Adaptive Control Using Multiple Parallel Dynamic Neural Networks

Chao Jia and Xiaoli Li
School of Automation and Electrical Engineering
University of Science and Technology Beijing
Key Laboratory of Advanced Control of Iron and Steel Process (Ministry of Education)
Beijing, 100083, P. R. C
E-mail: jiachaocharles@gmail.com
Corresponding author: lixiaoli@hotmail.com

Dexin Liu and Dawei Ding
School of Automation and Electrical Engineering
University of Science and Technology Beijing
Key Laboratory of Advanced Control of Iron and Steel Process (Ministry of Education)
Beijing, 100083, P. R. C
E-mail: bjldxin@126.com
E-mail: ddaweiauto@163.com

***Abstract*—The control problem of an unknown nonlinear dynamic system which contains the abrupt changes of parameters is concerned. Multiple models based on dynamic neural networks are used to approximate the dynamic character of unknown system. Different controllers based on these models and an effectively switching mechanism are applied to an unknown system to trace a reference trajectory. Further, we propose different switching and turning schemes for adaptive control which combine fixed and adaptive models. From the simulation, it can be shown that the multiple model adaptive control method proposed in this paper can improve the control performance greatly compared with the conventional adaptive control.**

I. Introduction

It is well known that the neural networks have the merits of massive parallelism, fast adaptability, and approximation ability for any complicated nonlinear systems accurately, thus neural networks have concentrated a lot of researches ([1]-[4]), especially in the area of identification and control a complex nonlinear system. The neural networks always has great advantage for the system without complete model information, or the system regarded as 'black-box' [7]. According to the connection of the neural networks, it can be classified as static (feedforward) and dynamic (recurrent) nets. The shortcoming of the static nets for identification is too long time for the convergence of parameters of identified system and the function approximation is sensitive to training data. In contrast, dynamic neural networks can successfully overcome this disadvantage, because of their structure corporate feedback [8]. There exists two kinds of structure of identification model: a 'series-parallel' model and a 'parallel' model. The output of the parallel identification model is a combination of its past values as well as those of the input. In the series-parallel model, the output is a combination of the past values of the input and output of the plant [9].

With the development of the neural networks, the results of adaptive control research in linear systems had been extended to nonlinear systems. However, the conventional adaptive control systems are usually based on a fixed or slowly adaptive model. This implicitly assumes that the operating environment is either time invariant, or varies slowly with time. So when the parameters or structure or the set-point value of system change abruptly from one context to another, the conventional adaptive control will react slowly, the output of the system will change abruptly and may be out of control at this time.

One way to solve this problem is to use multiple model adaptive control (MMAC). The approach first introduced in [5] and [6] for improving the transient response. In recent years, multiple model adaptive control using neural networks has been paid more attention in control theory and its applications ([14]-[18]). When the environment of a system changes abruptly, the original model (hence controller) is no longer valid. If models are available for different environments, controllers corresponding to them can be designed *a priori*. During system operation, one has to identify the existing environment to determine the correct controller. Such identification can again be achieved if a model for each environment is known in advance. Based on these two ideas, the control stately proposed is to determine the best model for the current environment at every instant and activate the corresponding controller [10].

To the best of our knowledge, most of the MMAC is based on linear system. In this paper, we established different kinds of multiple neural networks with different initial weights in many cases respectively. Different controllers based on these models and an effectively switching mechanism are applied to an unknown system to trace a reference trajectory. The switching scheme determine when to switch and which one model can be switched to. From the simulation, the different adaptive control strategies proposed using multiple models will be tested for the improvement of the performance of adaptive control systems.

II. Identification and Control Using Parallel Dynamic Neural Networks

A. Identification

The nonlinear system to be identified is given as

$$\begin{aligned}&\dot{x}(t) = f(x(t), u(t), t)\\ &x(t) \in \Re^n, u(t) \in \Re^m\end{aligned} \tag{1}$$

978-1-4673-5769-2/13/$31.00 ©2013 IEEE

For the series-parallel dynamic neural network used in ([11], [12]) we construct the following parallel dynamic neural network:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + W_1(t)\sigma(\hat{x}(t)) + W_2(t)\phi(\hat{x}(t))\gamma(u(t))$$

where $\hat{x}(t)\in\Re^n$ is the state of the neural network, $u(t)\in\Re^m$, $W_1(t)\in\Re^{n\times n}$ is the matrix for nonlinear state feedback, $W_2(t)\in\Re^{n\times n}$ is the input matrix and $A\in\Re^{n\times n}$ is a stable matrix. The vector functions $\sigma(\hat{x}(t)) \in \Re^n$ is assumed to be n-dimensional with the elements increasing monotonically. The matrix function $\phi(\cdot)$ is assumed to be $\Re^{n\times n}$ diagonal: $\phi(\hat{x}_t) = diag(\phi_1(\hat{x}_1)...\phi_n(\hat{x}_n))$. The typical presentation of the elements $\sigma_i(\cdot)$ and $\phi_i(\cdot)$ are as sigmoid functions, that is,

$$\sigma_i(x_i) = \frac{a_i}{1+e^{-b_i x_i}} - c_i \tag{2}$$

This neural network is the simplest one and does not contain any hidden layers.

Because $\sigma(\cdot)$ and $\phi(\cdot)$ are chosen as sigmoid functions, clearly they satisfy the following assumptions.

Assumption 1: Functions $\sigma(\cdot)$ and $\phi(\cdot)$ satisfy the following sector conditions (Fig. 1):

$$\sigma^T(x)Z_\sigma\sigma(x) \leq x^T C_\sigma x$$

$$\phi^T(x)Z_\phi\phi(x) \leq x^T C_\phi x$$

and their differences $\tilde{\sigma}$ and $\tilde{\phi}$ fulfill the generalized Lipshitz condition:

$$\tilde{\sigma}^T\Lambda_\sigma\tilde{\sigma} \leq \Delta^T D_\sigma\Delta$$

$$\tilde{\phi}^T\Lambda_\phi\tilde{\phi} \leq \Delta^T D_\phi\Delta$$

where $\tilde{\sigma}$:=$\sigma(\hat{x})-\sigma(x)$, $\tilde{\phi}$:=$\phi(\hat{x})-\phi(x)$, Λ_σ, Λ_ϕ, D_σ, D_ϕ, Z_σ, Z_ϕ, C_σ, C_ϕ are known positive constants. We assume the control input function $\gamma(\cdot)$ is differentiable.

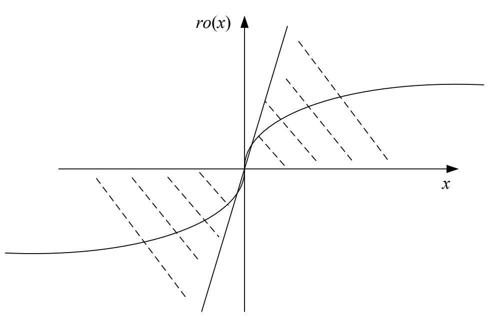


Fig. 1. *Shaded part satisfies the sector condition*

Assumption 2: The nonlinear function $\gamma(\cdot)$ is a differentiable input function and $\gamma(\cdot) : \Re^m \to \Re^n$.

To simplify our further analysis, we select $\phi(\cdot) = I$, throughout of this paper we will consider these dynamic NN's

$$\dot{\hat{x}}(t) = A\hat{x}(t) + W_1(t)\sigma(\hat{x}(t)) + W_2(t)\gamma(u(t)) \tag{3}$$

Let us first we assume that an exact neural network model of the plant without the unmodelled dynamics, that is, there exist weights W_1^* and W_2^* such that the nonlinear system (eqn. 1) is complete described by following neural network:

$$\dot{x}(t) = Ax(t) + W_1^*\sigma(x(t)) + W_2^*\gamma(u(t)) \tag{4}$$

where W_1^* and W_2^* are bounded as

$$\begin{array}{c} W_1^*\Lambda_1^{-1}W_1^{*T} \leq \bar{W}_1 \\ W_2^*\Lambda_2^{-1}W_2^{*T} \leq \bar{W}_2 \end{array} \tag{5}$$

here Λ_1, Λ_2, $\bar{W}_1$ and $\bar{W}_2$ are already known matrices.

In fact, the system (eqn. 1) cannot be completely described by eqn. 4, and there is always an approximation error Δf:

$$-\Delta f = Ax(t) + W_1^*\sigma(x(t)) + W_2^*\gamma(u(t)) - f(x(t), u(t), t)$$

Where W_1^* and W_2^* are fixed weights satisfying conditions eqns. 5. As for the unmodeled dynamics Δf, we assume that it is bounded as [11].

Assumption 3: There exist diagonal matrices $\eta_\sigma \in \Re^n$ and normalising matrices Λ_1 such that

$$\begin{array}{c} ||\Delta f||^2 \leq \bar{\eta}_\sigma \\ \bar{\eta}_\sigma = ||\eta_\sigma||^2_{\Lambda_1} := \eta_\sigma^T\Lambda_1\eta_\sigma \end{array}$$

In this paper, we just consider the neural network model without unmodeled dynamics. In subsequent research work, we will give a more detailed and comprehensive paper with unmodeled dynamics.

It is well known if the pair $(A, R^{1/2})$ is controllable, the pair $(Q^{1/2}, A)$ is observable and a local frequency condition

$$A^T R^{-1} A - Q \geq \frac{1}{4}[A^T R^{-1} - R^{-1}A]R[A^T R^{-1} - R^{-1}A]^T$$

is fulfilled [13], then the following matrix Riccati equation:

$$A^T P + PA + PRP + Q = 0 \tag{6}$$

has a solution $P = P^T > 0$. Thus we can make following assumption:

Assumption 4: There exists a strictly positive defined matrix Q_0 such that if $R := \bar{W}_1$, $Q := Q_0 + Q_\sigma$, the matrix Riccati equation (eqn. 6) has a positive solution.

This condition is easily fulfilled if we select A as stable diagonal matrix. The following theorem states the learning procedure of parallel dynamic neural network.

Theorem 1: Consider the unknown nonlinear system (eqn. 1) and a model matching neural network (eqn. 4) whose weights are adjusted as

$$\begin{array}{c} \dot{W}_1(t) = -k_1 P\Delta(t)\sigma^T(\hat{x}(t)) \\ \dot{W}_2(t) = -k_2 P\Delta(t)\gamma^T(u(t)) \end{array} \tag{7}$$

where k_1 and k_2 are positive constants, P is the solution of the matrix Riccati equation (eqn. 6).

□

Assume that assumptions 1-4 hold. Then,

$$\begin{array}{c} \lim\limits_{t\to\infty} \Delta(t) = 0 \\ W_1(t) \in L_\infty, W_2(t) \in L_\infty \end{array} \tag{8}$$

where, $\Delta(t) := \hat{x}(t) - x(t)$.

The reader is referred to [8] for more detailed proof of the updating law.

B. Control

The unknown nonlinear dynamic system (eqn. 1) is identified by a dynamic neural network (eqn. 4). The control goal is to force the system states to track an optimal trajectory $x_t^* \in \Re^r$ which is assumed to be sufficiently smooth. This trajectory is regarded as a solution of a nonlinear reference model:

$$\dot{x}^*(t) = \varphi(x^*(t), t) \tag{9}$$

with a fixed and known initial condition. In the case of the regulation problem $\varphi(x^*(t), t) = 0$, $x^*(0) = c$, c is constant.

Define the error between the identifier states and the reference model states as

$$\Delta^*(t) = \hat{x}(t) - x^*(t) \tag{10}$$

Differentiating (eqn. 10) we obtain

$$\dot{\Delta}^*(t) = \dot{\hat{x}}(t) - \dot{x}^*(t)$$

or

$$\begin{aligned} \dot{\Delta}^*(t) &= A\hat{x}(t) + W_1(t)\sigma(\hat{x}(t)) \\ &+ W_2(t)\gamma(u(t)) - \varphi(x^*(t), t) \end{aligned} \tag{11}$$

Taking $\gamma(u(t))$ to be equal to

$$\begin{aligned} \gamma(u(t)) &= W_2^{-1}(t)[\varphi(x^*(t), t) \\ &- Ax_t^*(t) - W_1(t)\sigma(\hat{x}(t))] \end{aligned} \tag{12}$$

and substituting it to eqn. 11 we finally obtain

$$\dot{\Delta}^*(t) = A\Delta^*(t) \tag{13}$$

To apply the control law eqn. 12, we have to assure the existence of $W_2^{-1}(t)$. This can also be guaranteed by a projection algorithm [20], [21], [22].

III. IDENTIFICATION AND CONTROL USING MULTIPLE PARALLEL DYNAMIC NEURAL NETWORKS

The conventional adaptive control systems are usually based on a fixed or slowly adaptive model. It reacts too slowly to abrupt changes, resulting in large transient errors before convergence. Therefore, we use multiple model adaptive control to solve this problem, in which multiple models are used to cover the uncertainty of the controlled plant. Meanwhile, at least one model should be close to the unknown controlled plant adequately. The structure of the multiple model adaptive control is shown as Fig. 2.

In this section, we consider two different combinations of fixed and adaptive models. They correspond to i) multiple adaptive models, ii) multiple fixed models and one adaptive model. We want to demonstrate that the multiple model adaptive control method proposed in this paper can improve the control performance greatly compared with conventional adaptive control.

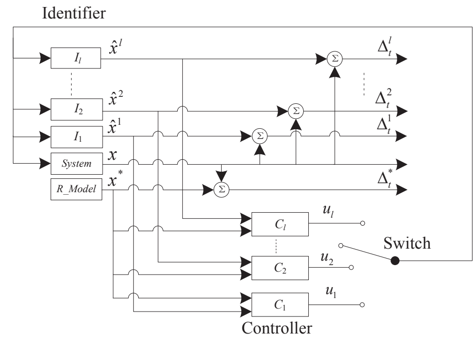


Fig. 2. *Structure of the multiple model adaptive control*

A. Multiple adaptive models

Multiple adaptive models can be regard as an extension of conventional indirect adaptive control. Reference to the section II-A, we construct the multiple adaptive identification models as

$$\dot{\hat{x}}^l(t) = A\hat{x}^l(t) + W_1^l(t)\sigma(\hat{x}^l(t)) + W_2^l(t)\gamma(u^l(t)) \tag{14}$$

where, $l \in \{1, 2 \cdots M\}$. Then the weights are adjusted as

$$\begin{aligned} \dot{W}_1^l(t) &= -k_1 P\Delta^l(t)\sigma^T(\hat{x}^l(t)) \\ \dot{W}_2^l(t) &= -k_2 P\Delta^l(t)\gamma^T(u^l(t)) \end{aligned} \tag{15}$$

where,

$$\Delta^l(t) = \hat{x}^l(t) - x(t). \tag{16}$$

Multiple adaptive models with different initial weights adjust dynamically in any instant. When the models and environments are parameterized suitably, the model with the smallest error, according to some criterion, is selected rapidly and then its parameters are adjusted.

B. Multiple fixed models and one adaptive model

In section III-A, we adopt multiple adaptive models. Because of each adaptive model need to adjust dynamically, it will produce massive calculation. Furthermore, when the parameters change, the change has to be detected and the weights of neural networks must be initialized to adapt the new environment with choosing the optimal model again. Fixed models do not have these drawbacks. But for all fixed models, the requirement that the parameter error of at least one model be small enough is difficult to be satisfied. This condition is automatically satisfied by an adaptive model after finite time [6]. If we combine with the fixed model and the adaptive model, then the efficiency can be improved by the multiple fixed models and the accuracy can be increased by the adaptive model.

C. The switching rule

The switching scheme determine when to switch and which one model can be switched to. On the basis of the identification error, the specific performance index has the form

$$J_l(t) = \alpha(\Delta^l(t))^2 + \beta\int_0^t (\Delta^l(\tau))^2 d\tau \tag{17}$$

where, $\Delta^l(t) = \hat{x}^l(t) - x(t)$, $\alpha > 0$, $\beta > 0$, and $l \in \{1, 2 \cdots M\}$.

The detailed discussions on the choice of this index can be find in [5], [6] and [19].

D. Multiple model adaptive control

The model set and switching rule in section III-A, B, C will be the main elements for multiple model control. Considering the switching instants are $\{T_i\}, i = 1, 2, \cdots$, where, $T_{i+1} - T_i > T_{\min} > 0$, $T_{\min}$ is a nonzero lower bound for switching interval. The multiple model adaptive controller can be given as the following steps:

Step 1: For the system to be controlled, multiple parallel dynamic neural network models with weights $W_1^l(t)$ and $W_2^l(t)$ are given. $W_1^l(t)$ and $W_2^l(t)$, $l \in \{1, 2 \cdots M\}$ can be calculated adaptively or $W_1^l(t)$ and $W_2^l(t)$, $l \in \{1, 2 \cdots M-1\}$ are fixed, only $W_1^M(t)$ and $W_2^M(t)$ are calculated adaptively.

Step 2: For the interval between two successive switches $[T_i, T_{i+1})$, calculating

$$l'(t) = \arg\min_{1 \leq l \leq M} J_l(t) \qquad t \in [T_i, T_{i+1}) \tag{18}$$

Let $W_1(t) = W_2^{l'}(t)$, $W_2(t) = W_2^{l'}(t)$, and the controller can be set up as eqn. 12. The following theorem states the stability of multiple adaptive models controller.

Theorem 2: Based on multiple adaptive models and *step 1, 2*, the application of the multiple model adaptive controller for unknown system (eqn. 1) will lead to the following results

$$\lim_{t \to \infty} \Delta'(t) = 0$$
$$W_1(t) \in L_\infty, W_2(t) \in L_\infty$$

where, $\Delta'(t) := x(t) - x^*(t)$ is defined as control error. □

Proof: For $t \in [T_i, T_{i+1})$, from eqn. 18 we have

$$\begin{gathered} \hat{x}(t) = \hat{x}^{l'}(t) \\ W_1(t) = W_1^{l'}(t), W_2(t) = W_2^{l'}(t) \end{gathered} \tag{19}$$

The control error can be rewritten as

$$\begin{aligned} \Delta'(t) &= x(t) - \hat{x}(t) + \hat{x}(t) - x^*(t) \\ &= x(t) - \hat{x}^{l'}(t) + \hat{x}(t) - x^*(t) \end{aligned}$$

then, from eqns. 10 and 16, we obtain

$$\Delta'(t) = -\Delta^{l'}(t) + \Delta^*(t) \tag{20}$$

where, $l' \in \{1, 2 \cdots M\}$ and satisfies eqn. 18. This holds over any arbitrary interval $[T_i, T_{i+1})$. From eqns. 8, 14 and 15,

$$\begin{gathered} \lim_{t \to \infty} \Delta^l(t) = 0 \\ W_1^l(t) \in L_\infty, W_2^l(t) \in L_\infty \end{gathered} \tag{21}$$

where, $l \in \{1, 2 \cdots M\}$.

So,

$$\begin{gathered} \lim_{t \to \infty} \Delta^{l'}(t) = 0 \\ W_1^{l'}(t) \in L_\infty, W_2^{l'}(t) \in L_\infty \end{gathered} \tag{22}$$

In addition, according to the eqn. 13, we have

$$\lim_{t \to \infty} \Delta^*(t) = 0 \tag{23}$$

Then, from eqns. 19-23, we have

$$\lim_{t \to \infty} \Delta'(t) = 0$$
$$W_1(t) = W_1^{l'}(t) \in L_\infty$$
$$W_2(t) = W_2^{l'}(t) \in L_\infty$$

■

According to the performance index (eqn. 17), the actual controller for the system is set up on the model with the minimum index at every switching instant. When the parameters change abruptly, the change has to be detected, then the weights and the index functions of neural networks must be initialized to adapt the new environment by choosing the optimal model again.

As the theorem 2, the similar results for the stability of the MMAC with fixed and adaptive models can also be proved. Due to space limitations, the proof is omitted.

IV. SIMULATION

In this section, the following nonlinear system which contains the abrupt changes of parameters will be considered

$$\dot{x}_1 = -5x_1 + 3sign(x_2) + u_1$$
$$\dot{x}_2 = -i(t)x_2 + 2sign(x_1) + u_2$$

where, $x_1(0) = 0$, $x_2(0) = 0$,

$$i(t) = \begin{cases} 10 & 0 \leq t \leq 50 \\ 3 & 50 < t \leq 100 \end{cases}$$

The control goal is to force the system states to track a nonlinear reference model trajectory. We select the reference model as

$$\dot{x}_1^* = x_2^*$$
$$\dot{x}_2^* = \sin(x_1^*)$$

where $x_2^*(0) = 0$, $x_1^*(0) = 1$. We select the identification model as eqn. 3. The sigmoid function are chosen as

$$\sigma(x_i) = \frac{2}{1 + e^{-2x_i}} - 0.5$$

The initial parameters are

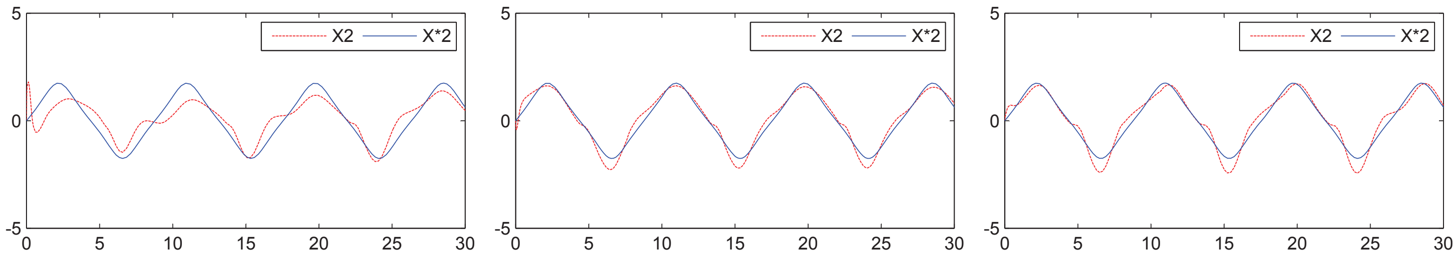
$$A = \begin{bmatrix} -15 & 0 \\ 0 & -10 \end{bmatrix} \quad \hat{x}_0 = [-5, -5]^T$$

$$W_{1,0} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \quad W_{2,0} = \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \end{bmatrix}$$

$Q_0{=}I$, $R{=}\begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix}$ and $Q{=}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. The solution of the Riccati equation (eqn. 6) is

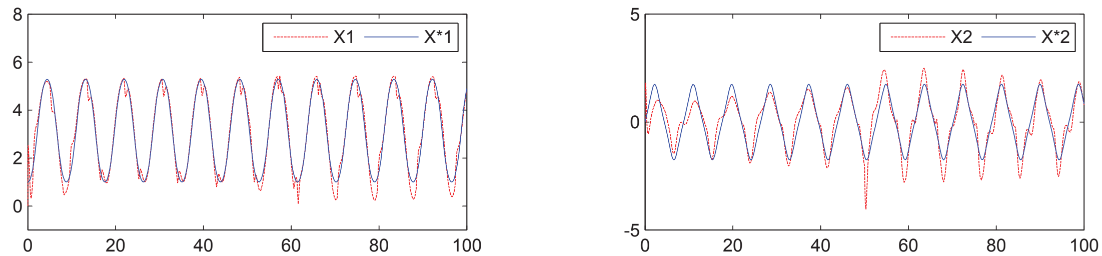
$$P = \begin{bmatrix} 0.28 & 0.09 \\ 0.09 & 0.11 \end{bmatrix}$$

and adaptive gains are $k_1{=}k_2{=}20$. The weights are updated according to eqn. 7. From the simulation, we want to



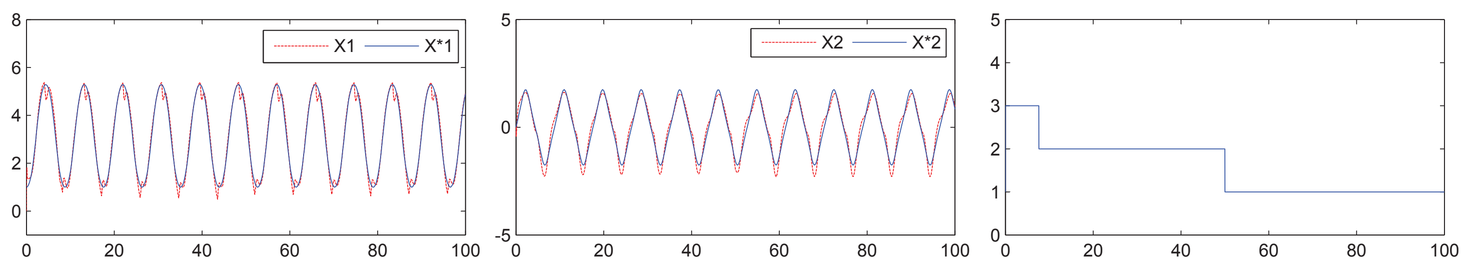
(a) Adaptive model only (b) Three adaptive model (c) Three fixed model and one adaptive model

Fig. 3. *Transient response*



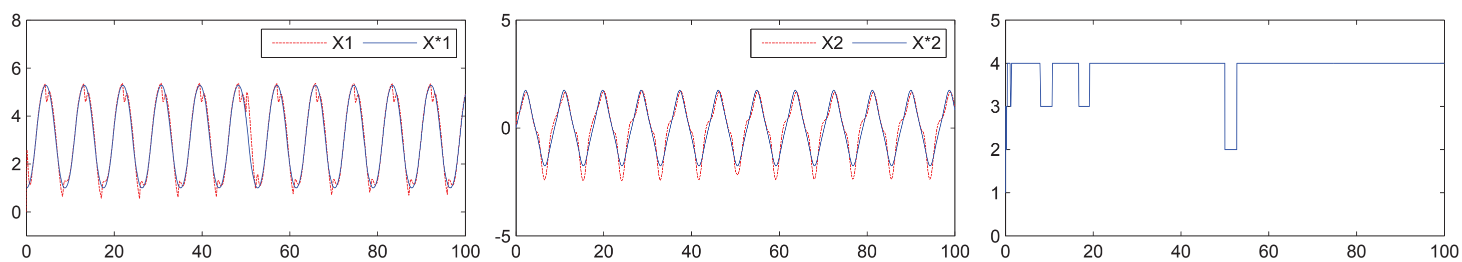
(a) (b)

Adaptive model only



(c) (d) (e)

Three adaptive models



(f) (g) (h)

Three fixed models and one adaptive model

Fig. 4. *Control for the system with abrupt change in parameter*

demonstrate that the multiple model adaptive control method proposed in this paper can improve the control performance.

Adaptive model only: In the process of parameter identification, since the identified initial values of weights are far away from the truth values, the conventional adaptive model always will lead to a poor transient response (Fig. 3a). When the parameters change at 50 seconds, the overshoot of the system is bigger and the weights need to be adjusted again. For quite a long time, the nonlinear system cannot track the reference model trajectory (Fig. 4a, b).

Multiple adaptive models: In this section, we establish three adaptive models (I_{a1}, I_{a2}, I_{a3}) with different initial weights. The multiple models based on neural networks are chosen as eqn.14. Fig. 4c, d present the plant responses. Switching sequence of controllers is shown in Fig. 4e. Obviously this method can track the reference trajectory fast and improve the transient response during the early control period (Fig. 3b). According to the index function, the system can choose an approximate model to identify the unknown plant. Once the parameters change, the change can be detected, then the weights and the index functions of neural network models will be initialized and the system will choose the optimal model again to conduct identification. In this way, the overshot of the system can be decreased and the reference trajectory can be tracked fast at the same time (Fig. 4c, d).

multiple fixed models and one adaptive model: Considering each adaptive model need to adjust dynamically, then it will produce massive calculation. We use three fixed models to improve the efficiency and one adaptive model to increase the accuracy. We establish three fixed models (I_{f1}, I_{f2}, I_{f3}) with different initial weights, and I_{a4} is the adaptive model. In the process of parameter identification, it could improve the transient response compared with the conventional adaptive control (Fig. 3c). Switching sequence of controllers is shown in Fig. 4h. In the early period, the controller will switch between the fixed models. After a period of time, the system will be switched to the adaptive model (I_{a4}) due to the convergence of weight identified. Once the parameters change abruptly at 50 seconds, the controller will switch to the nearest fixed model (I_{f2}) to reduce the error. When the adaptive model gradually converge to the true value, the system will switch to the adaptive model (I_{a4}) again (Fig. 4 f, g). Multiple fixed models play an excessive role in the process of identification.

V. CONCLUSION

The purpose of this paper is to control unknown nonlinear dynamic systems adaptively using dynamic neural networks. Because of the unknown dynamic character of the system, firstly we introduce the structure of the neural networks identification model and establish the controller with one adaptive model. Further, based on the conventional adaptive controller, we propose the MMAC methods. Our main contributions are that we proposes different switching and turning schemes for adaptive control which combine fixed and adaptive models. We give a mathematical proof of the stability of switching. Finally, from the simulation, it can be shown that the multiple model adaptive control method proposed in this paper can improve the control performance greatly compared with the conventional adaptive neural networks controller.

ACKNOWLEDGMENT

This work is partially supported by the Fund of National Natural Science Foundation of P. R. China (Grant No. 61074055, 61104013), Program for New Century Excellent Talents in Universities (NCET-11-0578) and the Fundamental Research Funds for the Central Universities (FRF-TP-12-005B).

REFERENCES

[1] G. P. Liu, V. Kadirkamanathan, *Predictive control for non-linear systems using neural networks* International Journal of Control, Vol. 71, No. 6, pp. 1119-1132, 1998

[2] P. M. MILLS, A. Y. ZOMAYA, M. O. TAD, *Adaptive model-based control using neural networks* International Journal of Control, Vol. 60, No. 6, pp. 1163-1192, 1994.

[3] Yan-Jun Liu, Shaocheng Tong, Yongming Li, *Adaptive neural network tracking control for a class of non-linear systems* International Journal of Systems Science, Vol. 41, No. 2, pp. 143-158, 2010

[4] K. S. Narendra, Osvaldo A. Driollet Matthias Feiler, Koshy George, *Adaptive control using multiple models, switching and tuning* International Journal of Adaptive Control and Signal Processing, Vol. 17, No. 2, pp. 87-102, 2003.

[5] K. S. Narendra and J. Balakrishnan, *Performance improvement in adaptive control systems using multiple models and switching* in Proc. Seventh Yale Wrkshp. Adaptive Learning Syst., Center for Systems Sicience, Yale University, New Haven, CT, May 1992, pp. 27-33.

[6] K. S. Narendra and J. Balakrishnan, *Improving transient response of adaptive control systems using multiple models and switching* IEEE Transactions on Automatic Control, Vol. 39, pp. 1861-1866, 1994.

[7] K. J. Hunt, D. Sbarbaro, R. Zbikowski, and P. J. Gawthrop, *Neural Networks for control systems–A survey* Automatica, Vol. 28, pp. 1083-1112, 1992.

[8] W. Yu, A. S. Poznyak, *Indirect adaptive control via parallel dynamic neural networks* IEE Proceedings Control Theory and Applications, Vol. 46, pp. 25-30, 1999.

[9] K. S. Narendra and K. Parthasarathy, *Identification and control of dynamic systems using neural networks* IEEE Transactions on Neural Networks, pp. 4-27, 1990.

[10] K. S. Narendra and J. Balakrishnan, *Adaptive control using multiple models* IEEE Transactions on Automatic Control, Vol. 42, No. 2, pp. 171-187, 1997.

[11] G. A. Rovithakis and M. A. Christodoulou, *Direct adaptive regulation of unknown nonlinear dynamical system via dynamical neural networks* IEEE Transactions on Systems, Man, and Cybernetics, pp. 1578-1594, 1995.

[12] A. S. Poznyak and E. N. Sanchez, *Nonlinear system approximation by neural networks:error stability analysis* International Journal of Intelligent Automation and Soft Computing, pp. 247-258, 1995.

[13] J. C. Willems, *Least squares optimal control and algebraic Riccati equations* IEEE Transactions on Automatic Control, pp. 621-634, 1971.

[14] Yu-Ying Guo, Bin Jiang, *Multiple model-based adaptive reconfiguration control for actuator fault* Acta Automatica Sinica, Vol. 35, No. 11, pp. 1452-1458, 2009.

[15] K. S. Yeung, W. D. Chung, H. Yu, *Identifier-tracking model reference adaptive control*, International Journal of Control, Vol. 81, No. 6, pp. 976-988, 2008.

[16] Weitian Chen, Anderson, B. D. O., *A combined multiple model adaptive control scheme and its application to nonlinear systems with nonlinear parameterization* IEEE Transactions on Automatic Control, Vol. 57, No. 7, pp. 1778-1782, July 2012.

[17] Zhuo Han, K. S. Narendra, *New concepts in adaptive control using multiple models* IEEE Transactions on Automatic Control, vol. 57, No. 1, pp. 78-89, Jan. 2012.

[18] Xudong Ye, *Nonlinear adaptive control using multiple identification models* Systems and Control Letters, Vol. 57, No. 7, pp. 578-584, July 2008

[19] T. Autenrieth and E. Rogers, *Performance enhancements for a class of multiple model adaptive control schemes* International Journal of Adaptive Control and Signal Processing, Vol.13, No. 2, pp. 105-127, 1999.

[20] K. S. Narendra and A. M. Annaswamy, *Stable adaptive systems* Englewood Cliffs, NJ: Prentice Hall, 1989.

[21] G. C. Goodwin and D. Q. Mayne, *A parameter estimation perspective of continuous time model refernce adaptive control* Automatica, Vol.23, No. 2, pp. 57-70, 1987.

[22] P. A. Ioannou and A. Datta, *Robust adaptive control: Design, analysis and robustness bounds* in Foundations of Adaptive Control, P. V. Kokotovic, ed. Berlin: Springer-Verlag, 1977.