Formation Control of Multiple Robots Using Constrained Motion Formulation

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Abstract: *This paper presents a novel approach for formation control of multiple robots by representing the desired formation as motion constraints and transferring the problem to cooperative hybrid position/force control of the robots. The desired formation is achieved by controlling the virtual forces of the robots induced on the constraint manifolds. It is shown that a simple PD plus virtual force feedback controller yields asymptotic stability in the leader-follower formation regulation of multiple robots. To avoid planning desired trajectories for all the robots in the formation tracking problem, a new concept called reference propagation is proposed to propagate the desired motion of the group leader to all the other robots. Based on this new concept, a new controller is also developed for formation tracking of multiple robots in a leader-follower formation. The stability of the controllers is rigorously proved by Lyapunov theory. Simulations were performed to validate the proposed approaches.*

Keywords: **Formation Control, Multiple Robots, Virtual Force.**

I. INTRODUCTION

Formation control of multiple robots or agents has received extensive attention in robotics and control community [1]-[5]. A number of approaches, including the behavior-based approach [2][5][6][11], the consensus formulation [4][14][15], the leader-follower formulation [7][8][12][13], the synchronization method[1][9][10], the virtual structure approach [20][21], to name a few, have been proposed. Behavior-based approaches control the robots reactively but are hard to guarantee the stability of the system. The virtual structure methods are not distributed control. The consensus methods formulated the formation control problem by the graph theory and control the states or outputs of all the robots to approach the common trajectories. A large number of consensus controllers have been developed for linear multi-agent systems under different network topologies [15][22][23], but there is few work consensus for nonlinear systems like robots whose dynamics are nonlinear in general.

The leader-follower approach provides a good formulation to formation control of multiple robots because robots usually carry out tasks under the leadership of a robot, like the group behaviors of animals, fish, and birds. In the leader-follower formation, there is a group leader leading motion of all the other robots. In [12][13], Desai et. al. and Das et. al. proposed distributed formation controllers for two and three robots to control their separations and bearings using the input/output linearization technique. In the controllers, the inputs of the followers depend on that of their leaders. When there are a large number of robots, all the robots must know the input of the group leader. In this sense, this control is not truly distributed. The controller in [18] deals with the unit-speed motion of nonholonomic robots in circular or parallel formations, which are special cases. The work [8] coped with vision-based localization and control of two mobile robots. The synchronization method, proposed by Sun et al. [9][10], applied motion synchronization to formation control of robots. However, the method is not suitable for cases when the desired trajectory of the leader is not known to the followers before the motion. The approach in [1] is more focused on motion planning of mobile robots for formation control. The cooperative controller of non-holonomic robots in [17] needs the desired trajectories of the robots.

In this paper, we formulate leader-follower formation control of multiple robots as a constrained motion problem of robots. Both the formation regulation and tracking problems are addressed. We use the constraint manifolds to represent the desired formation of the robots. Any derivation from the desired formation is considered as a virtual force. As a result, the formation control problem can be transferred into a cooperative hybrid position/force control problem of robots [24][25]. Since only the group leader knows its desired trajectory, we propose a new method called "reference propagation" to propagate the desired motion of the robots from the leader to the followers. Using the reference propagation, it is not necessary to plan the desired trajectories for all the robots, as conducted in most of existing formation controllers. It has been proved by Lyapunov theory that the proposed method yields asymptotic stability of the system. To relax the communication requirement in the reference propagation, we also developed a controller with delayed reference propagation, which propagates the reference information of the leader at the previous sampling time to the follower. The formation controller with delayed reference propagation results in bounded formation errors whose upper bound is determined by the sampling period. We have performed a simulation on 5 planar manipulators to validate the proposed controllers. The contribution of this work can be summarized as follows. First, to avoid planning of desired motion for all the robots, the new concept of reference propagation is developed, which makes it possible for all the followers to follow motion of the group leader without knowing its desired motion. Second, we formulate the formation control problem as constrained problems of robots and applied the cooperative hybrid controller to the formation control problem. Third, the proposed controller is developed based on nonlinear robotic dynamics, while most existing controllers are for linear systems.

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II. Robot Dynamics and Problem Formulation

A. Robot Dynamics

Consider a number of robots moving in a formation. The robots could be holonomic mobile robots, or manipulators, or free-flying robots, etc. Denote the number of the robots by *N*. The position (including orientation) of the robot *i* is represented by $\mathbf{q}_i(t)$. The dynamics of the robot can be represented as the following general form:

$$\mathbf{H}_i(\mathbf{q}_i(t)) + \left\{ \frac{1}{2}\dot{\mathbf{H}}_i(\mathbf{q}_i(t)) + \mathbf{S}_i(\mathbf{q}_i(t), \dot{\mathbf{q}}_i(t))\dot{\mathbf{q}}_i(t) \right\} + \mathbf{G}_i(\mathbf{q}_i(t)) = \mathbf{u}_i(t) \tag{1}$$

where $\mathbf{H}_i(\mathbf{q}_i(t))$ is the inertia matrix. The second term represents the nonlinear centrifugal and Coriolis forces, and $\mathbf{S}_i(\mathbf{q}_i(t), \dot{\mathbf{q}}_i(t))$ is a screw-symmetric matrix. $\mathbf{G}_i(\mathbf{q}_i(t))$ is the gravity force and $\mathbf{u}_i(t)$ denotes the input of the robot.

B. Formation and Motion Constraints

Assume that for robot *i* there are *n*(*i*) robots in its neighborhood, which can communicate with the robot *i* bi-directionally via wireless links. Denote the set of the robots in the neighborhood of the robot *i* by $Neig(i)$. In the desired motion formation, the positions of the robot *i* and its neighborhood robots must satisfy a desired function:

$$\mathbf{f}_{ij}(\mathbf{q}_i(t), \mathbf{q}_j(t)) = \mathbf{0} \qquad j \in Neig(i) \tag{2}$$

Eq. (2) represents the formation constraints. Assume that there is a robot, called the group leader and denoted by *L*, to lead motion of the group of robots. Furthermore, for simplicity we assume that all the robots have the same number of degrees of freedom (DOF) and their motion is fully constrained by the desired formation constraints (2).

Assumption 1: The formation constraint functions $\mathbf{f}_{ij}(\mathbf{q}_i(t), \mathbf{q}_j(t))$ in eq. (2) are differentiable. The functions and their derivatives are Lipschitz continuous in $\mathbf{q}_i(t)$ and $\mathbf{q}_j(t)$.

In this paper, we will use bold capital letters to represent matrices, bold lower case letters to represent vectors, and italic letters to denote scalars.

C. Problem Statement

Initially, the robots are not in the desired formation (2). We consider the following two types of problems in this paper:

Formation Regulation Problem: *Given a desired position* $\mathbf{q}_{Ld}$ *of the group leader L, design a distributed controller for each robot so that the group leader asymptotically approaches the desired position and the robots maintain the desired formation defined in eq. (2).*

Formation Tracking Problem: *Given a desired trajectory, denoted by* $(\mathbf{q}_{Ld}(t), \dot{\mathbf{q}}_{Ld}(t), \ddot{\mathbf{q}}_{Ld}(t))$, *of the group leader, design a distributed controller for each robot so that the group leader asymptotically follows the desired trajectory while all the robots maintain the desired formation defined in eq. (2).*

It should be noted that the distributed controller means that the controller uses feedback from itself or its neighborhood. No communication more than 1-hop is allowed.

III. Formation Regulation Using Virtual Force

A. Definition of Virtual Force

The desired formation eq. (2) imposes constraints on motion of the robots. The control objective is to ensure the robots to move on the surfaces (manifolds) corresponding to eq. (2). The problem is similar to cooperation of multiple robots on constrained surfaces. Therefore, the formation control problem can be transferred to cooperative hybrid position and force control problem of multiple robots. During motion of the robots, the formation equation (2) does not hold before the convergence, i.e., in general

$$\mathbf{f}_{ij}(\mathbf{q}_i(t), \mathbf{q}_j(t)) = \mathbf{f}_{ij}(t) \neq \mathbf{0} \qquad j \in Neig(i) \tag{3}$$

where $\mathbf{f}_{ij}(t)$ is the formation error, which is considered as a virtual force attracting the two robots. $\mathbf{f}_{ij}(t)$ is the resultant due to deviation from the constraint manifolds. The control objective here is to enforce the virtual force to zero. Let the error of the virtual force be $\Delta\mathbf{f}_{ij}(t)$, and

$$\Delta\mathbf{f}_{ij}(t) = \mathbf{f}_{ij}(t) - \mathbf{0} = \mathbf{f}_{ij}(t) \tag{4}$$

Differentiating equation (3) leads to

$$\mathbf{J}_{ij}(t)\dot{\mathbf{q}}_i + \mathbf{J}_{ji}(t)\dot{\mathbf{q}}_j(t) = \Delta\dot{\mathbf{f}}_{ij}(t) \tag{5}$$

where $\mathbf{J}_{ij}(t)$ and $\mathbf{J}_{ji}(t)$ represent the Jacobian matrices:

$$\mathbf{J}_{ij}(t) = \partial\mathbf{f}_{ij}(\mathbf{q}_i(t), \mathbf{q}_j(t)) / \partial\mathbf{q}_i(t) \tag{6}$$

$$\mathbf{J}_{ji}(t) = \partial\mathbf{f}_{ij}(\mathbf{q}_i(t), \mathbf{q}_j(t)) / \partial\mathbf{q}_j(t) \tag{7}$$

For *n(i)* neighborhood robots of the robot *i*, there are *n(i)* virtual forces, which correspond to the formation errors. Let

$$\Delta\mathbf{f}_i(t) = \left(\Delta\mathbf{f}_{ij_1}(t), \Delta\mathbf{f}_{ij_2}(t), ..., \Delta\mathbf{f}_{ij_{n(i)}}(t)\right)^T \qquad j_k \in Neig(i) \tag{8}$$

$$\mathbf{J}_i(t) = \left(\mathbf{J}_{ij_1}(t), \mathbf{J}_{ij_1}(t), ..., \mathbf{J}_{ij_{n(i)}}(t)\right)^T \tag{9}$$

By introducing the virtual force, the formation control problem can be changed to a problem of cooperatively controlling a number of robots to move a set of constrained surfaces in a decentralized manner in which both the force and position of the robot must be controlled. Therefore, existing approaches [24][25] for decentralized cooperation control can be used.

B. Control for the Group Leader

In the formation regulation problem, the desired position of the group leader *L* is given as a constant vector $\mathbf{q}_{Ld}$. The desired velocity and acceleration of the robot are equal to zero. It should be noted that the desired positions of the other robots are not given. The position error of the group leader is given by

$$\Delta\mathbf{q}_L(t) = \mathbf{q}_L(t) - \mathbf{q}_{Ld} \tag{10}$$

To control the position of the group leader, we adopt the classical the PD feedback plus gravity compensation controller. To maintain the formation with the neighboring robots, we include a feedback of the virtual forces in the controller. In detail, the controller is given as follows:

$$\mathbf{u}_L(t) = \mathbf{G}_L(\mathbf{q}_L(t)) - \mathbf{B}_L\dot{\mathbf{q}}_L(t) - \mathbf{A}_L\Delta\mathbf{q}_L(t) - \mathbf{K}_L\mathbf{J}_L^T(t)\Delta\mathbf{f}_L(t) \quad (11)$$

where $\mathbf{B}_L$, $\mathbf{A}_L$, and $\mathbf{K}_L = \mathrm{diag}\{\mathbf{K}_{Lj}\}, j \in Neig(L)$ are positive-definite gain matrices. By submitting this into eq. (1), we have the following closed-loop dynamics:

$$\mathbf{H}_L(\mathbf{q}_L(t))\ddot{\mathbf{q}}_L(t) + \left\{\frac{1}{2}\dot{\mathbf{H}}_L(\mathbf{q}_L(t)) + \mathbf{S}_L(\mathbf{q}_L(t),\dot{\mathbf{q}}_L(t))\right\}\dot{\mathbf{q}}_L(t) = -\mathbf{B}_L\dot{q}_L(t) - \mathbf{A}_L\Delta\mathbf{q}_L(t) - \mathbf{K}_L\mathbf{J}_L^T(t)\Delta\mathbf{f}_1(t) \quad (12)$$

C. Controller for following Robots

For the robots other than the group leader, their desired positions are not known. The control objective is to enforce the formation errors to zero. We introduce the following controller:

$$\mathbf{u}_i(t) = \mathbf{G}_i(\mathbf{q}_i(t)) - \mathbf{B}_i\dot{\mathbf{q}}_i(t) - \mathbf{K}_i\mathbf{J}_i^T(t)\Delta\mathbf{f}_i(t) \quad (13)$$

The first term is the gravity compensator, and the second term is the velocity feedback. The third term is the virtual force feedback. $\mathbf{B}_i$ and $\mathbf{K}_i = \mathrm{diag}\{\mathbf{K}_{ij}\}, j \in Neig(i)$ are positive-definite gain matrices. By substituting (13) into the robot dynamics (1), we have the following closed-loop dynamics:

$$\mathbf{H}_i(\mathbf{q}_i(t))\ddot{\mathbf{q}}_i(t) + \left\{\frac{1}{2}\dot{\mathbf{H}}_i(\mathbf{q}_i) + \mathbf{S}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\right\}\dot{\mathbf{q}}_i(t) = -\mathbf{B}_i\dot{\mathbf{q}}_i(t) - \mathbf{K}_i\mathbf{J}_i^T(t)\Delta\mathbf{f}_i(t) \quad (14)$$

D. Stability Analysis

Theorem 1: *The proposed controllers (11) and (13), respectively for the group leader and the followers, will give rise to convergence of the position error of the group leader and the virtual forces between the robots to zero as time approaches the infinity, i.e.*

$$\lim_{t\to\infty}\Delta\mathbf{q}_L = \mathbf{0},\ \lim_{t\to\infty}\dot{\mathbf{q}}_i = 0,\ \textit{and}\ \lim_{t\to\infty}\Delta\mathbf{f}_i(t) = \mathbf{0}. \quad (15)$$

Proof: Consider the following positive-definite function:

$$V(t) = \frac{1}{2}\Delta\mathbf{q}_L^T(t)\mathbf{A}_L\Delta\mathbf{q}_L(t) + \frac{1}{2}\sum_{i=1}^{N}\dot{\mathbf{q}}_i^T(t)\mathbf{H}_i(\mathbf{q}_i(t))\dot{\mathbf{q}}_i(t) + \frac{1}{2}\sum_{i=1}^{N}\Delta\mathbf{f}_i^T(t)\mathbf{K}_i\Delta\mathbf{f}_i(t) \quad (16)$$

By multiplying $\dot{\mathbf{q}}_L^T(t)$ to the closed-loop equation (12) of the group leader from the left, we have

$$\dot{\mathbf{q}}_L^T(t)\mathbf{H}_L(\mathbf{q}_L(t))\ddot{\mathbf{q}}_L(t) + \frac{1}{2}\dot{\mathbf{q}}_L^T(t)\dot{\mathbf{H}}_L(\mathbf{q}_L(t))\dot{\mathbf{q}}_L(t) = -\dot{\mathbf{q}}_L^T(t)\mathbf{B}_L\dot{\mathbf{q}}_L^T(t) - \dot{\mathbf{q}}_L^T(t)\mathbf{A}_L\Delta\mathbf{q}_L^T(t) - \dot{\mathbf{q}}_L^T(t)\mathbf{K}_L\mathbf{J}_L^T(t)\Delta\mathbf{f}_L(t) \quad (17)$$

Note that $\dot{\mathbf{q}}_L^T(t) = \Delta\dot{\mathbf{q}}_L^T(t)$. Multiplying $\dot{\mathbf{q}}_i^T(t)$ to eq. (14) of the robot i from the left leads to

$$\dot{\mathbf{q}}_i^T(t)\mathbf{H}_i(\mathbf{q}_i(t))\ddot{\mathbf{q}}_i(t) + \frac{1}{2}\dot{\mathbf{q}}_i^T(t)\dot{\mathbf{H}}_i(\mathbf{q}_i(t))\dot{\mathbf{q}}_i(t) = -\dot{\mathbf{q}}_i^T(t)\mathbf{B}_i\dot{\mathbf{q}}_i^T(t) - \dot{\mathbf{q}}_i^T(t)\mathbf{K}_i\mathbf{J}_i^T(t)\Delta\mathbf{f}_i(t) \quad (18)$$

By summarizing eqs. (17) and (18) for all the robots, we have

$$\frac{d}{dt}\left\{\frac{1}{2}\Delta\mathbf{q}_L^T(t)\mathbf{A}_L\Delta\mathbf{q}_L(t) + \frac{1}{2}\sum_{i=1}^{N}\dot{\mathbf{q}}_i^T(t)\mathbf{H}_i(\mathbf{q}_i(t))\dot{\mathbf{q}}_i(t)\right\} = -\sum_{i=1}^{N}\dot{\mathbf{q}}_i^T(t)\mathbf{B}_L\dot{\mathbf{q}}_i(t) - \sum_{i=1}^{N}\dot{\mathbf{q}}_i^T(t)\mathbf{K}_i\mathbf{J}_i^T(t)\Delta\mathbf{f}_i(t) \quad (19)$$

Note that when $\mathbf{K}_{ij} = \mathbf{K}_{ji}$ from eq. (5),

$$\mathbf{K}_{ij}\mathbf{J}_{ij}(t)\dot{\mathbf{q}}_i(t) + \mathbf{K}_{ji}\mathbf{J}_{ji}(t)\dot{\mathbf{q}}_j(t) = \mathbf{K}_{ij}\Delta\dot{\mathbf{f}}_{ij}(t) \quad (20)$$

Therefore,

$$\sum_{i=1}^{N}\dot{\mathbf{q}}_i^T(t)\mathbf{K}_i\mathbf{J}_i^T(t)\Delta\mathbf{f}_i(t) = \sum_{i=1}^{N}\sum_{k=1}^{n(i)}\Delta\dot{\mathbf{f}}_{ij}^T(t)\mathbf{K}_{ij}\Delta\mathbf{f}_{ij}(t) \quad (21)$$

Consequently,

$$\dot{V}(t) = -\sum_{i=2}^{N}\dot{\mathbf{q}}_i^T(t)\mathbf{B}_i\dot{\mathbf{q}}_i(t) \quad (22)$$

Therefore, $V(t)$ never increases. From LaSalle theorem, we have convergence of the velocity $\dot{\mathbf{q}}_i(t)$ to zero, and

$$\lim_{t\to\infty}\mathbf{A}_L\Delta\mathbf{q}_L + \mathbf{K}_L\mathbf{J}_L^T(t)\Delta\mathbf{f}_1(t) = \mathbf{0} \quad (23)$$

$$\lim_{t\to\infty}\mathbf{J}_i^T(t)\Delta\mathbf{f}_i(t) = \mathbf{0} \qquad i = 1,2,\ldots,N(i \neq L) \quad (24)$$

To further prove the convergence of the position error and formation error, it is important to note that $\Delta\mathbf{f}_{ij}(t) = \Delta\mathbf{f}_{ji}(t)$. Let the degrees of freedom of the robot be *m*, so the total number of degrees of freedom of the system is *Nm*. Let *R* denote the total number of independent variables in the formation errors $\Delta\mathbf{f}_i(t)$, which is $R = 0.5\sum_{i=1}^{N}n(i)$. We consider two possible cases. First, when $Nm \geq R + m$, eqs. (23) and (24) imply that there are more linear equations than the unknown variables or the same number of equations and unknowns. Obviously, $\Delta\mathbf{q}_L(t)$ and $\Delta\mathbf{f}_{ij}(t)$ will be convergent to zero in this case. Second, when $Nm < R + m$, there are more formation constraints than the degrees of freedom of the robots. Assume that there exists a unique desired position $\mathbf{q}_{id}$ for each robot corresponding to the desired position of the group leader and the desired formation.

$$\Delta\mathbf{f}_{ij}(t) = \mathbf{f}_{ij}(\mathbf{q}_i(t),\mathbf{q}_j(t)) - \mathbf{f}_{ij}(\mathbf{q}_{id},\mathbf{q}_{jd}) \quad (25)$$

By applying Taylor's expansion derives, we have

$$\Delta\mathbf{f}_{ij}(t) = \mathbf{J}_{ij}(\tilde{\mathbf{q}}_i,\tilde{\mathbf{q}}_j)\Delta\mathbf{q}_i(t) + \mathbf{J}_{ji}(\tilde{\mathbf{q}}_i,\tilde{\mathbf{q}}_j)\Delta\mathbf{q}_j(t) \quad (26)$$

Here $(\tilde{\mathbf{q}}_i^T,\tilde{\mathbf{q}}_j^T)$ is a point on the line connecting to $(\mathbf{q}_{id}^T,\mathbf{q}_{jd}^T)$ and $(\mathbf{q}_i^T(t),\mathbf{q}_j^T(t))$. By substituting eq. (26) into eqs. (23) and (24),

$$\lim_{t\to\infty}\mathbf{Q}(\tilde{\mathbf{q}}_1,\tilde{\mathbf{q}}_2,\ldots,\tilde{\mathbf{q}}_N)\begin{pmatrix}\Delta\mathbf{q}_1(t)\\ \Delta\mathbf{q}_1(t)\\ \cdots\cdots\\ \Delta\mathbf{q}_N(t)\end{pmatrix} = 0 \quad (27)$$

It should be noted that $\mathbf{Q}(\tilde{\mathbf{q}}_1,\tilde{\mathbf{q}}_2,\ldots,\tilde{\mathbf{q}}_N)$ is a square matrix. It is reasonable to assume that $\mathbf{Q}(\tilde{\mathbf{q}}_1,\tilde{\mathbf{q}}_2,\ldots,\tilde{\mathbf{q}}_N)$ is not singular. Therefore, we can conclude that all the robots will be convergent to desired positions, and hence the convergence of the formation error to zero. The proof of Theorem 1 is completed.

IV. FORMATION TRACKING CONTROL

This section considers the formation tracking problem of a group of robots. For simplicity, we consider only the case when the robots form a tree structure in the formation (Fig. 1). The group leader L is at the root node of the tree. For a robot i, it has one leader only, which is its parent.

The desired trajectory of the group leader is given as $\{\mathbf{q}_{Ld}(t), \dot{\mathbf{q}}_{Ld}(t), \ddot{\mathbf{q}}_{Ld}(t)\}$. It is important to note that the desired trajectories of all the follower robots are not known. The controller proposed in this paper does not plan the desired trajectories for the other robots. The control objective here is to enforce that the group leader follows the desired trajectory and the other robots move in the desired formation.

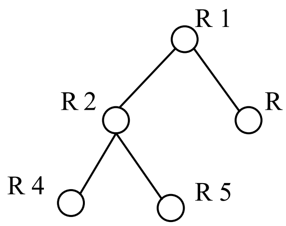


Fig. 1 The robots in a tree topology used in the simulation.

Our controller is designed on the basis of a new concept called *reference propagation* which propagates the reference information from the leaders to the followers. The core idea is the introduction of a *propagated velocity*, which will be propagated from the leaders to the followers.

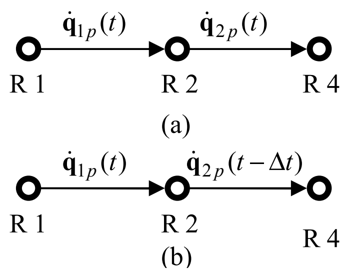


Fig. 2 The reference propagation (a) and the delayed reference propagation (b) in the topology of Fig. 1.

A. Controller Design of the Group Leader

We introduce the following vector:

$$\dot{\mathbf{q}}_{Lp}(t) = \dot{\mathbf{q}}_{Ld}(t) - \lambda_L \Delta \mathbf{q}_{Ld}(t) \tag{28}$$

where λ_L is a positive constant. $\dot{\mathbf{q}}_{Lp}(t)$ is called *propagated velocity* of the group leader. The proposed controller propagates $\dot{\mathbf{q}}_{Lp}(t)$ to the immediate followers of the group leader. The nominal reference of the group leader is given by

$$\dot{\mathbf{q}}_{Lr}(t) = \dot{\mathbf{q}}_{Lp}(t) - \beta \mathbf{J}_L^T(t)\Delta \mathbf{f}_L(t) \tag{29}$$

where β_L is a positive-definite constant. The nominal acceleration can be derived from eq. (29) by differentiation. The error vector of the group leader is given by

$$\mathbf{s}_L(t) = \dot{\mathbf{q}}_L(t) - \dot{\mathbf{q}}_{Lr}(t) = \Delta\dot{\mathbf{q}}_L(t) + \lambda \Delta \mathbf{q}_L(t) + \beta \mathbf{J}_L^T(t)\Delta \mathbf{f}_L(t) \tag{30}$$

The controller is similar to the cooperative controller [24][25] for hybrid position and force in the joint space:

$$\begin{aligned}\mathbf{u}_L(t) &= \mathbf{G}_L(\mathbf{q}_L) + \mathbf{H}_L(\mathbf{q}_L(t))\ddot{\mathbf{q}}_{Lr}(t) \\ &\quad + \left\{\frac{1}{2}\dot{\mathbf{H}}_L(\mathbf{q}_L(t)) + \mathbf{S}_L(\mathbf{q}_L(t), \dot{\mathbf{q}}_L(t))\right\}\dot{\mathbf{q}}_{Lr}(t) \\ &\quad - \mathbf{B}_L \mathbf{s}_L(t) - \mathbf{K}_L \mathbf{J}_L^T(t)\Delta \mathbf{f}_L(t)\end{aligned} \tag{30}$$

The first three terms on the right side of eq. (30) represents compensation of nonlinear forces of robot dynamics using the nominal references. By substituting the controller into eq. (1), we have following closed-loop dynamics:

$$\begin{aligned}&\mathbf{H}_L(\mathbf{q}_L(t))\dot{\mathbf{s}}_L(t) + \left\{\frac{1}{2}\dot{\mathbf{H}}_L(\mathbf{q}_L(t)) + \mathbf{S}_L(\mathbf{q}_L(t), \dot{\mathbf{q}}_L(t))\right\}\mathbf{s}_L(t) \\ &\quad = -\mathbf{B}_L \mathbf{s}_L(t) - \mathbf{K}_L \mathbf{J}_L^T(t)\Delta \mathbf{f}_L(t)\end{aligned} \tag{31}$$

B. Controller of Following Robots

For a robot i, it has only one leader, denoted by robot *l(i)*.The propagated velocity of the robot i is given as follows:

$$\dot{\mathbf{q}}_{ip}(t) = -\mathbf{J}_{il(i)}^{+}(t)\mathbf{J}_{l(i)i}(t)\left\{\dot{\mathbf{q}}_{l(i)p}(t) + \varepsilon_i \mathbf{J}_{l(i)i}^T(t)\Delta \mathbf{f}_{il(i)}(t)\right\} \tag{32}$$

where ε_i is a positive constant. In eq. (32), $\dot{\mathbf{q}}_{l(i)p}(t)$ is the propagated velocity of the leader robot *l(i)* . Fig. 2 (a) shows the scheme. The nominal reference of the robot is defined by

$$\dot{\mathbf{q}}_{ir}(t) = \dot{\mathbf{q}}_{ip}(t) - \beta_i \mathbf{J}_i^T(t)\Delta \mathbf{f}_i(t) \tag{33}$$

The nominal acceleration is calculated by differentiating eq. (33). The error vector of the robot is given by

$$\mathbf{s}_i(t) = \dot{\mathbf{q}}_i(t) - \dot{\mathbf{q}}_{ir}(t) = \dot{\mathbf{q}}_i(t) - \dot{\mathbf{q}}_{ip}(t) + \beta_i \mathbf{J}_i^T(t)\Delta \mathbf{f}_i(t) \tag{34}$$

The controller has a similar form to that for the group leader, i.e.

$$\begin{aligned}\mathbf{u}_i(t) &= \mathbf{G}_i(\mathbf{q}_i(t)) + \mathbf{H}_i(\mathbf{q}_i(t))\ddot{\mathbf{q}}_{ir}(t) \\ &\quad + \left\{\frac{1}{2}\dot{\mathbf{H}}_i(\mathbf{q}_i(t)) + \mathbf{S}_i(\mathbf{q}_i(t), \dot{\mathbf{q}}_i(t))\right\}\dot{\mathbf{q}}_{ir}(t) + \\ &\quad - \mathbf{B}_i \mathbf{s}_i(t) - \mathbf{K}_i \mathbf{J}_i^T(t)\Delta \mathbf{f}_i(t)\end{aligned} \tag{35}$$

Substituting the controller (35) into eq. (1) leads to the following closed-loop dynamics of the system:

$$\begin{aligned}&\mathbf{H}_i(\mathbf{q}_i(t))\dot{\mathbf{s}}_i(t) + \left\{\frac{1}{2}\dot{\mathbf{H}}_i(\mathbf{q}_i(t)) + \mathbf{S}_i(\mathbf{q}_i(t), \dot{\mathbf{q}}_i(t))\right\}\mathbf{s}_i(t) = \\ &\quad -\mathbf{B}_i \mathbf{s}_i(t) - \mathbf{K}_i \mathbf{J}_i^T(t)\Delta \mathbf{f}_i(t)\end{aligned} \tag{36}$$

C. Stability Analysis

Theorem 2: *The controller leads to asymptotic formation tracking of the robots, i.e.*

$$\lim_{t\to\infty} \Delta \mathbf{q}_L(t) = 0, \ \lim_{t\to\infty} \Delta \dot{\mathbf{q}}_L(t) = 0, \text{ and } \lim_{t\to\infty} \Delta \mathbf{f}_{ij}(t) = 0, \forall i, j$$

Proof: Consider the following positive-definite function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} \mathbf{s}_i^T(t)\mathbf{H}_i(\mathbf{q}_i(t))\mathbf{s}_i(t) + \frac{1}{2}\sum_{i=1}^{N} \Delta \mathbf{f}_i^T(t)\mathbf{K}_i \Delta \mathbf{f}_i(t) \tag{37}$$

By multiplying $\mathbf{s}_i(t)$ from the left to eq. (36), we have

$$\begin{aligned}&\mathbf{s}_i^T(t)\mathbf{H}_i(\mathbf{q}_i(t))\dot{\mathbf{s}}_i(t) + \frac{1}{2}\mathbf{s}_i^T(t)\dot{\mathbf{H}}_i(\mathbf{q}_i(t))\mathbf{s}_i(t) \\ &\quad = -\mathbf{s}_i^T(t)\mathbf{B}_i \mathbf{s}_i(t) - \mathbf{s}_i^T(t)\mathbf{K}_i \mathbf{J}_i^T(t)\Delta \mathbf{f}_i(t)\end{aligned} \tag{38}$$

The last term in eq. (38) can be extended as follows:

$$
\begin{aligned}
\mathbf{s}_i^T(t)\mathbf{K}_i\mathbf{J}_i^T(t)\Delta\mathbf{f}_i(t) &= \dot{\mathbf{q}}_i^T(t)\mathbf{K}_{il(i)}\mathbf{J}_{il(i)}^T(t)\Delta\mathbf{f}_{il(i)}(t) \\
&+\dot{\mathbf{q}}_{l(i)p}^T(t)\mathbf{K}_{il(i)}\mathbf{J}_{l(i)i}^T(t)\Delta\mathbf{f}_{il(i)}(t) \\
&+\Delta\mathbf{f}_{il(i)}^T(t)\mathbf{J}_{l(i)i}(t)\varepsilon_i\mathbf{K}_{il(i)}\mathbf{J}_{l(i)i}^T(t)\Delta\mathbf{f}_{il(i)}(t) \\
&+\sum_{j\in Neigh(i),j\neq l(i)}\left\{\dot{\mathbf{q}}_i^T(t)\mathbf{K}_{ij}\mathbf{J}_{ij}^T(t)\Delta\mathbf{f}_{ij}(t)-\dot{\mathbf{q}}_{ip}^T(t)\mathbf{K}_{ij}\mathbf{J}_{ij}^T(t)\Delta\mathbf{f}_{ij}(t)\right\} \\
&-\sum_{j\in Neigh(i),j\neq l(i)} +\beta_i\Delta\mathbf{f}_i^T(t)\mathbf{J}_i(t)\mathbf{K}_i\mathbf{J}_i^T(t)\Delta\mathbf{f}_i(t)
\end{aligned}
\tag{39}
$$

Let gain $\mathbf{K}_{ij}=\mathbf{K}_{ji}$. By summarizing eq. (39) for all the robots

$$
\begin{aligned}
\frac{1}{2}\frac{d}{dt}\sum_{i=1}^{N}\mathbf{s}_i^T(t)\mathbf{H}_i(\mathbf{q}_i(t))\mathbf{s}_i(t) &= -\sum_{i=1}^{N}\mathbf{s}_i^T(t)\mathbf{B}_i\mathbf{s}_i(t) \\
&-\sum_{i=1}^{N}\Delta\mathbf{f}_i^T(t)\mathbf{J}_i(t)\beta_i\mathbf{K}_i\mathbf{J}_i^T(t)\Delta\mathbf{f}_i(t) \\
&-\sum_{i=1,i\neq L}^{N}\Delta\mathbf{f}_{il(i)}^T(t)\mathbf{J}_{l(i)i}(t)\varepsilon_i\mathbf{K}_{il(i)}\mathbf{J}_{l(i)i}^T(t)\Delta\mathbf{f}_{il(i)}(t) \\
&-\sum_{i=1,i\neq L}^{N}\left\{\dot{\mathbf{q}}_i^T(t)\mathbf{J}_{il(i)}^T(t)+\dot{\mathbf{q}}_{l(i)}^T(t)\mathbf{J}_{l(i)i}^T(t)\right\}\mathbf{K}_{il(i)}\Delta\mathbf{f}_{il(i)}(t)
\end{aligned}
\tag{40}
$$

From eq. (5), we have

$$
\begin{aligned}
\left\{\dot{\mathbf{q}}_i^T(t)\mathbf{J}_{il(i)}^T(t)+\dot{\mathbf{q}}_{l(i)}^T(t)\mathbf{J}_{l(i)i}^T(t)\right\}\mathbf{K}_{il(i)}\Delta\mathbf{f}_{il(i)}(t) \\
=\Delta\dot{\mathbf{f}}_{il(i)}^T(t)\mathbf{K}_{il(i)}\Delta\mathbf{f}_{il(i)}(t)
\end{aligned}
\tag{41}
$$

Therefore,

$$
\begin{aligned}
\dot{V}(t) &= -\sum_{i=1}^{N}\mathbf{s}_i^T(t)\mathbf{B}_i\mathbf{s}_i(t) \\
&-\sum_{i=1}^{N}\Delta\mathbf{f}_i^T(t)\mathbf{J}_i(t)\varepsilon_i\mathbf{K}_i\mathbf{J}_i^T(t)\Delta\mathbf{f}_i(t) \\
&-\sum_{i=1,i\neq L}^{N}\Delta\mathbf{f}_{il(i)}^T(t)\mathbf{J}_{l(i)i}(t)\mathbf{J}_{l(i)i}^T(t)\mathbf{K}_{il(i)}\Delta\mathbf{f}_{il(i)}(t)
\end{aligned}
\tag{42}
$$

Then, we can conclude that

$$\lim_{t\to\infty}\Delta\mathbf{f}_{ij}(t)=0 \qquad \forall i \text{ and } \forall j\in Neigh(i) \tag{43}$$

$$\lim_{t\to\infty}\mathbf{s}_i(t)=0 \qquad i=1,2,\ldots,N \tag{44}$$

From definition of the for the group leader, we have

$$\lim_{t\to\infty}\Delta\dot{\mathbf{q}}_L(t)+\lambda\Delta\mathbf{q}_L(t)=0 \tag{45}$$

This equation leads to the convergence of the trajectory of the group leader to the desired one. The proof is completed.

The idea of the proposed controller is the introduction of the propagated velocity, which propagates the desired motion of the robots from the group leader to the robots along the formation tree. We call the process "reference propagation". By the reference propagation, the desired trajectory for each of the robots is not necessary in the formation control. As the result, planning of the desired trajectories of the robots in formation can be avoided. This is one of the most distinct features from other existing ones.

On the other hand, in the reference propagation process the controller of a robot i must use the information of all its ancestors in the formation tree. This can be interpreted as on-line planning of the desired motion of the robots other than the group leader. One solution to this problem is to use delayed reference propagation, as discussed in next section.

V. Formation Control with Delayed Reference Propagation

In the proposed reference propagation scheme, the desired motion is propagated from the group leader to all the robots following the paths in the formation tree. By this scheme, the difficult formation planning can be avoided. A problem due to this is that the controller of a robot must use the information of all its ancestors. In this sense, the proposed controller is not truly distributed.

To solve this problem, this section proposes another controller that delays the reference propagation by one sampling time Δt. In detail, the propagated velocity of the robot i is given by

$$\dot{\mathbf{q}}_{ip}(t)=-\mathbf{J}_{il(i)}^{+}(t)\mathbf{J}_{l(i)i}(t)\left\{\dot{\mathbf{q}}_{l(i)p}(t-\Delta t)+\varepsilon_i\mathbf{J}_{l(i)i}^T(t)\Delta\mathbf{f}_{il(i)}(t)\right\} \tag{46}$$

It should be noted that $\dot{\mathbf{q}}_{l(i)p}(t-\Delta t)$ represents a Δt–delayed propagated velocity of the leader. Fig. 2(b) shows the delayed reference propagation. The nominal reference has the same form as that in eq. (33), and hence the error vector is the same as that in eq. (34). The controller has the same form to that in (35) except for the propagated velocity is given by eq. (46).

By using the delayed reference propagation, distributed calculation of the controller (35) can be realized. In other words, only one-hop communication is needed in the controller, so truly distributed formation control is achieved.

Theorem 3: *The controller with the delayed reference propagation in (45) is a fully distributive and yields bounded trajectory tracking errors and formation errors:*

$$\lim_{t\to\infty}\|\Delta\mathbf{q}_L(t)\|\le c_1\Delta t$$

$$\lim_{t\to\infty}\|\Delta\dot{\mathbf{q}}_L(t)\|\le c_2\Delta t$$

$$\lim_{t\to\infty}\|\Delta\mathbf{f}_{ij}(t)\|\le c_3\Delta t, \quad i \text{ and } j \text{ are in a neighborhood}$$

where $c_i\,(i,=1,2,3)$ *are constants and* Δt *is the time-delay in the reference propagation.*

This theorem can be proved using the same positive-definite function in eq. (37). The proof procedure is similar to that of Theorem 2. By selecting very small Δt, it is possible to guarantee small tracking and formation errors of the robots.

VI. Simulations

To verify the performance of the proposed controllers, we have carried out a simulation on formation control of five 2-DOF planar manipulators. The robots follow the tree-topology shown in Fig. 1. In the desired formation, the motion of the manipulators must be synchronized, i.e. for two robots i and j in neighborhood

$$q_i(t)-q_j(t)=0 \tag{47}$$

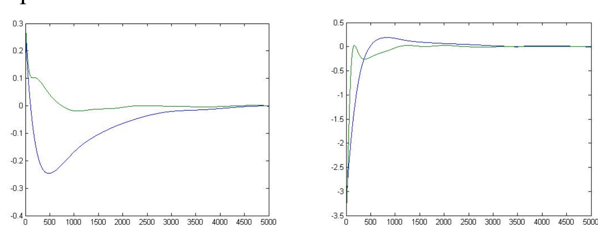
Only the group leader, i.e. robot 1, is given the following desired trajectory:

$$\begin{pmatrix}\theta_1(t)\\ \theta_2(t)\end{pmatrix}=\begin{pmatrix}\pi/4\\ \pi/4\end{pmatrix}+\begin{pmatrix}\pi/3\\ \pi/3\end{pmatrix}\sin\pi t \ . \tag{48}$$

All the other robots do not know the desired trajectory. Initially, all the robots are at rest at position (0,0). The proposed controller with delayed reference propagation was used to control the robots. Fig. 3 shows the trajectory error of the group leader robot. The horizontal axis represents time in *ms*. The units of the vertical axis are *radian* and *radian/s*. Fig. 4(a) plots the profiles of the formation errors of the group leader with its followers, i.e. robots 2 and 3. The formation errors of robot 2 with its followers, i.e. robot 4 and 5 are shown in Fig. 4(b). The results ascertained the convergence of both the trajectory tracking error of the group leader and the formation errors, even though only the boundedness of the errors can be achieved by the controller with the delayed reference propagation.

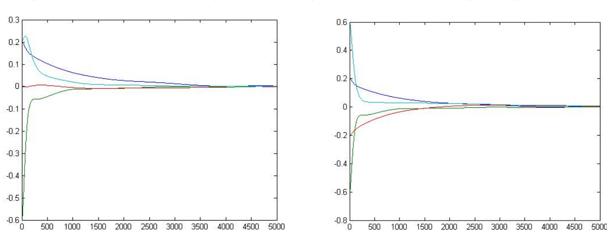
VII. CONCLUSIONS

In this paper, we proposed a new controller for formation control of multiple robots on the basis of the constrained motion formulation. The desired formation is represented as constraint surface so that the control objective is to enforce the virtual force to zero. As a result, existing controllers for cooperative hybrid position/force control can be used for formation control of multiple robots. The proposed controller does not require desired trajectory for all the robots, but the group leader. The core idea is the development of the reference propagation scheme which propagates the motion of the group leader to the followers one by one. Based on the nonlinear dynamics of the robot, we prove that the proposed controller gives convergence of the trajectory tracking error of the group robot and formation errors among the robots to zero. The asymptotic stability of the proposed controller is further verified by simulations of formation control of five 2 DOF manipulators.



Position error Velocity error

Fig. 2 The trajectory tracking errors of the group leader.



(a) (b)

Fig. 3 The formation errors.

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