$$x_{k+1} = \phi_k x_k + w_k \qquad 5.5.1$$

$$z_k = H_k x_k + v_k \qquad 5.5.2$$

$w_k \quad v_k$

$$\mathrm{E}\left[w_k w_i^{\ T}\right] = \begin{cases} Q_k, i = k \\ 0, i \neq k \end{cases} \qquad 5.5.3$$

$$\mathrm{E}\left[v_k v_i^{\ T}\right] = \begin{cases} R_k, i = k \\ 0, i \neq k \end{cases} \qquad 5.5.4$$

$$\mathrm{E}\left[w_k v_i^{\ T}\right] = 0,$$

$$e_k^- = x_k - \hat{x}_k^- \qquad 5.5.6$$

$$\mathrm{P}_k^- = E[e_k^- e_k^{-T}] = E[\left(x_k - \hat{x}_k^-\right)\left(x_k - x_k^-\right)^T] \qquad 5.5.7$$

$$\hat{x}_k = x_k^- + K_k\left(z_k - H_k x_k^-\right) \qquad 5.5.8(****)式$$

$$\mathrm{P}_k = E[e_k e_k^T] = E[\left(x_k - \hat{x}_k\right)\left(x_k - x_k\right)^T] \qquad 5.5.9式$$

把5.5.2式，再把5.5.8式入式5.5.9，可得： 5.5.9

$$\mathrm{P}_k = E\{\left[\left(x_k - \hat{x}_k\right) - K_k\left(H_k x_k + v_k - H_k x_k^-\right)\right]$$

$$\left[\left(x_k - \hat{x}_k\right) - K_k\left(H_k x_k + v_k - H_k x_k^-\right)\right]^T \qquad 5.5.10式$$

式中x_k与量$\hat{x}_k$差是不相v_k的关，于是得到：

$$=E\left[(I - K_k H_k)\left(x_k - \hat{x}_k\right) - K_k v_k\right]\left[(I - K_k H_k)\left(x_k - x_k^-\right) - K_k v_k\right]^T$$

$$= (I - K_k H_k)\,\mathrm{P}_k^-(I - K_k H_k)^T + K_k R_k K_k^T \qquad 5.5.11式$$

$$= \mathrm{P}_k^- - K_k H_k\,\mathrm{P}_k^- - \mathrm{P}_k^- H_k^{\ T} K_k^T + K_k(H_k \mathrm{P}_k^- H_k^{\ T} + R_k)K_k^T \qquad 5.5.12式$$

$$\frac{d(trace\mathrm{AB})}{d\mathrm{A}} = \mathrm{B}^{\mathrm{T}}$$ (AB)

$$\frac{d(trace\mathrm{ACA}^T)}{d\mathrm{A}} = 2AC$$ (C)

$$\frac{d(traceP_k)}{dK_k} = -2(H_k P_k^-)^T + 2\mathrm{K}_k(H_k P_k^- H_k^T + R_k) \qquad 5.5.16$$

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \qquad 5.5.17(***)$$

5.5.17 5.5.12

$$\begin{aligned}
\mathrm{P}_k &= \mathrm{P}_k^- - K_k H_k \mathrm{P}_k^- - \mathrm{P}_k^- H_k{}^T K_k^T + K_k (H_k \mathrm{P}_k^- H_k{}^T + R_k) K_k^T \\
&= \mathrm{P}_k^- - K_k H_k \mathrm{P}_k^- \\
&= \mathrm{P}_k^- - P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} H_k \mathrm{P}_k^- && 5.5.20 \\
&= (I - K_k H_k)\mathrm{P}_k^- && 5.5.21(*****)
\end{aligned}$$

$$\hat{x}^-_{k+1} = \phi_k x_k \qquad 5.5.23$$

$$\begin{aligned}
e^-_{k+1} &= x_{k+1} - \hat{x}^-_{k+1} \\
&= (\phi_k x_k + w_k) - \phi_k \hat{x}_k \\
&= \phi_k e_k + w_k && 5.5.24
\end{aligned}$$

$$\begin{aligned}
P^-_{k+1} &= E[e^-_{k+1} e^{-T}_{k+1}] = E[(\phi_k e_k + w)(\phi_k e_k + w)^T] \\
&= \phi_k P_k \phi_k{}^T + Q_k && **
\end{aligned}$$