Dominant Three Pole Placement in PID Control Loop with Delay

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Abstract — **In applying the pole assignment to tuning the PID controllers the usual presence of delay in the control loop brings about an infinite order character of the system dynamics, i.e. an infinite spectrum of poles. Therefore any pole placement can result in the desired tuning of the control loop only if the prescribed and placed poles really become the dominant poles of the control loop dynamics. With respect to three parameters of PID controller just three poles can be placed by the assignment and a dominance guarantee of their prescription is crucial in this way of tuning. A novel method of selecting a trio of numbers with an equal real part to make them the dominant poles of the control loop is dealt with in the paper with an additional minimizing the absolute error integral. An original assessment is introduced to check the dominance of the pole placement and an optimum of relative damping of the response is assessed to minimize the control error integral. The quality of the disturbance rejection response is taken as the decisive criterion in the presented design of the time delay plant control.**

Keywords —system dominant pole placement;PID controller tuning; ultimate frequency; absolute error integral criterion

I. Introduction

The pole assignment is widely applied not only to the state space system design but also to tuning PID controllers in the last two decades. Since the PID controller principle admits an assignment of just three poles only, assumptions of sophisticated, detailed or higher-order process models turn out to be unrealistic. On the other hand the time delay effect can be considered as a general process property, but its presence leads to the appearance of an infinite spectrum of control loop poles, and to a need to investigate their dominance as a crucial issue in the pole placement.

The dependence of the dominant pair of closed-loop poles on the controller parameters was first investigated by Hwang and Chang [1] by means of the Taylor series expansion about the critical gain. Instead of dominant, the term "leading poles" was used in that paper. Dominant pole placement design was introduced somewhat differently by Persson and Åström [2] and was further explained in Åström and Hägglund [3]. At about the same time, Hwang and Fang [4] published an extensive optimization study on dominant pole placement for first and second order time delay plants. Numerous methods with modified specifications of tuning conditions were presented subsequently, and a survey was presented by O'Dwyer [5].

Applying the pole assignment approach to systems with a delay (i.e. with an infinite spectrum of poles) has led to the specific problem of how to select a proper prescription of pole positions so that they will be capable candidates for becoming dominant poles that really do determine the behavior of the system. Any pole placement in a time delay system is always connected with a risk that, although the prescribed poles are achieved in the system spectrum, they may lose any meaning because some other poles spontaneously assume the dominant role in the infinite system spectrum. Consequently, any result of a pole assignment of this kind can be approved as valid only after checking that the placed poles really have assumed the dominant positions. To the best of the authors' knowledge, no general theorem is yet available that guarantees in advance that a chosen prescription of poles for a time delay system will reliably result in the group of system dominant poles.

The pole placement approach in a control loop with delay is to be considered only for placing a small group of dominant poles – either a complex conjugate pair or a three-pole group, usually one pair with a real pole. The key issue for this design is to select the prescribed poles in a way that guarantees their dominant position [6]. Because of the number of three-controller parameters, only the three-pole $p_{1,2,3}$ option can reasonably be prescribed in applying the pole assignment to tuning the PID controller for a time delay plant. Besides, in a considerable number of papers, the dominant pole placement in PID tuning has also been considered for a single pair of complex conjugate poles $p_{1,2} = -\alpha \pm j\Omega$, $\alpha > 0$, assigned to take up the rightmost position in the system spectrum and satisfying an additional requirement of the frequency response specification of control synthesis [7], [8]. The case of placing three prescribed poles, a complex conjugate pair $p_{1,2}$ and a real pole $p_3 = -\beta$, as dominant was solved by Hwang and Fang [4]. A guarantee of dominance in pole placement based on the root locus and Nyquist plot applications was presented by Wang et al. [6]. A minimization of IAE criterion by finding a maximum of controller integration gain is presented in [10].

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The contribution of our work we envisage in the following three aspects. First, unlike the usual approach the tuning is focused on the disturbance rejection as usually the primary task in industrial process control. Secondly, absolute error integral (IAE) minimization is applied simultaneously with checking the dominance of the assigned poles. Finally, the use of the same value of real parts for all the three poles is proved as well-preventing the assignment from the loss of dominance.

The rest of the paper is organized as follows. In Section II the selection of the prescribed candidate trio of dominant poles is discussed. The issue how the used plant model hits off the real process properties is dealt with in Section III and the employment of ultimate frequency in the control loop design is presented in Section IV. The procedure of placing the poles itself is described in Section V and a relative damping optimization with a novel approach to dominance proof as well as an application example are presented in Section VI. The concluding remarks are added in Section VII.

II. SELECTING THE CANDIDATE TRIO OF DOMINANT POLES

In order really to achieve a dominant positions of the poles to be placed the prescribed values of $p_{1,2,3}$ are to be selected with a careful respect to the specific dynamic properties of the plant controlled. As Åström and Hägglund [3] revealed particularly the ultimate frequency ω_k of the control loop has to be taken into account. The ultimate frequency at which the relay feedback control loop is oscillating determines the bounds within which the attainable frequency of the control response can be expected. Let the following three poles be considered for the pole assignment

$$p_{1,2} = -\alpha \pm j\Omega = \Omega(-\delta \pm j), \qquad p_3 = -\kappa\alpha = -\delta\kappa\Omega \tag{1}$$

where α, β, Ω are supposed positive and introducing the ratios $\delta = \alpha / \Omega$ (relative damping) and $\kappa = \beta / \alpha$ helps to characterize the $p_{1,2,3}$ group. The assignment of $p_{1,2,3}$ can be accepted as valid only if none of the infinitely many poles of the rest of system spectrum appears to the right of them. Note that the following consideration serves only for a preliminary comparison of various versions of $p_{1,2,3}$ group, not for a particular assessment of the response. Let a simplified plant model be introduced in the following form of second-order differential equation with delay

$$\frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = K\, a_0\, u(t-\tau) \tag{2}$$

$a_{0,1} > 0, \tau > 0$, fitting only the dominant modes of the plant. The ability of this model to hit off the dynamics of the plant is discussed in the next section and the number of its parameters well corresponds to the second-order principle of PID.

Lemma 1. Consider a *stable* time delay plant as in (2) and assume that by means of a PID controller setting such a pole placement of $p_{1,2,3}$ prescribed as in (1) has been achieved that none of the other poles substantially influence the control loop response. On this assumption of a *perfect dominance* of $p_{1,2,3}$ the *disturbance rejection* step response of the PID control loop on (2) may be preliminarily considered in the following three-term reference form

$$h(t+\tau) = \exp(-\alpha t)\left[C_1 \cos \Omega t + C_2 \sin \Omega t\right] + C_3 \exp(-\beta t) \tag{3}$$

for $t \geq \tau$, where the coefficients $C_{1,2,3}$, with respect to the disturbance derivative in the control loop equation, satisfy the initial conditions $h(0) = 0,\ h'(0) = 0,\ h''(0) = -p_1 p_2 p_3$. Then if $\kappa \leq 1$, the absolute error integral $I_{AE} = \int_0^\infty |h(t)|\, dt$ of response (3) is independent of the parameters δ, Ω, κ and equal to 1, and if $\kappa > 1$, it is *larger than* 1.

Proof. A simple proof of this Lemma can be found in [9].■

The value of κ influences the response (3) in the way illustrated in Fig. 1. The choice of $\kappa < 1$ brings about an over-damped response with dominating role of p_3 with a rather sluggish rejection of the disturbance. On the other hand values of $\kappa > 1$ bring about an enhanced risk of worsening of dominance of $p_{1,2,3}$ or even of its loss. Consequently the choice of $\kappa = 1$ can be preferred for the three pole assignment from the viewpoint of the disturbance rejection.

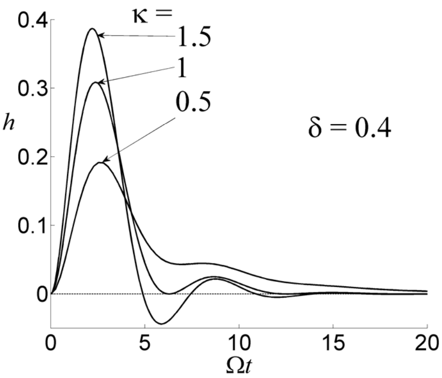

Fig. 1. Influence of κ on the shape of disturbance rejection response (3).

In fact however, due to the delay in the loop the disturbance rejection cannot start until $t = 2\tau$ and function (3) holds only for $t \geq 2\tau$, while in the interval $t \in \langle 0, \tau \rangle$ $h(t+\tau)$ in (3) is given only by the step response $h_0(t+\tau)$ of the plant itself. Then the "*perfectly dominant*" $h(t+\tau)$ is to be considered with nonzero initial conditions in $t = \tau$, given by the step response $h_0(t+\tau)$ of the plant itself (without the controller impact). These initial conditions are as follows: $h(2\tau) = h_0(2\tau) = a$, $h'(2\tau) = h_0'(2\tau) = b$, $h''(2\tau) = h_0''(2\tau) = c$. This more realistic version of the response with *perfectly dominant* $p_{1,2,3}$ will be used in the rest of the paper.

Concluding this section one has to realize the very approximate character of the above speculation. In spite of a successful tuning of the PID controller the control loop keeps its time delay and therefore the infinite nature of its spectrum. Hence the assumed response (3) can be considered as only a rough approximation of the true behavior of the control loop assuming the fictitious and irreversible dominance of $p_{1,2,3}$.

III. Identifying the Plant Parameters with a Step Response

Model (2) was taken as an ad hoc option with only four parameters corresponding to the number of PID parameters. Consider this model as an approximate description of the dynamical properties of the real process given by the unit step response. This section demonstrates which features of the step response of a process with delay τ can be specified by numbers $a_{0,1}$ and *K*. To identify their values we apply the method of the successive improper integrals of the process step response. The following Lemma holds for this identification.

Lemma 2. Suppose that a stable SISO time delay process is given by its unit-step response *h*(*t*). Its model (2) with parameters $a_0 > 0, a_1 > 0, \tau > 0$ and *K* is to be identified with this response so that the following integrals

$$h_I(t) = \int_0^t [h(\infty) - h(t)]dt\,, \qquad h_{II}(t) = \int_0^t [h_I(\infty) - h_I(t)]dt \quad (4)$$

have the same limits for $t \to \infty$, both for the plant and the model. The model parameters satisfying this criterion are as follows

$$K = h(\infty)\,,\; a_0 = \left(A_1(\tau + A_1) - A_2\right)^{-1},\; a_1 = A_1 a_0 \quad (5)$$

where $A_1 = h_I(\infty)/h(\infty) - \tau$ and $A_2 = h_{II}(\infty)/h(\infty) - \tau^2/2$.

Proof. Consider model (2) divided by a_0 and suppose integrations (4) to be performed in the Laplace transform. Let the plant step response be $h(t)$ and its transform $h(t) \to H(s)$

$$H(s) = \frac{K\exp(-s\tau)}{s\left(\frac{1}{a_0}s^2 + \frac{a_1}{a_0}s + 1\right)} \quad (6)$$

with the limit $t \to \infty$ as $h(\infty) = \lim_{s\to 0} s\,H(s) = K$. The Laplace transform of integral $h_I(t)$ of step response $h(t)$ results from division by s

$$H_I(s) = \frac{1}{s}\left[\frac{K(1-\exp(-s\tau)) + K(a_1 s/a_0 + s^2/a_0)}{s(s^2/a_0 + a_1 s/a_0 + 1)}\right] \quad (7)$$

The whole area between $h(t)$ and $h(\infty)$ given by $H(0)$, and using the well-known limit $\lim_{s\to 0}\left(1-\exp(-s\tau)\right)/s = \tau$ we obtain

$$h_I(\infty) = \lim_{s\to 0} s\,H_I(s) = h(\infty)\left(\tau + \frac{a_1}{a_0}\right) \quad (8)$$

The second integral of (4) is performed similarly by dividing by s again

$$H_{II}(s) = \frac{1}{s}\left[\frac{sK\left(\tau + \frac{a_1}{a_0}\right)\left(1 + s\frac{a_1}{a_0} + \frac{s^2}{a_0}\right) - K\left(1 - \mathrm{e}^{-s\tau} + s\frac{a_1}{a_0} + \frac{s^2}{a_0}\right)}{s^2\left(1 + s\frac{a_1}{a_0} + \frac{s^2}{a_0}\right)}\right] \quad (9)$$

and the value of $h_{II}(\infty) = \lim_{s\to 0} sH_{II}(s)$ is to be evaluated. This limit can be assessed only after using the second derivatives of both the numerator and the denominator of $sH_{II}(s)$. In this manner we obtain that

$$\lim_{s\to 0} sH_{II}(s) = K\left[\frac{\tau^2}{2} + \frac{a_1}{a_0}\tau + \left(\frac{a_1}{a_0}\right)^2 - \frac{1}{a_0}\right] = h_{II}(\infty) \quad (10)$$

From the limits (8) and (10) we obtain the set of equations

$$\begin{aligned} &\frac{a_1}{a_0} = \frac{h_I(\infty)}{h(\infty)} - \tau = A_1 \\ &\frac{a_1}{a_0}\tau + \left(\frac{a_1}{a_0}\right)^2 - \frac{1}{a_0} = \frac{h_{II}(\infty)}{h(\infty)} - \frac{\tau^2}{2} = A_2 \end{aligned} \quad (11)$$

where τ can be considered as assessed directly from $h(t)$. Then the solution of this set is identical with (5). ■

Concluding this section it is to realize that model (2) is applicable only for stable plants without any RHP zero effect.

IV. Ultimate Frequency Assessment

In order really to achieve the dominant position for the prescribed poles it is necessary to select their positions with careful respect to the specific dynamic properties of the plant that is to be controlled. Åström and Hägglund [3] revealed that particularly the ultimate frequency ω_K of the control loop has to be taken into account. The ultimate frequency at which the relay feedback control loop oscillates determines the bounds within which the attainable frequency Ω of the control loop can be expected.

In order to assess the ultimate gain and the corresponding ultimate frequency the proportional gain K_K is assumed as feedback of plant (2). From the characteristic equation of this feedback loop the ultimate gain K_K is obtained at the stability margin, i.e. if undamped oscillations arise, at frequency ω_K given by the following condition

$$-\omega_K^{\,2} + a_1 j\omega_K + a_0 + a_0 K_K \exp(-j\omega_K\tau) = 0 \qquad (12)$$

The decomposition into the equalities of the real and imaginary parts, $a_0 K_K \cos\omega_K\tau = \omega_K^{\,2} - a_0$, $a_0 K_K \sin\omega_K\tau = a_1\omega_K$. The ultimate gain K_K multiplied by a_0 can be excluded when evaluating the tangent function

$$\tan\omega_K\tau = \frac{a_1\omega_K}{\omega_K^{\,2} - a_0} \qquad (13)$$

where the ultimate frequency ω_K can be determined from. Owing to the periodicity of the tangent function this equation has infinitely many real solutions. However, with respect to the physical meaning of ω_K only the smallest of the positive roots of (13) can represent the ultimate frequency.

V. Three-pole Placement to Set a PID Control Loop with Time Delay

The idea of dominant pole placement itself was introduced by Hwang and Chang [1]. They documented that the closed loop response is primarily dominated in most PID control systems by a trio of poles: a complex conjugate pair and a real pole. However, for each particular plant a standard pole placement application requires a suitable option of the candidate trio of numbers to be selected that will really be its dominant poles. Let the above description of the control loop be applied as if the proper candidate trio of characteristic quasi-polynomial zeros $p_{1,2,3}$ were already chosen. The selection of $p_{1,2,3}$ with the warranty of dominance will be dealt with in the next section.

Consider a PID control loop on the time delay plant (2) with the controller

$$\frac{du}{dt} = r_0\frac{de}{dt} + r_D\frac{d^2e}{dt^2} + r_I e \qquad (14)$$

where r_0, r_D, r_I are the proportional, derivative and integration gains respectively and e is the control error. As soon as the controller feedback is closed each of the parameters r_0, r_D, r_I is multiplied by static gain K and the final control parameters of the loop may be considered modified as follows

$$\rho_0 = K\,r_0, \quad \rho_D = Kr_D, \quad \rho_I = K\,r_I \qquad (15)$$

The pole placement is a matter of forming the characteristic quasi-polynomial of the control loop. After joining the plant (2) to the controller (14) the third order differential equation of the control loop is obtained with a characteristic quasi-polynomial of the following form

$$M(s) = \frac{1}{a_0}s^3 + \frac{a_1}{a_0}s^2 + s + \exp(-s\tau)\left[\rho_0 s + \rho_D s^2 + \rho_I\right] \qquad (16)$$

After selecting a suitable trio of numbers $p_{1,2,3}$ one can easily assess the PID control loop parameters ρ_0, ρ_D, ρ_I with which $p_{1,2,3}$ become the control loop poles by means of satisfying conditions $M(p_i) = 0$, $i = 1,2,3$. The PID loop parameters ρ_0, ρ_D, ρ_I are then given by the plant parameters a_0, a_1, τ and by the coordinates of $p_{1,2,3}$. In case of selecting a specific trio of poles as in (1) the following theorem holds for ρ_0, ρ_D, ρ_I assessment.

Theorem 1. Consider a plant (2) with PID controller (14), i.e. a loop with characteristic quasi-polynomial (16) with given a_0, a_1, τ and assign $p_{1,2,3}$ with given κ, δ, Ω as in (1) to be the zeros of $M(s)$. Then the control loop parameters ρ_0, ρ_D, ρ_I to achieving these zeros (without guaranteeing their dominance) have to be adjusted to the values given by the following explicit formulae

$$\rho_0 = \frac{1}{1+\delta^2(\kappa-1)^2}\begin{vmatrix} B_1, & -(1-\delta^2), & 1 \\ B_2, & -2\delta, & 0 \\ B_3, & \kappa^2\delta^2, & 1 \end{vmatrix} \qquad (17)$$

$$\rho_D = \frac{1}{\Omega\left[1+\delta^2(\kappa-1)^2\right]}\begin{vmatrix} -\delta, & B_1, & 1 \\ 1, & B_2, & 0 \\ -\kappa\delta, & B_3, & 1 \end{vmatrix} \qquad (18)$$

$$\rho_I = \frac{\Omega}{1+\delta^2(\kappa-1)^2}\begin{vmatrix} -\delta, & -(1-\delta^2), & B_1 \\ 1, & -2\delta, & B_2 \\ -\kappa\delta, & \kappa^2\delta^2, & B_3 \end{vmatrix} \qquad (19)$$

where

$$B_1 = \exp(-\delta\Omega\tau)\left[b_R\cos\Omega\tau - b_I\sin\Omega\tau\right] \qquad (20)$$

$$B_2 = \exp(-\delta\Omega\tau)\left[b_R\sin\Omega\tau + b_I\cos\Omega\tau\right] \qquad (21)$$

$$B_3 = \exp(-\kappa\delta\Omega\tau)\left[\kappa\delta - \frac{a_1}{a_0}\kappa^2\delta^2\Omega + \frac{1}{a_0}\kappa^3\delta^3\Omega^2\right] \qquad (22)$$

$$b_R = \delta + \frac{a_1}{a_0}(1-\delta^2)\Omega - \frac{1}{a_0}(3\delta - \delta^3)\Omega^2 \tag{23}$$

$$b_I = -1 + \frac{a_1}{a_0}2\delta\Omega + \frac{1}{a_0}(1-3\delta^2)\Omega^2 \tag{24}$$

Proof. After inserting $p_1 = (-\delta + j)\Omega$ into equality $M(s) = 0$ and dividing by $\exp(-s\tau)$ we obtain

$$\begin{aligned}
&\rho_0(-\delta + j)\Omega + \rho_D(\delta^2 - 1 - j2\delta)\Omega^2 + \rho_I = \\
&= \exp(-\delta\Omega\tau)(\cos\Omega\tau + j\sin\Omega\tau)\times \\
&\left[(\delta - j)\Omega + \frac{a_1}{a_0}(1-\delta^2 + j2\delta)\Omega^2 - \right. \\
&\left. -\frac{1}{a_0}(3\delta - \delta^3 - j(1-3\delta^2))\Omega^3\right] = \tilde{B}
\end{aligned} \tag{25}$$

Using the expressions b_R and b_I, as in (23) and (24), the real and imaginary parts of $\tilde{B}$, respectively, may be expressed as follows

$$\begin{aligned}
\mathrm{Re}(\tilde{B}) = B_1\Omega = \exp(-\delta\Omega\tau)[b_R\cos\Omega\tau - b_I\sin\Omega\tau]\Omega \\
\mathrm{Im}(\tilde{B}) = B_2\Omega = \exp(-\delta\Omega\tau)[b_R\sin\Omega\tau + b_I\cos\Omega\tau]\Omega
\end{aligned} \tag{26}$$

Inserting the third pole $p_3 = -\kappa\delta\Omega$ into $M(s) = 0$ we obtain

$$\begin{aligned}
&-\rho_0\kappa\delta\Omega + \rho_D(\kappa\delta\Omega)^2 + \rho_I = \\
&\exp(-\kappa\delta\Omega\tau)\left[\kappa\delta\Omega - \frac{a_1}{a_0}(\kappa\delta\Omega) + \frac{1}{a_0}(\kappa\delta\Omega)^3\right]
\end{aligned} \tag{27}$$

From (25), (26) and (27) the set of equations $\mathbf{A}\cdot\mathbf{P} = \mathbf{B}$ results where the matrices are as follows

$$\mathbf{A} = \begin{bmatrix} -\delta\Omega, & -(1-\delta^2)\Omega^2, & 1 \\ \Omega, & -2\delta\Omega^2, & 0 \\ -\kappa\delta\Omega, & \kappa^2\delta^2\Omega^2, & 1 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} \rho_0 \\ \rho_D \\ \rho_I \end{bmatrix} \tag{28}$$

$$\mathbf{B} = \begin{bmatrix} \Omega\exp(-\delta\Omega\tau)(b_R\cos\Omega\tau - b_I\sin\Omega\tau) \\ \Omega\exp(-\delta\Omega\tau)(b_R\sin\Omega\tau + b_I\cos\Omega\tau) \\ \Omega\exp(-\kappa\delta\Omega\tau)\left[\kappa\delta - \frac{a_1}{a_0}\kappa^2\delta^2\Omega + \frac{1}{a_0}\kappa^3\delta^3\Omega^2\right] \end{bmatrix} = \begin{bmatrix} \Omega B_1 \\ \Omega B_2 \\ \Omega B_3 \end{bmatrix}$$

The solution of this set of equations $\mathbf{P} = [\mathbf{A}]^{-1}\mathbf{B}$ may be expressed by Cramer's rule, and then after reducing the determinant fractions for ρ_0, ρ_D, ρ_I by the powers $\Omega^3, \Omega^2, \Omega^3$, respectively, formulae (17), (18) and (19) are directly obtained. ■

VI. Application Example of the Placement and Relative Damping Optimization

Consider a plant (2) with the parameters $a_0 = a_1 = 1$, K=1 in several versions with delays: $\tau_i = 0.5 + 0.25\cdot(i-1)$, $i = 1,2,3,4,5$. From the solution of (13) the ultimate frequencies for the considered versions are obtained, namely $\omega_{k1} = 1.5984\ \mathrm{s}^{-1}$, $\omega_{k2} = 1.3559\ \mathrm{s}^{-1}$, $\omega_{k3} = 1.2078\ \mathrm{s}^{-1}$, $\omega_{k4} = 1.1024\ \mathrm{s}^{-1}$ and $\omega_{k5} = 1.0203\ \mathrm{s}^{-1}$. For the pole placement the poles are selected according to $p_{1,2i} = -\delta\omega_{ki} \pm j\omega_{ki}$ and $p_{3i} = -\delta\omega_{ki}$, $i = 1,2,3,4,5$ where relative damping ratios δ_i are to be selected to minimize the IAE criterion. In each of the versions the control parameters ρ_{0i}, ρ_{Di} and ρ_{Ii} are assessed repeatedly for a sequence of δ_i using (17), (18) and (19). The IAE plots are shown in Fig. 2. From the form of the disturbance rejection transfer function it is apparent that the integral value of $h(t)$ is equal to K/ρ_I. Therefore almost the same δ_i optimum values are obtained, namely $\delta_1 = 0.44$, $\delta_2 = 0.37$, $\delta_3 = 0.34$, $\delta_4 = 0.33$ and $\delta_5 = 0.33$, if the integration gain maximization instead of the IAE minimization in Fig. 3 is applied.

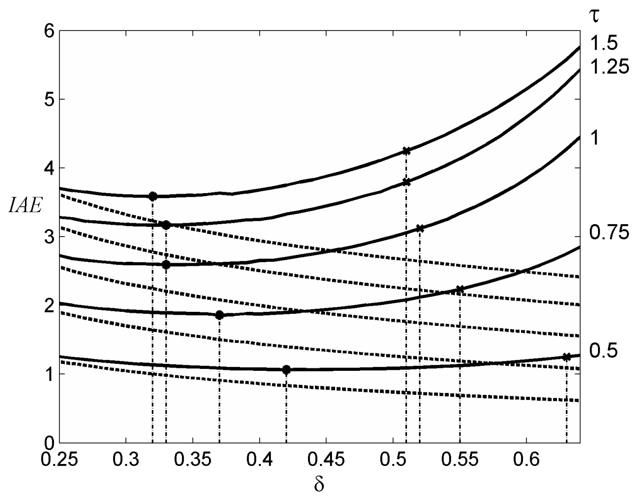

Fig. 2. Optimal setting of relative damping δ_i with respect to minimal IAE.

In Fig. 2 the IAE plots for both the true disturbance rejection responses (full lines) and their "*perfectly dominant*" versions (dashed lines) are compared. While in the "*perfectly dominant*" version the influence of growing δ_i is weak and negative in the true version its impact is strong and positive.

Growing difference between them indicates the worsening or even the loss of dominance of $p_{1,2,3}$.

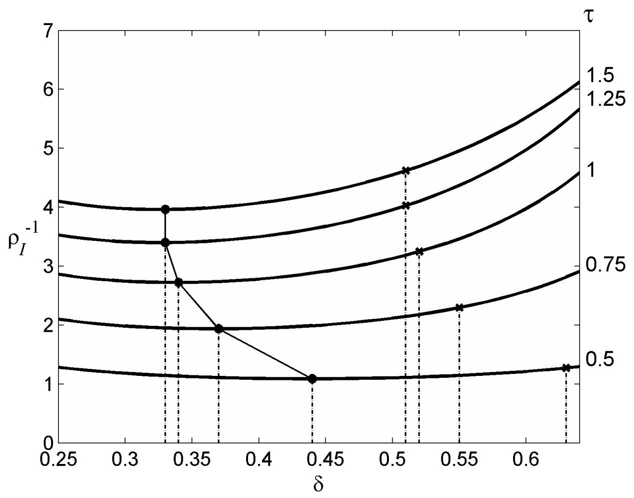

Fig. 3. Optimal setting of relative damping δ_i with respect to maximal ρ_I.

The optimum of relative damping δ_i is to be assessed not only from the minimum of true IAE but also from a minimum difference between both these versions. The optimum values of δ_i (round dots) are shown in Fig. 2 for several options τ_i of the delay as well as the boundary δ_i values at which the dominance of $p_{1,2,3}$ is lost (depicted by the cross mark).

From wide investigations of various control loops the authors have proved that δ optimization results in almost the same recommendation $\delta \in \langle 0.3 \div 0.4 \rangle$ for a wide range of coefficients a_0 and a_1. Also for various delays ranging from $\tau = 0.5/a_1$ to $\tau = 3/a_1$ this recommendation holds uniformly (δ_i values near 0.4 belong to lower values of delay). Let be noted that the software presented in [11] has been used in the above computations.

VII. Conclusions

The presented method of dominant pole placement is specific in its aim to the disturbance rejection that is critical in control loops with time delay. As any approach using the ultimate gain to identifying the process dynamics it can be solely applied to stable processes both aperiodical and oscillating. Although obviously the final response is different from the response of fictitious system with "*perfectly dominant*" $p_{1,2,3}$ its employment in selecting the prescribed roots is justifiable. Comparing the *true* IAE criterion with the auxiliary one obtained for the "*perfectly dominant*" case serves as an indicator of $p_{1,2,3}$ dominance. Only for settings where both the criteria are close to each other a satisfactory dominance is achieved and, on the contrary, the growing difference between them indicates a worsening or even a loss of $p_{1,2,3}$ dominance. Moreover, due to the use of integration action the IAE minimization can be substituted by a maximization of the integration gain ρ_I. From our IAE investigations it also turned out that the optimum damping ratio leading to the best level of dominance warranty is close to $\delta \cong 0.35$. So the values $\kappa = 1$ and $\delta = 0.35 \pm 0.05$ can universally be recommended for any stable plant (2) with an essential delay and free of RHP zero effect. Although the presented method is not directly based on the phase margin prescription as e.g. [7] it results in very satisfactory values of phase margin higher than 60° for all considered options of PID control loops.

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