

Using Closed-Loop Detection to Improve Homography Estimation and Mosaicing

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1 Introduction and Overview

The goal of this project is to improve an automatically mosaiced image by detecting a closed loop in a sequence of images and re-computing the calculated image homographies using nonlinear optimization. First homographies will be discussed, with some discussion about factoring homography matrices to determine the various components of a homography. Next the techniques for detecting a closed loop in an image sequence are discussed. Then the optimization problem for improving the homography matrices is described in terms of formulation and the constraints. Lastly, further considerations and ideas are discussed.

The high-level structure for the process of correcting an image mosaic using closed loop information is as follows:

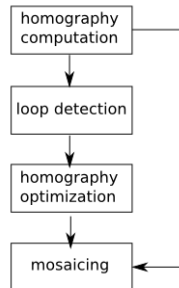


Figure 1: The high-level work flow for optimizing the image mosaic.

2 Homographies

2.1 In General

A homography H in this context is a 3 by 3 matrix $H_{i,j}$ that describes the relationship between image i and image j in an image sequence. In this application, the homographies $H_{i,i+1}$ are calculated between consecutive images i and $i+1$. When calculating an image mosaic, the cumulative homography for image k in the sequence is calculated by multiplying each of the previous homography matrices together until the current image:

$$H_{1,k} = H_{1,2} \cdot H_{2,3} \cdots H_{k-1,k}$$

In the application of creating a 2-D mosaic from a video, the homography is a 2D homography, transforming a point $[u, v, 1]^T$ to $[u', v', 1]^T$ up to a scale factor, normalized in OpenCV to be 1.

2.2 Factoring a Homography Matrix

As discussed by Sonka, et. al in [1], homographies form a group under multiplication as shown above. There are various important subgroups that can be used to factor any homography matrix H into it's components. As discussed in [1], any homography can be decomposed as $H = H_P H_A H_S$ where

$$H_P = \begin{bmatrix} I & \mathbf{0} \\ \mathbf{a}^T & b \end{bmatrix}, \quad H_A = \begin{bmatrix} K & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}, \quad H_S = \begin{bmatrix} R & -R\mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix}$$

And K is upper triangular. Page 557 in [1] gives more information about these subgroups.

Using the factorization above as a starting point, a homography matrix is therefore written as follows:

$$H = H_P H_A H_S = \begin{bmatrix} KR & -KR\mathbf{t} \\ \mathbf{a}^T KR & -\mathbf{a}^T KR\mathbf{t} + b \end{bmatrix}$$

Where t is the translation vector, R is the rotation matrix for rotation in the xy -plane, and K is the dilation matrix in the xy -directions. \mathbf{a} and \mathbf{b} are coefficients related to the rotation and dilation in the z direction.

2.3 Computing the Homography Matrices in Code: Overview

The homography matrices can be computed using an OpenCV method called `cvFindHomography`, described in [2], which needs to be tuned according to the application you are working with. For instance, the window size that is needed to find a correct homography depends on how close the camera is to the scene in question.

To use the `cvFindHomography` method you already need to have points picked out on both images that are known to be correspondent. In applications with few images, these points can be picked out by hand and visual inspection. Because the goal of this project was to create a fully automated mosaicing algorithm, manually picking points was not an option. Instead, because the image sequence is taken from a video, the video can be sequenced in such a way to make the images overlapped by a significant amount. Given this information, we use optical flow in the OpenCV method `cvCalcOpticalFlowPyrLK` to calculate the new positions of interest points from the first image in the second image, described in [4]. The optical flow method has parameters to tune as well.

The interest points in the first image are found using the OpenCV methods `cvGoodFeaturesToTrack` and `cvFindCornerSubPix`, described in [3]. The parameters on these methods must be tuned as well.

The `cvFindHomography` method then uses correspondences in those points to compute an image-wide homography that is then used to construct the mosaic.

The code is given in B and the information about adjusting parameters in all the functions is given in C.

3 Mosaicing

Mosaicing is the process of stitching images together to form a larger image. The homographies that were calculated between consecutive images are applied to each of the images in turn on a black canvas, overlapping the images in the same frame. The method `drawMosaic3` from the DOLBA project code was used for the image mosaicing step.

4 Detecting a Closed Loop

4.1 Overview

The general idea is to detect the presence of a closed loop in an image sequence using information from the images alone, that is not very computationally intensive. To mosaic images, the homography between them must be computed, so finding a way to use the already-computed homography matrices to determine the presence of a closed loop is a good use of information that is already computed.

4.2 Method

Based on the factorization of a homography discussed in section 2.2, various components of a homography matrix can be factored out and used in an algorithm to detect a closed loop. There are a number of approaches you can take, all of which I will describe here.

1. Submatrix of homography matrix

According to the factorization described in section 2.2, the subvector in the top right corner of the homography matrix (entries (1,3) and (2,3)) represents $-KRt$ and could be used as an approximation to the translation.

2. Cumulative translation vector

The general idea is this: when you factor each homography matrix $H_{i,i+1}$ as you compute it between two consecutive images, you compute the translation change $t_{i,i+1}$. Keep track of a cumulative translation vector $t_{1,k}$ by

adding each of the previously-computed translation vectors. The translation vector computed is the “general translation” for that particular homography. It does not necessarily represent the translation of the center of the image - it represents the “average” (for some sense of the word average) translation between the image and the previous image. If the translation component of each pixel in the image were to be computed, the translation vector would represent the average of all these vectors.

To compute the translation vector \mathbf{t} use the following steps:

- First compute $-KR\mathbf{t}$ by taking the (1,3) and (2,3) entries in the $H_{i,i+1}$ matrix.
- Next compute KR by taking the 2 by 2 submatrix of $H_{i,i+1}$ that starts in the top left corner.
- Compute the inverse of KR .
- Using this vector and matrix, $\mathbf{t} = -(KR)^{-1}(-KR\mathbf{t})$.

To see the code excerpt for computing \mathbf{t} , refer to Appendix D.

The translation for the first image is (0,0) (since the homography for the first image is the identity matrix), so the other translation vectors can be thought of as the new position of the center of the image. Really it is the new position of the center of mass of the image, but thinking of it as the translation of the center is a good approximation.

3. Translation vector from cumulative homography

Instead of factoring the homography matrices between images i and $i + 1$, use the computed cumulative homography matrix in the same way (factoring) to extract the translation estimation from the cumulative homography, instead of needing to keep track of the cumulative translation. Much like in the previous option, factor \mathbf{t} from the submatrices of the cumulative homography matrix.

4.3 Closed loop metric

Regardless of which method is used to compute the “translation”, the method and metric for determining whether there is a closed loop in the image is the same.

To determine whether there is a closed loop in the image, use the translation vector approximation determined using one of the techniques described above. If the new center of the image falls within the maximally-inscribed ellipse inscribed in the first image, then it is possible that enough of the second image overlaps with the first image to compute a new homography. That is, if

$$\mathbf{t} = \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

And if

$$\frac{t_x^2}{\left(\frac{\text{imgWidth}}{2}\right)^2} + \frac{t_y^2}{\left(\frac{\text{imgHeight}}{2}\right)^2} \leq 1$$

Then image $i + 1$ overlaps with the first image in the sequence.

4.3.1 Problems and a correction factor

There are some problems with this approach of attempting to determine the presence of a closed loop from the computed image homographies.

The problem that this approach is initially meant to solve makes the estimation of the true translation vector difficult, mainly because the homography computations inherently are error-prone. The summation of translation vectors from image to image just compounds the error present in each of the individual homography computations.

This means that the estimation of the center of the image from the cumulative translation vector is not completely accurate, so there needs to be some kind of correction factor in the elliptical equation to account for the drift in the homographies. So the actual equation used to determine whether there is an overlap between the current image and the first image is

$$\frac{t_x^2}{\left(\frac{\text{imgWidth} \cdot \text{scaleFactor}}{2}\right)^2} + \frac{t_y^2}{\left(\frac{\text{imgHeight} \cdot \text{scaleFactor}}{2}\right)^2} \leq 1$$

Using a scale factor of 1.5 yields good results, yielding many false positives. However, having strict conditions to enter the optimization phase alleviates the need to minimize false positives in closed loop detection. In fact, the more potential closed loops that are detected, the more opportunity for optimization and adjustment. So if an image is prematurely detected as overlapping with the original image and ends up satisfying the criteria for entering the optimization phase, then the mosaic will be adjusted earlier.

4.4 Translation tracking examples

The following are two examples of tracking the centers of the images using the algorithms detected above.

In this example, there are 49 images in this closed loop sequence.

From these three images it is evident that any of these three methods can be possible valid ways of estimating the translation vector from the homography matrices. Determining the most effective method for determining closed loops would require more test data sets and running many tests, comparing against a standard metric.

4.5 Another closed-loop estimation possibility

A potential possibility for determining the presence of a closed-loop is to use the same overlap metric as described in Section 4.3 but using a different technique altogether for estimating the translation component of the images.

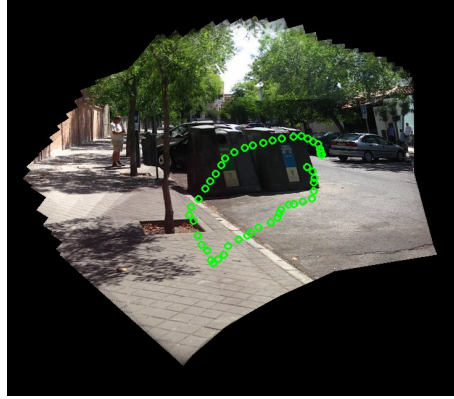


Figure 2: Computed using the submatrix of the cumulative homography matrix



Figure 3: Computed using the cumulative translation vector

In this technique, instead of using homography decomposition and factoring, the idea would be to use computation to estimate the positions of the centers of the images using direct transformation. The center of the first image would be without loss of generality assumed to be at $(0, 0, 1)$ (because all vectors are in 3D space). When the cumulative homography for image k ($H_{1,k}$) is calculated, it will be multiplied to this center vector to determine the position of the new center. However, this technique is actually the same as using the submatrix from the cumulative homography matrix as described in Section 4.2, because multiplying the homography matrix by $(0, 0, 1)$ will just yield the $(1, 3)$ and $(2, 3)$ entries of the matrix. So there is no need to try this method as a way of estimating image translation vectors.



Figure 4: Computed using the translation vector from the cumulative homography

5 Optimization Problem Formulation

This is the problem formulation for the optimization problem in closed-loop homographies.

There is a closed loop detected between image 1 and image n . Each homography computed by the mosaicing algorithm, $H_{i,i+1}$, is the homography from image i to the next image $i+1$, computed using OpenCV (and RANSAC). Once the closed loop is detected, the homography $H_{1,n}$ is computed between image 1 and the overlap image n .

There are two possible formulations of the optimization that I will discuss. I have tried both of them to varied success, as I will also describe with each section.

5.1 Objective Function Minimizing the Cumulative Error

Overall, the goal of the optimization algorithm is to minimize the error between

$$H_{1,2} \cdot H_{2,3} \cdots H_{n-2,n-1} \cdot H_{n-1,n} - H_{1,n} = 0$$

Which can be written as

$$H_{1,n}^{\text{cumulative}} - H_{1,n} = 0.$$

5.1.1 Objective function

Using a scalar formulation of the problem, the error is calculated by calculating the sum of the squares of the differences between the cumulative and new homographies for each component in the matrix.

$$\sum_{i,j} ((H_{1,n}^{\text{cumulative}})_{ij} - (H_{1,n})_{ij})^2.$$

The variables given to the optimizer are the 8 parameters in each of the matrices $H_{i,i+1}$ (each entry of the homography matrix except for the 3,3 entry, which is assumed to be 1. The values of the $H_{1,n}$ matrix are not variables in the optimization function, but are taken as truth.¹ The optimizer will yield n new homography matrices $H_{i,i+1}^{\text{optimized}}$.

5.1.2 Constraints

The optimizer tries to minimize this nonlinear multivariable function according to the following constraints:

1. The variables in each matrix can't change "too much." There are a few ways to implement this:
 - (a) The individual variables can only change within a certain range (with this range being different depending on which component in the homography it is):

$$\begin{aligned}
((H_{i,i+1})_{1,1} - (H_{i,i+1}^{\text{optimized}})_{1,1})^2 &\leq \text{change_threshold} \\
((H_{i,i+1})_{1,2} - (H_{i,i+1}^{\text{optimized}})_{1,2})^2 &\leq \text{change_threshold} \\
((H_{i,i+1})_{2,1} - (H_{i,i+1}^{\text{optimized}})_{2,1})^2 &\leq \text{change_threshold} \\
((H_{i,i+1})_{2,2} - (H_{i,i+1}^{\text{optimized}})_{2,2})^2 &\leq \text{change_threshold} \\
((H_{i,i+1})_{1,3} - (H_{i,i+1}^{\text{optimized}})_{1,3})^2 &\leq \text{pixel_threshold_x} \\
((H_{i,i+1})_{2,3} - (H_{i,i+1}^{\text{optimized}})_{2,3})^2 &\leq \text{pixel_threshold_y} \\
((H_{i,i+1})_{3,1} - (H_{i,i+1}^{\text{optimized}})_{3,1})^2 &\leq \text{small_threshold} \\
((H_{i,i+1})_{3,2} - (H_{i,i+1}^{\text{optimized}})_{3,2})^2 &\leq \text{small_threshold}
\end{aligned}$$

The reason each entry (or group of entries) should have it's own threshold is that each component of the homography matrix is related to a different transformation and has different similarity tolerances.

- (b) The total change in all the variables of a particular matrix can't exceed a certain value.

$$\sum_{i,j} |(H_{k,k+1})_{i,j}| \leq \text{sum_thresh}$$

I don't like this method very much because the components more related to the translation can change more than the components related to rotation (and sometimes will) but this method will not take that into effect.

¹It is possible to use the components of the closed-loop matrix as variables too, but for now I've implemented the code to consider it as truth. I will try using those components as variables as well.

- (c) Factor the new homography matrix and allow not “too much” of a change from the new translation and rotation components. The drawback of this is that it is much more computationally intensive and most likely will not be able to be used for real-time applications further down the line. The factorization would be done as described above in Section 2.2. However, it is probably more accurate in terms of potential thresholds of change for the individual matrix components, since it will be related to the actual components instead of the components in the matrix.
- (d) Don’t care about changes between each of the individual homographies but care about the changes in all the cumulative homographies. This is more computationally intensive for the optimizer, since it has to do many many matrix multiplications with every iteration of the optimizing function.

Right now my code uses the 1a metric of “too much change.”

2. The new matrices that are computed must be homography matrices (i.e. still have determinant close to 1).

$$(\det(H_{i,i+1}) - 1)^2 \leq \text{determinant_threshold} \quad \forall i \in (1, n - 1)$$

3. The 3, 3 entry of all the intermediate homographies computed with $H^{\text{optimized}}$ cannot be very different from 1.

It is possible that this condition becomes obsolete if the “too much change” condition is already instituted.

$$\begin{aligned} \text{Let } H_{1,k}^{\text{optimized}} &= H_{1,2}^{\text{optimized}} \dots H_{k-1,k}^{\text{optimized}} \\ ((H_{1,k}^{\text{optimized}})_{3,3} - 1)^2 &\leq \text{entry33_threshold} \quad \forall k \in (2, n) \end{aligned}$$

It is possible that this condition becomes obsolete if the “too much change” condition is already instituted.

5.1.3 Results

The original mosaic, consisting of 16 images, is shown here:

Trying the minimization function with a variety of different constraint thresholds yielded mixed results, none of which were better than the original mosaic.

The mosaic that performed the best was the optimization problem using only constraint 1a with the following parameters:

- `change_threshold` = .10 (used for components (1, 1), (1, 2), (2, 1), and (2, 2), related to the rotation.)
- `small_threshold` = .001 (used for components (3, 1) and (3, 2), related to the dilation.)



Figure 5: The original mosaic without optimization

- `pixel_threshold_x = 300` (used for component (1,2), related to the x-coordinate of translation.)
- `pixel_threshold_y = 225` (used for component (1,3), related to the y-coordinate of translation.)

The result was this mosaic:

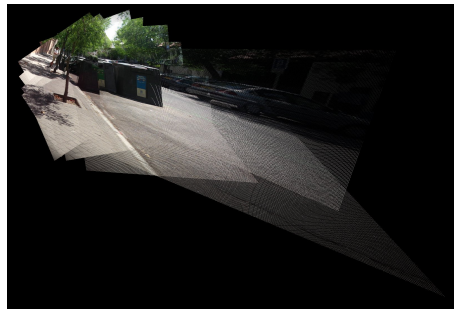


Figure 6: One example of optimized mosaic

5.2 Objective Function Minimizing Error in Individual Homographies

The goal of this optimization algorithm is to minimize the error between the initial and optimized homography matrices. That is, the error between $H_{i,i+1}$ and $H_{i,i+1}^{\text{optimized}}$ is minimized.

5.2.1 Objective Function

Using the scalar formulation of the problem, the objective function is calculated by taking the sum of square difference between the initial homography estimates and the optimized homography estimates.

$$\sum_i \sum_{j,k} ((H_{i,i+1})_{j,k} - (H_{i,i+1}^{\text{optimized}})_{j,k})^2$$

The variables given to the optimizer are the 8 parameters in each of the matrices $H_{i,i+1}$. The values of the $H_{1,n}$ matrix are used as initial estimates. The optimizer will yield n new homography matrices $H_{i,i+1}^{\text{optimized}}$.

5.2.2 Constraints

The optimizer tries to minimize the objective function according to the following constraints:

1. The sum of squared differences between the cumulative homography calculated using the optimized homographies is the same as the closed-loop homography $H_{1,n}$ is zero.

$$\sum_{j,k} ((H_{1,n})_{j,k} - (H_{1,n}^{\text{optimized}})_{j,k})^2$$

The potential problem with this constraint is that it might be too strict - it says that there must be no difference between the closed-loop homography and the new optimized cumulative homography. Another possibility is to instead of having the error be zero, allowing the error to be below a threshold.

5.2.3 Results

The constraints with this formulation of the problem yield no feasible solutions.

6 Discussion

The most immediate result that can be seen from this work is a new closed-loop detection scheme. I have proposed three main ways of estimating translation vectors using the factorization of the homographies, but a large-scale test with

more datasets needs to be conducted to determine which estimation is more accurate.

7 Potential Further Work

While the objective of the work here is to use closed-loop homographies to better create image mosaics, closed-loop techniques are not the only possible solution.

For example, the use of masks to determine which features to track during the optical flow/new homography calculation metric may yield better results than using the entire image. Using only a subset of the image for interest points will decrease the confounding of the homography matrix from the distortion effects at the edge of the field of vision.

Additionally, another potential solution is to break the image into sub-images, and compute homographies between sections of the image to other sections of the previous image. This also gets rid of distortion effects by tiling the images according to the most appropriate homography estimation.

A problem with this entire project is the lack of metrics for determining the accuracy of an image mosaic. To better be able to write automated tests of both various optimization parameters and closed-loop detection methods,

8 Conclusion

Right now, the highlight of this work is the new detection of a closed loop in a sequence of images. The optimization function is not yielding good results and still needs to be improved.

References

- [1] M. Sonka, V. Hlavac, and R. Boyle, *Image Processing, Analysis, and Machine Vision*, 3rd ed., Thomson, USA, 2008.
- [2] http://opencv.willowgarage.com/documentation/camera_calibration_and_3d_reconstruction.html
- [3] http://opencv.willowgarage.com/documentation/c/feature_detection.html
- [4] http://opencv.willowgarage.com/documentation/c/video_motion_analysis_and_object_tracking.html

A All code

All of the code I've worked on (python scripts, MATLAB code, C++/OpenCV) are all on my github account: <https://github.com/mprat/closedloophomographies>.

B Homography Calculation Code

The following is code to compute the features to track in the first image.

```
IplImage* imgfirstBW = cvCreateImage(cvSize(imgWidth, imgHeight), IPL_DEPTH_8U, 1);
CvPoint2D32f* cornersA = new CvPoint2D32f[MAX_CORNERS];
IplImage* eig_image = cvCreateImage(cvSize(imgWidth, imgHeight), IPL_DEPTH_8U, 1);
IplImage* tmp_image = cvCreateImage(cvSize(imgWidth, imgHeight), IPL_DEPTH_8U, 1);
const int MAX_CORNERS = 500;
int corner_count = MAX_CORNERS;

//not shown: saving image to imgfirstBW

//compute the features to track from the first image
cvGoodFeaturesToTrack(
    imgfirstBW, eig_image, tmp_image, cornersA,
    &corner_count, 0.01, 5.0, NULL, 3, 1, 0.04);
cvFindCornerSubPix(
    imgfirstBW,
    cornersA,
    corner_count,
    cvSize(10, 10),
    cvSize(-1, -1),
    cvTermCriteria(CV_TERMCRIT_ITER|CV_TERMCRIT_EPS, 20, 0.03));
```

This code is used to calculate the optical flow between two images and convert the computed array of points into a matrix that can be used for the homography method.

```
IplImage* imgBW = cvCreateImage(cvSize(imgWidth, imgHeight), IPL_DEPTH_8U, 1);
IplImage* pyr1 = cvCreateImage(cvSize(imgWidth, imgHeight), IPL_DEPTH_8U, 1);
IplImage* pyr2 = cvCreateImage(cvSize(imgWidth, imgHeight), IPL_DEPTH_8U, 1);
CvPoint2D32f* cornersB = new CvPoint2D32f[MAX_CORNERS];
char features_found[MAX_CORNERS];
float feature_errors[MAX_CORNERS];
CvMat* PointImg1;
CvMat* PointImg2;

//not shown: saving the image to imgBW

//compute homography with first image and the new image
cvCalcOpticalFlowPyrLK(
    imgfirstBW,
    imgBW,
    pyr1,
    pyr2,
    cornersA,
```

```

        cornersB,
        corner_count,
        cvSize(30, 30),
        3,
        features_found,
        feature_errors,
        cvTermCriteria(CV_TERMCRIT_ITER | CV_TERMCRIT_EPS, 20, 0.03),
        0
    );

    countfound = 0;

    //count how many matched features you get from the optical flow
    for (int i = 0; i < corner_count; i++)
    {
        if (features_found[i] == 0 || feature_errors[i] > point_num_limit) {continue; }
        countfound++;
    }
    if (countfound <= 0) {countfound = 1;}
    PointImg1 = cvCreateMat(countfound, 2, CV_32F);
    PointImg2 = cvCreateMat(countfound, 2, CV_32F);
    countfound = 0;
    matched_features = 0;
    for (int i = 0; i < corner_count; i++)
    {
        if (features_found[i] == 0 || feature_errors[i] > point_num_limit) {continue; }
        CvPoint p0 = cvPoint(cvRound(cornersA[i].x), cvRound(cornersA[i].y));
        CvPoint p1 = cvPoint(cvRound(cornersB[i].x), cvRound(cornersB[i].y));
        cvmSet(PointImg1, countfound, 0, p0.x);
        cvmSet(PointImg1, countfound, 1, p0.y);
        cvmSet(PointImg2, countfound, 0, p1.x);
        cvmSet(PointImg2, countfound, 1, p1.y);
        countfound++;

        matched_features++;
    }
}

```

The output of the above code is then used to calculate the homography between the images:

```

CvMat* H = cvCreateMat(3, 3, CV_32FC1);
// get a singular homography if less than 5 points, but use
// 10 to guarantee that you can at least find 10 of them in
// the next image
if (matched_features >= 10) {
    cvFindHomography(PointImg2, PointImg1, H, CV_RANSAC, 1.0, NULL);
}

```

```

    }
    else{
        cvSetIdentity(H);
    }

```

After this code is executed, H contains the homography between the two images.

C Parameters in OpenCV Methods

The various parameters tuned for the OpenCV methods are as follows (written in order as they appear on [2, 3]):

- **cvGoodFeaturesToTrack**: the goal of this function is to detect interest points in the first image

const CvArr* image black and white image you're calculating the homography *from*

CvArr* eigImage temporary image the same size as **image**

CvArr* tempImage another temporary image the same size as **image**

CvPoint2D32f* corners an array of **CvPoint2D32f** where the location of corner points will be stored

int cornerCount variable where the corner count will be stored

double qualityLevel set to 0.01 in the code; denotes the minimum quality parameter for accepting points in the corner count. Setting this higher means there will be more interest points and setting this lower means there will be fewer interest points.

double minDistance set to 5.0 in the code; specifies the minimum distance between interest points. Setting this lower means there is the possibility for interest points to be clustered together more.

const CvArr* mask = NULL set to NULL in the code; denotes the mask used to determine interest points in the image. I experimented with using different masks (i.e. the whole image, not including a 50-pixel border around the edge of the image) but it didn't yield good results. I did no formal tests of this with the entire pipeline, so there needs to be more thought put into the use of masks. See 7 for more details.

int blockSize = 3 set to 3 in the code (the default in OpenCV); I did not play with this parameter, so I'm not sure on the effect it has on the detected features.

int useHarris=0 set to 1 in the code, so the function uses the Harris detector to detect corners in the image.

double k = 0.04 set to 0.04, the default value; Used by the Harris detector and I did not adjust this parameter.

- `cvFindCornerSubPix`: the goal of this function is to improve on estimates from the Harris detector in determining the location of interest points in the first image

`const CvArr* image` the first image in black and white (the same image the Harris detector was run)

`CvPoint2D32f* corners` the same array of `CvPoint2D32f` that was passed to `cvGoodFeaturesToTrack` that contains the locations of the detected corners. At the end of the method will contain the updated positions of the interest points.

`int count` the same `cornerCount` variable that was returned by the `cvGoodFeaturesToTrack` method for the number of detected corners

`CvSize win` set to `cvSize(10, 10)` in the code; this defines the search space for the coordinate-refining in the method. I did not try different values of this parameter.

`CvSize zero_zone` set to `cvSize(-1, -1)`; a parameter used to calibrate false negatives, but setting it to `(-1, -1)` does not try to exclude singularities. I did not try different values of this parameter.

`CvTermCriteria criteria` set to `cvTermCriteria(CV_TERMCRIT_ITER|CV_TERMCRIT_EPS, 20, 0.03)` as was set for another application and I did not try different values for this parameter; this specifies termination criteria for the algorithm

- `cvCalcOpticalFlowPyrLK`: this function calculates the optical flow using the Lucas-Kanade method with pyramids between two images

`const CvArr* prev` the image at time t in black and white

`const CvArr* curr` the image at time $t + dt$ in black and white

`const CvArr* prevPyr` buffer pyramid for the first image. Initialized to the same size as the `prev` image.

`const CvArr* currPyr` same as `prevPyr` but for the second image

`const CvPoint2D32f* prevFeatures` the returned `corners` variable returned by the `cvFindCornerSubPix` method; these are the interest points for which the flow will be calculated in the second image

`const CvPoint2D32f* currFeatures` an array of `CvPoint2D32f` types of the same size as `prevFeatures`; will contain the positions of the new points at the end of the method

`int count` the same variable as the output `cornerCount` variable from `cvFindCornerSubPix` that contains the number of feature points

`CvSize winSize` set to `cvSize(30, 30)`; defines the size of the window in which to look for the new position of the interest point. Setting this higher will give the ability to potentially find optical flow between two images that are less overlapping, but will also lead to more errors. I tried different values of the window size, but not against any formal tests.

`int level` set to 3; the number of pyramid layers to use. I did not adjust this parameter.

`char* status` an empty array needs to be passed of the size of the maximum number of features possibly found; keeps track of which features were found in the second image

`float* track_error` an empty array needs to be passed the size of the maximum number of features possibly found; keeps track of the error in detection.

`CvTermCriteria criteria` set to `cvTermCriteria(CV_TERMCRIT_ITER | CV_TERMCRIT_EPS, 20, 0.03)`. I did not try different parameters.

`int flags` set to 0.

- `cvFindHomography`: Determines the homography matrix between two images and normalizes it to scale so that the 3,3 entry is 1.

`const CvMat* srcPoints` The concatenated matrix of points output from `prevFeatures` output from `cvCalcOpticalFlowPyrLK`. The code for transforming from `prevFeatures` to `srcPoints` is given in B.

`const CvMat* dstPoints` The same as `srcPoints` except for `currFeatures`.

`CvMat* H` A 3 by 3 matrix that will store the calculated homography.

`int method=0` Set to `CV_RANSAC`; use the RANSAC method to calculate the homography to get rid of outliers in the detected points.

`double ransacReprojThreshold=0` Set to 1.0; used for the RANSAC threshold in the algorithm.

`CvMat* status=NULL` Set to `NULL`; I did not experiment with this parameter.

D Code for computing the translation vector

The following is the OpenCV C++ code used to compute the translation vector for a particular image in a sequence.

```

CvMat* KR = cvCreateMat(2, 2, CV_32FC1);
CvMat* KRinv = cvCreateMat(2, 2, CV_32FC1);
CvMat* nKRt = cvCreateMat(2, 1, CV_32FC1);
CvMat* tcur = cvCreateMat(2, 1, CV_32FC1);
CvMat* tcum = cvCreateMat(2, 1, CV_32FC1);

cvGetSubRect(H, KR, cvRect(0, 0, 2, 2));
cvInvert(KR, KRinv, CV_SVD);
cvGetSubRect(H, nKRt, cvRect(2, 0, 1, 2));
cvGEMM(KRinv, nKRt, -1.0, NULL, 0, tcur, 0);
cvAdd(tcur, tcum, tcum);

```

`H` is a 3 by 3 matrix holding the homography between the previous image i and the current image $i + 1$. Now `tcum` holds the cumulative translation vector to the current image $i + 1$. The vector `tcum` must be set to zero before the execution of multiple iterations of this code.