Performance Analysis of MPC Based on Structures Subject to No-Model Input/Output Combinations

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***Abstract*—An analysis of the benefits over MPC performance obtained through the use of a method to detect no-model IO (input/output) combinations for open and closed-loop in multiple input multiple output (MIMO) systems is performed. Traditional approaches to IO selection are usually performed after the plant model is already characterized, which can lead to model-plant mismatch. The approach herein presented is applied during the pre-identification stage, in order to provide previous information to the following stages. A study case involving identification of a 2×2 MIMO system is discussed.**

I. Introduction

Model Predictive Control (MPC) is an advanced control technique based on the optimal control theory which has been in use by process industries, such as refineries, chemicals and petrochemicals plants, since the early 80s.

The objective of the optimal control theory is to determine the control signals (inputs) that will cause a process to satisfy physical constraints and at the same time minimize (or maximize) some performance criterion [1]. Model predictive controllers, such as the optimal controllers, require the following three points to be addressed:

1) A mathematical description (or model) of the process to be controlled.
2) A statement of the physical constraints.
3) Specification of a performance criterion.

The mathematical model of the process is composed by expressions which describe the relationships among the system variables. The performance of such advanced control structures relies on the accuracy of these models, so modelling the process is an important stage of the Control System Design.

Control System Design usually involves six stages: definition of control objectives, derivation of a nominal model G_0, Control Structure Design, controller design, control system evaluation and tuning, and finally, control implementation [2].

The definition of the nominal models is a nontrivial part of any control problem. The objective of this stage is to obtain the simplest mathematical expressions that will be able to predict the response of the physical system to all anticipated inputs. Modeling a MIMO process is usually based on building a statistical model, also known as system identification. The building of a statistical model is characterized by the iteration of the following steps: Identification, Fitting and Diagnostic Checking [3].

Generally, issues related to IO (Input/Output) selection are part of the Control Structure Design stage [2], after the plant model has already been characterized. IO selection is described as the procedure of selecting suitable variables u to be manipulated by the controller (plant inputs) and suitable variables y to be supplied to the controller (plant outputs). This approach can lead to model-plant mismatch and poor controller performance, since models can be identified in IO combinations that do not present relationships (IO pairings with no-model). An extensive survey of methods for IO selection can be found in [2].

Some system identification packages focused on process applications, such as TaiJi, developed by Y. Zhu [4] and Profit Stepper [5], use previous knowledge of plant operation personnel to acquire no-model IO information. Approaches such these are effective, but they rely on personnel experience and not on process data. It should be avoided in new plants or after plant modifications, since the previous operator knowledge does not apply any more.

In order to reduce the number of iterations, a pre-identification step can be applied. The typical purpose of this step is to provide previous information regarding system order and time delay estimation.

This paper analyses the method proposed in [6], which detects no-model IO combinations for open-loop MIMO systems during the pre-identification step. It also extents the method for closed-loop applications. The method is based on cross-correlation analysis. Although this one could also be applied during the identification step, it provides important advantages if proceeded previously. No-model IO combination knowledge can reduce experiment time, decrease model parameter variance and improve the accuracy of the remaining models.

Theoretically, if the model structure exactly matches the structure of the actual system, then the model estimated from a one-step ahead predictor is equivalent to the maximum likelihood estimate, which also provides optimal multi-step ahead predictions. However, in practice, even around an operating point, it is not possible to propose a linear model structure that exactly matches the system to be identified. Consequently, any estimated model has modeling errors associated with the identification algorithm. Then, the objective of the proposed method is to reduce the error associated with the identified model structure.

The outline of the paper is the following: Section II presents a brief description of the method proposed in [6] for open-loop. In Section III the closed-loop method is presented. Section IV consists in the application of the algorithm to detect no-model IO combinations in a 2×2 MIMO system

(Shell Benchmark Plant [7] and [8]). In Section V, the MPC performance analysis is discussed. Finally, conclusions are drawn in Section VI.

II. The No-Model IO Combination Method for Open-Loop

The method proposed in [6] is a variation of the one proposed in [9] for MIMO $m \times n$ systems and is applied to open-loop systems. The algorithm is based on two steps and it is applied in a pre-identification stage. Usually, in this first approximation, the process is excited by pulses to obtain preliminary characteristics of the system, such as steady-state gain, settling time and delay time.

A. Step 1

The first step of the method consists in determining the linear correlation between the input and output signals used in the pre-identification stage. Herein the cross-correlation used is the Pearson product-moment correlation coefficient, or simply Pearson's correlation defined by:

$$\rho = \mathtt{corr}(u, y) = \frac{E[(u - \mu_u)(y - \mu_y)]}{\sigma_u \sigma_y} \tag{1}$$

where μ_u and μ_y are the expected values and σ_u and σ_y are the standard deviations of the signals u and y, respectively. For a MIMO system may be used as:

$$\rho^{(1)}(i,j) = \mathtt{corr}(u_i, y_j) \tag{2}$$

where $\rho^{(1)}$ is the $m \times n$ matrix that represents the linear correlation between the *i*-th input and *j*-th output signals.

Next it is found the maximum linear correlation for each output:

$$\rho_{max}(j) = \max\{\rho^{(1)}(i,j)\} : \quad i = 1, \ldots, n \tag{3}$$

and then it is calculated a first estimate of the no-model IO combination matrix according to the following criterion:

$$\begin{aligned} &\text{if } \ (\rho^{(1)}(i,j) < \alpha_j \rho_{max}(j)) \\ &\text{then } \ \chi^{(1)}(i,j) = 0 \ \text{ else } \ \chi^{(1)}(i,j) = 1 \end{aligned} \tag{4}$$

where α_j is the tolerance of the problem, usually chosen as $\alpha_j = 0.1, \quad j = 1, \ldots, m$ and $\chi^{(1)}$ is the no-model IO combination matrix.

B. Step 2

If the process is subject to disturbances and measurement noise, another step is required. This step consists in a preliminary estimate of the model. In this sense, an identification by a high order FIR is performed:

$$\hat{y} = G_P u \tag{5}$$

where $\hat{y}$ is the estimate output column vector, u is the input column vector and G_P is the process model, respectively.

Under open-loop, the bias errors in the deterministic part of the model can generally be minimized by using high order FIR models. Under these circumstances, the modeling errors are dominated by the variance errors. In practical applications, any identified model has bias and variance errors associated with the identification algorithm. It is assumed that the true process consists of a deterministic part driven by the manipulated variables and a stochastic part driven by white noise. The quality of the predictor depends on the quality of the deterministic and the stochastic parts of the model [10].

Then, with the input signals filtered by the high order FIR, it is calculated again the linear correlation, but now between the input signal and the estimated output.

$$\rho^{(2)}(i,j) = \mathtt{corr}(u_i, \hat{y}_j) \tag{6}$$

The objective of this procedure is to obtain a filtered estimate of the linear correlation matrix.

Finally, it is defined a relationship between matrices $\rho^{(1)}$ and $\rho^{(2)}$.

$$R(i,j) = \frac{\rho^{(2)}(i,j)}{\rho^{(1)}(i,j)}, \ \ i = 1, \ldots, n \ \ j = 1, \ldots, m \tag{7}$$

If the value of $R(i,j)$ is close to 1, this means that the linear correlation obtained by the estimate of the high order FIR and the true outputs are independent of the disturbances and noise. Then, if $R(i,j) \approx 1$ and the linear correlation $\rho(i,j)$ is low, this fact suggests that there exists a model in that position, but its gain is very low. On the other hand, when $R(i,j) \not\approx 1$ and $\rho^{(2)}(i,j)$ is low, this may indicate that the difference between linear correlations $\rho^{(1)}(i,j)$ and $\rho^{(2)}(i,j)$ is due to unmeasured disturbances or noise.

This proposition can be summarized as:

$$\begin{aligned} &\text{if } \ ((\chi^{(2)}(i,j) == 0) \ \text{ and } \ (R(i,j) \approx 1)) \\ &\text{then } \ \chi^{(2)}(i,j) = 1 \ \text{ else } \ \chi^{(2)}(i,j) \ \text{ is kept.} \end{aligned} \tag{8}$$

III. The Extension of the No-Model IO Combination Method for Closed-Loop

Limitations of cross-correlation methods when employed to detect input/output effects in closed-loop have been stated in [3]. However, good results can be obtained if the input signal is dithered. In [9] a closed-loop cross-correlation method is employed to detecting model mismatch in MIMO model-based controllers. In this proposal, IO subset pairings of a linear MIMO system which demand re-identification are detected. The method is based on the comparison of the correlation between the prediction error and input (u). A dithering in the setpoint (r) is required to use this signal as excitation.

The use of a dithering signal is not an option during the pre-identification stage. In order to detect no-model IO combinations for closed-loop systems, an adaptation of the method proposed in Section II was performed.

To excite the process in closed-loop systems, pulses are superposed over the controller setpoints (r) during the pre-identification stage. In [11], it was shown that methods for detecting no-model based on the analysis of correlations between the setpoints (r) and the plant outputs (y) do not lead to accurate results, due to the action of the controller. The behavior of the system with closed-loop regulatory control "hides" information about IO combinations with no-model.

The solution for this problem is the application of a cross-correlation analysis based on the setpoints (r) and the plant inputs (u). The algorithm for closed-loop systems is also based on a single step. This step of the method consists in determining the linear correlation between the setpoint and input signals.

$$\rho^{(1)}(i,j) = \mathtt{corr}(r_i, u_j) \tag{9}$$

where $\rho^{(1)}$ is the $m \times n$ matrix that represents linear correlation between the setpoint and input signals.

Next, it is found the maximum linear correlation for each input:

$$\rho_{max}(j) = \max\{\rho^{(1)}(i,j)\} : \quad i = 1, \ldots, n \tag{10}$$

and then it is calculated a first estimate of the no-model IO combination matrix according to the following criterion:

$$\begin{gathered} \text{if } \left(\rho^{(1)}(i,j) < \alpha_j \rho_{max}(j)\right) \\ \text{then } \chi^{(1)}(i,j) = 0 \text{ else } \chi^{(1)}(i,j) = 1 \end{gathered} \tag{11}$$

where α_j is the tolerance of the problem, usually chosen as $\alpha_j = 0.01, \quad j = 1, \ldots, m$ and $\chi^{(1)}$ is the no-model IO combination matrix.

IV. Application Of The Method

To test the proposed method, the plant employed in the simulations is the distillation column described in [7] and [8]. A simplified diagram of this 2×2 system is shown in Figure 1.

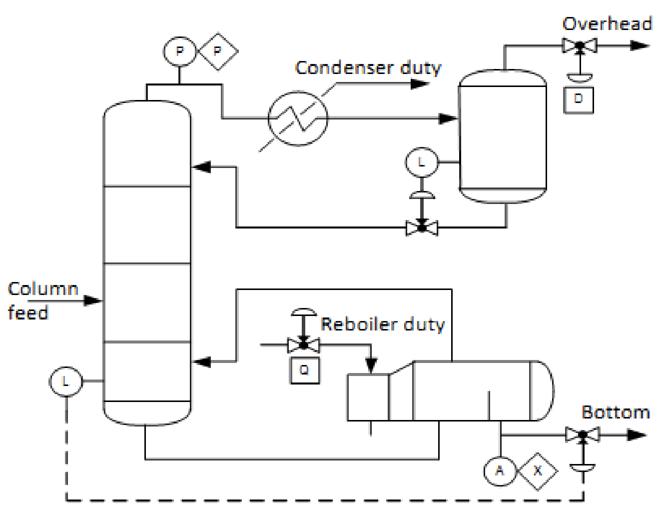

Fig. 1. Diagram of the Distillation Column [7].

The plant discrete time transfer matrix is shown in (12).

$$G(q) = \begin{bmatrix} \frac{-0.61+0.40q^{-1}}{1-1.53q^{-1}+0.57q^{-2}} & \frac{-0.11+0.092q^{-1}}{1-1.53q^{-1}+0.57q^{-2}} \\ 0 & 0.076\frac{5x10^5}{q^{-7}-1500} + 0.9235q^{-1} \end{bmatrix} \tag{12}$$

The manipulated variables of the plant are the overhead vapor flow MV-1 (D) and reboiler flow MV-2 (Q). The controlled variables are the top pressure CV-1 (P) and the reboiler outlet composition CV-2 (X), as shown in Figure 1. Note that input u_2 (MV-2) only affects the output y_2 (CV-2). All outputs are affected by white noise and unmeasured disturbances. Signal-to-Noise Ratio (SNR), specified as the ratio of output and noise variances, was set to 3.

Figure 2 presents the control structure for the open-loop pre-identification and Fig. 3 for the closed-loop one. The PID controller is composed of two PIDs, one between u_1 and y_1 and another between u_2 and y_2.

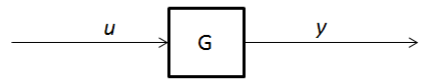

Fig. 2. Regulatory Open-loop.

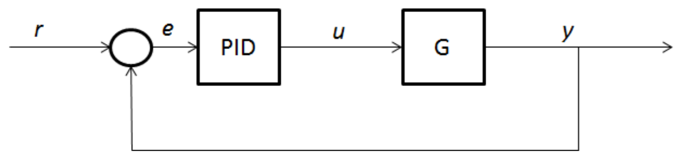

Fig. 3. Regulatory Closed-loop.

A first pre-identification test was performed using the open-loop configuration. The signal used to excite each input of the plant was composed of two pulses (positive and negative). The pulse duration corresponded to two settling times (T_s) and similar pulses shifted in time ($2T_s$) are applied to each input, as shown in Fig. 4. The respective outputs y_1 and y_2 are presented in Fig. 5.

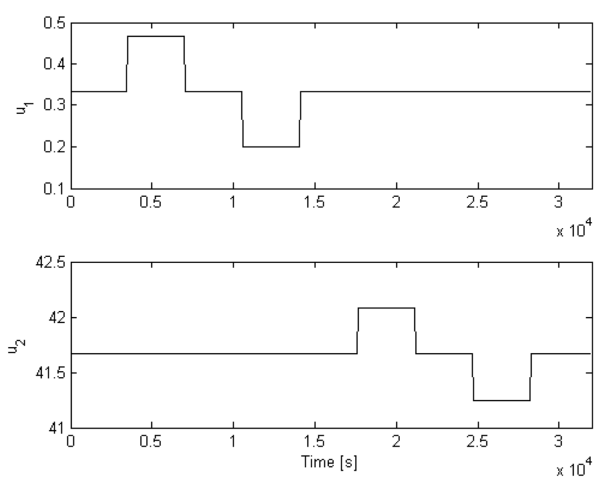

Fig. 4. Inputs of the plant in the open-loop pre-identification.

A closed-loop pre-identification test was performed and Fig. 6 shows the inputs u_1 and u_2 whereas Fig. 7 shows the outputs y_1 and y_2.

The result of the application of the linear correlation $\rho^{(1)}$ between the inputs u and the outputs y for the open-loop pre-identification is shown in (13). The result of the correlation matrix $\rho^{(2)}$ obtained with the filtered signals in the second step of the method is given by (14).

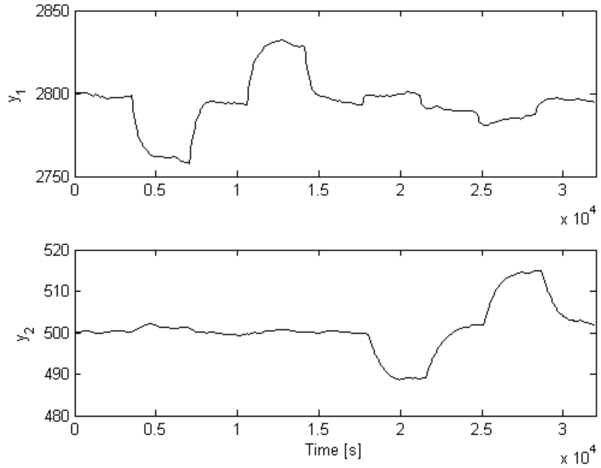

Fig. 5. Outputs of the plant in the open-loop pre-identification.

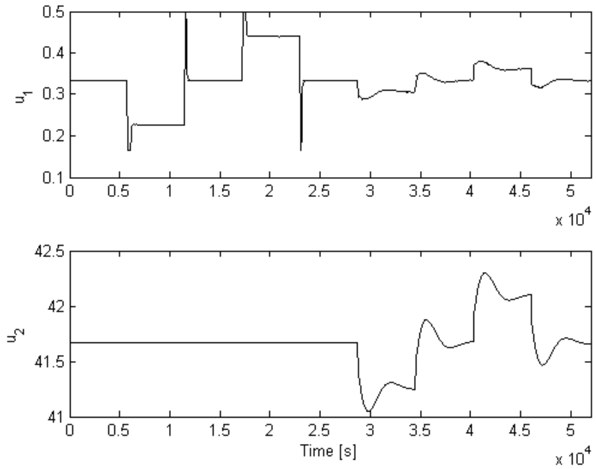

Fig. 6. Inputs of the plant in the closed-loop pre-identification.

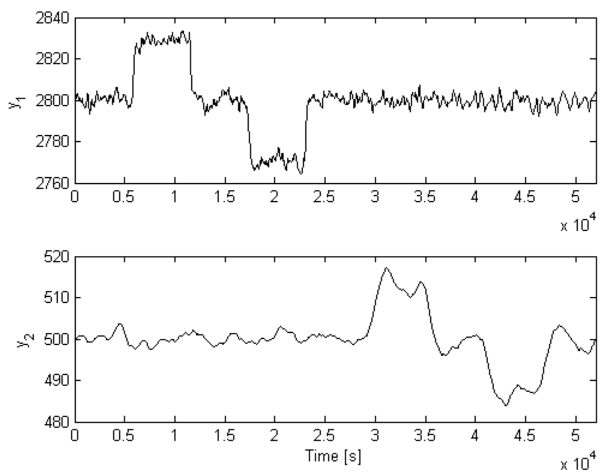

Fig. 7. Outputs of the plant in the closed-loop pre-identification.

$$\rho^{(1)} = \begin{bmatrix} -0.89 & 0.22 \\ 0.04 & -0.77 \end{bmatrix} \tag{13}$$

$$\rho^{(2)} = \begin{bmatrix} -0.43 & 0.32 \\ 0.01 & -0.76 \end{bmatrix} \tag{14}$$

The relation matrix R is given by (15), which is exactly the relation presented in the model in (12).

$$R = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \tag{15}$$

For the closed-loop systems, if the same algorithm for open-loop systems is applied, the results will be wrong. The result of the application of the linear correlation between the inputs u and the outputs y for the closed-loop pre-identification is shown in (16). The result of the correlation matrix $\rho^{(2)}$ obtained with the filtered signals at the second step is given by (17).

$$\rho^{(1)} = \begin{bmatrix} 0.96 & 0.01 \\ -0.04 & 0.84 \end{bmatrix} \tag{16}$$

$$\rho^{(2)} = \begin{bmatrix} 0.35 & -0.01 \\ 0.27 & 0.06 \end{bmatrix} \tag{17}$$

The relation matrix R is given by (18), which is different from the relation presented in the model in (12). The reason for this mistake is the action of the controller, as quoted by [11].

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{18}$$

But, if the modified algorithm is applied to the closed-loop configuration, the results are accurate. The result of the application of the linear correlation between the setpoints r and the inputs u for the closed-loop pre-identification is shown in (19).

$$\rho = \begin{bmatrix} -0.92 & -0.26 \\ 0 & -0.95 \end{bmatrix} \tag{19}$$

The relation matrix R is given by (20), which is exactly the relation presented in the model in (12).

$$R = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \tag{20}$$

V. MPC Performance Analysis

In order to test the benefits obtained by the detection of no-model input/output combinations, some identification tests using the same system were done. The objective is to show that this approach can bring benefits to the identification process. The following structures were adopted for the identification tests:

- High Order ARX
- Box-Jenkins

- MPC Relevant Identification by Gopaluni (MRI Gopaluni - [12])
- MPC Relevant Identification by Huang (MRI Huang - [13])
- MPC Relevant Identification EMPEM ([14])

To identify the plant, two GBN signals were used. The following combinations were tested:

- Identification in open-loop with no-model information
- Identification in open-loop without no-model information
- Identification in closed-loop with no-model information
- Identification in closed-loop without no-model information

The first analysis was performed based on the validation index FIT. This index is based on the percentage of difference between the real and estimated outputs k-step ahead FIT(k) and may be expressed by (21):

$$\mathrm{FIT}(k) = 100\left(1 - \frac{\sqrt{\sum_{t=0}^{t_f-k}\left(\hat{y}(t+k|t) - y(t+k|t)\right)^2}}{\sqrt{\sum_{t=0}^{t_f-k}\left(y(t+k|t) - \bar{y}(t+k|t)\right)^2}}\right) \tag{21}$$

where $\hat{y}(t+k|t)$ is the output prediction estimated k-step ahead, $y(t+k|t)$ is the system output k-step ahead, $\bar{y}(t+k|t)$ is the mean of $y(t+k|t)$ and t_f is the final time of the experiment. The FIT(k) analysis was performed with prediction for 10, 20 and 30 steps ahead. Equation (22) shows the difference between the FIT(k) index of two estimated models

$$\Delta\mathrm{FIT}(k) = \mathrm{FIT}^{zd}(k) - \mathrm{FIT}^{nd}(k) \tag{22}$$

where $\mathrm{FIT}^{zd}(k)$ and $\mathrm{FIT}^{nd}(k)$ are the FIT index of the model at the prediction horizon k for the model estimated with the no-model information and without this information, respectively.

Table I presents the ΔFIT(k) analysis for the open-loop pre-identification and for the five structures. Table II presents the ΔFIT(k) analysis for the closed-loop pre-identification and for the five structures.

The results herein presented show that the use of the no-model information is not relevant for the FIT(k) index in closed-loop pre-identification. For the open-loop pre-identification, there is only a beneficial over the MRI EMPEM structure.

The second analysis was performed based on the MPC validation. This validation analyses the MPC based on the performance index J, which is the performance criterion that the MPC controller minimizes. The MPC used here is the Quadratic Dynamic Matrix Control (QDMC - [15]) and may be expressed by (23):

$$J = min_{\Delta u}((E - A\Delta u)^T\Gamma(E - A\Delta u) + (\Delta u)^T F^2 \Delta u) \tag{23}$$

subject to:

$$\begin{array}{rcl} u_{min} & \leq u_i \leq & u_{max} \\ \Delta u_{min} & \leq \Delta u_i \leq & \Delta u_{max} \\ y_{min} & \leq y_i \leq & y_{max} \end{array}$$

where Δu is the variation input signal matrix, A is the dynamic matrix consisting of step response coefficients, Γ is the weight matrix and F is the move suppression matrix. E is the error vector defined by (24):

$$E = y_{sp} - y^P - \epsilon) \tag{24}$$

where y_{sp} is the setpoint, ϵ is the error between measured and predicted value of $y(t_0)$. y^P is the prediction vector, which consists of the effects of past manipulated variable changes on future controlled variable values.

Fig. 8 presents the MPC analysis for the plant without regulatory control and for the five structures whereas Fig. 9 presents it for the plant with regulatory control.

The results show that the use of the no-model information is not relevant for the MPC performance index J with regulatory control. For the case without regulatory control, the MPC performance index J has a beneficial improvement over the following structures:

- Box-Jenkins
- MPC Relevant Identification by Huang (MRI Huang - [13])

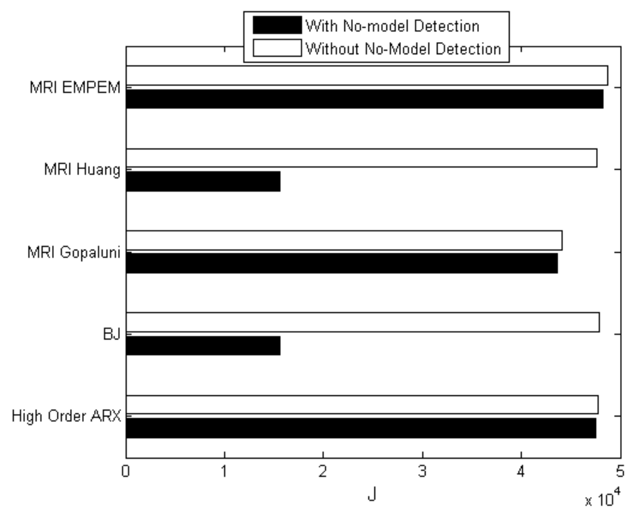

Fig. 8. MPC analysis - Without Regulatory Control.

VI. Conclusion

Herein, an adaptation for closed-loop estimation of the method proposed in [6] was discussed. The adaptation is based on the linear correlation between the setpoints and the inputs in the MIMO models and its use is also suggested in a pre-identification stage. It has only one stage.

To test and validate the algorithm, the Shell Benchmark Plant was used. Twenty MIMO models were identified by different structures, with the no-model IO combination matrix given by the proposed no-model IO combination algorithm and without this information, for open and closed-loop. Using the FIT(k) index to validate the prediction of the models, it was shown that the no-model IO combination information does not provide a better estimate of the model for closed-loop systems. For open-loop systems, the FIT(k) index has only benefits over the MRI EMPEM structure. Using the MPC

TABLE I. Δ FIT(k) ANALYSIS - OPEN-LOOP.

	Output y_1			Output y_2		
	$k = 10$	$k = 20$	$k = 30$	$k = 10$	$k = 20$	$k = 30$
High Order ARX	1.45	1.32	1.95	−3.48	−4.62	−4.25
BJ	1.43	1.40	1.28	−5.18	−7.46	−7.37
MRI Gopaluni	−1.46	−2.40	−3.27	−3.07	−2.70	−2.76
MRI Huang	1.46	1.15	0.94	−5.22	−7.46	−7.22
MRI EMPEM	5.24	5.24	5.24	17.52	17.52	17.52

TABLE II. Δ FIT(k) ANALYSIS - CLOSED-LOOP.

	Output y_1			Output y_2		
	$k = 10$	$k = 20$	$k = 30$	$k = 10$	$k = 20$	$k = 30$
High Order ARX	0	0	0	1.07	0.82	0.65
BJ	0	0	0	0.04	0.28	0.26
MRI Gopaluni	0	0	0	0.35	0.37	0.33
MRI Huang	0	0	0	0.12	0.52	0.59
MRI EMPEM	0	0	0	0.17	0.23	0.28

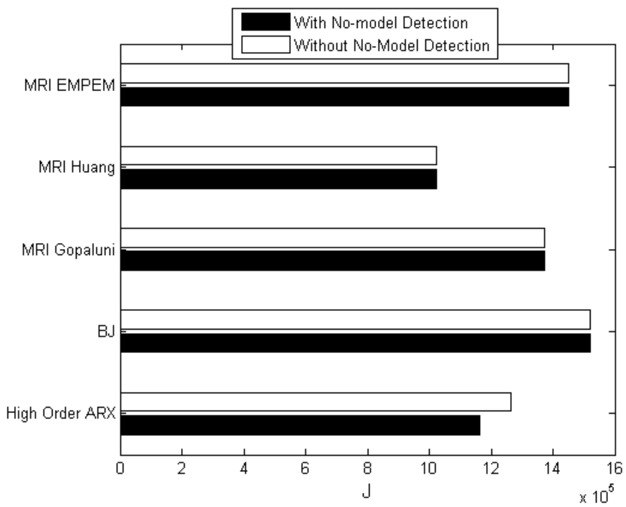

Fig. 9. MPC analysis - With Regulatory Control.

analysis to validate the models, it was shown that the no-model IO combination information provided a better estimate of the model for 40 percent of the structures, but only for systems without regulatory control.

Although the FIT index is widely employed in model validation, apparently, in this case, the results are not conclusive. In this paper, the MRI methods (MRI EMPEM and MRI Gopaluni) and the High Order ARX are more robust then the ones based on Box-Jenkins structure (Box-Jenkins and MRI Huang) when dealing with model-mismatch and this can be observed in the MPC analysis results. Further analysis must be performed in order to confirm the results herein presented.

In addition, if the identification is relevant to MPC, it can be favored in terms of computational effort, once no-model IO combinations are unconsidered. These improvements will be better noted for large systems, where more than one no-model IO combinations are detected.

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