# A Decentralized Control Algorithm Based on the DC Power Flow Model for Avoiding Cascaded Failures in Power Networks

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***Abstract*—A distributed random algorithm for controlling electrical power flow after a failure in (or attack on) a power transmission grid is proposed. The aim is to minimize load shedding and to avoid a cascaded failure in the network. The DC power flow model is used to simulate the power flow in the network. The algorithm is based only on the information about the closest neighbours of each node. A mathematically rigorous proof of convergence with probability 1 of the proposed algorithm is provided.**

## I. INTRODUCTION

Cascaded failures in electrical power networks have been attracting the attention of researchers due to the catastrophic impact they have on electrical power systems [1]–[3]. Cascaded failures were the reason for large blackouts throughout the world (e.g. USA in 1996 [4] and 2003 [5], Italy in 2003 [6], and Europe in 2006 [7])

A cascaded failure is initiated when a heavily loaded bus or transmission line in the power network is lost due to a random failure or a targeted attack, and the power flow passing through the lost bus or line is re-routed to other buses and lines in the network. If not executed properly, the re-routing may cause other buses and lines to become overloaded and therefore disconnected from the network. This effect may propagate throughout the network and the entire network maybe affected [8]. As a consequence, researchers have been developing control algorithms for power re-routing with minimum load loss [8]–[11].

For the purpose of applying the proposed algorithm, the DC power flow model is used to simulate the power network. The DC power flow equations are widely used in the literature for cascading failure analysis due to the low computational cost compared to the AC power flow equations [12], [13]. A detailed discussion of the DC power equations with examples are found in [14].

Typically, the minimum load shed in the DC power flow model is calculated by applying linear programming which has been used since the 1970s until now [9], [10], [15]–[17]. However, linear programming requires the status of all the nodes and the links in the network to be available for a centralized control system.

In this paper a new algorithm that drives the power network is proposed aiming to minimize load shedding and to avoid cascaded failure in the network. The novel decentralized randomized control algorithm is implemented locally at each node using simple local control rules that are based only on information about the closest neighbours of each bus in the network. As a result, the computational costs are reduced and less information needs to be transferred on the network.

The proposed algorithm is theoretically verified. In particular, this paper provides a mathematically rigorous proof of convergence of the algorithm with probability 1 for any initial power values. Moreover, while the algorithm is simulated using the DC power flow model in this paper, the algorithm is model independent and future research will investigate adapting the algorithm to more accurate AC power flow models.

The randomized algorithm used in this paper is inspired by the distributed random algorithm used in [18] for blanket coverage self-deployment of a network of mobile wireless sensors.

## II. PROBLEM STATEMENT

Consider a power network consisting of $n$ buses connected by $m$ transmission lines. For a given time $k$, each bus is assigned a power rating value $N_i(k)$, $i = \{1, 2, \cdots, n\}$. The bus is considered a generator if

$$0 \leq N_i(k) \leq N_{i_{max}} \tag{1}$$

where $N_{i_{max}}$ is a positive value representing the maximum power generation capacity of the generator. The bus is considered a load if

$$N_{i_{min}} \leq N_i(k) \leq 0 \tag{2}$$

where $N_{i_{min}}$ is a negative value representing the full-loading capacity of the load.

For the transmission lines, each line connecting buses $N_i(k)$ and $N_j(k)$ has a power flow $F_{i,j}(k)$ that should not exceed a specific limit

$$|F_{i,j}(k)| \leq F_{i,j_{max}} \tag{3}$$

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otherwise the line will be overloaded and may be disconnected from the network. The two buses $N_i(k)$ and $N_j(k)$ connected through $F_{i,j}(k)$ communicate with each other at the discrete time instance $k = 1, 2, \cdots$ for the coordination of their operation.

This paper considers ideal power transmission networks with no power losses. In ideal networks, the power produced by the generator buses must equal the power consumed by the fully-loaded load buses, and the power delivered by the transmission lines must not exceed the maximum flow capacities of the lines.

A random failure or an intentional attack on the power network may cause one or more transmission lines to be disconnected. Moreover, the failure or attack may target one or more buses disconnecting those buses and the lines connecting them from the network. In such cases it is important to re-route the power in the network such that the minimum load is lost and no transmission lines are overloaded. Otherwise overloaded lines may fail and be disconnected from the network, triggering a cascaded failure in the entire network.

The goal of the proposed algorithm is to select power ratings of the buses $N_i(k)$ such that the maximum load is maintained and equations (1), (2) and (3) are satisfied. For a computerized controller to operate properly, a discretizing parameter $d$ is defined to be the smallest change in bus power ratings. Thus, the algorithm can increment or decrement the power ratings of the buses $N_i(k)$ by multiples of $d$.

*Assumption 2.1:* The power rating values of the buses $N_i(k)$ and their limits $N_{i_{min}}$ and $N_{i_{max}}$ are multiples of the discretizing parameter $d$.

### *A. The DC power flow model*

In the DC power flow model, one bus is selected as the reference bus (also called the slack bus). The reference bus is assumed to be linearly dependent on the other buses in the system and is not included in the model. The power rating of the reference bus is such that the balance between the generated power and the consumed power is maintained.

$$\sum_{i=1}^{n} N_i(k) = 0 \tag{4}$$

To model the power network an adjacency matrix $\mathbf{A}$ with dimensions $m \times n$ is constructed to reflect the relation between the buses and the transmission lines. If the direction of flow in the $r^{\text{th}}$ transmission line $F_{i,j}(k)$ is assumed to be from bus $N_i(k)$ to bus $N_j(k)$, then the $r^{\text{th}}$ row of the matrix $\mathbf{A}$ is zeros except for $A(r, i) = 1$ and $A(r, j) = -1$. In addition, the model requires a matrix $\mathbf{B}$ which is a $m \times m$ diagonal matrix with the susceptance of the transmission lines on the diagonal.

In the DC power flow model, the power flow in the transmission lines is calculated using the equation

$$\mathbf{F}(k) = \mathbf{H}\mathbf{N}(k) \tag{5}$$

where $\mathbf{F}(k)$ is the vector of the power flow in the transmission lines at time $k$, $\mathbf{N}(k)$ is the vector of power ratings of the buses except the reference bus, and

$$\mathbf{H} = \mathbf{BA}(\mathbf{A}'\mathbf{BA})^{-1} \tag{6}$$

is a constant matrix.

*Lemma 2.1:* For a power network modelled by (5) and satisfying the power rating limits (1) and (2) and Assumption 2.1, there exists a finite non-empty set of solutions that satisfy the power flow limit (3) and the power balance equation (4).

*Proof:* Assumption 2.1 along with the power rating limits equations (1) and (2) imply that each bus in the network has a finite set of values that can be assigned for its power rating.

$$\begin{aligned} N_i(k) \in \quad & \{N_{i_{min}}, N_{i_{min}} + d, N_{i_{min}} + 2d, \cdots, \\ & -d, 0, d, \cdots, N_{i_{max}} - d, N_{i_{max}}\} \end{aligned} \tag{7}$$

The number of elements in the set in (7) is $1 + \frac{1}{d}(N_{i_{max}} - N_{i_{min}})$. Therefore, the number of solutions obtained from different combinations of bus power rating values has a theoretical upper limit of

$$\prod_{i=1}^{n} \left(1 + \frac{1}{d}(N_{i_{max}} - N_{i_{min}})\right) \tag{8}$$

It follows that the number of solutions that satisfy (3) and (4) is finite and less than the theoretical upper limit (8). One solution that satisfy (3) and (4) is the trivial worst-case solution when all the loads in the network are shed. In this case we have

$$\begin{aligned} N_i(k) &= 0 \quad \forall i = \{1, 2, \cdots, n\} \\ F_{i,j} &= 0 \quad \forall i, j = \{1, 2, \cdots, n\} \end{aligned} \tag{9}$$

Hence there is at least one solution that satisfies (3) and (4). ■

The aim of the proposed algorithm is to converge to a solution $\bar{\mathbf{N}}(k)$ that satisfy Lemma 2.1 such that

$$\bar{\mathbf{N}}(k) = \arg\max_{\mathbf{N}(k)} \sum_{i=1}^{n} |N_i(k)| \tag{10}$$

for all the solutions $\mathbf{N}(k)$ that satisfy Lemma 2.1. Equation (10) may have one unique solution or a finite number of solutions at which the maximum possible load ratings are maintained. The proposed algorithm will converge to one of these solutions.

This paper proposes a distributed algorithm based only on information about the adjacent neighbours of each bus in the network. At this stage the algorithm does not include a way for a bus to detect overloaded lines not connected to it. The following assumption ensures that an overload in a transmission line is locally detected and removed by the buses on the two ends of the line.

*Assumption 2.2:* For the two buses $N_i(k)$ and $N_j(k)$ connected by a transmission lines carrying a power flow $F_{i,j}(k)$, the line power flow limits in the network (3) are such that an increase in the power rating difference between the two buses $N_i(k)$ and $N_j(k)$ causing the line flow to reach its maximum $|F_{i,j}(k)| \to F_{i,j_{max}}$ does not cause overload in other lines in the network.

Future research is to investigate the relaxation of Assumption 2.2.

## III. The Decentralized power re-routing algorithm

The goal of the proposed algorithm is to select power ratings of the buses such that the maximum load is maintained without overloading the transmission lines. The buses in the network are categorized into four sets of buses:

- the set of loads fully-connected to the network

$$\mathcal{N}_L(k) = \{N_i(k) : N_{i_{min}} = N_i(k) < 0\} \tag{11}$$

- the set of loads that are not fully-connected to the network

$$\mathcal{N}_l(k) = \{N_i(k) : N_{i_{min}} < N_i(k) \leq 0\} \tag{12}$$

- the set of under-loaded generators

$$\mathcal{N}_g(k) = \{N_i(k) : 0 \leq N_i(k) < N_{i_{max}}\} \tag{13}$$

- the set of generators operating at their maximum capacities

$$\mathcal{N}_G(k) = \{N_i(k) : 0 < N_i(k) = N_{i_{max}}\} \tag{14}$$

Let $\mathcal{Z}(N_i(k))$ be the set of buses connected to the bus $N_i(k)$ through non-overloaded transmission lines. Then the neighbourhood of $N_i(k)$ is defined as:

$$\mathcal{Z}(N_i(k)) = \{N_j(k) : \exists F_{i,j}(k),\ -A(r,i)F_{i,j}(k) < F_{i,j_{max}}\} \tag{15}$$

where $r$ is the row in $\mathbf{A}$ corresponding to $F_{i,j}$. The utilization of the coefficient $-A(r,i)$ is to account for the fact that if the flow direction is out of the load bus, then connecting more load produces an opposing flow towards the bus and the net flow in the transmission line decreases.

Also, let $\mathcal{P}(N_i(k), N_j(k))$ denote a set of buses excluding $N_j(k)$ that form a non-overloaded path in the network connecting buses $N_i(k)$ and $N_j(k)$, where all the buses in $\mathcal{P}(N_i(k), N_j(k))$ are connected along the path through non-overloaded transmission lines.

*Definition 3.1:* A bus $N_i(k)$ is said to be a **demanding** bus if it is a load bus not fully-connected to the network, or a generator bus operating at maximum capacity and connected to a not fully-connected load bus through a non-overloaded path composed of generator buses operating at maximum capacity. The set of demanding buses $\mathcal{D}(k)$ is defined as

$$\begin{aligned}\mathcal{D}(k) = \quad & \{N_i(k) : N_i(k) \in \mathcal{N}_l(k) \text{ OR} \\ & \exists \mathcal{P}(N_i(k), N_j(k)) \subset \mathcal{N}_G(k), N_j(k) \in \mathcal{N}_l(k)\}\end{aligned} \tag{16}$$

Since the objective is to maintain the maximum load ratings in the network, the algorithm allows the power ratings in the network to be driven by the demanding buses in $\mathcal{D}(k)$, mainly the set of load buses not fully-connected to the network. These load buses try to reach their maximum power ratings (and thus minimize load shedding). In addition, a generator bus operating at maximum capacity and connected to a demanding bus through a non-overloaded line is also considered to behave as a demanding buses. The reason is to allow such a generator to pass power from neighbouring generators to neighbouring loads (even though it is not able to generate more power locally).

*Definition 3.2:* A bus $N_i(k)$ is said to be a **supplying** bus if it is an under-loaded generator bus, or a fully-connected load bus that is connected to an under-loaded generator bus through a non-overloaded path composed of fully-connected load buses. The set of supplying buses $\mathcal{S}(k)$ is defined as

$$\begin{aligned}\mathcal{S}(k) = \quad & \{N_i(k) : N_i(k) \in \mathcal{N}_g(k) \text{ OR} \\ & \exists \mathcal{P}(N_i(k), N_j(k)) \subset \mathcal{N}_L(k), N_j(k) \in \mathcal{N}_g(k)\}\end{aligned} \tag{17}$$

The supplying buses $\mathcal{S}(k)$ are used to maintain the power balance changed by the demanding bused. Under-loaded generator buses can increase their power generation to accommodate the power consumption of the demanding buses. A fully-connected load bus that is connected to a supplying bus is also considered to behave as a supplying bus in order to pass power from neighbouring generators to neighbouring loads (even though it does not require more power locally).

The generation/loading power balance equation (4) must be maintained in the power network, and therefore an increase in the loading by the demanding buses is possible only when it can be balanced by an increase in the generation by the supplying buses. This is constrained by the power rating limits of the buses (1),(2) and the power flow limits in the transmission lines (3).

For each demanding bus $N_i(k) \in \mathcal{D}(k)$ a neighbourhood of candidate supplying buses $\mathcal{C}(N_i(k))$ is defined such that each candidate supplying bus is connected to the demanding bus through a non-overloaded transmission line.

$$\mathcal{C}(N_i(k)) = \{N_j(k) : N_j(k) \in \mathcal{S}(k) \cap \mathcal{Z}(N_i(k))\} \cup \{N_i(k)\} \tag{18}$$

This definition adds the demanding bus $N_i(k)$ to its neighbouring candidate supplying buses in order to avoid the situation where $\mathcal{C}(N_i(k))$ is empty.

The randomized algorithm operates as follows:
For each demanding bus $N_i(k) \in \mathcal{D}(k)$ the set of neighbouring candidate buses $\mathcal{C}(N_i(k))$ is found. A weight $w_j$ is assigned to each supplying bus $N_j(k) \in \mathcal{C}(N_i(k))$

$$w_j = N_{j_{max}} + N_{j_{min}} - N_j(k) + 2, \quad j \neq i \tag{19}$$

whereas the demanding bus $N_i(k)$ is assigned a weight $w_i = 1$.

A bus $N_j(k) \in \mathcal{C}(N_i(k))$ is randomly selected using weighted probabilities to balance the change in the network power according to the algorithm

$$\begin{aligned}& N_i(k+1) = N_i(k) - d \\ & \text{Select } N_j(k) \in \mathcal{C}(N_i(k)) \text{ with probability } \tfrac{w_j}{W} \\ & N_j(k+1) = N_j(k) + d\end{aligned} \tag{20}$$

where $W$ is the sum of all weights $W = \sum_j w_j$ including $w_i$, and $d$ is the discretizing parameter satisfying Assumption 2.1. This scheme gives the priority to under-loaded generators to be selected as supplying buses, followed by fully-connected loads, and finally the demanding bus in consideration.

The power rating of the demanding bus $N_i(k)$ is decremented by $d$ (increasing the load or reducing the generation), and the power rating of the randomly selected candidate bus $N_j(k)$ is incremented by $d$ (increasing the generation or reducing the load) such that the network power balance is maintained. If the selected candidate bus is $N_i(k)$ itself, then its value is not changed.

For a network where (3), (4) and Assumption 2.1 hold, the vector of power ratings assigned to the buses $\mathbf{N}(k)$ belongs to the finite non-empty set of solutions defined by Lemma 2.1. The the algorithm aims to remove the demanding buses from the set $\mathcal{D}(k)$ by fully-connecting the loads to the network. Assuming $\mathcal{C}(N_i(k))$ contains at least one supplying bus, the power rating of the demanding bus $N_i(k)$ is decremented by $d$ in each iteration getting the value closer to $N_{i_{min}}$.

$$N_i(k) \rightarrow N_{i_{min}}, \quad |\mathcal{C}(N_i(k))| > 1 \tag{21}$$

and the process continues until $N_i(k) = N_{i_{min}}$ at which the load is fully connected to the network and the bus is removed from the set $\mathcal{D}(k)$.

*Theorem 3.1:* For a power network modelled by (5), suppose that equations (3) and (4) and Assumption 2.1 hold and the buses in the network behave according to the algorithm (20). Then with probability 1 there exists a time $k_0 \geq 0$ such that the solution satisfies (10) for all $k > k_0$.

*Proof:* The proposed algorithm defines an absorbing Markov chain which includes absorbing states (that are impossible to leave). The absorbing states in the Markov chain are categorized into two groups. The first group of absorbing states is reached when all the loads are fully-connected to the network and no load shedding is required. In this case the generators are producing enough power to supply the loads and the transmission lines are able to carry the required power flow. The set of demanding buses is empty

$$\mathcal{D}(k) = \varnothing \tag{22}$$

and there are no buses to drive the algorithm so the network is in steady state.

The second group of absorbing states is reached when all the remaining demanding buses have no paths to connect them to a supplying bus and balancing a change in power is not possible. In this case all the demanding buses do not have candidate supplying buses in their neighbourhoods

$$\mathcal{C}(N_i(k)) = \{N_i(k)\}, \quad \forall N_i(k) \in \mathcal{D}(k) \tag{23}$$

and the probability for each bus $N_i(k)$ to maintain its current value is $\frac{w_i}{w_i} = \frac{1}{1} = 1$.

By attempting to remove the load buses form the set $\mathcal{D}(k)$, the algorithm drives the network towards one of the absorbing states. In any of these states it is impossible to increase the loads in the network and hence the value of any existing load shed is minimum and the solution satisfies (10). ∎

### *A. The reference bus in the DC power flow model*

In the DC power flow model a reference (slack) bus is nominated and its power rating value is assumed to be linearly dependent on the other buses in the network [9]. Hence it is important to assign a value to the reference bus such that the network power balance equation (4) holds.

Define $\mathcal{L}(N_i(k))$ to be the set of power flow values of the transmission lines connected to the bus $N_i(k)$. The network power balance implies that

$$\sum \mathcal{L}(N_i(k)) = N_i(k), \quad \forall N_i(k),\ i = \{1, 2, \cdots, n\} \tag{24}$$

A random failure or a targeted attack on the network disturbs the generation/loading power balance, and the power rating assigned to the reference bus must be modified to reflect the new net power flow in the bus as calculated by the DC power flow model. In some cases, however, the new required value may exceed the power rating limits of the reference bus or it may cause some transmission lines to be overloaded. Therefore in our simulation the reference bus is continuously changed in order to be able to maintain the generation/loading balance.

### *B. Overloaded transmission lines*

Each transmission line has a maximum power transfer capacity limit. If this limit is exceeded, the line is overloaded and may be disconnected from the network. Such case must be avoided as it may lead to a cascaded power failure.

In the proposed algorithm, if two buses are connected by an overloaded bus the power ratings of the two buses are changed such that the power flow is reduced below the line capacity. The power flows in a transmission line from the higher rating power bus to the lower rating power bus (note that loads have negative power ratings). When two buses detect that they are connected by an overloaded line, the higher power rating is decremented and the lower power rating is equally incremented. This way, the difference between the two power ratings is reduced and hence the power flow in the line is also reduced. In the same time the generation/loading balance in the network is maintained.

## IV. Simulation Results

### *A. Five-bus network*

A simple five-bus network [10], [19] is used to illustrate the proposed algorithm. The topology of the network and the bus load demands are shown in Fig. 1. Transmission lines reactance is expressed on a base of $100MVA$ and $138kV$ with a power flow capacity $F_{max} = 100MW$. All generators have a maximum generation limit of $150MW$. Assuming that each generator supplies local load demand at its bus before providing power to the network, the buses can be considered as network generators or network loads as shown in Table I.

Table II shows the power loss if one transmission line is disconnected from the network due to a failure or an attack. Verifying that the algorithm results in minimum load shed is trivial for this simple network. For example, it is clear that if any of the lines connected to Bus 5 is targeted, then the load loss is $50MW$ as the bus will be connected to the network through one line that is capable of delivering $100MW$ only. Otherwise, the network can tolerate the failure and re-route

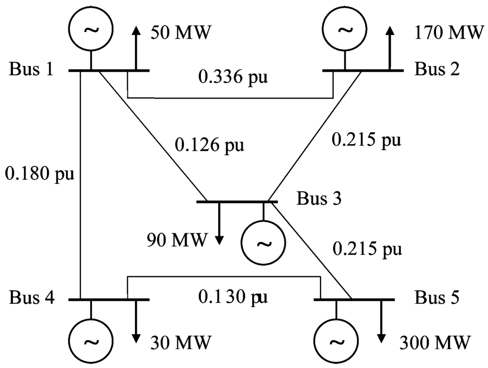


Fig. 1. Five-Bus System Example [19]

TABLE I
FIVE-BUS EXAMPLE BUS POWER RATINGS

| Bus Number | Bus Type | Power rating limits |
|---|---|---|
| Bus 1 | Generator | $0 \leq N_1 \leq 100$ |
| Bus 2 | Load | $-20 \leq N_2 \leq 0$ |
| Bus 3 | Generator | $0 \leq N_3 \leq 60$ |
| Bus 4 | Generator | $0 \leq N_4 \leq 120$ |
| Bus 5 | Load | $-150 \leq N_5 \leq 0$ |

TABLE II
FIVE-BUS EXAMPLE: ONE LINE TARGETED

| Line Targeted | Power Loss (MW) | Iterations ($d$ = 10) |
|---|---|---|
| $F_{3,5}$ | 50 | 4 |
| $F_{4,5}$ | 50 | 4 |
| $F_{1,2}$ | 0 | 1 |
| $F_{1,3}$ | 0 | 9 |
| $F_{1,4}$ | 0 | 1 |
| $F_{2,3}$ | 0 | 1 |

the power through the remaining lines. However, targeting the transmission line $F_{1,3}$ initially causes the line $F_{4,5}$ to exceed its power flow capacity and the algorithm immediately deals with this situation by reducing the power ratings of the generator Bus 4 and the load Bus 5. Once the overload is avoided, the algorithm re-routs the power flow to deliver the required power to Bus 5.

Table III shows the power loss if two lines are disconnected from the network due to a failure or an attack. The worst case is when the two lines connecting Bus 5 are targeted and the Bus is disconnected from the network.

Table IV shows the power loss if one bus is the target of a failure or an attack and disconnected from the network.

### B. Fourteen-bus network

To study the effect of the discretizing parameter $d$ on the speed of convergence of the algorithm, a Fourteen-bus system is used. The network is similar to the IEEE 14-bus system [20], [21] where the connections and line reactances are the same, but the power ratings of the buses are changed. All the transmission lines are assigned a maximum capacity of

TABLE III
FIVE-BUS EXAMPLE: TWO LINES TARGETED

| Lines Targeted | Power Loss (MW) | Notes |
|---|---|---|
| $F_{3,5}$, $F_{4,5}$ | 150 | bus 5 disconnected |
| $F_{1,2}$, $F_{3,5}$ | 50 | |
| $F_{1,2}$, $F_{4,5}$ | 50 | |
| $F_{1,3}$, $F_{3,5}$ | 50 | |
| $F_{1,3}$, $F_{4,5}$ | 50 | |
| $F_{1,4}$, $F_{3,5}$ | 50 | two networks |
| $F_{1,4}$, $F_{4,5}$ | 50 | bus 4 disconnected |
| $F_{2,3}$, $F_{3,5}$ | 50 | |
| $F_{2,3}$, $F_{4,5}$ | 50 | |
| $F_{1,2}$, $F_{2,3}$ | 20 | bus 2 disconnected |
| $F_{1,2}$, $F_{1,3}$ | 10 | |
| $F_{1,2}$, $F_{1,4}$ | 0 | |
| $F_{1,3}$, $F_{1,4}$ | 0 | |
| $F_{1,3}$, $F_{2,3}$ | 0 | |
| $F_{1,4}$, $F_{2,3}$ | 0 | |

TABLE IV
FIVE-BUS EXAMPLE: ONE BUS TARGETED

| Bus targeted | Power Loss (MW) | Disconnected Lines |
|---|---|---|
| Bus 5 | 150 | $F_{3,5}$, $F_{4,5}$ |
| Bus 3 | 50 | $F_{1,3}$, $F_{2,3}$, $F_{3,5}$ |
| Bus 4 | 50 | $F_{1,4}$, $F_{4,5}$ |
| Bus 2 | 20 | $F_{1,2}$, $F_{2,3}$ |
| Bus 1 | 10 | $F_{1,2}$, $F_{1,3}$, $F_{1,4}$ |

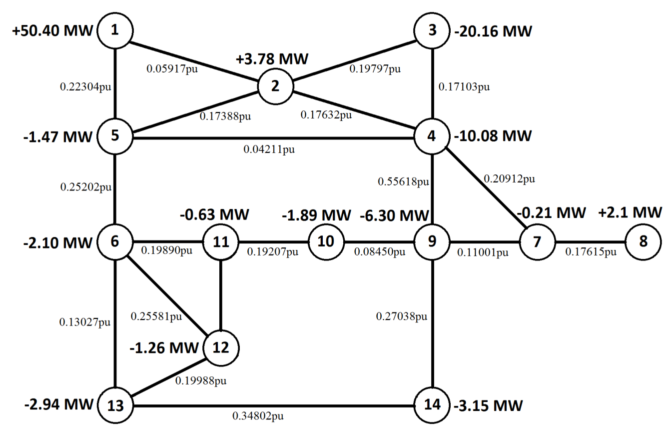


Fig. 2. Fourteen-Bus System Example

$F_{i,j_{max}} = 40MW$. The network is presented in Fig. 2.

The network is assumed to be at steady state with all load buses fully connected, then the transmission line $F_{1,2}$ is targeted. The results for targeting the transmission line are the average of running the test 100 times and the results for different values of $d$ are presented in Table V.

In another test, the network is assumed to be at steady state with all load buses fully connected, then the generator Bus 2 and the transmission line $F_{10,11}$ are both targeted. The results for different values of $d$ that are presented in Table VI are the average of running the test 100 times.

TABLE V
FOURTEEN-BUS EXAMPLE: TRANSMISSION LINE $F_{1,2}$ TARGETED

| $d$ (kW) | Iterations | Power Loss (MW) |
|---|---|---|
| 35 | 71 | 4.305 |
| 42 | 61 | 4.326 |
| 70 | 40 | 4.340 |
| 105 | 28 | 4.305 |
| 210 | 18 | 4.410 |

TABLE VI
FOURTEEN-BUS EXAMPLE: BUS 2 AND TRANSMISSION LINE $F_{10,11}$ TARGETED

| $d$ (kW) | Iterations | Power Loss (MW) |
|---|---|---|
| 35 | 66 | 8.085 |
| 42 | 56 | 8.106 |
| 70 | 34 | 8.120 |
| 105 | 23 | 8.085 |
| 210 | 13 | 8.190 |

The results in Tables V and VI show that the number of iterations required to reach steady state is inversely proportional to the discretizing parameter $d$. It can be seen that a larger discretizing parameter $d$ reaches steady state faster, but may result in a small increase in load shed due to the larger change in power ratings. Another interesting observation is that the algorithm reaches steady state faster when the required load shed is larger. This is due to the fact that for recovery from a failure with less load shed, the generators have to increase their power rating to higher values requiring more iterations.

## V. CONCLUSION AND FUTURE WORK

A distributed power redistribution algorithm for power transmission networks after a failure/attack was proposed. A convergence with probability 1 of this algorithm with minimum load shed was proved.

Simulation results using a five-bus network and a fourteen-bus network verified the performance of the algorithm and showed the effect of the discretizing parameter on the speed of convergence.

Some improvements to this algorithm are to be investigated in future research. First, the algorithm does not account for buses with no power rating (i.e. $N_{i_{min}} = N_{i_{max}}$=0). For the current algorithm to operate with such buses, a bus is considered to be a very small load with $N_{i_{min}} = -d$. Second, the algorithm is to be improved to allow a bus to anticipate any line overloading that would result from adjusting the power rating of the bus in any line and not only in adjacent lines. Finally, while the DC power flow model provides a fast linear approximation of the active power flow in the network, it does not include the reactive power flow and thus a more detailed AC power flow model is to be investigated.


## ACKNOWLEDGMENT

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