Distributed Tracking for Networked Euler-Lagrange Systems Using Only Relative Position Measurements

Qingkai Yang, Hao Fang, Yutian Mao, Jie Huang and Jian Sun

Abstract— **In this paper, the problem of distributed coordinated tracking control for networked Euler-Lagrange systems using only relative position measurements is studied. Under the condition that only a subset of followers have access to the leader, sliding mode estimators are developed to estimate the states of the dynamic leader accurately in finite time. A set of distributed observers which only uses relative position information is designed to deal with the unavailability of the followers' velocities. Using the observer outputs, distributed control laws are proposed such that the objective of tracking a dynamic leader under a spanning tree is achieved. The asymptotic stability of the proposed distributed observer-controller is proved through Lyapunov method. Numerical simulation results are also provided to show the effectiveness of the control laws.**

I. Introduction

Distributed control of multi-agent systems has received increasing concern due to its broad application in various fields, ranging from physics, engineering to biology; see [1]-[2]. Currently, most research work on MAS mainly concentrates on integrator models [3]-[6], however, for some kinds of mechanical systems, such as autonomous vehicles, robotic manipulators, and rigid bodies, the nonlinear dynamics can not be neglected in practice. Hence it is of great significance to study coordinated control of Euler-Lagrange systems which can generally describe action of mechanical systems [7].

The problem of distributed control for networked Euler-Lagrange systems has recently been studied in [8]-[13]. In [8]-[9], the case of leaderless consensus for multiple Euler-Lagrange systems has been investigated. Min in [8] proposed adaptive consensus protocols in the case of time-delay and switching topology with the balanced graph assumption. In [9], the output synchronization of the multiple Euler-Lagrange systems has been achieved under both fixed and switching topology based on the passivity property of mechanical systems. The problem of tracking a leader or equivalently a reference for a class of mechanical systems modeled by Euler-Lagrange equations has been studied in [10]-[13]. In [10], the authors applied the nonlinear contraction theory to prove the stability for multiple robotic manipulators. A distributed adaptive control law in [11] was designed to track the reference trajectory for multiple uncertain mechanical systems. Mei in [12] coped with the tracking problem for networked Euler-Lagrange systems under the condition that the leader's generalized coordinate derivative was constant and varying, respectively, an adaptive control law together with a distributed continuous estimator and a model-independent sliding mode control algorithm were proposed. In [13], the containment problem (i.e., tracking with multiple leader) was studied in the presence of parametric uncertainties under a directed graph. Similarly, a distributed adaptive controller was proposed such that the followers could converge into the convex hull spanned by leaders. The followers could achieve synchronization with the dynamic leader in [10], however, it is required that all the followers should have access to the dynamic leader, which is rather restrictive. Although [11]-[13] have realized distributed control, the interaction graph has to be undirected in [11]. Instead of undirected graph, Mei in [12]-[13] has investigated coordinated control problems under directed graph.

It should be noticed that all the above mentioned research works are conducted under the condition that all the followers' information has to be available in order to implement the controller design. Nevertheless, in practice, velocity and acceleration measurements may not always be measurable due to the strict constraints on the cost and space for installing the devices. In [14], the containment control algorithms via only position measurements were proposed based on the super twisting algorithm. The coordinated tracking problem with a dynamic leader were investigated in [15]-[16]. The authors in [17] studied consensus problem without velocity measurements. However, [14]-[17] only consider the systems with linear integrator models. Up to now, there are very few papers dealing with distributed control of Euler-Lagrange system without velocity measurements. Ren in [18] proposed distributed algorithms for networked Euler-Lagrange systems without velocity and relative velocity information. However, the topology should be undirected connected.

In this paper, only the relative position measurements among the followers can be used to achieve distributed coordinated tracking for networked Euler-Lagrange systems, while moreover only a subset of followers have access to the leader in a directed graph. To overcome this challenging problem, a novel observer and estimators are designed with the help of followers' position measurements, then a distributed tracking control law is proposed based on the outputs of the observer and estimators.

The subsequent sections are organized as follows: Section

All the authors are with School of Automation, Beijing Institute of Technology Beijing100081; Key Laboratory of Intelligent Control and Decision of Complex Systems, Beijing, 100081

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II introduces the models to be controlled and graph theory. In section III, distributed sliding-mode estimators are developed in first part. In second part, distributed velocity observer is presented, followed by the main result(Theorem 1) and the proof using Lyapunov method. A numerical example is carried out to illustrate the effectiveness of the proposed control algorithm in Section IV. Section V gives the conclusion.

II. Background

A team of n mechanical systems labeled as agents 1 to n are considered as followers, and a leader is labeled as agent 0. They are all described by Euler-Lagrange equations of as follows:

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, \qquad i = 1,\cdots,n \quad (1)$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{p\times p}$ is the symmetric positive-definite inertia matrix, $C_i(q_i,\dot{q}_i)\dot{q}_i \in \mathbb{R}^p$ is the vector of coriolis and centrifugal torques, $g_i(q_i)$ is the vector of gravitational torques, and $\tau_i \in \mathbb{R}^p$ is the vector of control torque on the ith agent.

In the following, we use $M(q) \triangleq diag[M_1(q_1),\cdots, M_n(q_n)], C(q,\dot{q}) = diag[C_1(q_1,\dot{q}_1),\cdots,C_n(q_n,\dot{q}_n)], G(q) \triangleq [g_1^T(q_1),\cdots,g_n^T(q_n)]^T$ as the vector form of $M_i(q_i)$, $C_i(q_i,\dot{q}_i), G_i(q_i)$.

For (1), the following properties are held [19],[20].

1) For any i, there exist positive constants $k_{\underline{m}}$, $k_{\overline{m}}$, k_C and k_{g_i} such that $0 < k_{\underline{m}} I_p \le M_i(q_i) \le k_{\overline{m}} I_p$, $\|C_i(x,y)z\| \le k_C\|y\|\|z\|$ for all vectors $x,y,z \in R^p$, and $\|g_i(q_i)\| \le k_{g_i}$.
2) $C_i(q_i,\dot{q}_i)$,$M_i(q_i) - 2C_i(q_i,\dot{q}_i)$ is skew symmetric.

$\mathscr{G} \triangleq (\mathscr{V},\mathscr{E})$ is used to represent the interactions among the agents 1 to n with the node set $\mathscr{V} \triangleq \{1,\cdots,n\}$ and the edge set $\mathscr{E} \subseteq \mathscr{V}\times\mathscr{V}$. The edge (i,j) denotes the agent j can obtain information from agent i in a directed graph, but not vice versa. In undirected graph, an edge $(i,j) \in \mathscr{E}$ if vehicle i and j can receive information from each other. Here, we assume that there is no loop in the graph, i.e., $(i,i) \notin \mathscr{E}$. If an edge $(i,j) \subseteq \mathscr{E}$, then we call node i is a neighbor of node j. Thus, the neighbor set of agent i is defined as $\mathscr{N}_i \triangleq \{j|(j,i)\in\mathscr{E}\}$. A root is the one that has directed paths to all other nodes in a directed tree. Note that the directed graph has a directed spanning tree if and only if the directed graph has at least one root. The adjacency matrix $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is defined such that $a_{ij} > 0$ if $(j,i) \in \mathscr{E}$, and $a_{ij} = 0$ otherwise. Note that, $a_{ij} = a_{ji}$ in an undirected graph. Let the Laplacian matrix $L = [l_{ij}] \in R^{n\times n}$, with $l_{ii} = \sum_{j=1,j\neq i}^n a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$.

Lemma 1: [21] Let $\overline{\mathscr{G}} \triangleq (\overline{\mathscr{V}},\overline{\mathscr{E}})$ be the directed graph characterizing the interaction among the n followers and the leader, accordingly, $a_{i0} > 0$ if $(i,0) \in \mathscr{E}$ and $a_{i0} = 0$ otherwise. Considering the extended graph $\overline{\mathscr{G}}$, the generalized Laplacian matrix $H = L + diag(a_{10},\cdots,a_{n0})$ is positive stable if the leader has directed paths to all the followers.

Assumption 1: The desired trajectory q_0 is differentiable, and its derivatives are bounded, i.e., $\|\dot{q}_0\|_\infty \le \eta_a, \|\ddot{q}_0\|_\infty \le \eta_b$ and $\|\dddot{q}_0\|_\infty \le \eta_c$.

III. Main Results

To deal with the problem of distributed tracking control without velocity measurement, in this section, a novel distributed tracking control algorithm is proposed for the networked Euler-Lagrange systems underlying the interaction topology where only a subset of followers have access to the leader.

A. Estimator Design

Since the measurements of the leader is known to parts of the followers, inspired by [13], distributed sliding-mode estimators are designed to estimate the information of the leader as

$$\dot{\overline{v}}_{0i} = -\alpha_1 \mathrm{sgn}[\sum_{j\in\mathscr{N}_i} a_{ij}(\overline{v}_{0i} - \overline{v}_{0j}) + a_{i0}(\overline{v}_{0i} - \dot{q}_0)], \qquad (2a)$$

$$\dot{\overline{a}}_{0i} = -\alpha_2 \mathrm{sgn}[\sum_{j\in\mathscr{N}_i} a_{ij}(\overline{a}_{0i} - \overline{a}_{0j}) + a_{i0}(\overline{a}_{0i} - \ddot{q}_0)], \qquad (2b)$$

where $\overline{v}_{0i},\overline{a}_{0i}$ are the estimations of the leader's generalized coordinate derivative and generalized coordinate acceleration(i.e., $\dot{q}_0$ and $\ddot{q}_0$). $a_{ij}, i,j = 1,\cdots,n,$ is the entry of the adjacency matrix. α_1 and α_2 are positive constants, and sgn$(\cdot)$ is the signum function accordingly. Next, it will be shown that the outputs of the distributed estimators (2) will converge to the leader's corresponding states in finite time.

Lemma 2: Suppose that in $\overline{\mathscr{G}}$, the leader has directed paths to all the followers. Then, $\|\overline{v}_{0i} - \dot{q}_0\| \to 0$ in finite time, if $\alpha_1 > \|\ddot{q}_0\|$. Similarly, $\|\overline{a}_{0i} - \ddot{q}_0\| \to 0$ in finite time, if $\alpha_2 > \|\dddot{q}_0\|, i = 1,\cdots,n$.

Proof: The proof is similar to that of Lemma 3.1 in [13]. Equation (2a) can be written in vector form as

$$\dot{\overline{v}}_0 = -\alpha_1 \mathrm{sgn}[(H\otimes I_p)(\overline{v}_0 - \dot{q}_0)], \qquad (3)$$

where $\overline{v}_0$ is the column vector form of $\overline{v}_{0i}$. Then, equation (3) can be transformed to

$$\dot{\overline{v}}_0 = -\alpha_1 \mathrm{sgn}[(H\otimes I_p)\tilde{v}_0], \qquad (4)$$

i.e.,

$$\dot{\tilde{v}}_0 = -\alpha_1 \mathrm{sgn}[(H\otimes I_p)\tilde{v}_0] - \ddot{q}_0, \qquad (5)$$

where $\tilde{v}_0 \triangleq \overline{v}_0 - \dot{q}_0$. Let $\tilde{v}_{00} \triangleq \mathbf{0}_p$, then equation (5) can be written as

$$\dot{\tilde{v}}_0 = -\alpha_1 \mathrm{sgn}[\sum_{j=0}^n a_{ij}(\tilde{v}_{0i} - \tilde{v}_{0j})] - \ddot{q}_0, \qquad (6)$$

Then from [22], if $\alpha_1 > \|\ddot{q}_0\|$, $\|\overline{v}_{0i} - \dot{q}_0\| \to 0$ in finite time. The upper bound of the convergent time denoted by T_1 is $\frac{\max\|\tilde{v}_{0i}(0)\|_\infty}{\alpha_2 - \sup_{t\ge0}\|\ddot{q}_0\|}$. Similarly, if $\alpha_2 > \|\dddot{q}_0\|$, $\|\overline{a}_{0i} - \ddot{q}_0\| \to 0$, when $t > T_2 = \frac{\max\|\tilde{a}_{0i}(0)\|_\infty}{\alpha_3 - \sup_{t\ge0}\|\dddot{q}_0\|}$. ■

B. Observer-Controller Design

Define the following auxiliary variables as follows:

$$s_i \triangleq \dot{e}_i + \sum_{j\in\mathscr{N}_i} a_{ij}(q_i - q_j) + a_{i0}(q_i - q_0) \qquad (7a)$$

$$s_{oi} \triangleq \dot{e}_{oi} + \sum_{j\in\mathscr{N}_i} a_{ij}(e_{oi} - e_{oj}) \qquad (7b)$$

where $\dot{e}_i = \dot{q}_i - \dot{q}_0, \dot{e}_{oi} = \dot{q}_i - \dot{\hat{q}}_i$, and $\dot{\hat{q}}_i$ is the estimation of generalized coordinate derivatives of agent i, $a_{ij}, i, j = 1, \cdots, n$, is the entry of the adjacency matrix. Hence, (7) can be written in vector form as

$$s \triangleq \dot{e} + (H \otimes I_p)e \tag{8a}$$

$$s_o \triangleq \dot{e}_o + (L \otimes I_p)e_o \tag{8b}$$

Then, (1) can be transformed into

$$M_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i + \Delta W_i = \tau_i \tag{9}$$

The corresponding vector form is

$$M(q)\dot{s} + C(q, \dot{q})s + \Delta W = \tau \tag{10}$$

where

$$\Delta W = M(q)(\ddot{q} - \dot{s}) + C(q, \dot{q})(\dot{q} - s) + G(q) \tag{11}$$

Lemma 3: There exist positive definite functions $b_{11}, b_{12}^{'}, b_{12}^{''}, b_{13}^{'}$, $b_{21}, b_{21}^{'}, b_{22}^{''}, b_{23}^{'}$ such that the following inequations hold,

$$\begin{aligned} &-s^T \Delta W \leq s^T M(q)(H \otimes I_p)s - s^T M(q)(H \otimes I_p)^2 e \\ &+ b_{11}\|s\|^2 + b_{12}\|s\|\|e\| + b_{13}(\|s\|^2\|e\| + \overline{\lambda}_H \|s\|\|e\|^2) \end{aligned} \tag{12}$$

$$\begin{aligned} &-s_o^T \Delta W \leq s_o^T M(q)(L \otimes I_p)s - s_o^T M(q)(L \otimes I_p)^2 e \\ &+ b_{21}\|s\|\|s_o\| + b_{22}\|s_o\|\|e\| + b_{23}(\|s\|\|s_o\|\|e\| \\ &+ \overline{\lambda}_L \|s_o\|\|e\|^2) \end{aligned} \tag{13}$$

where $b_{12} = b_{12}^{'} + \overline{\lambda}_H b_{12}^{''}, b_{13} = \overline{\lambda}_H b_{13}^{'}, b_{22} = b_{22}^{'} + \overline{\lambda}_H b_{22}^{''}, b_{23} = \overline{\lambda}_H b_{23}^{'}$, ΔW is defined in (11), $\underline{\lambda}_X = \min \sqrt{\lambda(X^T X)}, \overline{\lambda}_X = \max \sqrt{\lambda(X^T X)}$, $\lambda(X)$ is the eigenvalue of the matrix X. Please refer to [23] for the details of the proof of Lemma 3.

First, the distributed velocity observer is proposed as follows

$$\begin{cases} \dot{\hat{q}}_i = z_i - \sum\limits_{j \in \mathcal{N}_i} a_{ij}(q_i - q_j) - a_{i0}(q_i - q_0) \\ \qquad + \sum\limits_{j \in \mathcal{N}_i} a_{ij}(e_{oi} - e_{oj}) + \eta_1 e_{oi} \\ \dot{z}_i = \overline{a}_{0i} - K_i(\overline{s}_i - s_{oi}) + \eta_1 \sum\limits_{j \in \mathcal{N}_i} (e_{oi} - e_{oj}) \end{cases} \tag{14}$$

Where the gains η_1, η_2 are positive constants and K_i are positive definite matrix in appropriate dimension. $\overline{s}_i = \dot{q}_i - \overline{v}_{0i} + (H \otimes I_p)e_i$ Then, (14) can be written in vector form as

$$\begin{cases} \dot{\hat{q}} = z - (H \otimes I_p)e + (L \otimes I_p)e_o + \eta_1 e_o \\ \dot{z} = \overline{a}_0 - K(\overline{s} - s_o) + \eta_1 (L \otimes I_p)e_o \end{cases} \tag{15}$$

where $\dot{\hat{q}}, z, e, e_o, \overline{a}_0, \overline{s}$ and s_o are, respectively, the column stack vectors of $\dot{\hat{q}}_i, z_i, e_i, e_{oi}, \overline{a}_{0i}, \overline{s}_i$ and s_{oi}, $i = 1, 2, \cdots, n$.

Based on the outputs of the designed estimator and velocity observer, we then design the control algorithm as

$$\tau_i = -\beta K_i(\overline{s}_i - s_{oi}) - \eta_2 e_i, i = 1, \cdots, n. \tag{16}$$

In (16), it is noted that the term $\overline{s}_i - s_{0i} = \dot{\hat{q}}_i - \overline{v}_{0i} + \sum_{j \in \mathcal{N}_i} a_{ij}(e_{oi} - e_{oj}) - \sum_{j \in \mathcal{N}_i} a_{ij}(q_i - q_j) - a_{i0}(q_i - q_0)$, therefore the control input does not need any information of derivatives of generalized coordinate [24].

Theorem 1: Assume that the generalized graph contains a directed spanning tree, in which the dynamic leader is the root, and $\alpha_1 > \|\ddot{q}_0\|$, $\alpha_2 > \|\dddot{q}_0\|$. Then using (2), (14),(16), for (1), there exist parameters β, K, η_1, η_2 such that $\|q_i(t) - q_0(t)\| \to 0$ and $\|\dot{q}_i(t) - \dot{q}_0(t)\| \to 0, i = 1, \cdots, n$, as $t \to \infty$.

Proof: First, in order to solve the problem that only a subset of followers have access to the leader, the estimator (2) is used from Lemma 2 it can be guaranteed that all the estimations of leader's states may converge to the real values in finite time. Hence $\overline{v}_{0i} = \dot{q}_0$, and $\overline{a}_{0i} = \ddot{q}_0$ when $t \geq T \triangleq \max\{T_1, T_2\}$. Then (15) become

$$\begin{cases} \dot{\hat{q}} = z - (H \otimes I_p)e + (L \otimes I_p)e_o + \eta_1 e_o \\ \dot{z} = \ddot{q}_0 - K(s - s_o) + \eta_1 (L \otimes I_p)e_o \end{cases} \tag{17}$$

Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}s^T M(q)s + \frac{1}{2}s_o^T M(q)s + \frac{1}{2}e^T \eta_2 e, \tag{18}$$

Taking the derivative of V follows that

$$\begin{aligned} \dot{V}(t) \ &= \ s^T M(q)\dot{s} + \frac{1}{2}s^T \dot{M}(q)s + s_o^T M(q)\dot{s}_o \\ &+ \ \frac{1}{2}s_o^T \dot{M}(q)s_o + e^T \eta_2 \dot{e}, \end{aligned} \tag{19}$$

Combining (10) and (16), when $t \geq T$, the error dynamics of the system can be written as

$$M(q)\dot{s} = -\beta K(s - s_o) - \eta_2 e - \Delta W - C(q, \dot{q})s \tag{20}$$

From the observer (17), we have

$$\dot{s_o} = \dot{s} + Ks - Ks_o - \eta_1 s_o \tag{21}$$

Then, substituting (20) and (21) into (19), and considering the skew symmetric property, we can get

$$\begin{aligned} \dot{V}(t) = &- \ \beta s^T K(s - s_o) - s^T \Delta W + s_o^T C s_o - s_o^T C(q, \dot{q})s \\ &- \ \beta s_o^T K(s - s_o) - \eta_2 s_o^T e - s_o^T \Delta W + s_o^T MKs \\ &- \ s_o^T MKs_o - \eta_1 s_o^T M s_o - \eta_2 s^T e + \eta_2 e^T \dot{e} \end{aligned} \tag{22}$$

According to Lemma 3, $\dot{V}(t)$ can be written as

$$\begin{aligned} \dot{V}(t) \leq &- \ \beta s^T Ks + s^T M(H \otimes I_p)s + b_{11}\|s\|^2 + s_o^T C(q, \dot{q})s_o \\ &+ \ \beta s_o^T K s_o - \eta_1 s_o^T M s_o - s_o^T MKs_o + b_{13}\|s\|^2\|e\| \\ &+ \ b_{13}\overline{\lambda}_H \|s\|\|e\|^2 + b_{12}\|s\|\|e\| + b_{21}\|s\|\|s_o\| \\ &+ \ b_{22}\|s_o\|\|e\| + b_{23}\|s\|\|s_o\|\|e\| + b_{23}\overline{\lambda}_L \|s_o\|\|e\|^2 \\ &- \ s_o^T C(q, \dot{q})s + s_o^T MKs + s_o^T M(L \otimes I_p)s \\ &- \ s_o^T M(L \otimes I_p)^2 e - \eta_2 s^T e - \eta_2 s_o^T e + \eta_2 e^T \dot{e} \\ &- \ s^T M(H \otimes I_p)^2 e \end{aligned} \tag{23}$$

When $\beta - \frac{1}{2}\overline{\lambda}_M > 0$, we have

$$\begin{aligned} \dot{V}(t) \leq &- \ \Big[(\beta - \frac{1}{2}\overline{\lambda}_M)\underline{\lambda}_K - \frac{b_{13}}{2}\overline{\lambda}_H \|e\|^2 - (\frac{b_{23}}{2} + b_{13})\|e\| \\ &+ \ P_1\Big]\|s\|^2 - \Big[(\underline{\lambda}_H - \frac{1}{2})\eta_2 + P_2\Big]\|e\|^2 - \Big[\eta_1 \underline{\lambda}_M \\ &- \ \frac{b_{23}}{2}\overline{\lambda}_L \|e\|^2 - \frac{b_{23}}{2}\|e\| + P_3\Big]\|s_o\|^2 + D \end{aligned} \tag{24}$$

where

$$\begin{aligned} P_1 &= -\frac{1}{2}\overline{\lambda}_M\overline{\lambda}_L - \frac{1}{2}\overline{\lambda}_M\overline{\lambda}_H^2 - \overline{\lambda}_M\overline{\lambda}_H - b_{11} \\ &\quad -\frac{b_{12}}{2} - \frac{b_{21}}{2} \\ P_2 &= -\frac{1}{2}\overline{\lambda}_M\overline{\lambda}_H^2 - \frac{1}{2}\overline{\lambda}_M\overline{\lambda}_L^2 - \frac{b_{12}}{2} - \frac{b_{13}}{2}\overline{\lambda}_H \\ &\quad -\frac{b_{22}}{2} - \frac{b_{23}}{2}\overline{\lambda}_H \\ P_3 &= \underline{\lambda}_M\underline{\lambda}_K - \frac{3}{2}\overline{\lambda}_C - \beta\overline{\lambda}_K - \frac{b_{21}}{2} - \frac{b_{22}}{2} \\ &\quad -\frac{1}{2}\overline{\lambda}_M\overline{\lambda}_L - \frac{1}{2}\overline{\lambda}_M\overline{\lambda}_K - \frac{1}{2}\overline{\lambda}_M\overline{\lambda}_L^2 - \frac{\eta_2}{2} \end{aligned}$$

$$\begin{aligned} D = &- d_1(\|s\| - \|e\|)^2 - d_2(1-\|s\|)^2 - d_3(\|s\| \\ &- \|s_o\|)^2 - d_4(\|s_o\| - \|e\|)^2 - d_5(1-\|s_o\|)^2 \end{aligned}$$

Here

$$\begin{cases} \mathrm{d}_1 = \frac{b_{12}}{2} + \frac{1}{2}\bar{\lambda}_M\bar{\lambda}_H^2 \\ d_2 = \frac{b_{13}}{2}\bar{\lambda}_H \\ d_3 = \frac{b_{21}}{2} + \frac{b_{23}}{2}\|e\| + \frac{1}{2}\bar{\lambda}_M\bar{\lambda}_L + \frac{1}{2}\bar{\lambda}_M\bar{\lambda}_K + \frac{1}{2}\bar{\lambda}_C \\ d_4 = \frac{b_{22}}{2} + \frac{1}{2}\bar{\lambda}_M\bar{\lambda}_L^2 + \frac{\eta_2}{2} \\ d_5 = \frac{b_{23}}{2}\bar{\lambda}_L\|e\|^2 \end{cases}$$

Note that $D \le 0$, $\dot{V}(t) \le 0$ if the following inequations hold

$$\begin{cases} \beta - \frac{1}{2}\overline{\lambda}_M > 0 & \text{(25a)} \\ (\beta - \frac{1}{2}\overline{\lambda}_M)\underline{\lambda}_K - \frac{b_{13}}{2}\overline{\lambda}_H\|e\|^2 - (\frac{b_{23}}{2} + b_{13})\|e\| + P_1 > 0 & \text{(25b)} \\ (\underline{\lambda}_H - \frac{1}{2})\eta_2 + P_2 > 0 & \text{(25c)} \\ \eta_1\underline{\lambda}_M - \frac{b_{23}}{2}\overline{\lambda}_L\|e\|^2 - \frac{b_{23}}{2}\|e\| + P_3 > 0 & \text{(25d)} \end{cases}$$

It is noted that (25b),(25d) contain the term $\|e\|$ and $\|e\|^2$, thus (25) is solved and it is concluded that (25) has solutions when

$$(\beta - \frac{1}{2}\overline{\lambda}_M)\underline{\lambda}_K + P_1 > 0 \qquad \text{(26a)}$$

$$\eta_1\underline{\lambda}_M + P_3 > 0 \qquad \text{(26b)}$$

Derived from (25) and (26), it can be proved that $V(t)$ is semi-definite negative when the parameters satisfy the following inequations

$$\begin{cases} \beta > \frac{1}{2}\overline{\lambda}_M & \text{(27a)} \\ \underline{\lambda}_K > \dfrac{\overline{\lambda}_M\overline{\lambda}_L + \overline{\lambda}_M\overline{\lambda}_H^2 + \overline{\lambda}_M\overline{\lambda}_H + b_{11} + b_{12} + b_{21}}{2\beta - \overline{\lambda}_M} & \text{(27b)} \\ \eta_1 > \dfrac{-2\underline{\lambda}_M\underline{\lambda}_K + 3\overline{\lambda}_C + 2\beta\overline{\lambda}_K + b_{21} + b_{22}}{2\underline{\lambda}_M} & \text{(27c)} \\ \eta_2 > \dfrac{\overline{\lambda}_M\overline{\lambda}_H^2 + \overline{\lambda}_M\overline{\lambda}_L^2 + b_{12} + b_{13}\overline{\lambda}_H + b_{22} + b_{23}\overline{\lambda}_H}{2\underline{\lambda}_H - 1} & \text{(27d)} \end{cases}$$

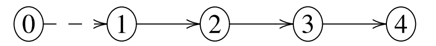

Fig. 1. Network topology associated with the leader 0 and four followers. Here, an arrow from i to j denotes agent j can receive information form agent i, $i, j = 0, 1, \cdots, 4$.

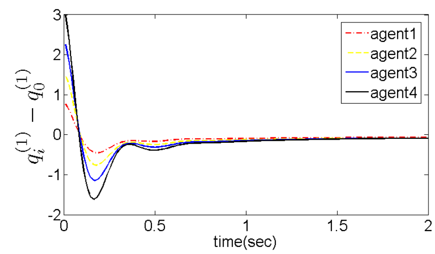

Fig. 2. Angle differences of the first joint between followers and the leader

where $\underline{\lambda}_H = 1$ corresponding to the directed spanning tree. $\dot{V}(t) \le 0$ implies that terms s, s_o, and e together with their derivatives are bounded. By differentiating (23), we can see that $\ddot{V}(t)$ is bounded. Thus $\dot{V}(t)$ is uniformly continuous. It can be concluded from Barbalat's Lemma [25] that $\dot{V}(t) \to 0$, as $t \to \infty$, *i.e.*, $s(t) \to 0, s_o(t) \to 0$, and $e(t) \to 0$, as $t \to \infty$. Therefore, $\|q_i(t) - q_0(t)\| \to 0$, and $\|\dot{q}_i(t) - \dot{q}_0(t)\| \to 0, i = 1, \cdots, n$, as $t \to \infty$ ■

Remark 1: (8b) can be written as $\dot{e}_o = -(L \otimes I_p)e_o + s_o$. Because the agents form a directed spanning graph, the state e_o converges to zero from Lasalle Invariance Principle and the theory of ISS(input-to-state stable) [25], if s_o converges to zero. Therefore, we also have $\|\dot{q}_i(t) - \hat{\dot{q}}_i(t)\| \to 0$, as $t \to \infty$, which means that we can effectively observe the velocities only using relative position measurements.

IV. Simulation Results

Numerical simulations are presented in this section to demonstrate the effectiveness of proposed control algorithm. Consider four networked two-link manipulators modeled by Euler-Lagrange equations. For simplicity, we choose identical joint arms for the four followers. Let the masses of link 1 and link 2 be, respectively, $m_1 = 0.5$kg, and $m_2 = 0.4$kg, the lengths of link1 and link 2 be, respectively, $l_1 = 0.4$m, and $l_2 = 0.3$m, the distances of the mass center of link 1 and link 2 between neighbors be, respectively, $l_{c1} = 0.2$m, and $l_{c2} = 0.15$m. In addition, the moments of inertia of link 1 and link 2 are, respectively, $J_1 = 0.0067\text{kg}\cdot\text{m}^2$, and $J_2 = 0.003\text{kg}\cdot\text{m}^2$.

The interaction topology between followers and the leader are showed in Fig. 1, in which only agent 1 can reach the leader(i.e., agent 0). The initial positions of the followers are chosen as $q_i(0) = [(\pi/4)i, (\pi/5)i]^T$rad, and the velocity estimations of the four followers as $[0,0]^T$rad/s. Let the angles of the leader be $q_0(t) = [\sin(t), -\sin(t)]$rad, and hence the angular velocity be $\dot{q}_0(t) = [\cos(t), -\cos(t)]$rad/s. The control parameters are selected as $\alpha_1 = \alpha_2 = 1.5$, $\beta = 2, \eta_1 = 6, \eta_2 = 20$ and $K_i = 13I_2$.

Fig. 2 and Fig. 3 show the angle differences of the two joints between followers and the leader. Accordingly,

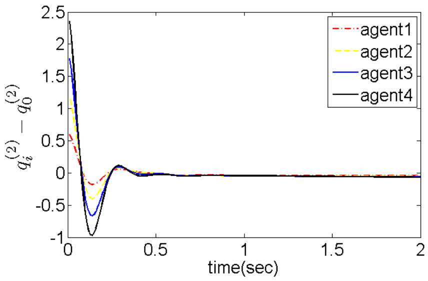

Fig. 3. Angle differences of the second joint between followers and the leader

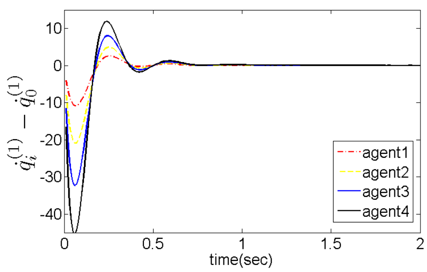

Fig. 4. Angular velocity differences of the first joint between followers and the leader

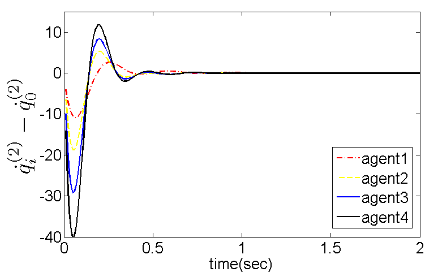

Fig. 5. Angular velocity differences of the second joint between followers and the leader

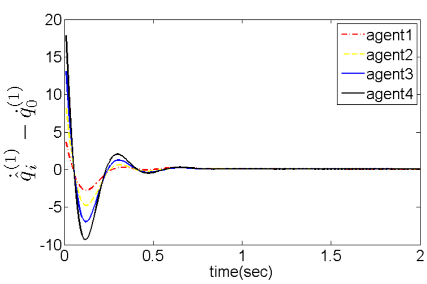

Fig. 6. Differences between estimations of followers' angular velocity and the leader's angular velocity of the first joint

Fig. 4 and Fig. 5 show the angular velocity differences between followers and the leader. The differences between estimations of followers' angular velocity and the leader's angular velocity are shown in Fig. 6 and Fig. 7. It can be seen from the figures that all the errors converge to zero, which implies that the tracking objective is achieved using (2), (16), (14).

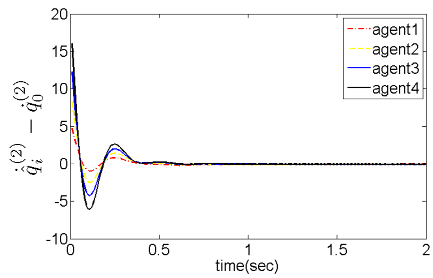

Fig. 7. Differences between estimations of followers' angular velocity and the leader's angular velocity of the second joint

V. Conclusion

The problem of distributed coordinated tracking for multiple networked Euler-Lagrange systems has been investigated under the constraint that the leader is a neighbor of only part of the followers in a directed spanning tree. Compared with the previous research work, the distributed observer-controller is proposed. Since only position measurements between followers are available, the advantage of the proposed controller is that the followers can precisely track a dynamic leader without velocity measurements. The convergence of the position errors has been illustrated through the numerical simulations.

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