Passivity-Based Observer Design and Robust Output Feedback Control for Nonlinear Uncertain Systems

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Abstract—**This paper presents a new method for a passivity-based observer design and robust output feedback control of a class of nonlinear uncertain systems. The uncertainties satisfy the Lipschitz-type constraints. Firstly, the passivity condition, which assures the existence of an observer, is expressed in terms of a linear matrix inequality (LMI). Then, for the output feedback control, a sufficient condition in terms of LMI is given for input-to-state stability (ISS) with regard to the observer error. Meanwhile, it is shown that the observer error decays to zero. Therefore, the asymptotic stability is guaranteed by ISS. This proposed method is much less conservative. Finally, a simulation example is illustrated the effectiveness of the proposed results.**

Keywords—passivity; observer; output feedback; LMI; ISS

I. INTRODUCTION

In many control systems, not all the state variables can be available for measurement, or we may choose not to measure some of them due to technical or economic reasons. In this case, it is necessary to design a state observer to reconstruct the state. Recently, much effort has been devoted to design an observer, see [1-13] and the references therein. In [3], the observer design was studied for a class of nonlinear systems with discrete-time measurements. The observer error was shown to decay to zero exponentially by using a continuous Newton method for the map inversion. The authors of [7] present a synthesized hybrid observer such that the continuous state estimation error converges exponentially. In [10], a sufficient condition in terms of LMI is presented such that observer error can converge to zero exponentially.

The majority of the above works have a common feature. Whether the observer exists or not is determined by analyzing an observer error system. We know that, for a linear system, the existence of an observer can be judged by the features of the system itself. While for a nonlinear system, due to the complexity of its structure, the observer design problem has no systematic solution. Is it possible to obtain the existence condition for an observer from the system itself for a nonlinear uncertain system? This is one of our concerns of this paper. We, in this paper, will consider the observer design from a passivity point of views for a class of nonlinear systems. Passivity is a particular type of dissipativity, which was introduced by Willems in 1972 in his seminal two-part papers ([14-15]), and generalized by Hill and Moylan in 1980 ([16]). Passivity ideas emerged in the electrical networks, from the phenomenon of dissipation of energy across resistors. In [14], Willems systematized passivity by introducing the notion of a storage function and a supply rate. Recently, it has played a major role due to the advantage of less conservativeness in robust stability, which has received much attention [17-18]. However, observer design based on passivity has seldom been investigated. In this paper, we will confirm that the passive condition can guarantee the existence of a high-gain observer under some certain conditions.

For an output feedback control, some sufficient conditions for static output feedback and dynamic output feedback have been presented to guarantee the stability of systems [19-29]. In [19], a separation principle for a class of nonlinear systems is proposed such that semi-global stability can be achieved by means of dynamic output feedback. Jiang et al. in [22] use the reduced-order high-gain observer to obtain semi-global stabilization for a benchmark example. [10] and [28] present a sufficient condition for the stability of observer-based control systems in terms of LMI with an equality constraint. The obtained result is novel and interesting. However, owing to the equality constraint, the search of such a controller gain is difficult and complicated. Can we remove this equality constraint? This paper focuses on the passivity-based observer design and the robust output feedback control for a class of systems in Lure's form. Although such a structure in discrete-time case has been studied in [10], the result is still not easy to apply for practical problems by such a method. In this paper, we propose a new method for analyzing an observer design and the output feedback control. Comparing with the existing results, the newly developed method has the following advantages: 1. For the observer design, we present a sufficient condition such that the existence of the observer can be determined by the feature of the system itself. 2. For an output feedback, a sufficient condition in terms of LMI is presented such that the system is ISS with respect to the observer error. The asymptotical stability of the system can be concluded with the decay property of the observer error. In addition, the equality constraint in solving LMI can also be avoided. The advantage of proposed method is shown clearly.

II. PROBLEM FORMULATION

Consider the following system with nonlinear perturbation:

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$$\dot{x} = Ax + Bu + f(x), \quad (1)$$

$$y = Cx, \quad (2)$$

where $x \in R^n$ is a state, $u \in R^p$ is an input, A, B and C are all known constant matrices with appropriate dimensions. $y \in R^p$ is an output. $f \in R^n$ is a nonlinear uncertainty with $f(0) = 0$ and satisfies a Lipschitz type condition:

$$\frac{\partial f(x)}{\partial x} = GF(x)H, \quad (3)$$

where G, $H \in R^{n\times n}$ are real matrices with appropriate dimensions, and $F(x)$ satisfies

$$F^T(x)F(x) \le I.$$

Remark 1: As stated in [10], this Lipschitz structure in the form of (3) does not involve any approximation of nonlinearities by their norms. Thus, it is expected that our method can lead to a less conservative in an observer design.

We first give some basic lemmas and definitions before continuing our discussion, which will be used in the derivations of the main results.

Lemma 1 (Schur's Complement): Let S be a square matrix partitioned as

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{pmatrix},$$

where S_{11} and S_{22} are both symmetric matrices. Then the following three statements are equivalent:

$$1)\ S < 0;\ 2)\ S_{11} < 0,\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0;$$

$$3)\ S_{22} < 0,\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0.$$

Lemma 2[30]: Assume that H, D and E are real matrices with appropriate dimensions. $F(t)$ is a real matrix function satisfying $F^T(t)F(t) \le I$. Then

(1) $H + DF(t)E + (DF(t)E)^T < 0$ holds if and only if there exists a scalar $\varepsilon > 0$ satisfying

$$H + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0.$$

(2) For any $\varepsilon > 0$, one has

$$DF(t)E + (DF(t)E)^T \le \varepsilon DD^T + \varepsilon^{-1} E^T E < 0.$$

Definition 1[15]: The system (1)-(2) is said to be passive, if there exist a C^1 positive semi definite storage function $S(x)$ and a bilinear supply rate $u^T y$ satisfying the following inequality

$$\dot{S}(x) \le u^T y \quad (4)$$

for all $x \in R^n$ and all $y \in R^p$. The system (1)-(2) is said to be input feed forward passive (IFP), if, instead of (4), the inequality

$$\dot{S}(x) \le u^T y - u^T \varphi(u) \quad (5)$$

holds for some function φ.

Remark 2: The concept of system passivity is related to the output signal and external input signal. It is the property that the rate of increase of storage is not higher than the supply rate. This means that the motion of a passive system is always accompanied by energy dissipation. For IFP, we can see that positive sign of $u^T\varphi(u)$ means that system has an excess of passivity. Conversely, the negative sign of $u^T\varphi(u)$ implies the system to have the shortage of passivity.

The motivation of this paper is to design a suitable observer-based feedback controller such that the system (1) is made ISS with the observer error as an imaginary input.

III. Main Results

In this section, we first present a sufficient condition in terms of LMI such that system (1)-(2) is IFP and then derive a criterion for the existence of an observer based on passivity such that observer error converges to zero exponentially.

Theorem 1: If there exists a scalar $\varepsilon > 0$, and positive definite matrices $P > 0$ and $Q > 0$ such that the following LMI holds:

$$\Omega = \begin{pmatrix} A^T P + P^T A + \varepsilon H^T H & P^T B - C^T & P^T G \\ B^T P - C & -2Q & O \\ G^T P & O & -\varepsilon I \end{pmatrix} < 0, \quad (6)$$

then, the system (1)-(2) is input feed forward passive.

Proof: To prove that the system (1)-(2) is input feed forward passive, we take the storage function as follows

$$S(x) = \frac{1}{2} x^T P x.$$

Then, one has

$$\dot{S} - u^T Q u - u^T y =$$

$$\frac{1}{2}[(Ax + Bu + \int_0^1 GF(s)|_{s=\lambda x} H x\, d\lambda)^T P x$$

$$+x^T P(Ax + Bu + \int_0^1 GF(s)|_{s=\lambda x} H x\, d\lambda) - 2u^T Q u - 2u^T Cx].$$

Applying Lemma 2 yields that for any $\varepsilon > 0$,

$$\dot{S}-u^TQu-u^Ty\le\frac{1}{2}(x^T \quad u^T)\Omega_0\begin{pmatrix}x\\u\end{pmatrix},$$

where

$$\Omega_0=\begin{pmatrix}A^TP+P^TA+\varepsilon^{-1}PGG^TP+\varepsilon H^TH & P^TB-C^T\\ B^TP-C & -2Q\end{pmatrix}. \quad (7)$$

By Schur's Complement Lemma, $\Omega_0<0$ can be guaranteed by $\Omega<0$. Then, $\dot{S}\le u^Ty+u^TQu$, which implies that the system (1)-(2) is input feed forward passive. Thus the proof of Theorem 1 is completed. ■

As for the observer design, we restrict ourselves to seeking an observer in the following form:

$$\dot{\hat{x}}=A\hat{x}+Bu+f(\hat{x})+P^{-1}Y(\hat{y}-y), \quad (8)$$

$$\hat{y}=C\hat{x}, \quad (9)$$

where P is given in Theorem 1 and $Y\in R^{n\times p}$ is an undetermined matrix.

Denote $e(t)=\hat{x}-x$ is an observation error between the controlled system and the observer. Then, the observation error system is given by

$$\dot{e}=(A+P^{-1}YC)e+f(\hat{x})-f(x). \quad (10)$$

The following result presents a sufficient condition which guarantees the existence of an observer.

Theorem 2: Suppose that the conditions of Theorem 1 hold. If there exists a matrix $Y\in R^{n\times p}$ such that the following LMI holds

$$YC+C^TY^T-(P^TB-C^T)(2Q)^{-1}(B^TP-C)<0, \quad (11)$$

Then, the observation error decays to zero exponentially.

Proof: Choose a Lyapunov function candidate as follows

$$V(x)=e^TPe,$$

where P is given in Theorem 1. By the condition (3) and Lemma 2, for any $\varepsilon>0$, the derivative of V along the trajectories of the system (10) yields

$$\begin{aligned}\dot{V}&=\dot{e}^TPe+e^TP\dot{e}\\&\le e^T(A^TP+P^TA+YC+C^TY^T+\varepsilon^{-1}PMM^TP+\varepsilon N^TN)e.\end{aligned}$$

Using the inequality (11), it yields

$$\dot{V}\le e^T\Pi e,$$

where

$$\begin{aligned}\Pi&=A^TP+P^TA+\varepsilon^{-1}PMM^TP+\varepsilon N^TN\\&\quad+(P^TB-C^T)(2Q)^{-1}(B^TP-C).\end{aligned}$$

By Schur's Complement Lemma, $\Pi<0$ is guaranteed by the conditions of Theorem 1. So we obtain

$$\dot{V}\le-\lambda_{\min}(-\Pi)\lambda_{\max}^{-1}(P)V.$$

This implies that the observation error system is exponentially stable. ■

Remark 3: In fact, Theorem 2 presents a criterion to guarantee the existence of a robust observer against disturbance. It's noted that (11) can be solved easily and there is almost no any limitation from the LMI because of the negative semi-definiteness of the third term of the left-hand side of (11). Then, we can say that input feed forward passivity of the system guarantees the existence of an observer. In the other words, we can judge the observer existence by the system itself.

Next, we will consider the observer-based feedback control. We will give an observer-based feedback controller of the form

$$u=K\hat{x}, \quad (12)$$

where K is a control gain matrix such that closed-loop system

$$\dot{x}=(A+BK)x+f(x)+BKe \quad (13)$$

is made ISS with the observation error as an input.

Before stating the result, we first recall the definition that will be used for the proof of the main result [31].

Definition 2: Consider the system:

$$\dot{x}(t)=f(x(t),u(t)),$$

where the state $x(t)$ is in R^n, and control input $u(t)$ in R^m. $f:R^n\times R^m\to R^n$ is continuous and locally Lipschitz in x and u. The input u is a bounded function for all $t\ge0$. Then the system is said to be input-to-state stable (ISS) if there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that for any initial state $x(t_0)$, the solution $x(t)$ exists for all $t\ge t_0$ and satisfies:

$$|x(t)|\le\beta(|x(t_0)|,t-t_0)+\gamma(\sup_{t_0\le\tau\le t}|u(\tau)|).$$

Remark 4: The last inequality guarantees that for any bounded input $u(t)$, the state $x(t)$ will also be bounded, and as t increases, the state $x(t)$ will be ultimately bounded by a class $\mathcal{K}$ function of $|u|$. Furthermore, the inequality also shows that if $u(t)$ converges to zero as $t\to\infty$, so does $x(t)$.

Theorem 3: If there exist a scalar $\varepsilon>0$, a positive definite matrix $X\in R^{n\times n}$ and $Z\in R^{r\times n}$ such that

$$\Omega_1=\begin{pmatrix}AX+X^TA^T+BZ+Z^TB^T+\varepsilon GG^T & X^TH^T\\ HX & -\varepsilon I\end{pmatrix}, \quad (14)$$

then, the system (1) is ISS with respect to the observation error e. Moreover, the controller gain matrix can be chosen as

$$K = ZX^{-1}. \tag{15}$$

Proof: By Schur's Complement Lemma, (14) is equivalent to

$$X^T A^T + AX + Z^T B^T + BZ + \varepsilon^{-1} X^T H^T H X + \varepsilon G G^T < 0,$$

which is in turn equivalent to

$$A^T X^{-1} + X^{-T} A + X^{-T} Z^T B^T X^{-1} + X^{-T} B Z X^{-1}$$
$$+\varepsilon^{-1} H^T H + \varepsilon X^{-T} G G^T X^{-1} < 0.$$

Set $P = X^{-1}$. Then, by (15) one has

$$\Theta = A^T P + P^T A + K^T B^T P + P^T BK$$
$$+\varepsilon P^T G G^T P + \varepsilon^{-1} H^T H < 0. \tag{16}$$

Define a Lyapunov function candidate for the system (13) as follows.

$$V(x) = x^T P x.$$

Then, for any $\varepsilon > 0$, taking the derivative of V along the system (13) yields:

$$\dot{V} = ((A+BK)x + \int_0^1 GF(s)|_{s=\lambda x} H x\, d\lambda + BKe)^T P x + 2x^T PBKe$$
$$+x^T P((A+BK)x + \int_0^1 GF(s)|_{s=\lambda x} Hx d\lambda + BKe)$$
$$\le x^T \Theta x + 2x^T PBKe.$$

Therefore, it yields

$$\dot{V} \le -\alpha_1 V + 2\sqrt{V}\sqrt{\lambda_{\max}(P)}\,\|BK\|\cdot|e|,$$

where $\alpha_1 = \dfrac{\lambda_{\min}(-\Theta)}{\lambda_{\max}(P)} > 0$. Then, by the comparison principle, one has

$$|x| \le \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\exp(-\frac{\alpha_1}{2}(t-t_0))|x_0|$$
$$+2\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\frac{1-\exp(-\frac{\alpha_1}{2}(t-t_0))}{\alpha_1}\|BK\|\sup_{t_0\le\tau\le t}|e(\tau)|$$
$$\le \beta(|x_0|, t-t_0) + \gamma(\sup_{t_0\le\tau\le t}|e(\tau)|), \tag{17}$$

where

$$\beta(r,s) = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\exp(-\frac{\alpha_1}{2}s)r$$

and

$$\gamma(s) = \frac{2}{\alpha_1}\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\|BK\|s.$$

That is, the system (1) is ISS with respect to the observation error. This completes the proof. ■

Remark 5: Theorem 3 provides an ISS criterion for the system (1). Note that the observation error is globally exponentially stable by Theorem 2. Therefore, it follows from (17) and Theorem 2 that the closed-loop system (13) is asymptotically stable. In addition, it's noted that the above theorem does not involve any equality constraint; its corresponding numerical problem can be avoided when using the LMI toolbox.

IV. A Numerical Example

In this section, we will present a numerical example to illustrate the effectiveness of the proposed results.

$$\begin{cases} \dot{x} = Ax + Bu + f(x), \\ y = Cx, \end{cases} \tag{18}$$

where

$$A = \begin{pmatrix} -2 & 1 & 1 \\ 0 & -2 & 1 \\ 1 & -2 & -5 \end{pmatrix},\ B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix},\ f(x) = \begin{pmatrix} 0 \\ 0 \\ -3.33\sin(x_2) \end{pmatrix}.$$

The Jacobian of $f(x)$ can be written as

$$\frac{\partial f(x)}{\partial x} = G\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\cos(x_2) \\ 0 & 0 & 0 \end{pmatrix}H,$$

where

$$G = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{3.33} & 0 \end{pmatrix},\ H = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{3.33} & 0 \end{pmatrix}.$$

We find the feasible solutions of LMIs (6) and (11) as follows:

$$P = \begin{pmatrix} 1.0311 & 0.1145 & 0.2450 \\ 0.1145 & 2.8801 & 0.3288 \\ 0.2450 & 0.3288 & 0.4420 \end{pmatrix},$$

$$Q = \begin{pmatrix} 1.9440 & 0 \\ 0 & 1.9440 \end{pmatrix}, \ \tau = 2.9261,$$

$$Y = \begin{pmatrix} -1.1344 & -0.8452 \\ 1.6266 & -0.1446 \\ 0.6173 & -0.7814 \end{pmatrix}.$$

Thus, it is shown that the system (1)-(2) is input feed forward passive and the observer gain matrix can be obtained as follows

$$P^{-1}Y = \begin{pmatrix} -1.6188 & -0.4486 \\ 0.4014 & 0.1542 \\ 1.9953 & -1.6340 \end{pmatrix}.$$

Solving the LMI (14), we get some parameters for the output feedback control.

$$X = \begin{pmatrix} 1.2873 & 0 & 0.0672 \\ 0 & 0.1681 & -0.0311 \\ 0.0672 & -0.0311 & 0.6149 \end{pmatrix},$$

$$Z = \begin{pmatrix} 1.8637 & -0.1330 & -1.4004 \\ -0.0713 & -0.2765 & -0.4962 \end{pmatrix}, \ \varepsilon = 1.3619 .$$

By (15), we can obtain the control gain matrix as follows.

$$K = ZX^{-1} = \begin{pmatrix} 1.5791 & -1.2558 & -2.5137 \\ -0.0085 & -1.8111 & -0.8975 \end{pmatrix}.$$

Given the initial condition $x(0) = \begin{pmatrix} -0.5 & 0.5 & -1 \end{pmatrix}^T$ and $\hat{x}(0) = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T$, then the simulation for the observation error system (15) is shown in Fig.1. As shown in the simulation, the observation error decays to zero exponentially. Fig.2 is a simulation of the closed-loop system, which shows that the controlled system (1) is asymptotically stable under the designed control.

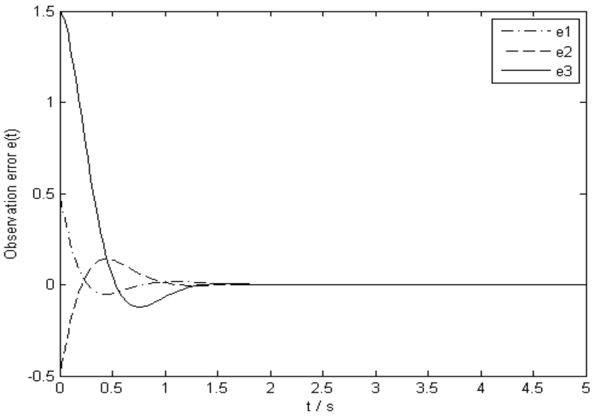

Fig.1 States of the observation error system.

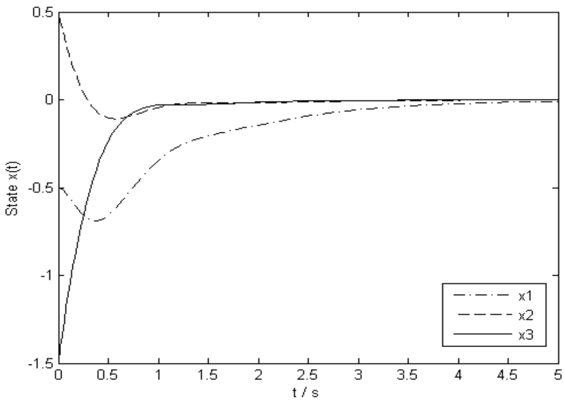

Fig.2 States of the closed-loop system.

From the numerical study, it is clear that our controller design method is simpler than the one adopted in [10]. There is no equality constraint involved in solving LMI. Thus, the numerical problem can be avoided when computing LMI. The effectiveness of the proposed method is shown clearly.

V. CONCLUSION

This paper has investigated an observer design and robust output feedback control of a class of nonlinear uncertain systems. Based on passivity, a new approach has been proposed to guarantee the existence of a robust observer. For an output feedback control, a sufficient condition in terms of LMI for ISS of the system with regard to the observer error has been presented. It is shown that the proposed method is simpler and more effective than the existing method.

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REFERENCES

[1] D. Bestle, M. Zeitz, “Canonical form observer design for non-linear time-variable systems”. *Int. J. Control*, vol. 38, no. 2, pp. 419-431, 1983.

[2] K. Busawon, M. Farza, H. Hammouri, “Observer design for a special class of nonlinear systems”. *Int. J. Control*, vol. 71, no. 3, pp. 405-410, 1998.

[3] P. E. Moraal, J. W. Grizzle. “Observer design for nonlinear systems with discrete-time measurements”. *IEEE Transactions on Automatic Control*, vol. 40, no. 3, pp. 396-404, 1995.

[4] T.-L. Liao, N.-S. Huang, “An observer-based approach for chaotic synchronization with application to secure communications”, *IEEE Transactions on Circuits and systems-1: Fundamental theory and applications*, vol. 46, pp. 1144-1150, 1999.

[5] J. P. Barbot, H. Saadaoui, M. Djemai, N. Manamanni, “Nonlinear observer for autonomous switching systems with jumps”, *Nonlinear Analysis: Hybrid System*, vol. 1, pp. 537-547, 2007.

[6] W. M. Xiang, J. Xiao, M. N. Iqbal, “Robust observer design for nonlinear uncertain switched systems under asynchronous switching”, *Nonlinear Analysis: Hybrid System*, vol. 6, no. 754-773, 2012.

[7] A. Balluchi, L. Benvenuti, M.D. Di Benedetto and A.L. Sangiovanni-Vincentelli. “Design of observers for hybrid systems”, in *proceedings of Hybrid Systems: Computation and Control Conference*, California, pp. 76-89, 2002.

[8] F. Cacace, A. Germani, and C. Manes. "An observer for a class of nonlinear systems with time varying observation delay". *Systems & Control Letters*, vol. 59, pp. 305-312, 2010.

[9] A. Germani, C. Manes, and P. Pepe. "A new approach to state observation of nonlinear systems with delayed output". *IEEE Trans. Automatic Control*, vol. 47, no. 1, pp. 96-101, 2002.

[10] S. Ibrir, W. F. Xie, C.Y. Su, "Observer-based control of discrete-time Lipschitzian nonlinear systems: application to one-link flexible joint robot", *Int. J. Control*, vol. 78, no. 6, pp. 385-395, 2005.

[11] A. Y. Pogromsky, H. Nijmeijer, "Observer-based robust synchronization of dynamical systems", *International Journal of Bifurcation and Chaos*, vol. 8, pp. 2243-2254, 1998.

[12] Z. Gao, B. Jiang, R. Qi, Y. Xu, and Y. Cheng, Fuzzy observer design for near space vehicle with application to sensor fault estimation, ICIC Exp. Lett., vol. 4, no. 1, pp. 177-182, 2010.

[13] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, "State estimation of Takagi-Sugeno systems with unmeasurable premise variables", *IET Control Theory Appl.*, vol. 4, no. 5, pp. 897-908, 2010.

[14] J. C. Willems, "Dissipative dynamical systems-Part I: General theory", *Arch. Ration. Mech. Anal.*, vol. 45, no. 5, pp. 321-351, Jan. 1972.

[15] J. C. Willems, "Dissipative dynamical systems-Part II: Linear systems with quadratic supply rates", *Arch. Ration. Mech. Anal.*, vol. 45, no. 5, pp. 352-393, Jan. 1972.

[16] D. Hill, P. Moylan, "Stability results for non-linear feedback systems", *Automatica*, vol. 13, no. 4, pp. 337-382, 1977.

[17] S. Xie, L. Xie, and C. de Souza, "Robust dissipative control for linear systems with dissipative uncertainty", *Int. J. Control*, vol. 70, no. 2, pp. 169-191, 1998,.

[18] Z. Tan, Y. Soh, L. Xie, "Dissipative control for linear discrete-time systems", *Automatica*, vol. 35, pp. 1557-1564, 1999.

[19] A. N. Atassi, H. K. Khalil, "A separation principle for the stabilization of a class of nonlinear systems", *IEEE Trans. Autom. Control*, vol. 44, no. 9, pp.1672-1687, 1999.

[20] L. Praly and Z. P. Jiang, "Stabilization by output feedback for systems with ISS inverse dynamics", *Systems & Control Letters*, vol. 21, pp. 19-33, 1993.

[21] T. F. Liu, Z. P. Jiang, D. J. Hill, "Small-gain based output feedback controller design for a class of nonlinear systems with actuator dynamic quantization", *IEEE Trans. Autom. Control*, vol. 57, no. 5, pp.1326-1332, 2012.

[22] Z. P. Jiang, D. J. Hill, Y. Guo, "Semiglobal output feedback stabilization for the nonlinear benchmark example", in *Proc. European Control Conf.*, Brussels, Belgium, 1997.

[23] D. Liberzon, "Observer-based quantized output feedback control of nonlinear systems", in *Proceedings of the 17th IFAC World Congress*, pp. 8039-8043, 2008.

[24] V. Andrieu and L. Praly, "A unifying point of view on output feedback designs" *Automatica*, vol. 45, pp. 1789-1798, 2009.

[25] A. Isidori, L. Praly and L. Marconi, "About the existence of locally Lipschitz output feedback stabilizers for nonlinear systems", *SIAM J. Control Optim.*, vol. 48, pp. 3389-3402, 2010.

[26] C. A. R. Crusius, A. Trofino, "Sufficient LMI conditions for output feedback control problems", *IEEE Trans. Autom. Control*, vol. 44, pp.1053-1057, 1999.

[27] C. H. Kuo, C. H. Fang, "Stabilization of uncertain linear systems via static output feedback control", in *Proc. Automatic Control Conf.*, vol. 1, Taiwan, pp. 1607-1611, 2003.

[28] C. H. Lien, "Robust observer-based control of systems with state perturbations via LMI approach", *IEEE Trans. Autom. Control*, vol. 49, pp.1365-1370, 2004.

[29] A. M. Dabroom, H. K. Khalil, "Output feedback sampled-data control of nonlinear systems using high-gain observers", *IEEE Trans. Autom. Control*, vol. 46, no. 11, pp.1712-1725, 2001.

[30] Y. Wang, L. Xie and C. de Souza. "Robust control of a class of uncertain nonlinear systems", *Systems & Control Letters*, vol. 19, pp. 139-149, 1992.

[31] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, N. J. Prentice Hall, 2000.