Nonholonomic Control of Distance-based Cyclic Polygon Formation

Byung-Hun Lee[†], Seung-Ju Lee[†],Myoung-Chul Park[†], Kwang-Kyo Oh[†] and Hyo-Sung Ahn[†]

Abstract— **Inter-agent formation is the interesting issue and coped within previous literatures. We propose a control strategy based on inter-agent cyclic formulation of nonholonomic model. Graph is directed. Each agent maintains a desired distance with a neighbor agent. Under sliding control technique, sequence of control input forces configuration to equilibrium manifold. Unicycle-like model is used as nonholonomic agent model.**

I. INTRODUCTION

Formation control of autonomous agent systems is studied by many researchers in recent few years. Particular problems such as multi-agent formation control that agents typically characterize multiple robots of similar dynamics are considered in previous literature[2]-[15]. The aim of muti-agent formation is to coverge to a definite form in the state space. Formation of autonomous agent is categorized as stationary [11] [12] and moving with constant velocity [13] [14] of the desired formation.

The problems of controlling a mobile autonomous agents in a ”directed” formation containing a cycle are concerned in the previous literatures [2] [6] [10]. The notion of *directed* is that each agent follows their ”co-leader” to meet a desired distance. Directed formation is called persistent if and only if graph is *rigid* and *constraint consistent* [1] [5]. Necessary and sufficient conditions for persistence described in [5]. Stabilization of cyclic formation have appeared for single integrator agents and nonholonomic agents [3] [6]. Single integrator model follows intuitive control law based on orientation angle and position estimation. Consequently, with the interaction topology of the agents, exponential convergence of the estimated orientation and position can be ensured [3]. Cao *et. al.* have considered cyclic formultion of three agent case based on gradient control [6]. Even though Cao *et. al.* provides unique solution to system of nonlinear differential equation using straightforward arguments, analysis of the stability is limited in 3-agent case. Subsequently, analyis of general n-agent case is carried out via Lyapunov-based stability in further work [2].

Previous research for nonholonomic agent provides specific control strategy. Leader-following approachs was used in [11] [13].Using some chosen leader and follower agents, leader tracks desired trajectories, and followers chase the desired trajectory with respect to the leader. Since leaders are pre-defined, individual robot’s control laws guarantee the internal stability of formation [7].

[†]School of Mechatronics, Gwangju Institute of Science and Technology (GIST), 261 Cheomdan-gwagiro, Buk-gu, Gwangju, 500-712 Korea. E-mail: {bhlee, bamboohat, mcpark, kkoh, hyosung}@gist.ac.kr

In this paper, we cope with nonholonomic constraint which is not covered in previous cyclic formulation works [2] [6]. Using sequence of control inputs is associated with the flow of vector fields. We approach sliding mode control technique to restrict the motion of system. The rest of this paper is organized as follows: Section II describes problem formulation and system model which is treated in this paper. Section III presents a proposed control strategy. Analysis of stability and convergence of sliding surface are described in Section IV. Comparing 3-agent case with n-agent case is presented as a simulation result in Section V while the results are summarized and further work is mentioned in Secion VI.

II. PROBLEM STATEMENT

Cyclic formation using mobile autonomous n-agents represents that a i th agent follows $(i+1)$ th agent to satisfy desired distance and when i equals to n, $(i+1)$th agent equals to 1st agent(Fig 1) [2].

Each agent have their local coordinate$(o_1, o_2, \cdots, o_n)$ with

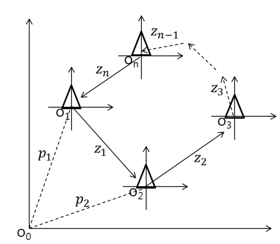

Fig. 1. A nonholonomic n-agent cyclic polygon formation

their heading angle. p_i is a vector that represents the base of i-local coordinate written in some fixed Cartesian coordinate. A z_i is a transitional vector from o_i to o_{i+1} based on i-local coordinate . There is a rotation matrix R_i such that

$$p_{i+1} = R_i z_i + p_i \tag{1}$$

it is rewritten as follows

$$R_i(t) z_i(t) = p_{i+1}(t) - p_i(t) \tag{2}$$

A pair of neighbor agent should satisfy a desired distance in finite time. Rotation matrix is canceled by eucldian norm($\|\cdot\|$) for difference between vector p_i and p_{i+1}.

$$\|p_{i+1} - p_i\|^2 = \|R_i z_i\|^2 = \|z_i\|^2 \quad , \quad \because R_i^T R_i = I \tag{3}$$

In the sequel, a goal of this system can be descibed by z_i

$$\lim_{t\to\infty} \|z_i(t)\| = d_i, \qquad d_i > 0, \quad \forall i \in \{1, 2, \cdots, n\} \tag{4}$$

d_i is a desired distance.

978-1-4673-5769-2/13/$31.00 ©2013 IEEE

A. Unicycle model

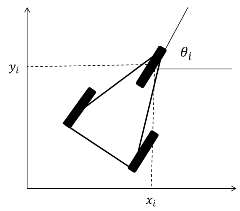

Fig. 2. Unicycle-like mobile robot

In this problem, we assume that all agents are unicycle models which are nonholonomic constraint systems. As shown in Fig. 2, configuration of unicycle model is $q = (x,y,\theta)$ where (x,y) are the Cartesian coordinates of ideal contact point and θ is the orientation of vehicle. Constraint system of unicycle model is derived as follows [3] [4]:

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \\ 0 \end{bmatrix} v_i + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega_i \qquad (5)$$
$$\forall i \in \{1,2,\cdots,n\}$$

Constraint system requires two control inputs (v_i, ω_i). From (4), equilibrium manifold can be written as

$$\mathscr{X} = \{p_i \in \mathbb{R}^2 \mid \|p_{i+1} - p_i\| = d_i\} \qquad (6)$$

where,

$$p_i = (x_i, y_i)$$

In the aspect of differential geometry, (5) is differential equation composed of vetor fields [4]. (5) is rewritten as

$$\dot{q}_i = g_1(q_i)v_i + g_2(q_i)\omega_i \qquad (7)$$

where,

$$g_1(q_i) = \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \\ 0 \end{bmatrix} \quad g_2(q_i) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad (8)$$

The flow of vector field $g_1(q_i)$ and $g_2(q_i)$ with input v_i and ω_i are the mapping of configurations. If the two inputs v_i and ω_i are never active at same instant, the soluion of (7) is obtained by composing the flows relative to g_1 and g_2. In other words, if we assume that input sequence within infinitesimal interval of time ε is follows:

$$\begin{aligned} v_i(t) = 1, w_i(t) = 0, &\quad t \in [0, \varepsilon) \qquad (9) \\ v_i(t) = 0, w_i(t) = 1, &\quad t \in [\varepsilon, 2\varepsilon) \end{aligned}$$

the solution of differential equation at time 2ε is derived by following the flow of g_1, then g_2 [4].

B. Sliding surface

We can design a control law that forces the motion of system to a sliding surface.

$$s = \theta_i - \gamma(x_i, y_i) = 0 \qquad (10)$$

where,

$$\gamma(x_i, y_i) = \angle R_i z_i$$

We assume that $\gamma(x,y)$ can be measured in each agent.On this manifold, reduced ordered model from (5) can be represented as

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} \cos(\gamma(x_i,y_i)) \\ \sin(\gamma(x_i,y_i)) \end{bmatrix} v_i \qquad (11)$$

Above equation can be handled by control input v_i. Choosing control input v_i may guarantee that $\|z_i\|$ tend to d_i as t tends to infinity and the rate of convergence can be controlled by choice of v_i. Following section desrcribes suitable approaches.

III. Control strategy

Control input v_i would be set as

$$v_i = e_i \|z_i\| \qquad (12)$$

where,

$$e_i(t) = \|z_i(t)\|^2 - d_i^2 \qquad (13)$$

Reduced ordered model(11) can be rewritten as

$$\dot{p}_i = \begin{bmatrix} \cos(\angle R_i z_i) \\ \sin(\angle R_i z_i) \end{bmatrix} v_i \qquad (14)$$

Combining (12) and (14), it is clear that motion control of this system is associated with only relative measurement which is not based on global coordinate system. Motion of ith agent can be written as

$$\dot{p}_i = e_i z_i^0 \qquad (15)$$
$$\because z_i^0 = \begin{bmatrix} \|z_i\| \cos(\angle R_i z_i) \\ \|z_i\| \sin(\angle R_i z_i) \end{bmatrix}$$

z_i^0 denotes $R_i z_i$ which is written in a fixed frame. Where $e_i z_i^0 = 0$ in (15) is the equilibrium point of p. From (12) and (2), (15) can be rewritten as,

$$\dot{p}_i = (\|p_{i+1}(t) - p_i(t)\|^2 - d_i^2)(p_{i+1}(t) - p_i(t)) \qquad (16)$$

Dynamics of relative position vector z_i can be described with combining (2) and (15).

$$\begin{aligned} \dot{z_i^0} &= \dot{p}_{i+1} - \dot{p}_i = e_{i+1} z_{i+1}^0 - e_i z_i^0 \\ &= (\|z_{i+1}^0\|^2 - d_{i+1}^2) z_{i+1}^0 - (\|z_i^0\|^2 - d_i^2) z_i^0 \qquad (17) \end{aligned}$$

What we focus on is the distances between all agent. Since every agent should follow only their neighbor to meet desired distance, persitent formation is not guaranteed except for the case of three agents [5].

IV. ANALYSIS

A. Convergence to equilibrium point

To analyze convergence characteristic of e, define a Lyapunov function as

$$V = \frac{1}{2}\sum_{i=1}^{n} e_i^2 \tag{18}$$

Then, time derivative of V is

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{n} e_i \dot{e}_i = \sum_{i=1}^{n} e_i (2(z_i^0)^T \dot{z}_i^0) \\
&= \sum_{i=1}^{n} 2e_i (z_i^0)^T (-e_i z_i^0 + e_{i+1} z_{i+1}^0) \\
&= -e_1^2 \|z_1^0\|^2 + 2e_1 e_2 (z_1^0)^T z_2^0 - e_2^2 \|z_2^0\|^2 \\
&\quad -e_2^2 \|z_2^0\|^2 + 2e_2 e_3 (z_2^0)^T z_3^0 - e_3^2 \|z_3^0\|^2 \\
&\quad \vdots \\
&\quad -e_{n-1}^2 \|z_{n-1}^0\|^2 + 2e_{n-1} e_n (z_{n-1}^0)^T z_n^0 - e_n^2 \|z_n^0\|^2 \\
&\quad -e_n^2 \|z_n^0\|^2 + 2e_n e_1 (z_n^0)^T z_1^0 - e_1^2 \|z_1^0\|^2
\end{aligned}$$

Therefore,

$$\dot{V} = -\sum_{i=1}^{n} \|e_i z_i^0 - e_{i+1} z_{i+1}^0\|^2 \leq 0 \tag{19}$$

As shown in (19), V does not increase on $[0, \infty)$ along trajectory of the solution of (16). V is radially unbounded. Therefore, e_i is bouned, which implies that z_i is also bounded due to (13). In particular

$$\begin{aligned}
\sum_{i=1}^{n} (\|z_i^0\|^2 - d_i^2)^2 &= \sum_{i=1}^{n} e_i^2 \\
\leq \sum_{i} e_i^2(0) &= \sum_{i=1}^{n} (\|z_i^0(0)\|^2 - d_i^2)^2
\end{aligned}$$

This shows that each solution is bounded wherever it exists [6].

B. Convergence of sliding surface

Sliding surface is set as $s = \theta - \gamma(x,y)$. What we want to achieve is to bring the trajectory to the manifold $s = 0$ and maintain it there. The variable s satisfies the equation

$$\dot{s} = \dot{\theta} - \dot{\gamma}(x,y) = \omega - \dot{\gamma}(x,y) \tag{20}$$

Assuming $\dot{\gamma}$ satisfies the inequality

$$\dot{\gamma}(x,y) \leq \alpha_0 < 0 \tag{21}$$

With $V_s = (1/2)s^2$ as a Lyapunov function candidate, time derivative of V_s is follows:

$$\begin{aligned}
\dot{V}_s &= s\dot{s} = s(\dot{\theta} - \dot{\gamma}(x,y)) \\
&= s(\omega - \dot{\gamma}(x,y)) \leq s\omega - |s|\alpha_0
\end{aligned} \tag{22}$$

By taking control input ω as

$$\omega = \beta(x,y) sgn(s) \tag{23}$$

With $\beta(x,y) \leq \alpha_0$, $\dot{V}_s$ can be rewritten as

$$\dot{V}_s \leq sw - |s|\alpha_0 = |s|(\beta(x,y) - \alpha_0) = -|s|g_0 \leq 0 \tag{24}$$

Where some $g_0 > 0$,

$$-g_0 = (\beta(x,y) - \alpha_0)$$

Thus,$W_s = \sqrt{2V_s} = |s|$ satisfies the differential inequality

$$D^+ W_s \leq -g_0 \tag{25}$$

Integration of (25) shows that

$$W_s(s(t)) \leq W_s(s(0)) - g_0 t \tag{26}$$

Therefore, the trajectory reaches the manifold $s = 0$ in finite time and, once on the manifold, it cannot leave it [15].

V. SIMULATION ANALYSIS

We conducted simulations two cases. General 3-agents and 5-agents cases are simulated. Formation shapes are not persistent, since a graph is not rigid except for three agent case. [5]. According to *Laman's criterion* [1], rigid graphs $G = (V, E)$ are equivalent for following conditions :

- *Laman's criterion* : There is a subset $E' \subseteq E$ satisfying the following two conditions :
  1) $|E'| = 2|V| - 3$.
  2) For all $E'' \subseteq E', E'' \neq \varnothing, |E''| \leq 2|V(E'')| - 3$, where $V(E'')|$ is the number of vertices that are end-vertices of the edges in E''.

More than three agents cannot satisfy Laman's criterion in this cyclic formation system, since every agent has only 1 out-degree. All initial positions and heading angles of agents are determined randomly. Proposed control law in Section III is implmented in each agent model. Fig. 3 and Fig. 4 show formation result and error change with time. Error is defined as difference between a square of current distance and a square of desired distance.

$$e_i = \|z_i\|^2 - d^2 \tag{27}$$

General n-agent system formation presents erratic shape as like Figure. 4. However, errors converge to zero in finite time as shown in figures.

VI. CONCLUSION AND FURTHER WORK

Control of inter-agent cyclic formulation with nonholonomic agent model is handled in this paper. We proposed sliding mode technique to control nonholonomic agent. Orientation and position are the configuration of unicycle model. Using sequence of control input, configurations converge to equilibrium point. However, since we only consider inter-agent cyclic formation, persistance of formation is not guaranteed. This issue is our further work. In this paper, we used measured direction of neighbor agent to control formation. Instead of direction, using single distance would be a interest in future work.

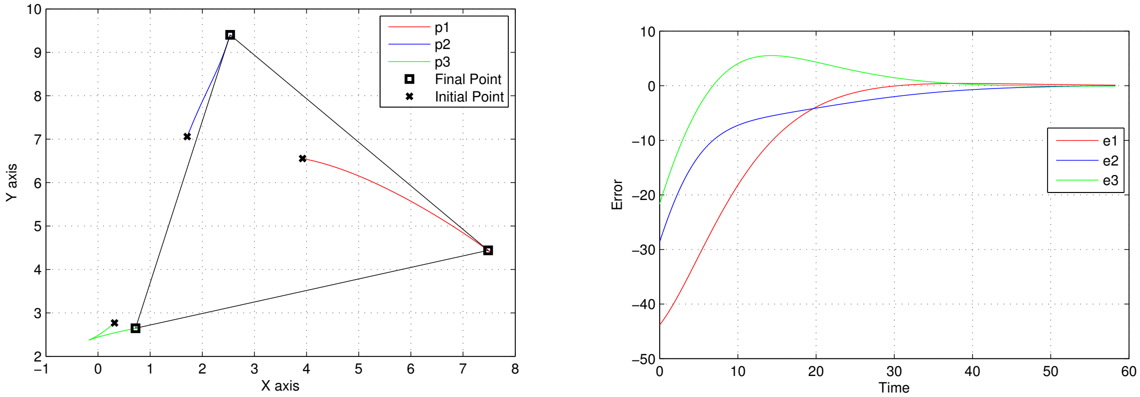

Fig. 3. 3-agent cyclic formation result(left) and error(right)

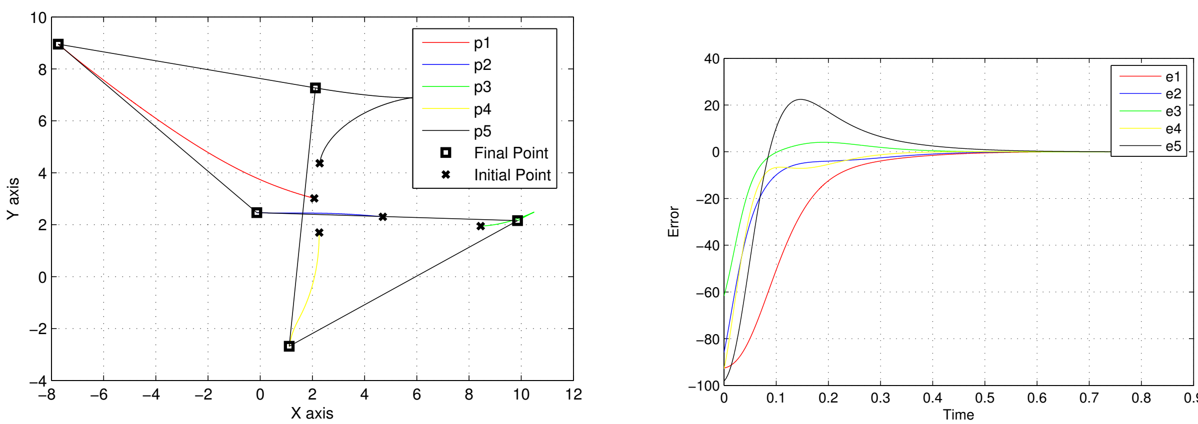

Fig. 4. 5-agent cyclic formation result(left) and error(right)

ACKNOWLEDGEMENT

The research of this paper was supported by Gwangju Institute of Science and Technology, and by the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(No.2012-043791)

REFERENCES

[1] B.Hendrickson, "Conditions for Unique Graph Realizations", *Society for Industrial and Applied Mathematics*, vol.21, No.1, pp.65-84, Feb. 1992.

[2] M.C Park, B.Y Kim and K.K Oh and H.S Ahn "Control of Inter-agent Distances in Cyclic Polygon Formation",*2012 IEEE Multi-Conference on Systems and Control*, October 3-5,2012.

[3] K.K Oh and H.S Ahn "Formation Control of Mobile Agents Based on Distributed Position Estimation" *IEEE Transactions on Automatic Control*, 2012

[4] A.De Luca and G.Oriolo "Modelling and Control of Nonholonomic Mechanical Systems"

[5] J.M. Hendrickx, B.D.O. Anderson, J.-C. Delvenne and V.D.Blondel,"Directed Graphs for the Anlysis of Rigidity and Persistence in Autonomous Agent systems", *International Journal of Robust and Nonlinear Control*, Vol.17,pp.960-981, 2007

[6] M. Cao, A. S. Morse, C. Yu, B. D. O. Anderson and S. Dasgupta,"Controlling a Triangular Formation of Mobile Autonomous Agents, *Proceedings of the 46th IEEE Conference on Decision and Control*, pp. 3603-3608, 2007.

[7] W.J. Dong, Yi Guo and J.A. Farrell, "Formation Control of Nonholonomic Mobile Robots",*American Control Conference*, June 14-16, 2006

[8] D.V. Dimarogonas and K.H. Johansson, "Further Results on the Stability of Distance-Based Multi-Robot Formations", *American Control Conference*, June 10-12, 2009

[9] Z. Lin, B. Francis and M. Maggiore, "Necessary and sufficient Graphical Conditions for Formation Control of Unicycles", *IEEE Transactions on Automatic Control*, 2005

[10] M. Cao, B.D.O. Anderson, A.S. Morse and C. Yu, "Control of Acyclic Formations of Mobile Autonomous Agents", *IEEE Conference on Decision and Control*, Dec. 9-11, 2008

[11] J.P. Desai, J. Ostrowski and V. Kumar "Controlling Formations of Multiple mobile robots." *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp 2864-2869, 1998

[12] D.V. Dimarogonas and K.J. Kyriakopoulos, "A connection between formation infeasibility and velocity alignment in kinematic multi-agent systems", *Automatica*, 44(10):26482654, 2008.

[13] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory", *IEEE Transactions on Automatic Control*, 51(3):401420, 2006.

[14] H.G. Tanner, A. Jadbabaie, and G.J. Pappas, "Flocking in fixed and switching networks", *IEEE Transactions on Automatic Control*,52(5):863868, 2007.

[15] H.K.Khalil, *Nonlinear System*, 3rd ed.,NJ: Prentice Hall, 2002.