

This is the problem formulation for the optimization problem in closed-loop homographies.

There is a closed loop detected between image 1 and image  $n$ . Each homography computed by the mosaicing algorithm,  $H_{i,i+1}$ , is the homography from image  $i$  to the next image  $i + 1$ , computed using OpenCV (and RANSAC). Once the closed loop is detected, the homography  $H_{1,n}$  is computed between image 1 and the overlap image  $n$ .

Overall, the goal of the optimization algorithm is to minimize the error between

$$H_{1,2} \cdot H_{2,3} \cdots H_{n-2,n-1} \cdot H_{n-1,n} - H_{1,n} = 0$$

which can be written as

$$H_{1,n}^{\text{cumulative}} - H_{1,n} = 0.$$

Using a scalar formulation of the problem, the error is calculated by calculating the sum of the absolute values of the differences between the cumulative and new homographies for each component in the matrix.

$$\sum_{i,j} |(H_{1,n}^{\text{cumulative}})_{ij} - (H_{1,n})_{ij}|.$$

The variables given to the optimizer are the 8 parameters in each of the matrices  $H_{i,i+1}$  (each entry of the homography matrix except for the 3,3 entry, which is assumed to be 1. The values of the  $H_{1,n}$  matrix are not variables in the optimization function, but are taken as truth.<sup>1</sup> The optimizer will yield  $n$  new homography matrices  $H_{i,i+1}^{\text{optimized}}$ .

The optimizer tries to minimize this nonlinear multivariable function according to the following constraints:

1. The new matrices that are computed must be homography matrices (i.e. still have determinant close to 1).

$$|\det(H_{i,i+1}) - 1| \leq \text{determinant\_threshold} \quad \forall i \in (1, n-1)$$

2. The variables in each matrix can't change "too much." There are a few ways to implement this:

- (a) The individual variables can only change within a certain range (with this range being different depending on which component in the homography it is):

$$|(H_{i,i+1})_{1,1} - (H_{i,i+1}^{\text{optimized}})_{1,1}| \leq \text{change\_threshold}$$

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<sup>1</sup>It is possible to use the components of the closed-loop matrix as variables too, but for now I've implemented the code to consider it as truth. I will try using those components as variables as well.

$$\begin{aligned}
|(H_{i,i+1})_{1,2} - (H_{i,i+1}^{\text{optimized}})_{1,2}| &\leq \text{change\_threshold} \\
|(H_{i,i+1})_{2,1} - (H_{i,i+1}^{\text{optimized}})_{2,1}| &\leq \text{change\_threshold} \\
|(H_{i,i+1})_{2,2} - (H_{i,i+1}^{\text{optimized}})_{2,2}| &\leq \text{change\_threshold} \\
|(H_{i,i+1})_{3,1} - (H_{i,i+1}^{\text{optimized}})_{3,1}| &\leq \text{pixel\_threshold} \\
|(H_{i,i+1})_{3,2} - (H_{i,i+1}^{\text{optimized}})_{3,2}| &\leq \text{pixel\_threshold} \\
|(H_{i,i+1})_{1,3} - (H_{i,i+1}^{\text{optimized}})_{1,3}| &\leq \text{small\_threshold} \\
|(H_{i,i+1})_{2,3} - (H_{i,i+1}^{\text{optimized}})_{2,3}| &\leq \text{small\_threshold}
\end{aligned}$$

The reason each entry (or group of entries) should have it's own threshold is that each component of the homography matrix is related to a different transformation and has different similarity tolerances.

- (b) The total change in all the variables of a particular matrix can't exceed a certain value.

$$|\sum_{i,j} (H_{k,k+1})_{i,j}| \leq \text{sum\_thresh}$$

I don't like this way because it does not account for two very large changes in two variables - i.e. if component (1,1) changes by -100 and component (3,3) changes by +99, it seems that there wasn't that much change in total whereas in reality the new homography matrix is very different from the old homography matrix.

- (c) Factor the new homography matrix and allow not "too much" of a change from the new translation and rotation components.
- (d) Don't care about changes between each of the individual homographies but care about the changes in all the cumulative homographies.

Right now my code uses the 2a metric of "too much change."

3. The 3,3 entry of all the intermediate homographies computed with  $H^{\text{optimized}}$  cannot be very different from 1.

$$\begin{aligned}
\text{Let } H_{1,k}^{\text{optimized}} &= H_{1,2}^{\text{optimized}} \dots H_{k-1,k}^{\text{optimized}} \\
|(H_{1,k}^{\text{optimized}})_{3,3} - 1| &\leq \text{entry33\_threshold } \forall k \in (2, n)
\end{aligned}$$