\bar{P}.$$

From (27) and (29) it follows that for $t \in (t_{l,k-1}, t_{l,k}] \subseteq [l\omega + \tau, (l+1)\omega]$,

$$\dot{V}(e(t))|_{(7)} \leq -\varepsilon_2 V(e(t)). \quad (19)$$

On the other hand,

$$\begin{aligned} &V(e(t_{l,k}^+)) = e(t_{l,k}^+)^T\bar{P}e(t_{l,k}^+) \\ &= e(t_{l,k})^T(I_N \otimes I_n + \bar{W})^T\bar{P}(I_N \otimes I_n + \bar{W})e(t_{l,k}) \\ &\leq \beta V(e(t_{l,k})) \end{aligned} \quad (20)$$

if

$$(I_N \otimes I_n + \bar{W})^T\bar{P}(I_N \otimes I_n + \bar{W}) - \beta\bar{P} < 0. \quad (21)$$

By Schur complement lemma, (21) is equivalent to

$$\begin{bmatrix} -\beta\bar{P} & (I_N \otimes I_n + \bar{W})^T\bar{P} \\ * & -\bar{P} \end{bmatrix} < 0$$

which are ensured by (27) and (12). Therefore, for $t \in [0, \tau]$,

$$V(e(t)) \leq V(e(0))e^{\varepsilon_1\tau}.$$

It follows from (30) that for $t \in [\tau, \omega]$,

$$V(e(t)) \leq \alpha V(e(0)).$$

In general, for $t \in [l\omega, l\omega + \tau]$,

$$V(e(t)) \leq V(e(l\omega))e^{\varepsilon_1\tau}. \quad (22)$$

It follows from (30) that for $t \in [l\omega + \tau, (l+1)\omega]$,

$$V(e(t)) \leq \alpha V(e(l\omega)). \quad (23)$$

From (22) and (23) it follows that

$$V(e(t)) \leq \begin{cases} \alpha^l V(e(0))e^{\varepsilon_1\tau}, t \in [l\omega, l\omega + \tau], \\ \alpha^{l+1}V(e(0)), t \in [l\omega + \tau, (l+1)\omega], \end{cases} \quad (24)$$

which implies that

$$\|e(t)\| \leq \begin{cases} c\|e(0)\|\alpha^l e^{\varepsilon_1\tau}, t \in [l\omega, l\omega + \tau], \\ c\|e(0)\|\alpha^{l+1}, t \in [l\omega, l\omega + \tau] \end{cases}$$

where $c = \frac{\max_{i \in \underline{N}}\{\lambda_{\max}(P_i)\}}{\min_{i \in \underline{N}}\{\lambda_{\min}(P_i)\}}$. Therefore, the error system (7) is globally asymptotically stable. This completes the proof.

IV. A SPECIAL CASE

If the effect of intermittent measurements is not considered in estimator (4), that is, $\tau = 0$, then (4) becomes

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + B\varphi(C\hat{x}_i(t)) \\ \quad + G_i \sum_{j \in \mathcal{N}_{ai}} a_{ij}(y_j(t) - H_j\hat{x}_j(t)), t \in (t_{k-1}, t_k], \\ \triangle\hat{x}_i(t) = K_i \sum_{j \in \mathcal{N}_{bi}} b_{ij}(y_j(t) - H_j\hat{x}_j(t)), t = t_k \end{cases} \quad (25)$$

Denoting $e_i(t) = x(t) - \hat{x}_i(t)$ and $e(t) = [e_1^T(t), \ldots, e_N^T(t)]^T$, then the estimation error dynamics is given by

$$\begin{cases} \dot{e}(t) = (I_N \otimes A + \bar{S})e(t) \\ \quad + (I_N \otimes B)\tilde{\Phi}(\tilde{C}e(t), x(t)), \ t \in (t_{k-1}, t_k], \\ \triangle e(t) = \bar{W}e(t), \ t = t_k, \end{cases} \quad (26)$$

where $\tilde{C} = I_N \otimes C$,

$$\bar{S} = (S_{ij})_{N \times N}, S_{ij} = \begin{cases} a_{ij}G_iH_j, \text{if } j \in \mathcal{N}_{ai}, \\ 0, \text{others}, \end{cases}$$
$$\bar{W} = (W_{ij})_{N \times N}, W_{ij} = \begin{cases} b_{ij}K_iH_j, \text{if } j \in \mathcal{N}_{bi}, \\ 0, \text{others}, \end{cases}$$
$$\tilde{\Phi}(\tilde{C}e(t), x(t)) = [\tilde{\Phi}_1(Ce_1(t), x(t))^T, \ldots, \tilde{\Phi}_N(Ce_N(t), x(t))^T]^T.$$

Corollary 1 The estimation error system (26) is globally asymptotically stable with

$$G_i = P_i^{-1}X_i, K_i = P_i^{-1}Y_i \quad (27)$$

if there exist matrices $\bar{P} = \text{diag}\{P_1, \ldots, P_N\} > 0$, $X_i, Y_i (i \in \underline{N})$, $\Gamma_i = \text{diag}\{\gamma_{i1}, \ldots, \gamma_{im}\} > 0 (i \in \underline{N})$ and scalars $\varepsilon > 0, \beta > 0, 0 < \mu < \varepsilon$ such that the following conditions hold.

$$\begin{bmatrix} \Xi_{11} & \bar{P}(I_N \otimes B) + \tilde{C}^T \Lambda \bar{\Gamma} \\ * & -2\bar{\Gamma} \end{bmatrix} < 0, \tag{28}$$

$$\begin{bmatrix} -\beta \bar{P} & (I_N \otimes I_n)\bar{P} + \bar{R}^T \\ * & -\bar{P} \end{bmatrix} < 0 \tag{29}$$

$$\ln\beta - \mu(t_k - t_{k-1}) \leq 0 \tag{30}$$

where $\bar{\Gamma} = \mathrm{diag}\{\Gamma_1, \ldots, \Gamma_N\}$,

$$\Xi_{11} = \mathrm{sym}\{\bar{P}(I_N \otimes A) + \bar{U}\} + \varepsilon \bar{P},$$

$$\bar{U} = (U_{ij})_{N\times N}, U_{ij} = \begin{cases} a_{ij} X_i H_j, \text{if } j \in \mathcal{N}_{ai}, \\ 0, \text{others}, \end{cases}$$

$$\bar{R} = (R_{ij})_{N\times N}, R_{ij} = \begin{cases} b_{ij} Y_i H_j, \text{if } j \in \mathcal{N}_{bi}, \\ 0, \text{others}. \end{cases}$$

Proof. Along similar lines of the proof of Theorem 1, the proof is straightforward and we omit it.

V. NUMERICAL EXAMPLES

To illustrate our main result in Theorem 1, we give the following example.

Example 1 Consider the following target

$$\dot{x}(t) = Ax(t) + B\varphi(Cx(t)) \tag{31}$$

where $x(t) = [x_1(t), x_2(t)]^T$,

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \ B = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.6 & 0.8 \end{bmatrix}, \varphi(s) = 0.5(|s+1| - |s-1|).$$

Suppose the system is observed using five sensors. The topology of the five sensors is represented by a graph $\mathcal{G}_a = (\mathcal{V}, \mathcal{E}_a, \mathcal{A}_a)$ with the set of nodes $\mathcal{V} = \{1,2,3,4,5\}$, the set of edges $\mathcal{E}_a = \{(1,1),(1,2),(1,3),(2,2),(2,5),(3,3),(3,4),(4,1),(4,4),(5,4),(5,5)\}$, and an adjacency matrix $\mathcal{A}_a = [a_{ij}]_{5\times 5}$ with adjacency elements $a_{ij} = 1$ if $(i,j) \in \mathcal{E}_a$; otherwise, $a_{ij} = 0$. It is assumed that the network topology at impulsive instants is represented by a graph $\mathcal{G}_b = (\mathcal{V}, \mathcal{E}_b, \mathcal{A}_b)$ $\mathcal{E}_b = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(3,1),(3,3),(4,1),(4,4),(5,1),(5,5)\}$, $\mathcal{A}_b = [b_{ij}]_{5\times 5}$ with adjacency elements $b_{ij} = 1$ if $(i,j) \in \mathcal{E}_b$; otherwise, $b_{ij} = 0$.

For each $i (i = 1,2,3,4,5)$, the model of sensor i is described by $y_i(t) = H_i x(t)$. It is assumed that

$$H_1 = \begin{bmatrix} 0.6 & 0.8 \end{bmatrix}, \ H_2 = \begin{bmatrix} 0 & 0.8 \end{bmatrix},$$

$$H_3 = \begin{bmatrix} 0.6 & 0 \end{bmatrix}, \ H_4 = \begin{bmatrix} 0.3 & 0.4 \end{bmatrix},$$

$$H_5 = \begin{bmatrix} 0.5 & 0.6 \end{bmatrix}.$$

For $\varepsilon_1 = 0.002, \varepsilon_2 = 0.06$ and $\beta = 1.005$, by solving a set linear matrix inequalities (28), (29) and (12) and using (27) the all parameters of the desired distributed estimators are obtained as follows.

$$G_1 = \begin{bmatrix} 0.5573 \\ 0.1 \end{bmatrix}, \ G_2 = \begin{bmatrix} 0.0256 \\ 0.1087 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} 0.8753 \\ 0.0086 \end{bmatrix}, \ G_4 = \begin{bmatrix} 0.3066 \\ 0.0704 \end{bmatrix},$$

$$G_5 = \begin{bmatrix} 0.6290 \\ 0.0015 \end{bmatrix}, \ K_1 = \begin{bmatrix} 0.0005 \\ 0.0015 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.0003 \\ -0.0027 \end{bmatrix}, \ K_3 = \begin{bmatrix} -0.0045 \\ 0.0002 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} -0.0016 \\ -0.0006 \end{bmatrix}, \ K_5 = \begin{bmatrix} -0.0045 \\ -0.0022 \end{bmatrix}.$$

Let $\omega = 1, \tau = 0.5, t_{l,k} - t_{l,k-1} = 0.1, t_{l,k} \in (l + 0.5, l + 1], l = 1,2,\cdots$, then it is easy to verify that the condition (30) is satisfied. By Theorem 1, the resulting estimation error dynamics is globally asymptotically stable. Denote $e_i(t) = [\ e_{i1}(t), e_{i2}(t)\ ]^T (i = 1,2,3,4,5)$, then estimation errors $e_{i1}(t)$ and $e_{i2}(t) (i = 1,2,3,4,5)$ are shown in Figs. 1-2, respectively.

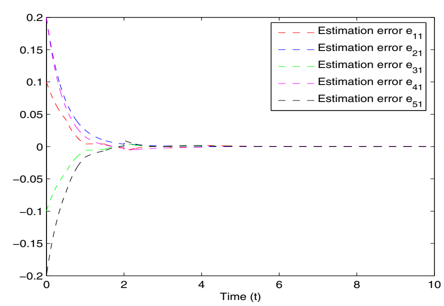

Fig. 1. Estimation error $e_{i1} (i = 1,2,3,4,5)$.

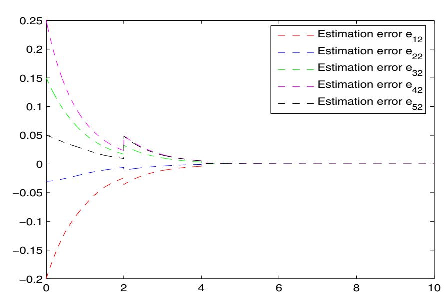

Fig. 2. Estimation error $e_{i2} (i = 1,2,3,4,5)$.

In the following example, we design distributed state estimators of hyperchaotic attractors in sensor networks where the effect of intermittent measurements is not taken into account.

Example 2 Consider the following hyperchaotic system which consists of two coupled Chua circuits [18]

$$\dot{x}(t) = Ax(t) + B\varphi(Cx(t)) \tag{32}$$

where $x(t) = [x_1(t), x_2(t), x_3(t), x_4(t), x_5(t), x_6(t)]^T$,

$$A = \begin{bmatrix} -18/7 & 9 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -14.28 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -18/7 & 9 & 0 \\ 0 & -0.01 & 0 & 1 & -0.99 & 1 \\ 0 & 0 & 0 & 0 & -14.28 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 27/7 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 27/7 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and $\varphi(Cx(t)) = [\varphi_1(x_1(t)), \varphi_2(x_4(t))]^T = [\ 0.5(|x_1+1|-|x_1-1|), 0.5(|x_4+1|-|x_4-1|)]^T$. The hyperchaotic behavior of system (32) is shown in Fig. 3 (projection onto the $x_1 - x_4$ plane), where the initial state is $x(0) = [\ -0.2, -0.2, -0.33, 0.2, 0.9, 0.33\ ]^T$.

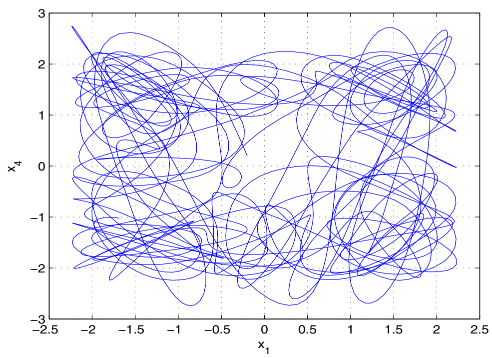

Fig. 3. Double-double scroll attractor (projection onto the $x_1 - x_4$ plane).

Suppose the system is observed using three sensors. $\mathcal{G}_a = \mathcal{G}_b = \mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, $\mathcal{V} = \{1, 2, 3\}$, $\mathcal{E} = \{(1,1), (1,3), (2,2), (2,3), (3,3)\}$, $\mathcal{A} = [a_{ij}]_{3\times 3}$ with adjacency elements $a_{ij} = 1$ if $(i,j) \in \mathcal{E}$; otherwise, $a_{ij} = 0$.

For each $i (i = 1, 2, 3)$, the model of sensor i is described by $y_i(t) = H_i x(t)$. It is assumed that

$$H_1 = \begin{bmatrix} 1 & 0 & 0.2 & 0 & 0 & 0.3 \\ 0.1 & 0 & 0 & 0.8 & 0.5 & 0 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0.1 & 0.1 & 0 & 0 & 0.3 & 0 \\ 0 & 0.2 & 0 & 0.5 & 0 & 0.4 \end{bmatrix},$$

$$H_3 = \begin{bmatrix} 0.3 & 0 & 0.1 & -0.1 & 0.2 & 0 \\ 0 & 0.1 & 0.2 & 0.4 & 0 & -0.1 \end{bmatrix}.$$

For $\varepsilon = 0.01$ and $\beta = 1.05$, by solving a set linear matrix inequalities (28) and (29) and using (27) the all parameters of the desired distributed state estimators are obtained as follows.

$$G_1 = \begin{bmatrix} 5.0072 & 0.6762 \\ -1.3099 & -0.2656 \\ 1.9691 & -1.8984 \\ 0.5315 & -7.6821 \\ 0.5813 & -1.1367 \\ 0.4409 & 3.9373 \end{bmatrix}, G_2 = \begin{bmatrix} -55.9316 & 1.3165 \\ -8.7175 & 0.9042 \\ 28.5593 & 0.1556 \\ -18.6043 & -22.8729 \\ 1.4881 & -1.6460 \\ -1.9449 & 0.1179 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} -39.3244 & 4.0852 \\ -4.3197 & 2.5900 \\ 10.1681 & 7.3714 \\ 6.3696 & -41.5416 \\ -0.4463 & -4.4644 \\ -7.3122 & 23.5263 \end{bmatrix}, K_1 = \begin{bmatrix} -0.8988 & -0.1667 \\ -0.0283 & 0.0031 \\ 0.3005 & -0.0399 \\ 0.0369 & -0.8024 \\ 0.0061 & -0.0648 \\ -0.0440 & 0.3121 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -3.5330 & -0.2126 \\ -0.5255 & -0.0578 \\ 1.5309 & 0.0999 \\ -0.1571 & -1.3966 \\ -0.0320 & -0.0166 \\ -0.2215 & -0.0972 \end{bmatrix}, K_3 = \begin{bmatrix} -3.2872 & -0.6557 \\ -0.2304 & 0.0303 \\ 0.1437 & 0.0938 \\ -0.0356 & -2.2391 \\ -0.2355 & -0.1995 \\ -0.2041 & 1.2626 \end{bmatrix}.$$

Let $t_k - t_{k-1} = \theta = 5$, then it is easy to verify that the condition (30) is satisfied. By Corollary 1, the resulting estimation error dynamics is globally asymptotically stable. Denote $e_i(t) = [\ e_{i1}(t), e_{i2}(t), e_{i3}(t), e_{i4}(t), e_{i5}(t), e_{i6}(t)\ ]^T (i = 1, 2, 3)$, then estimation errors $e_{ij}(t) (i = 1, 2, 3, j = 1, 2, 3, 4, 5, 6)$ are shown in Figs. 4-9.

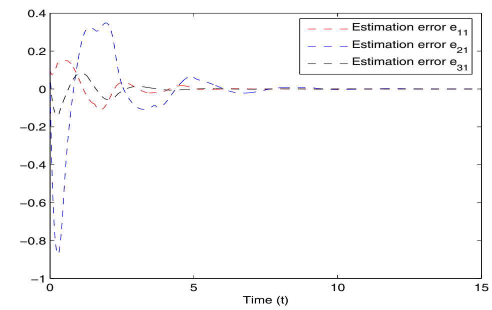

Fig. 4. Estimation error $e_{i1} (i = 1, 2, 3)$.

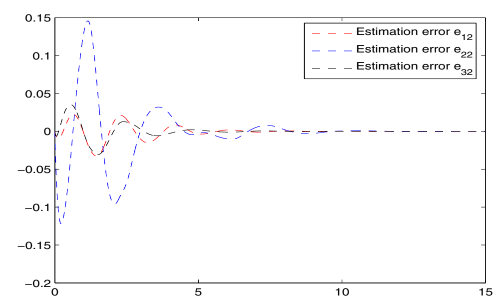

Fig. 5. Estimation error $e_{i2} (i = 1, 2, 3)$.

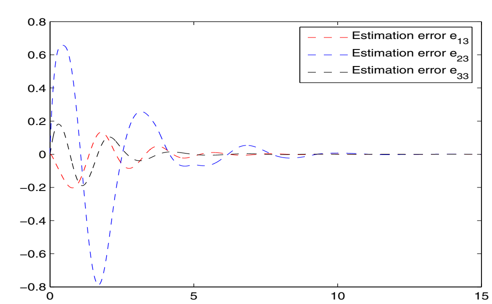

Fig. 6. Estimation error $e_{i3} (i = 1, 2, 3)$.

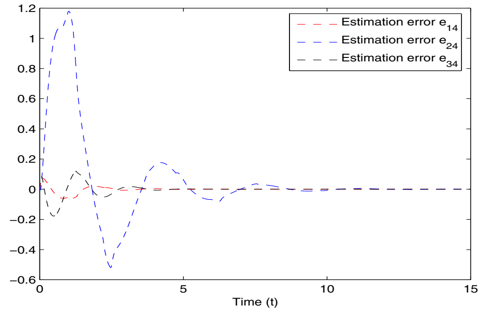

Fig. 7. Estimation error $e_{i4}(i = 1, 2, 3)$.

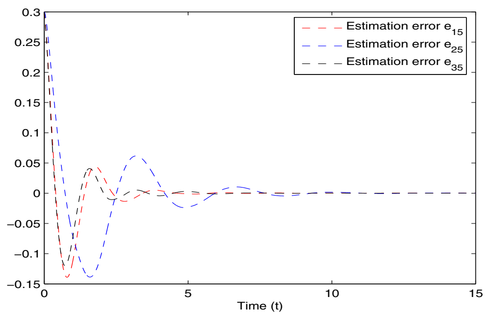

Fig. 8. Estimation error $e_{i5}(i = 1, 2, 3)$.

VI. CONCLUSIONS

A distributed state estimation scheme for Lur'e systems in sensor networks with impulsive effects and intermittent measurements has been investigated. Sufficient conditions have been given for global asymptotical stability of the estimation error dynamics in the network. An interesting problem for future research is to find a distributed state estimator design method when the sensor networks have switching interconnection topology.

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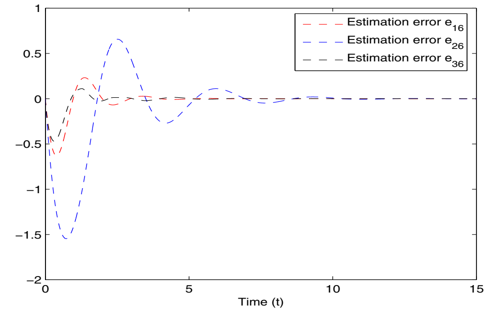

Fig. 9. Estimation error $e_{i6}(i = 1, 2, 3)$.

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