Numerical Analysis of a Reparable Multi-State Device

Houbao Xu[1], Weiwei Hu[2]

1. Department of Mathematics, Beijing Institute of Technology, Beijing 100081, China
E-mail: xuhoubao@bit.edu.cn

2. Department of Mathematics, University of Southern California, Los Angeles, CA 90089-2532, USA
E-mail: weiweihu@math.usc.edu

Abstract: A numerical scheme is formulated for approximating the dynamic behavior of a reparable multi-state device, which can be described as a distributed parameter system of coupled partial and ordinary hybrid equations. The convergence issues are established by applying the Trotter-Kato Theorem, and simulation results show the effectiveness of the proposed scheme.

Key Words: Multi-State, reparable system, Trotter-Kato Theorem

1 INTRODUCTION

The mathematical model of a reparable multi-state device with arbitrarily distributed repair time is formulated in [1] as follows:

$$\frac{dp_0(t)}{dt} = -\sum_{j=1}^{M} \lambda_j p_0(t) + \sum_{j=1}^{M} \int_0^{\infty} \mu_j(x) p_j(x,t) dx, \tag{1}$$

$$\frac{\partial p_j(x,t)}{\partial t} + \frac{\partial p_j(x,t)}{\partial x} = -\mu_j(x) p_1(x,t),\ j = 1, 2, \ldots, M \tag{2}$$

with boundary conditions,

$$p_j(0,t) = \lambda_j p_0(t),\ j = 1, 2, \ldots, M, \tag{3}$$

and initial conditions,

$$p_0(0) = 1,\ p_j(x,0) = 0,\ j = 1, 2, \ldots, M. \tag{4}$$

Where

- M Number of failure modes;
- j Failure of device due to failure mode j, $j = 1, 2, \ldots, M$;
- i ith state of the device (see the device transition diagram of Figure 1); $i = 0$, the device is in good state; $i = j$, the device is in the jth failure mode;
- λ_j Constant failure rate of the device for failure mode j;
- $\mu_j(x)$ Time-dependent nonnegative repair rate when the device is in state j and has an elapsed repair time of x; moreover, we assume that

$$0 < \mu = \sup_{0 \le x < \infty} \mu_j(x) < \infty;\ \int_0^x \mu_j(\tau) d\tau < \infty,\ x < \infty;$$

$$\int_0^{\infty} \mu_j(\tau) d\tau = \infty;\ 0 < \lim_{x \to \infty} \frac{1}{x} \int_0^x \mu_j(\tau) d\tau = \mu_j < \infty.$$

- $p_0(t)$ Probability that the device is in state 0 at time t;
- $p_j(x,t)$ Probability that the failed device is in state j at time t and has an elapsed repair time of x; Probability that the failed device is in state j at time t is

$$p_j(t) = \int_0^{\infty} p_j(x,t) dx.$$

The following assumptions are associated with the device:

- The failure rates are constant;
- All failures are statistically independent;
- All repair time of failed devices are arbitrarily distributed;
- There are M modes of failure, the state of the device is given by its failure mode number, 0 implies the good state;
- The repair process begins soon after the device is in failure state;
- The repaired device is as good as new;
- No further failure can occur when the device has been down.

The steady availability of the device was discussed in [1] by the method of the Laplace transform. Later on, by employing C_0-semigroup theory, the authors in [2] and [3] constructed a rigorous mathematical framework for proving the well–posedness and asymptotic stability of the model. However, the numerical approximation of the dynamical solution has not been addressed in these references. In recent years, how to analyze the instantaneous indexes of such multi-state system has drawn more and more attention. The Laplace transform seems to be one of the methods to solve it, but implementation of the algorithm to find the inverse Laplace transform is difficult. Moreover, the convergence of the method has not been discussed. Therefore, how to construct a suitable numerical strategy to analyze

This work is supported by National Nature Science Foundation under Grant 11001013.

978-1-4673-5769-2/13/$31.00 ©2013 IEEE

the instantaneous index of such systems becomes meaningful. This paper considers a multi-state reparable system and mainly focuses on the development of a practical numerical algorithm and its convergence issues.

2 WELL-POSEDNESS AND STABILITY OF THE MODEL

As shown in [2] and [3], the model equations (1)–(4) can be rewritten as an abstract Cauchy problem:

$$\begin{cases} \dot{P}(t) = \mathcal{A}P(t), \\ P(0) = (1, 0, \cdots, 0), \end{cases} \tag{5}$$

in the state space $\mathbf{X} = \mathbf{R} \times (L^1[0,\infty))^M$, with $\|\cdot\|_{\mathbf{X}} = |\cdot| + \sum_{j=1}^{M} \|\cdot\|_{L^1[0,\infty)}$. It is proved in [2] that the system operator $\mathcal{A}$ generates a positive C_0-semigroup of contraction, denoted as $\mathcal{T}(t), t \geq 0$, therefore the system has a unique nonnegative time-dependent solution. Moreover, 0 is shown as a simple eigenvalue of the system operator and also a unique spectral point on the imaginary axis. It can be derived that the time-dependent solution converges to the eigenvector corresponding to the eigenvalue 0. This eigenvector is called the steady-state solution of the system. Also, the system is conservative, i.e, $\|P(t)\|_{\mathbf{X}} = \|\mathcal{T}(t)P(0)\|_{\mathbf{X}} = 1$. Furthermore, $T(t)$ is shown to be quasi-compact and irreducible in [3]. As a result, the time-dependent solution exponentially converges to the steady-state solution.

3 THE MODEL WITH FINITE REPAIR TIME

We consider the case that the repair time x is finite, then equation (1) becomes

$$\frac{dp_0(t)}{dt} = -\sum_{j=0}^{M} \lambda_j p_0(t) + \sum_{j=1}^{M} \int_0^N \mu_j(x) p_j(x,t)dx. \tag{6}$$

Define the state space $X = \mathbf{R} \times (L^1[0,N])^M$ with $\|\cdot\|_X = |\cdot| + \sum_{j=1}^{M} \|\cdot\|_{L^1[0,N]}$. Operator A and its domain are defined by

$D(A) = \big\{Z(t) \in X | \frac{dz_j(t)}{dt} \in L^1[0,N],\ z_j(x)$ are absolutely continuous functions, $j = 1, 2, \ldots, M$, and $p_j(0) = \lambda_j p_0\big\}$,

$$A \begin{bmatrix} p_0 \\ p_1(\cdot) \\ \vdots \\ p_M(\cdot) \end{bmatrix} = \begin{bmatrix} -\sum_{j=1}^{M} \lambda p_0(t) + \sum_{j=1}^{M} \int_0^N \mu_j(x)p_j(x)dx \\ -(\frac{d}{dx} + \mu_1(x))p_1(x) \\ \vdots \\ -(\frac{d}{dx} + \mu_M(x))p_M(x) \end{bmatrix}.$$

Therefore, equations (2)–(4) and (6) can be rewritten as an abstract Cauchy problem:

$$\begin{cases} \dot{Z}(t) = AZ(t), \\ Z(0) = (1, 0, \cdots, 0). \end{cases} \tag{7}$$

Based on the proof in [2], it is easy to verify that operator A also generates a positive C_0-semigroup of contraction, denoted as $T(t), t \geq 0$, i.e, $A \in G(1,0)$, but 0 is not the eigenvalue of operator A (see 3.1, [3]). The following theorem can be derived by the same procedure in [4].

Theorem 3.1 *Denote $P(t)$ and $Z(t)$ be the solutions of systems* (5) *and* (7)*, respectively, then $Z(t) = T(t)Z(0)$ converges to the solution $P(t) = \mathcal{T}(t)P(0)$ strongly as $N \to \infty$.*

3.1 The Trotter-Kato Theorem

Let X and X_n be Banach spaces with norm $\|\cdot\|$, $\|\cdot\|_n$, $n = 1, 2, \ldots$, respectively, and assume that $T(t)$ is a C_0-semigroup on X. $A \in G(\mathcal{M}, \omega, X)$, $\mathcal{M} \geq 1$, $\omega \in R$, means that A is the infinitesimal generator of a C_0-semigroup $T(t)$, $t \geq 0$, satisfying $\|T(t)\| \leq \mathcal{M}e^{\omega t}$. Also assume that, for each $n = 1, 2, \ldots$, there exist bounded linear operators $P_n : X \to X_n$ and $E_n : X_n \to X$ satisfying

(A1) $\|P_n\| \leq M_1$, $\|E_n\| \leq M_2$, where M_1 and M_2 are independent of n;

(A2) $\|E_n P_n x - x\| \to 0$ as $n \to \infty$, for all $x \in X$;

(A3) $P_n E_n = I_n$, where I_n is the identity operator on X_n.

Theorem 3.2 *(Trotter-Kato). Assume that (A1) and (A3) are satisfied. Let $A \in G(\mathcal{M}, \omega, X)$ and $A_n \in G(\mathcal{M}, \omega, X_n)$, and let $T(t)$ and $T_n(t)$ be the semigroups generated by A and A_n on X and X_n, respectively. Then the following statements are equivalent:*
(a) There exists a $\lambda_0 \in \rho(A) \cap \bigcap_{n=1}^{\infty} \rho(A_n)$ such that, for all $x \in X$,

$$\|E_n(\lambda_0 I_n - A_n)^{-1} P_n x - (\lambda_0 I - A)^{-1} x\| \to 0 \ as\ n \to \infty.$$

(b) For every $x \in X$ and $t \geq 0$,

$$\|E_n T_n(t) P_n x - T(t)x\| \to 0 \ as\ n \to \infty$$

uniformly on bounded t-intervals.

If (a) or (b) is true, then (a) holds for all λ with $Re\lambda > \omega$. The proof of Trotter-Kato Theorem can be found in [5]. The assumption that, $A_n \in G(\mathcal{M}, \omega, X_n)$, $n = 1, 2, \ldots$, or equivalently, that $\|T_n(t)\|_n \leq \mathcal{M}e^{\omega t}$ is called the *stability property* of the approximation. Condition (a) is called the *consistency property* and condition (b) is called the *convergency property* of the approximation. In other words, the Trotter-Kato Theorem essentially states that, under the assumption of stability, consistency is equivalent to convergence. This is the functional analysis form of the Lax equivalent theorem (see [6]). However, it is usually difficult to compute the resolvent convergence. The authors in [7] showed that condition (a) is equivalent to (A2) and the following two statements:

(C1) There exists a subset $D \subset \mathcal{D}(A)$ such that $\overline{D} = X$ and $\overline{(\lambda_0 I - A)D} = X$ for a $\lambda_0 > \omega$;

(C2) For all $x \in D$ there exist a sequence $(\overline{x}_n)_{n\in N}$ with $(\overline{x}_n)_{n\in N} \in \mathcal{D}(A_n)$ such that

$$\lim_{n\to\infty} E_n \overline{x}_n = x, \ and\ \lim_{n\to\infty} E_n A_n \overline{x}_n = Ax$$

3.2 Finite Difference Scheme and Its Convergence

We will introduce the first order finite difference method to obtain the numerical approximation of system (7) and prove its convergence by using Trotter-Kato Theorem. Divide the interval $[0, N]$ into n equal subintervals $[x_{k-1}, x_k]$, $k = 1, 2, \ldots, n$, and set $\Delta x = \frac{N}{n}$, then $x_k = k\Delta x$ and $0 = x_0 < x_1 < \cdots < x_k < \cdots < x_n = N$. Applying a forward difference in space yields the difference equations corresponding to system (7)

$$\frac{dp_0(t)}{dt} = -\sum_{j=1}^{M}\lambda_j p_0(t) + \Delta x\sum_{j=1}^{M}\sum_{k=1}^{n}\mu_j(x_k)p_j(x_k,t), \quad (8)$$

$$\frac{dp_j(x_k,t)}{dt} = \frac{p_j(x_{k-1},t)-p_j(x_k,t)}{\Delta x} - \mu_j(x_k)p_j(x_k,t), \quad (9)$$

$$p_j(0,t) = \lambda_j p_0(t),\ j=1,2,\ldots,M, \quad (10)$$

where $p_j(t,x_k)$ is the value of $p_j(t,x)$ at the $k-th$ nodal point $x_k = k\Delta x$. Denote $X_n = \mathbf{R}\times(\mathbf{R}^n)^M$. For $P\in X_n$, $\|P\|_n = |p_0|+\Delta x\sum_{k=1}^{n}|p_1(x_k)|+\cdots+\Delta x\sum_{k=1}^{n}|p_M(x_k)|$. From equations (8)–(10), we can characterize the approximating generators A_n on X_n by matrix form

$$A_n = \begin{bmatrix} A_{00} & A_{01} & \cdots & A_{0,M} \\ \Lambda_1 & A_{11} & \cdots & 0 \\ & & \ddots & \\ \Lambda_M & 0 & \cdots & A_{M,M} \end{bmatrix},$$

where $A_{00} = -\sum_{j=1}^{M}\lambda_j$,

$$A_{0,j} = [\Delta x\mu_j(x_1)\cdots\ \Delta x\mu_j(x_n)]_{1\times n},$$

$$\Lambda_j = [\frac{\lambda_j}{\Delta x},0,\cdots,0]^T_{n\times 1},$$

and

$$A_{j,j} = \begin{bmatrix} u_{j1} & 0 & \cdots & 0 & 0 \\ \frac{1}{\Delta x} & u_{j2} & \cdots & 0 & 0 \\ & & \ddots & & \\ 0 & 0 & \cdots & \frac{1}{\Delta x} & u_{jn} \end{bmatrix}_{n\times n},$$

for $j=1,2,\ldots,M$ and $u_{jk} = -\frac{1}{\Delta x}-\mu_j(x_k)$, $k=1,2,\ldots,n$.

For $\varphi\in X_n$ and $\phi\in X$, we define

$$E_n\varphi = \big(\varphi_0,\sum_{k=1}^{n}\varphi_1(x_k)\chi_{I_k},\cdots,\sum_{k=1}^{n}\varphi_M(x_k)\chi_{I_k}\big)$$

and

$$P_n\phi = \Bigg(\phi_0,\big(\frac{1}{\Delta x}\int_{x_0}^{x_1}\phi_1(x)dx,\cdots,\frac{1}{\Delta x}\int_{x_{n-1}}^{x_n}\phi_1(x)dx\big), \cdots,\big(\frac{1}{\Delta x}\int_{x_0}^{x_1}\phi_M(x)\mathrm{d}x),\cdots,\frac{1}{\Delta x}\int_{x_{n-1}}^{x_n}\phi_M(x)\mathrm{d}x\big)\Bigg),$$

where χ_{I_k} is a characteristic function, $\chi_{I_k}=1$ if $x\in I_k=(x_{k-1},x_k]$, and $\chi_{I_k}=0$ elsewhere. It is straightforward to verify that conditions (A1)–(A3) are satisfied. For $\varphi\in X_n\setminus\{0\}$, we let $\varphi^* = \big(\frac{[\varphi_0]^+}{\varphi_0},(\frac{[\varphi_1(x_1)]^+}{\varphi_1(x_1)},\cdots,\frac{[\varphi_1(x_n)]^+}{\varphi_1(x_n)}),\cdots,(\frac{[\varphi_M(x_1)]^+}{\varphi_M(x_1)},\cdots,\frac{[\varphi_M(x_n)]^+}{\varphi_M(x_n)})\big)\in X_n^*$, the dual of X_n, where $[\varphi_i]^+=\varphi_i$ if $\varphi_i>0$ and $[\varphi_i]^+=0$ if $\varphi_i\le 0$, $i=0,1,2,\ldots,M$. Then it is easy to check that

$$(A_n\varphi,\varphi^*)\le 0,$$

which yields the *stability property*. Condition (C1) can be verified by taking $D=\mathcal{D}(A)$ with $\omega=0$. Now we only need to verify that condition (C2) holds.

For $\phi = (\phi_0,\phi_1(x),\cdots,\phi_M(x))\in\mathcal{D}(A)$, define $\overline{\phi}_n\in X_n$ by $\overline{\phi}_n = \big(\phi_0,(\phi_1(x_1),\cdots,\phi_1(x_n)),\cdots,(\phi_M(x_1),\cdots,\phi_{N+2}(x_n))\big)^T$, then

$$\begin{aligned}
&\|E_n\overline{\phi}_n-\phi\|_{L^1[0,N]} \\
=\ & \sum_{j=1}^{M}\int_0^N|\sum_{k=1}^{n}\phi_j(x_k)\chi_{I_k}-\phi_j(x)|dx \\
=\ & \sum_{j=1}^{M}\sum_{k=1}^{n}\int_{x_{k-1}}^{x_k}|\phi_j(x_k)-\phi_j(x)|dx \\
=\ & \sum_{j=1}^{M}\sum_{k=1}^{n}\int_{x_{k-1}}^{x_k}|\phi_j^{'}(\xi_k)(x_k-x)|dx \\
\le\ & \Delta x\sum_{j=1}^{M}\sum_{k=1}^{n}\int_{x_{k-1}}^{x_k}|\phi_j^{'}(\xi_k)|dx \\
=\ & \Delta x\sum_{j=1}^{M}\|\phi_j^{'}\|_{L^1[0,N]},
\end{aligned}$$

which proves that $\lim_{n\to\infty}E_n\overline{\phi}_n=\phi$. Further denote $A_n\overline{\phi}_n = (a_0,a_{1,1},\cdots,a_{1,n},\cdots,a_{M,1},\cdots,a_{M,n})^T$, where

$$\begin{aligned}
a_0 &= -\sum_{j=1}^{M}\lambda_j\phi_0(t)+\Delta x\sum_{j=1}^{M}\sum_{k=1}^{n}\mu_j(x_k)\phi_j(x_k), \\
a_{j,k} &= \frac{\phi_j(x_{k-1})-\phi_j(x_k)}{\Delta x}-\mu_j(x_k)\phi_j(x_k), \\
& \quad j=1,2,\ldots,M;\ k=1,2,\ldots,n,
\end{aligned}$$

and $A\phi=(b_0,b_1,\cdots,b_M)^T$, where

$$\begin{aligned}
b_0 &= -\sum_{j=0}^{M}\lambda_j\phi_0(t)+\sum_{j=1}^{M}\int_0^N\mu_j(x)\phi_j(x,t)dx, \\
b_j &= -\phi_j^{'}(x)-\mu_j(x)\phi_j(x), \quad j=1,2,...,M.
\end{aligned}$$

Note that

$$\begin{aligned}
&\|A_nE_n\overline{\phi}_n-A\phi\|_{L^1[0,N]} \\
=\ & |a_0-b_0|+\sum_{j=1}^{M}\int_0^N|\sum_{k=1}^{n}a_{j,k}\chi_{I_k}-b_j(x)|dx,
\end{aligned}$$

where

$$\begin{aligned}
&|a_0-b_0| \\
=\ & |\sum_{j=1}^{M}\sum_{k=1}^{n}\big[\Delta x\mu_j(x_k)\phi_j(x_k)-\int_{x_{k-1}}^{x_k}\mu_j(x)\phi_j(x)dx\big]| \\
\le\ & \sum_{j=1}^{M}\sum_{k=1}^{n}\sup_{|\xi|\le\Delta x}\int_{x_{k-1}}^{x_k}|\mu_j(x+\xi)\phi_j(x+\xi)-\mu_j(x)\phi_j(x)|dx \\
\le\ & \sum_{j=1}^{M}\sup_{|\xi|\le\Delta x}\int_0^N|\mu_j(x+\xi)\phi_j(x+\xi)-\mu_j(x)\phi_j(x)|dx \\
\le\ & \sum_{j=1}^{M}\omega(\mu_j(x)\phi_1(x);\Delta x)\to 0, \quad \text{as } \Delta x\to 0.
\end{aligned}$$

Here $\omega(\mu(x)\phi_j(x);\Delta x) = \sup_{|\xi|\le\Delta x}\int_0^N|\mu(x+\xi)\phi_j(x+\xi)-\mu(x)\phi_j(x)|dx$ is called the modulus of continuity for $\mu(x)\phi_j(x)$.

In addition,

$$
\begin{aligned}
& \sum_{j=1}^{M}\int_{0}^{N}\Big|\sum_{k=1}^{n}a_{j,k}\chi_{I_k}-b_j(x)\Big|dx \\
= & \sum_{j=1}^{M}\sum_{k=1}^{n}\int_{x_{k-1}}^{x_k}\Big|a_{j,k}-\big(-\phi_j^{'}(x)-\mu_j(x)\phi_j(x)\big)\Big|dx \\
\leq & \frac{1}{\Delta x}\sum_{k=1}^{n}\Big(\int_{x_{k-1}}^{x_k}\Big|\Delta x\phi_j^{'}(x)-\int_{x_{k-1}}^{x_k}\phi_j^{'}(\tau)d\tau\Big|dx\Big) \\
& +\sum_{k=1}^{n}\int_{x_{k-1}}^{x_k}\Big|\mu_j(x_k)\phi_j(x_k)-\mu_j(x)\phi_j(x)\Big|\mathrm{d}x \\
\leq & \frac{1}{\Delta x}\sum_{k=1}^{n}\int_{x_{k-1}}^{x_k}\int_{x_{k-1}}^{x_k}\Big|\phi_1^{'}(x)-\phi_1^{'}(\tau)\Big|d\tau dx \\
& +\sum_{k=1}^{n}\int_{x_{k-1}}^{x_k}\Big|\mu_j(x_k)\phi_j(x_k)-\mu_j(x)\phi_j(x)\Big|dx \\
\leq & \omega(\phi_j^{'}(x);\Delta x)+\omega(\mu_j(x)p_1(x);\Delta x) \\
\to & 0, \quad \text{as } \Delta x\to 0.
\end{aligned}
$$

Therefore $\lim_{n\to\infty} A_nE_n\overline{\phi}_n = A\phi$. Here $\omega(\phi_j^{'}(x);\Delta x)$ is called the modulus of continuity for $\phi_j^{'}(x)$. The following results hold immediately.

Theorem 3.3 *Let $T(t)$ and $T_n(t)$ be the semigroups generated by A and A_n on X and X_n, respectively, then for every $\phi \in X$ and $t \geq 0$, $\|E_nT_n(t)P_n\phi - T(t)\phi\| \to 0$ as $n \to \infty$ uniformly on bounded t-intervals.*

Corollary 3.4 *Denoted by $Z_n(t)$ the approximating solution of equations (8)–(10), then $Z_n(t) = E_nT_n(t)P_nZ(0)$ converges to the solution of system (7) strongly as $N \to \infty$.*

4 NUMERICAL EXPERIMENTS

To simplify the computation, we consider that $M = 1$, i.e, there is only one possible failure mode, which is sufficient to capture the features of dynamic behavior of the system. The numerical solutions are tested with different failure and repair rates. We set $N = 100$ and use equal interval of time $\Delta t = 0.5$. Fig.1 - Fig.3 show that the solutions of system converge with constant repair rate $\mu(x) = 0.8$. If we assume that the repair time follows Weibull distribution with hazard rate $\mu(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}$, where the shape parameters $\alpha = 2$ and $\beta = 10$, Fig.4 - Fig.6 also show the convergence of the solutions.

5 CONCLUSION

There are mainly two types of mathematical models for reparable systems, which are described by distributed parameter system of coupled partial and ordinary hybrid equations. The first type is with linear boundary conditions, like the model we present in the current paper, also see [8] and [9], etc. The second type is with integral boundary conditions presented in [10] and [11], etc. The above references point to the fact that although the qualitative behaviors of the models have been extensively studied, numerical issues have not been thoroughly addressed. In this paper, a finite difference scheme is constructed to simulate the dynamical solution of a specific reparable system with multi-state, which can also be applied to the the model with integral boundary conditions (see[12]). The convergence of this scheme is verified by Trotter-Kato Theorem. In the end, numerical experiments illustrate the effectivity of the scheme.

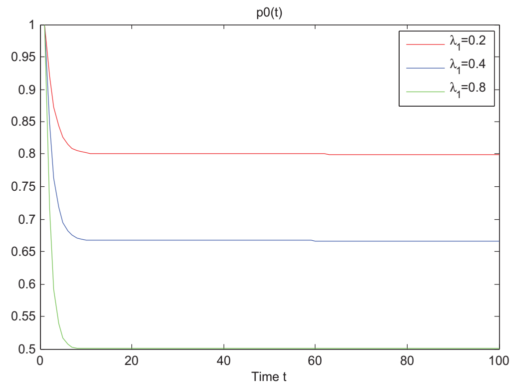

Figure 1: Probability $p_0(t)$ of the device in good state with different failure rates $\lambda_1 = 0.2, 0.4, 0.8$ and constant repair rate $\mu(x) = 0.8$.

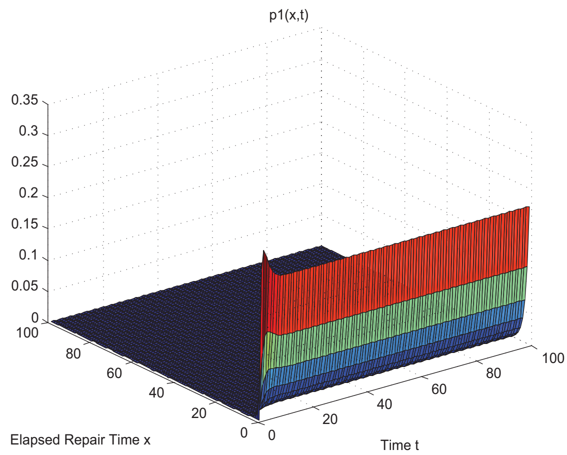

Figure 2: Probability $p_1(x,t)$ of the device in failed state with an elapsed repair time x; failure rate $\lambda_1 = 0.8$ and constant repair rate $\mu(x) = 0.8$.

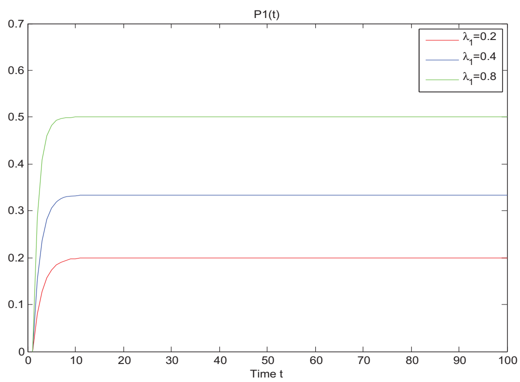

Figure 3: Probability $P_1(t)$ of the device in failed state with different failure rates $\lambda_1 = 0.2, 0.4, 0.8$ and constant repair rate $\mu(x) = 0.8$.

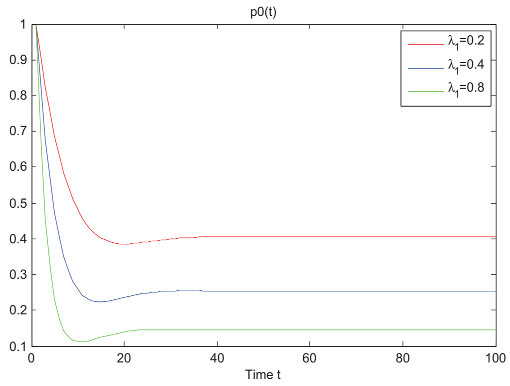

Figure 4: Probability $p_0(t)$ of the device in good state with different failure rates $\lambda_1 = 0.2, 0.4, 0.8$ and $\mu(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}, \alpha = 2, \beta = 10.$

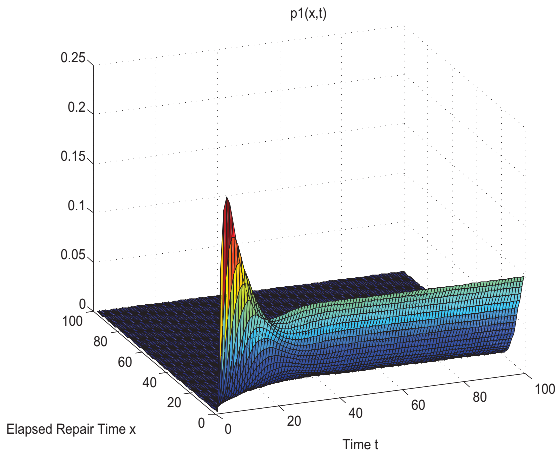

Figure 5: Probability $p_1(x,t)$ of the device in failed state with an elapsed repair time x; failure rate $\lambda_1 = 0.8$ and $\mu(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}, \alpha = 2, \beta = 10.$

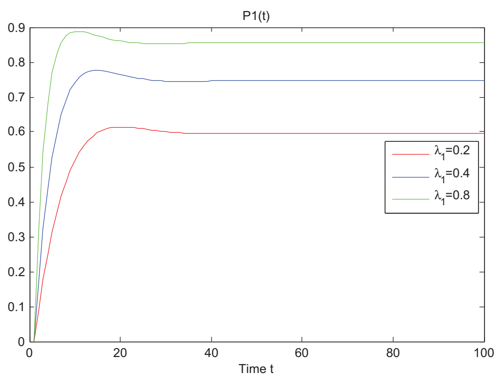

Figure 6: Probability $P_1(t)$ of the device in failed state with failure rate $\lambda_1 = 0.8$ and $\mu(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}, \alpha = 2, \beta = 10.$

ACKNOWLEDGEMENT

This paper is finished during the first author visiting ICAM, Virginia Polytechnic Institute and State University. The authors also expresses their gratitude to China Scholarship Council, and special thanks to professor J. A. Burns for his helpful suggestions.

REFERENCES

[1] W. K. Chung, A reparable multistate device with arbitrarily distributed repair times, Microelectronics Reliability, Vol.21, No.2, 255-256, 1981.

[2] H. B. Xu, J. Y. Yu and G. T. Zhu, Asymptotic property of a reparable multi-state device, Quarterly of Applied Mathematics, Vol.63, No.4, 779-789, 2005.

[3] W. W. Hu, H. B. Xu, J. Y. Yu and G. T. Zhu, Exponential stability of a reparable multi-state device, Journal of System Science and Complexity, Vol.20, No.3, 437-443, 2007.

[4] H. B. Xu, and W. W. Hu, Analysis and approximation of a reliable model, Applied Mathematical Modelling, Vol. 37, 3777-3788, 2013.

[5] A. Pazy, Semigoroups of Linear Operator and Applications to Partial Differential Equations, New York, Springer-Verlag, 1983.

[6] P. D. Lax and R. D. Richtmyer, Survey of the stability of linear finite difference equations, Commun. Pure Appl. Math., Vol.9, 267-293, 1956.

[7] K. Ito and K. Kappel, The Trotter-Kato theorem and approximation of PDEs, Mathematics of Computation, Vol.69, No.221, 21-44.1998.

[8] W. K. Chung, Stochastic analysis of N-out-of-N:G redundant systems with repair and multiple critical and non-critical errors, Microelectronics Reliability, Vol.35, No.11, 1429-1431, 1995.

[9] W. K. Chung, Reliability of imperfect switching of cold standby systems with multiple non-critical and critical errors, Microelectronics Reliability, Vol.35, No.12, 1479-1482, 1995.

[10] W. Li and J. H. Cao, Some performance measures of transfer line consisting of two unreliable machines with reprocess rule, Journal of Systems Science and Systems Engineering, Vol.7, 283-292, 1998.

[11] L. N. Guo, H. B. Xu, G. Cao, and G. T. Zhu, Stability analysis of a new kind series system, IMA Journal of Applied Mathematics, Vol,75, 439-460, 2010.

[12] J. A. Burns, W. W. Hu and H. B. Xu, Modeling and Approximation of a Software System with Rejuvenation Policy, preprint.