Self-Tuning Dynamic Matrix Control of Two-Axis Autopilot For Small Aeroplanes

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***Abstract*— An easy-to-implement method of self-tuning of 2-axis autopilot parameters using Dynamic Matrix Control (DMC) is presented. Based on step responses caused by a hysteresis relay feedback from roll/pitch angles, the inner (angular velocity) loop controllers are designed. Applying the step responses of the inner loop, the same method is used to design the outer (roll/pitch angle) controllers. Then the airspeed/vertical-speed controller is designed by recording the step responses to changes of thrust and pitch angle using the multivariable (2x2) DMC controller design method. Finally the directional (heading) and altitude control can be achieved by a simple controller. The simulations of an extremely sensitive (in pitch) and marginally stable airplane demonstrate the applicability of the method and confirm that the proposed system can perform advanced functions, which surpasses conventional 2-axis autopilots commonly used in light airplanes or UAVs by far, without increasing the complexity of the controller part.**

Keywords—Self-tuning conntollers; aerospace control; aerospace simulation

I. INTRODUCTION

In 1912, only 9 years after the first ever manned flight, Sperry Corporation developed and flew a gyro-compass based autopilot, which controlled the airplane. Through the years, an autopilot has become an invaluable tool for aviators, allowing pilots to focus on critical tasks while the piloting itself is handled by the machine. In fact, commuter aircraft nowadays cannot be certified without an elaborate autopilot system. However, it is Unmanned aerial vehicles (UAVs) that have the largest degree of autopilot functionality, since the system needs to react properly also in case of eventual control actuator or control surface failures. The trend is clear - to bring capable, but user friendly autopilot systems into personal, sport-class airplanes as well. Here, pilots/owners would benefit from the functionality if they could install the autopilot system themselves and be able to tune it "perfectly" as well.

Having gained experience with autopilot tuning, based on a 3-axis motion platform ([1,2,3,4]) and understanding the advantages as well as limitations of this approach, the research begun to focus on a software/hardware solution, which would be able to carry out the tuning procedure in-flight, automatically. While not being suitable for initial autopilot tuning of pilotless airplanes, the proposed principle brings clear advantage for remote-piloted and commercial applications, namely autopilot systems which traditionally require a technician with a-priori knowledge in order to set up the autopilot's functioning properly. With the proposed approach, based on [5], users themselves, without technical knowledge, would be able to perform autopilot tuning, including advanced control loop tuning with gain scheduling and fuzzy-calculation of parameters across the airplane's operational envelope with a simple, safe set of tasks and maneuvers. In section 1, the paper presents the proposed method of autopilot parameter self-tuning. Section 2 describes the self-tuning algorithm, followed by simulation results on a simplified aircraft- and full non-linear aircraft model in the Section 3. Analysis of results and future possibilities, as well as technological implications of the proposed approach to autopilot tuning, are given in Conclusion.

II. MODEL AND METHOD

A. Model of the aircraft

A simple two-axis autopilot is chosen, as this is the most commonly used autopilot architecture for small aircraft. There are two actuators - one for ailerons, controlling aircraft's roll, one for the elevator, controlling aircraft's pitch. This is enough to achieve aircraft's guidance for both dimensions of flight i.e. heading and altitude, by utilizing cascade control loops where the roll ultimately controls yaw (direction) and pitch controls altitude. It needs to be noted that commercially available autopilot systems feature relatively slow responses, to yield smooth flight, but this is not always desirable and usually arises as the effect of having tuned the autopilot to an aircraft only at one given set-point in the operational envelope (usually dictated by airspeed and load). By automated tuning the individual control loops a more precise, rapid response can be achieved, as multi-point tuning can be realized with ease.

The proposed method comprises two phases, the excitation phase where certain control inputs are performed and airplane's response observed, and the computational phase where the parameters for Dynamic Matrix Control are determined. There are no reservations towards using the method either in a simulated environment using an appropriate airplane dynamic model or on a real flying airplane.

This paper demonstrates the proposed method using simulations on an advanced six degrees-of-freedom aircraft dynamic model. Equations of aircraft motion were used to produce a non-linear model tailored to the proportions and specifications of a fast, heavy UAV design. A full set of equations of motion [3] are as follows in (1) through (12):

$$\dot{V} = \frac{1}{m} \cdot \left(F_x \cos\alpha\cos\beta + F_y \sin\beta + F_z \sin\alpha\cos\beta\right) \quad (1)$$

$$\dot{\alpha} = \frac{1}{Vm\cos\beta} \cdot \left(-F_x \sin\alpha + F_z \cos\alpha\right) + q - \\ -(p\cos\alpha + r\sin\alpha)\tan\beta \quad (2)$$

$$\dot{\beta} = \frac{1}{Vm} \cdot \left(-F_x \cos\alpha\sin\beta + F_y \cos\beta - F_z \sin\alpha\sin\beta\right) \\ + p\sin\alpha - r\cos\alpha \quad (3)$$

$$\dot{p} = P_{pp}p^2 + P_{pq}pq + P_{pr}pr + P_{qq}q^2 + \\ + P_{qr}qr + \mathrm{P}_{rr}r^2 + P_1 L + P_m M + P_n N \quad (4)$$

$$\dot{q} = Q_{pp}p^2 + Q_{pq}pq + Q_{pr}pr + Q_{qq}q^2 + \\ + Q_{qr}qr + Q_{rr}r^2 + Q_1 L + Q_m M + Q_n N \quad (5)$$

$$\dot{r} = R_{pp}p^2 + R_{pq}pq + R_{pr}pr + R_{qq}q^2 + \\ + R_{qr}qr + R_{rr}r^2 + R_1 L + R_m M + R_n N \quad (6)$$

$$\dot{\psi} = \frac{q\sin\varphi + r\cos\varphi}{\cos\theta} \quad (7)$$

$$\dot{\theta} = q\cos\varphi + r\sin\varphi \quad (8)$$

$$\dot{\varphi} = p + \dot{\psi}\sin\theta \quad (9)$$

$$\dot{x}_e = u\cos\theta\cos\psi + v(\sin\varphi\sin\theta\cos\psi - \\ -\cos\varphi\sin\psi) + w(\cos\varphi\sin\theta\cos\psi + \\ +\sin\varphi\sin\psi) \quad (10)$$

$$\dot{y}_e = u\cos\theta\sin\psi + v(\sin\varphi\sin\theta\sin\psi - \\ -\cos\varphi\cos\psi) + w(\cos\varphi\sin\theta\sin\psi + \sin\varphi\cos\psi) \quad (11)$$

$$\dot{H} = -(-u\sin\theta + v\sin\varphi\cos\theta + \\ + w\cos\varphi\cos\theta) \quad (12)$$

Where:

V	- aircraft velocity (speed)
α	- angle of attack
β	- sideslip angle
p	- angular velocity around x axis (roll rate)
q	- angular velocity around y axis (pitch rate)
r	- angular velocity around z axis (yaw rate)
φ, θ, ψ	- yaw, pitch, roll
F_x, F_y, F_z	- forces in direction of x, y, z axis
L, M, N	- torques around x, y, z axis
P_{ii}, Q_{ii}, R_{ii}	- constants which are derived from aircraft proportions and specifications.
x_e, y_e, H	– position of the aircraft in the Earth coordinate system

Aerodynamic properties are computed separately and inserted in equations in scope of forces.

B. DMC controller

Dynamic matrix control (DMC) algorithm was developed by Cutler and Ramaker [5] and is one of many Model Predictive Control algorithms [6, 7].

DMC uses finite step response model to predict plant output. The derivation of the algorithm is based on the optimization of the cost function which takes into account the sum of squared errors between the predicted plant output $\hat{y}(k+j|k)$ and the demanded reference trajectory $w(k+j)$ for prediction horizon (N_P) sampling intervals ahead. The cost function can also take into account weighted squared predicted moves of the controller output $\Delta u(k+j-1)$ for control horizon (N_C) sampling intervals ahead:

$$J = \sum_{j=1}^{N_P} (\hat{y}(k+j|k) - w(k+j))^2 + \lambda \sum_{j=1}^{N_C} (\Delta u(k+j-1))^2 \quad (13)$$

where k is current sampling interval. This optimization is analytically solvable as a least squares problem:

$$\boldsymbol{u}_\Delta = (\boldsymbol{G}^T\boldsymbol{G} + \lambda\boldsymbol{I})^{-1}\boldsymbol{G}^T(\boldsymbol{w} - \boldsymbol{f}) \quad (14)$$

where $\boldsymbol{G}$ is dynamic matrix formed by coefficients of step response model, $\boldsymbol{w}$ is vector of demanded reference trajectory, $\boldsymbol{f}$ is vector of predicted free response (response of plant if no input move would be applied) and $\boldsymbol{u}_\Delta$ is vector of optimal control output increments. At one sampling interval only the first control output increment is applied to the plant although N_C of moves are calculated. In next sampling interval a new control output increment vector is calculated. The λ parameter is weight for control increment in cost function.

The same idea for control can also be used for multiple input multiple output plants [8]. The multivariable algorithm is based on a similar cost function and it uses a finite step response models from each input to each output to construct the dynamic matrix $\boldsymbol{G}$ for the control law. Also in this case the optimization problem is solvable using least square method.

III. Self-tuning Procedure And Results Of Simulation

In this Section the self-tuning procedures for directional and altitude control respectively will be presented. The procedures are very similar within the first part: designing the inner and outer loop cascade control for angles (roll and pitch) as responses to the actuators (ailerons and elevator movements). As for the directional control, the roll angle indirectly effects the direction of the flight and only a simple controller is needed, as explained at the end of the section. Altitude control is more complex, it is multivariable and involves two inputs (thrust and pitch angle) and two outputs (airspeed and vertical speed). The altitude controller (computing vertical speed from desired and actual vertical positions) can be simple again.

A. Directional control

As stated in Section II.B, step responses are needed for DMC controller design. As the aircraft in question is open loop marginally stable process, the pilot can not afford to fly for longer times with the control surfaces deflected. Hence, a modified procedure for capturing step responses is presented in Fig. 1. The pilot establishes a stable horizontal flight. Then a 1 degree relay excitation of aileron deflection with respect to a roll angle with ±25 degrees hysteresis is performed.

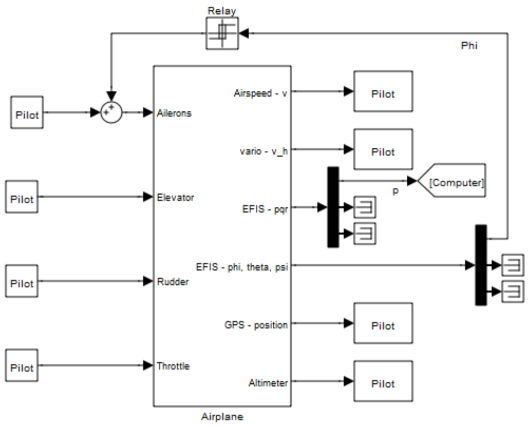

Fig. 1. Roll excitation scheme

The recorded open loop roll rate responses as obtained during relay excitation for different airspeeds are presented in the upper part of Fig. 2. One can see that with lower roll rates, the airplane takes longer to reach the 25 degree bank. The recorded open loop roll rate responses are used for the inner-loop DMC controller design. The closed inner-loop step responses are calculated numerically in two steps. In the first step, the step response of the open inner loop is calculated as the convolution of already recorded aircraft step responses and the designed controllers' impulse response. In the second step the closed loop response is calculated from the open-loop responses. So obtained close loop responses are presented in the lower part of Fig. 2 and are used for the design of the outer

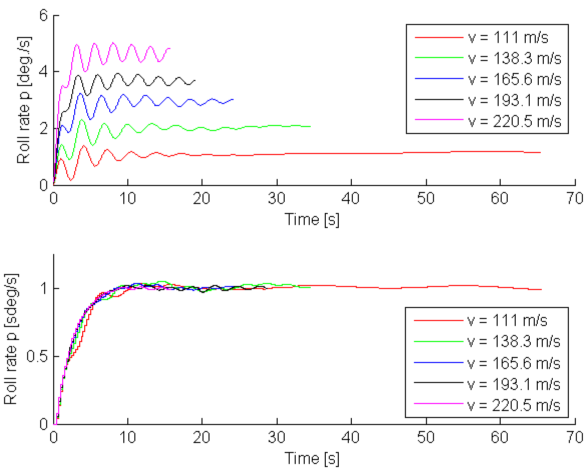

Fig. 2. Roll rate step responses for different airspeeds: open loop (upper) and closed inner loop (lower)

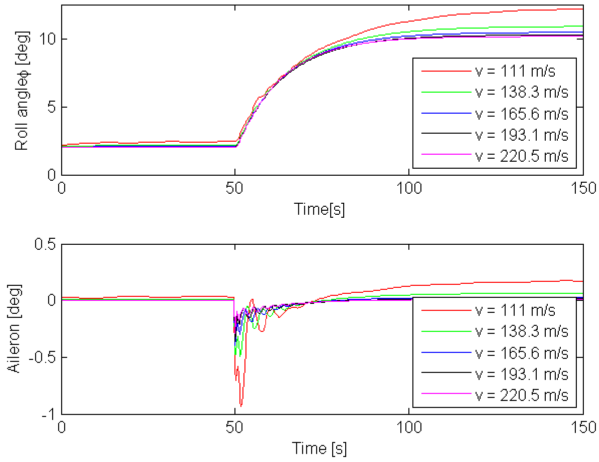

Fig. 3. Roll angle step responses (upper) and corresponding aileron movements (lower)

loop DMC controller. The results of the outer-loop roll controller design are presented in Fig. 3 (step response in the upper, corresponding ailerons movement in the lower part). After having achieved a stable aileron-roll angle behavior, directional control (heading) can be achieved by a simple controller as shown in Fig.11. This completes the design of the direction auto-tuning control.

One of the obvious advantages of such a self-tuning two-stage method, besides the fact that the operator does need to possess strong skills in control theory to tune the control loop parameters, is the ability to fine-tune the roll angle responses by forcing trajectories to track a certain reference model trajectory. Implicitly the operator can determine the character of the autopilot behavior (faster response, slower response with no overshoots!) and thus achieve constant airplane's responses to autopilot commands through the operational envelope.

B. *Altitude control*

With altitude control, the design of the pitch controller is analog to the roll controller design. The excitation scheme is given in Fig. 4 and corresponding results are presented in Fig. 5 (inner loop) and Fig. 6 (outer loop).

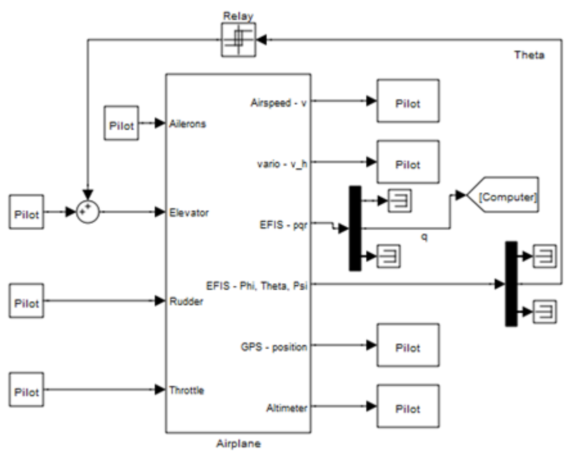

Fig. 4. Pitch excitation scheme

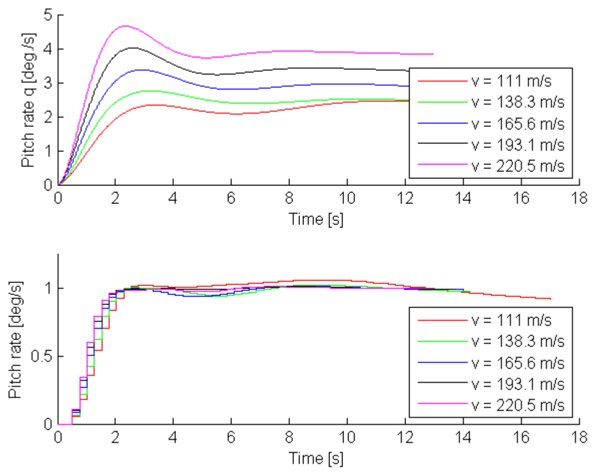

Fig. 5. Pitch rate step responses for different airspeeds: open loop (upper) and closed inner loop (lower)

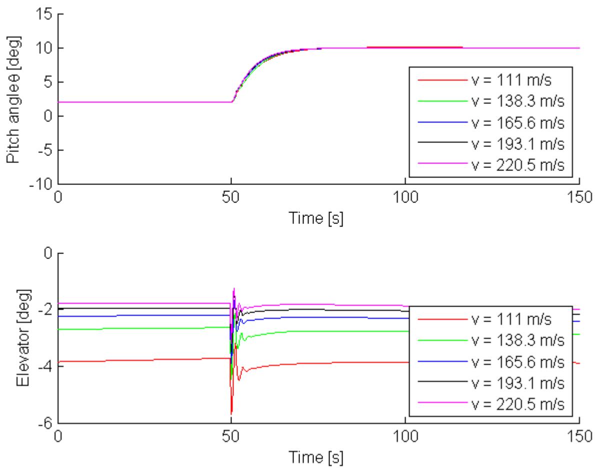

Fig. 6. Pith angle step responses (upper) and corresponding elevator movements (lower)

It needs to be stressed that altitude of an aircraft is influenced by both its pitch and thrust and that the maximal produced thrust of the engines is 15,000N. With two factors to the open loop response, a different open loop excitation scheme was produced, Figure 7. The procedure itself is however similar - as before, with the pilot having established stable flight, a change is made, but this time separately in thrust level (750N) as well as a in pitch angle (3 degrees). The corresponding responses for each are presented in Fig. 8. They are also used for the multivariable DMC controller design, which is able to control the airplane's vertical motion with or without airspeed changes. This is of vital importance as the airplane may lose lift after producing a strong climb, should this not be supported by increase of thrust from the engines. Similarly, when in a strong descent, the airplane could over-speed and damage itself with no decrease in trust through the maneuver. The ability to be able to control vertical speed and forward (airspeed) of the airplane at the same time, yields robust behavior in vertical maneuvers.

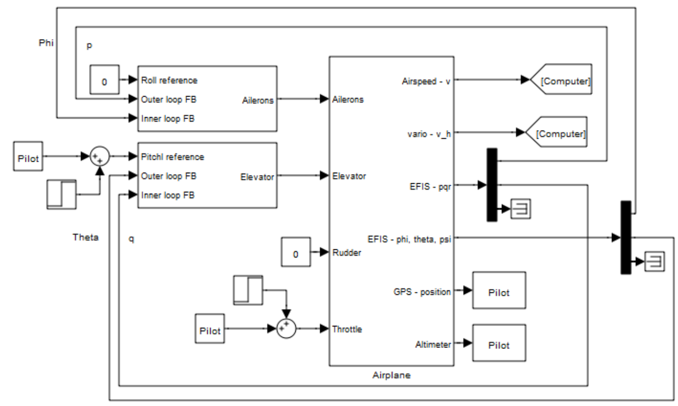

Fig. 7. Throttle and pitch angle excitation scheme

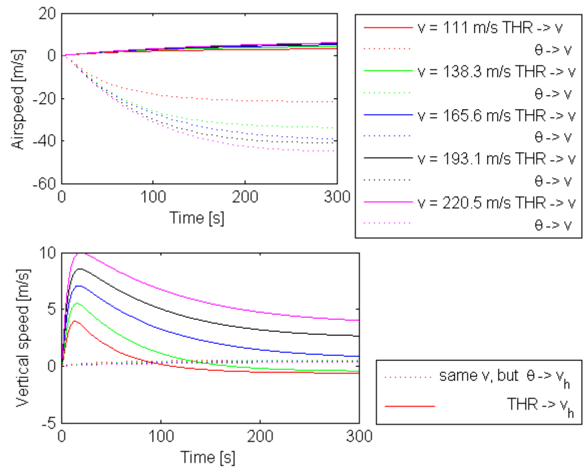

Fig. 8. Airspeed (uper) and vertcal speed (lower) step responses to 750 N throttle (solid) and 3 degree pich angle (dotted) for for different airspeeds

The results of this design are presented in Figs. 9 and 10. Fig. 9 depicts the airspeed (upper) and vertical speed (lower)

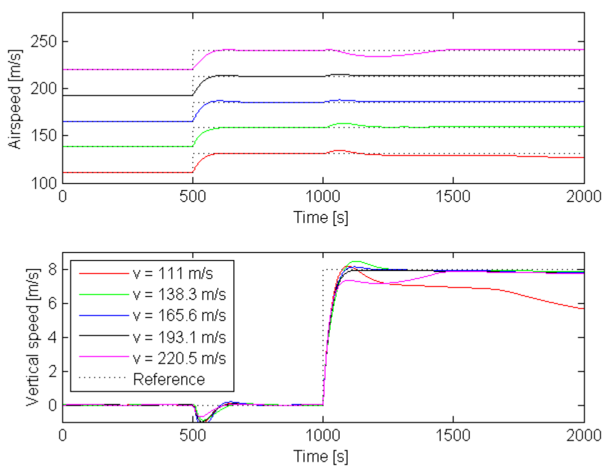

Fig. 9. Airspeed (upper) and vertical speed (lower) responses for a 20m/s airspeed reference change at 500s and a 8m/s vertical speed reference change at 1000 s.

responses to a 20m/s airspeed reference change at 500s and a 8m/s vertical speed reference change at 1000 s. In Fig. 10 the corresponding actuator (thrust, elevator) actions a can be seen. It should be noted that the steady state error of airspeed and vertical speed at airspeed 111m/s (Fig. 9) is due to the fact that even with full thrust (15,000 N), the commanded airspeed is too low to ensure enough lift force for the required vertical speed. All force provided by the engine is used to compensate the drag caused by a very high angle of attack. This is, of course not a favorable flight profile, and should not be commanded by appropriate reference values settings. The controller is certainly doing what it can, within the prescribed requirements and limitations.

The altitude control can be achieved by a simple controller as shown in Fig. 11. This figure represents the final 2 axis autopilot control scheme. Simple controllers for heading and altitude are P controllers with saturations which are defined by maximum bank and vertical speed. The only jobs of the pilot are the settings of heading, altitude and airspeed references and the observation of the position given by the GPS.

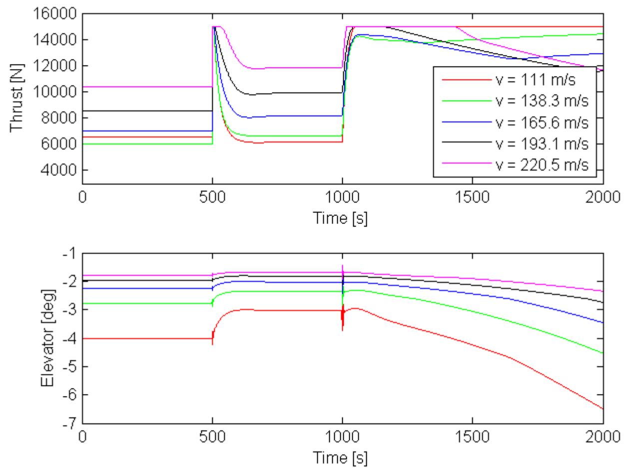

Fig. 10. Actuator's (thrust – upper, elevator –lower) responses for reference changes in Fig. 9

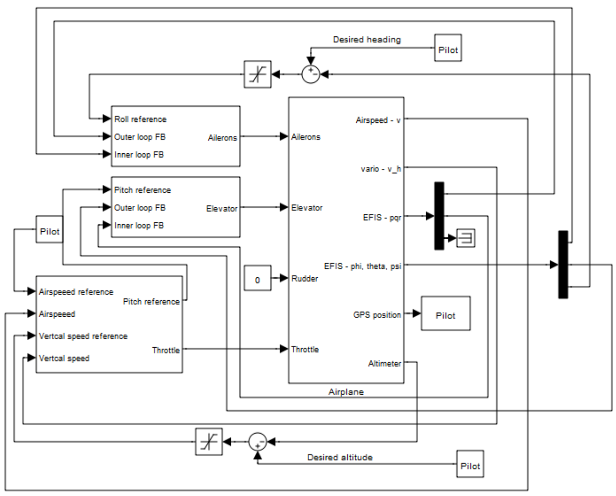

Fig. 11. The final 2 axis autopilot control scheme

The results of a manoeuver for various airspeeds are shown in Figs. 12, 13, 14 and 15. As 5 m/s maximum vertical speed (climbing velocity) and 20 degrees maximum bank were chosen, heading change of 180 degrees and altitude change of 450 m represent most critical choice since the heading and altitude manoeuvers start and end simultaneously. In this case the coupling between the longitudinal and lateral motions, not observed during the design procedure, comes to effect. It can be observed that at the start of the manoeuver the change in bank (depicted in Fig. 12) leads to the loss of lift and a small oscillation in vertical speed (Fig.13 lower). At the end of the manoeuver the opposite is the case: the reestablishment of the level flight increases the lift force and causes an overshoot of the altitude (Fig.13 upper).

In Fig.14 the corresponding airspeeds (which are supposed to remain constant) and thrust are depicted, while in Fig. 15 the corresponding ailerons and elevator movements can be seen.

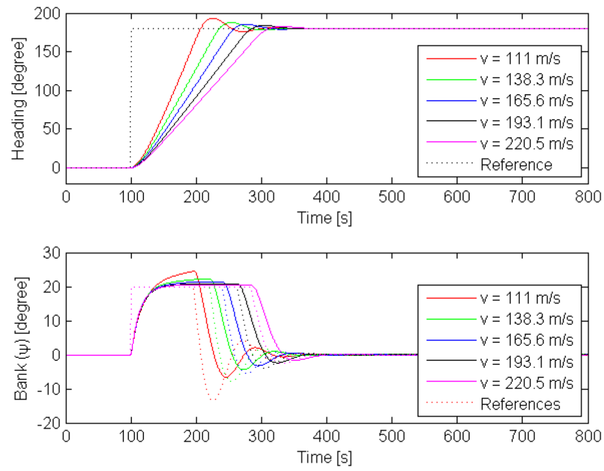

Fig. 12. A 180 degrees heading change maneouver: heding (upper) and bank (lower) – simultaneously with the altitude change depicted in Fig. 13.

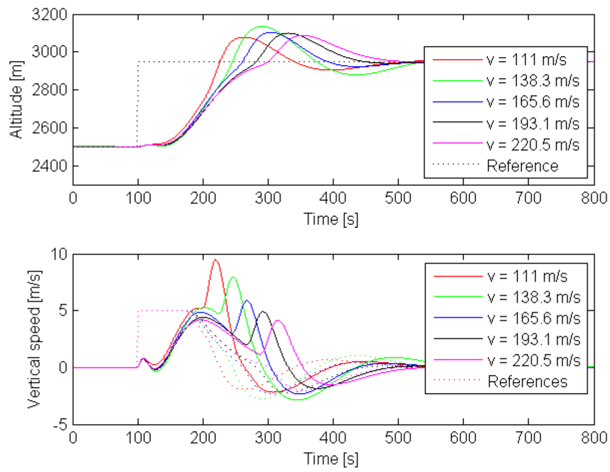

Fig. 13. A 450 m altitude change maneouver: altitude (upper) and vertical speed (lower) – simultaneously with the altitude change depicted in Fig. 12.

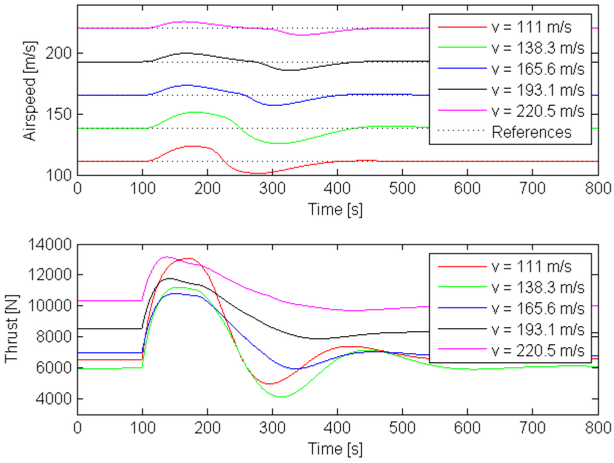

Fig. 14. The airspeed (upper) and thrust (lower) for the manoeuver depicted in Figs. 12 and 13.

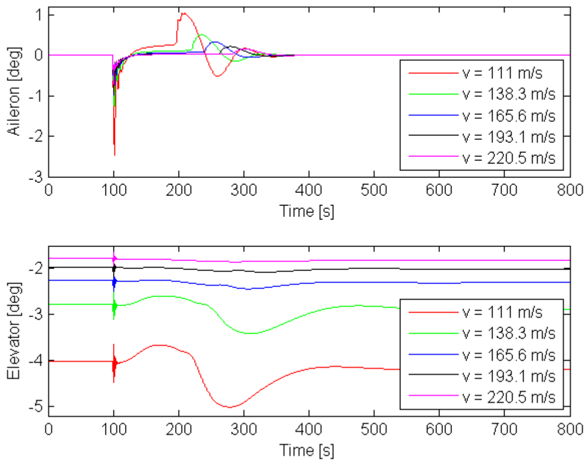

Fig. 15. The time course of ailerons (upper) and Elevator (lower) for the manoeuver depicted in Figs. 12 and 13.

As evident from the figures, the multivariable controller performs reasonably well, in managing forward speed and vertical speed (altitude) when it comes to commanded changes. It has to be noted the airplane modeled is extremely sensitive in pitch, and marginally stable. The controller is tuned to the degree where changes are not measurable any more by conventional off-the-shelf sensors. Light airplanes have much more humble dynamics; however the treated model was chosen to test the performances of the algorithm in a difficult case.

IV. Conclusion

A novel principle of autopilot self-tuning utilizing a two-phase approach and DMC control was demonstrated. Although a simple 2-axis autopilot by its architecture, the proposed system can perform advanced functions, which surpasses conventional 2-axis autopilots commonly used in light aeroplanes or UAVs by far, without increasing the complexity of the controller part.

Operator's experience and success when it comes to tuning controllers for marginally stable, multivariable systems, such as the discussed aeroplane, may vary significantly. Therefore, it is important to note that the proposed method decreses the importance of operators knowledge, and practically self-tunes the controller with operators intervening limited to monitoring aeroplanes behavior before and during the exctitaion phase, so as to prevent safe-flying envelope excursions. The rest of the procedure is carried out automatically by the routine.

The result is an autopilot, capable of flying the aeroplane along desired headings and vertical as well as horizontal airspeeds. What is more, the quality, speed and character of the response can be imposed on the autopilot by forcing a certain output trajectory to be tracked by the outside cascade loops.

Acknowledgment

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