
OPTIMIZING MIDDLE-MILE DELIVERY

Minimizing Cost Through Effective Shipping Strategies for Single Shipment

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March 2024

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1. Introduction

1.1. Research topic introduction and motivation

The landscape of global commerce is increasingly defined by the complexity and expansiveness of supply chain operations. Middle-mile delivery encompasses the transportation of goods from distribution centers or warehouses to regional hubs or fulfillment centers. It serves as the indispensable bridge between the initial production of goods and their eventual arrival at retail locations or last-mile delivery points, directly impacting the efficiency with which consumers receive products. With the relentless push towards faster delivery times, the resilience of supply chains, and the need to maintain or reduce operational costs, optimizing middle-mile delivery become not just a logistical necessity but a strategy imperative. This segment of the supply chain is rife with challenges, including but not limited to, fluctuating fuel costs, labor shortages, vehicular maintenance, and the complexities of fleet and inventory management. These challenges underscore the need for innovative strategies that can enhance efficiency and reliability while curtailing operational expenditures.

The motivation for this paper origins from the interest in addressing the inefficiencies in middle-mile delivery. There is a pressing need to explore and implement strategies capable of minimizing costs without compromising on the quality of service. This research is driven by the hypothesis that through shipment optimization, it is possible to achieve significant cost reductions. The exploration of this topic is timely and relevant, given the increasing pressure on supply chains to be more agile, resilient, and cost-effective. With the global economy still rebounding from the disruptions caused by recent global events, optimizing middle-mile delivery offers a pathway to build back better. By focusing on this often-neglected segment of the supply chain, this research aims to contribute valuable insights into how logistical operations can be reimaged and restructured for enhanced performance. The goal is to move beyond the conventional approaches to middle-mile delivery, paving the way for innovative solutions that can adapt to the changing dynamics of global trade and consumption patterns.

1.2. Methodology

To navigate the complexities of middle-mile delivery, we employ Integer Linear Programming models tailored for both Naïve and Consolidation methods. These models are developed to optimize the allocation and scheduling of shipments, ensuring maximum efficiency and cost-effectiveness. In addition, this paper also uses heuristic approach to optimize the use of truck space, aiming to approach the maximum capacity per truck. It prioritizes the consolidation of the largest possible orders into a single shipment, continuing until the addition of another order would surpass the truck's capacity threshold. Moreover, any portions of orders that are divided in the process are treated as

separate, individual orders for subsequent consolidation opportunities. This methodical approach facilitates the strategic assembly of shipments to maximize transportation efficiency.

Furthermore, an important aspect of the methodology is the performance evaluation. The objective is to identify which model gives optimal results under the constraints and variables provided by the provided dataset. This evaluation focuses on the total shipping cost incurred by each model, providing a quantitative basis for comparison.

2. Model construction

2.1. Problem description

Company A located in Indiana needs to ship its products to a local fulfilment center in Kentucky. The challenge lies in determining the most cost-effective method to transport these goods, considering the options: Naïve or Consolidation methods. Naïve method involves shipping each order individually using the most cost-effective mode (LTL for orders within capacity, and TL for larger orders). Consolidation method consolidates multiple order into fewer shipments to take advantages of TL's higher capacity. The decision hinges on how to optimally merge orders to either use TL, with a maximum capacity of 30,000 lbs., or LTL with a maximum capacity of 10,000 lbs.

The objective of this problem is to minimize shipping costs while adhering to the capacity constraints of TL and LTL shipping options. By evaluating the total shipping costs resulting from both methods, the company aims to identify the optimal shipping strategy that reduces expenses while fulfilling all orders efficiently. This problem draws inspiration from a case study by Opex Analytics, which, in turn, was based on experiences from their client [1]. In the case study, it discusses both Naïve and Consolidation methods, making this paper an exploration of how these approaches can be applied to optimize logistics and supply chain management for middle-mile delivery.

2.2. Models and Algorithms

2.2.1. Naïve method

For the Naïve method, where each order is shipped using the most cost-effective method while considering the capacity constraints, the objective function is defined:

Objective function

Minimize the total shipping cost by selecting the most cost-effective shipping method (TL or LTL) for each order. Let x_i be the decision variable indicating the fraction of order i shipped via TL, and y_i be the fraction of order i shipped via LTL. The objective function is:

$$\text{Minimize} \quad \sum_{i=1}^n (\text{Cost}_{\text{TL}} \cdot x_i + \text{Cost}_{\text{LTL}} \cdot y_i)$$

where Cost_{TL} and Cost_{LTL} are the costs of shipping a unit of product via TL and LTL, respectively, and n is the total number of orders.

Constraints

Capacity constraint

For each order i , the weight of the order shipped via TL ($W_{\text{TL},i}$) and LTL ($W_{\text{LTL},i}$) must not exceed the respective capacities:

$$\begin{aligned} x_i \cdot W_i &\leq \text{Capacity}_{\text{TL}} \\ y_i \cdot W_i &\leq \text{Capacity}_{\text{LTL}} \end{aligned}$$

where W_i is the weight of order i , $\text{Capacity}_{\text{TL}} = 30000$ lbs, and $\text{Capacity}_{\text{LTL}} = 10000$ lbs.

Order fulfilment constraint

The sum of fractions of order i shipped via TL and LTL must equal 1 to ensure the entire order is shipped:

$$x_i + y_i = 1 \quad \forall i$$

Non-negativity and integrality constraint

The fractions of order i shipped via TL and LTL must be non-negative and, in this scenario, should satisfy integrality since an order cannot be partially shipped via TL or LTL:

$$x_i, y_i \geq 0 \quad \text{and integer} \quad \forall i$$

2.2.2. Consolidation method

For the consolidation model, where focuses on using truckload shipping with the objective of minimizing the shipping cost by optimally consolidating orders. The preference for the TL method stems from its fixed cost structure, applicable across all

orders, contrasting with the LTL method's variable costs, which differ per order and are subject to variability influenced by various factors. The model looks like this:

Objective function:

The objective is to minimize the total shipping cost by efficiently consolidating orders and utilizing TL's capacity. Let's denote z_{ij} as a binary decision variable that equals 1 if order i is shipped with order j in the same TL shipment, and 0 otherwise. The objective function can be represented as:

$$\text{Minimize} \quad \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n \text{Cost}_{\text{TL}} \cdot z_{ij}$$

where Cost_{TL} is the cost of shipping a TL (regardless of its weight up to the maximum capacity), and n is the total number of orders.

Constraints

Truckload capacity constraint

The total weight of any group of orders consolidated into a TL shipment must not exceed the TL capacity:

$$\sum_{i=1}^n W_i \cdot z_{ij} \leq \text{Capacity}_{\text{TL}} \quad \forall j$$

where W_i is the weight of order i , and $\text{Capacity}_{\text{TL}} = 30000$ lbs.

Same product category constraint

Orders can only be consolidated with other orders of the same product category:

$$z_{ij} = 0 \quad \text{if } \text{Category}_i \neq \text{Category}_j$$

Fulfilment constraint

Each order must be shipped, either individually or as part of a consolidated shipment. This constraint ensures that every order is considered for shipment:

$$\sum_{\substack{j=1 \\ j \neq i}}^n z_{ij} \geq 1 \quad \forall i$$

Non-negativity and binary constraints

The decision variables z_{ij} must be non-negative and binary to indicate whether orders are consolidated:

$$z_{ij} \in \{0, 1\} \quad \forall i, j$$

3. Numerical Experiment

3.1. Simple numerical experiment

Order	Volume	TL cost	LTL cost
1	15000	500	120
2	20000	500	150
3	55000	500	230
4	63000	500	180
5	23000	500	210

Table 1. Sample data for simple numerical experiment

For naïve method, we only chose method with lower cost. Based on the objective function in section 2.2.1, the optimal solution is presented in the table below:

Order	Volume	TL truck (truck)	TL cost (\$)	LTL truck (truck)	TL cost (\$)
1	15000	0	0	2	240
2	20000	0	0	2	300
3	55000	2	1000	0	0
4	63000	2	1000	1	180
5	23000	1	500	0	0
Total cost: $Z = \$3,320$					

Table 2. Optimal solution for simple numerical experiment with Naïve method

For Consolidation method, we considered TL only and merged order to take use of maximum capacity of TL. We can merge order 1, 2 and 3 to ship within 3 truckload trucks and merge order 4 and 5 to ship within 3 other truckload trucks. Total cost when shipping all order with 6 truckload truck is \$ 3,000. When comparing results of Naïve and Consolidation methods, number of trucks in Naïve method is 10 in which there are 5 truckload trucks and 5 less-than-truckload truck. Its total cost for optimal solution is \$3,220 which is more expensive than total cost of consolidation method. Nevertheless, this straightforward numerical experiment does not encapsulate the entire spectrum of possibilities and scenarios. It serves as a preliminary analysis rather than a comprehensive evaluation of the methods' efficacy across diverse operational contexts.

3.2. Real-world numerical experiment

3.2.1. Data description

The dataset has 7,068 records detailing orders made by company A spanning from 2006 to 2015. It includes information about the product category, order volume (lbs.), number of items, truckload cost, and less than truckload cost for each order. The truckload cost remains constant for all order because the destination is fixed in Kentucky, while the less than truckload cost varies across different product categories and other factors in shipment that are not mentioned in the dataset.

order_id	Unique identifier for each order
warehouse_date	The date when the order arrived at the warehouse
delivery_due_date	The due date for shipping the order
product_group	Product categories
unit_of_measure	Number of items for each order
volume	The volume of the order
tl_cost	Total cost of truckload for each order
ltl_cos	Total cost of less than truckload for each order

Table 3. Dataset component description

The dataset comprises nine distinct product categories, with Eco-Friendly Apparel and Handcrafted Home Decor representing the majority of the entries.

	product_group	volume		unit_of_measure		ltl_cost	
		min	max	min	max	min	max
0	Customized Furniture	15443	75166	335	886	322	509
1	Eco-Friendly Apparel	10839	74278	897	1887	121	237
2	Electronics Components	27025	52945	1100	1900	117	229
3	Gardening and Outdoor	1123	49960	557	973	143	299
4	Handcrafted Home Decor	12796	71751	913	1575	131	277
5	Home Cleaning Supplies	1154	49955	732	1233	110	197
6	Home appliances	1131	49991	357	989	219	312
7	Kitchenware	1024	49970	450	1122	200	276
8	Organic Packaged Food	14761	71527	1001	2096	236	455

Table 4. Summary Statistics of Product Categories

3.2.2. Data visualization

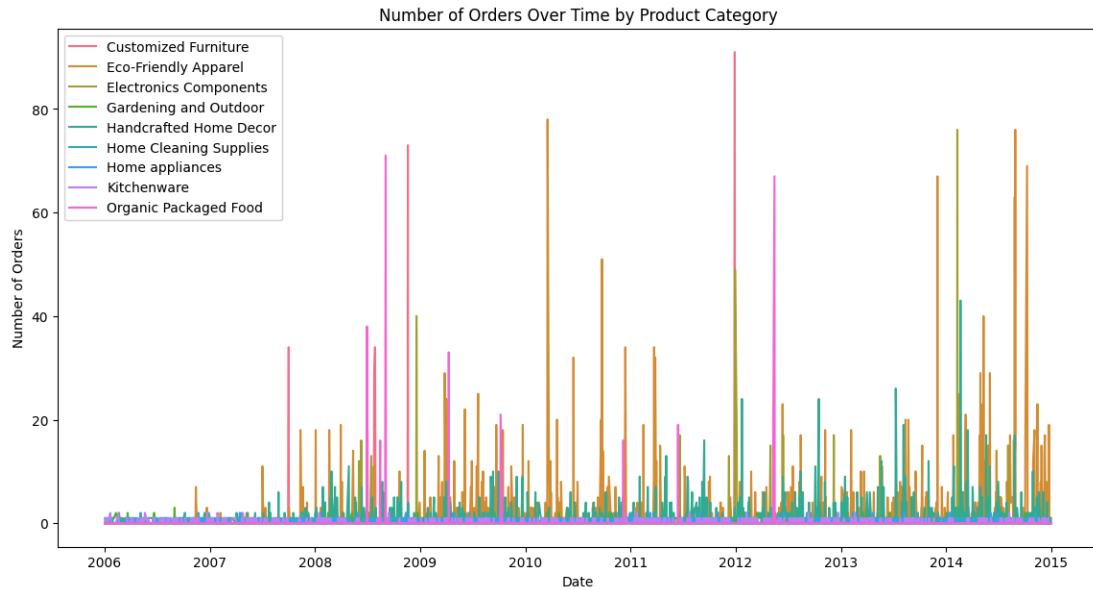


Figure 1. Number of orders overtime by product category

Figure 1 is time series analysis showing the number of orders over time, divided by product category spanning from 2006 to 2015. There is a fluctuation in the number of orders across all product categories throughout the years. Some categories show peaks that significantly stand out above the general trend, indicating periods of seasonal high demand in sales and middle-mile delivery for these product categories.

Eco-Friendly Apparel seems to have a couple of very high peaks around 2009 and subsequent years. Customized Furniture exhibits some notable peaks occasionally. From 2006 to mid-2008, all categories show a relatively steady or slightly increasing number of orders, but there isn't consistent pattern of growth or decline for any category throughout the entire time span. In addition, Electronics Components consistently shows a very low number of orders, indicating low demand for middle-mile delivery for this product category.

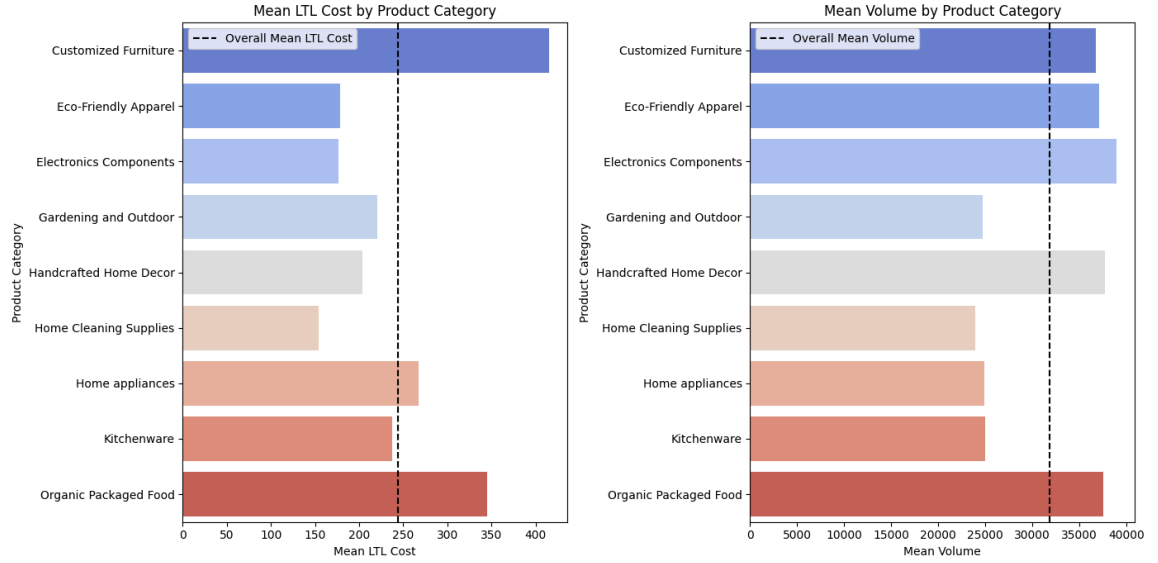


Figure 2. Mean LTL costs and order volumes by product category

While TL cost remains unchanged at \$850 for all shipments from Indiana’s warehouse to Kentucky’s local fulfilment center, the LTL costs varies significantly by category, with Customized Furniture and Organic Packaged Food having higher than average LTL costs. Electronic Components, Eco-friendly Apparel and Home Cleaning Supplies fall below the overall mean, suggesting that certain products are more expensive to transport on per-unit basis, which could be due to various factors such as product fragility, packaging need, or transportation requirements. Therefore, LTL cost is a random variable that the model has to address.

The volume graph shows Handcraft Home Decor, Organic Packaged Food and Electronic Components categories have higher mean volume than others, follow by Customized Furniture, Eco-friendly Apparel with volume higher than average volume of all orders within the dataset. This indicates that these categories may have more efficient space utilization in shipping due to product size, packaging efficiency, or order consolidation practices. For middle-mile delivery, which typically involves transportation from warehouses to distribution centers, optimizing the volume of shipments can lead to more efficient loads and reduced transportation costs. High-volume categories may benefit from strategies such as dedicated delivery routes or shipment frequency adjustments to maximize load efficiency. In addition, the volume of each order varies based on product category, therefore, it is a random variable that the model has to address.

3.2.3. Experiment Results and Interpretation

Method	No. of TL trucks	No. of LTL trucks	Total cost (\$)	Average cost (\$)
Naïve	6760	308	6,096,909	863
Eco-friendly Apparel	3059	0	2,722,510	890
Handcrafted Home Décor	1488	0	1,324,320	890
Gardening and Outdoor	379	67	355,644	797
Home Appliance	329	87	319,088	767
Kitchenware	302	79	291,111	764
Home Cleaning Supplies	287	75	268,996	743
Electronics Components	356	0	316,840	890
Organic Packaged Food	299	0	266,110	890
Customized Furniture	261	0	232,290	890

Table 5. Numerical experiment results for Naïve method

Method	No. of full shipment	No. of merged shipment	Total cost (\$)	Average cost (\$)
Consolidation	4460	3938	7,474,220	890
Eco-friendly Apparel	2106	1885	3,551,990	890
Handcrafted Home Décor	1063	894	1,741,730	890
Gardening and Outdoor	172	197	328,410	890
Home Appliance	163	183	307,940	890
Kitchenware	157	162	283,910	890
Home Cleaning Supplies	129	161	258,100	890
Electronics Components	303	161	412,960	890
Organic Packaged Food	186	226	366,680	890
Customized Furniture	181	147	291,920	890

Table 6. Numerical experiment results for Consolidation method without time constraint

Method	No. of full shipment	No. of merged shipment	Total cost (\$)	Average cost (\$)
Consolidation	3557	6740	9,968,000	890
Eco-friendly Apparel	2106	3053	4,591,510	890

Handcrafted Home Décor	1063	1483	2,265,940	890
Gardening and Outdoor	172	363	476,150	890
Home Appliance	163	336	444,110	890
Kitchenware	157	301	407,620	890
Home Cleaning Supplies	129	291	373,800	890
Electronics Components	303	355	585,620	890
Organic Packaged Food	186	298	430,760	890
Customized Furniture	181	260	392,490	890

Table 7. Numerical experiment results for Consolidation method with time constraint

Due to computational constraints, solving the model for the consolidation method proved complex with the full dataset, prompting the use of a subset for testing. Specially, data pertaining to Electronic Components was selected for this purpose. The outcomes, detailed in Table 5, reveal that the Naïve method predominantly favored the selection of TL trucks. This aligns with observations from section 3.2.2, where orders from product categories with average shipment volumes above the mean exclusively utilized TL for delivery. Conversely, categories with shipment volumes below the average displayed a mix of TL and LTL shipment. Notably, allocating shipment across both TL and LTL trucks resulted in lower average shipping costs compared to using TL trucks exclusively.

Comparing the results of 3 experiments for Naïve method, Consolidation method without time constraint and with time constraint, the following insights reveal the use of model in this optimization problem:

- The Naïve method shows the lowest total cost across all product categories compared to both the Consolidation methods with and without time constraints. This implies that, despite the lack of combined shipments, shipping each order individually or in full truckloads as soon as possible without waiting for consolidation may be more cost-effective.
- The Consolidation method with time constraints results in the highest total costs, suggesting that with the pressure of a time limit, resources may not be utilized as efficiently. There may be hidden costs or inefficiencies such as increased storage times which are not reflected in the cost per shipment.
- The time constraints in the Consolidation method seem to lead to a higher number of merged shipments compared to the Consolidation method without time constraints. This could suggest that the consolidation process without time constraints is more effective in combining orders to avoid requirements to meet shipping deadlines, potentially leading to better space utilization.

- In the Naïve method, the presence of LTL trucks indicates an attempt to optimize for smaller orders. However, given the variability of LTL costs and the simplicity of the Naïve method, it is notable that this method still results in a lower overall cost.
- The data suggests that logistics strategies should not assume consolidation will always lead to cost savings. It is crucial to consider the impact of both time constraints and the inherent costs of consolidation versus immediate shipment.

4. Conclusion and limitation

4.1. Conclusion

The analysis reveals that the Naïve shipping method, which avoids order consolidation, is unexpectedly the most cost-effective approach, challenging the conventional wisdom that consolidation always leads to cost savings. Consolidation without time constraints proved less efficient, possibly due to increased storage times and suboptimal resource utilization, while introducing time constraints improved consolidation efficiency but still didn't achieve the cost savings of the Naïve method. The higher number of merged shipments under time constraints suggests a more aggressive, yet not necessarily more cost-effective, consolidation strategy. This emphasizes the importance of assessing specific operational conditions and constraints before deciding on a logistics strategy.

4.2. Limitation

This paper acknowledges three primary limitations in its methodology and scope. Firstly, the reliance on synthesized data may not accurately capture the complexities and unpredictable variables present in real-world logistics. Therefore, while this provides a controlled environment to test the heuristic, it may limit the applicability of the findings to real-world scenarios, where data is messier and less predictable.

Secondly, the consolidation method considers the TL shipping model, owing to its fixed cost structure. However, this excludes the LTL shipping model, where costs are not fixed and are influenced by a variety of factors such as weight, distance, and freight class. In future research, a more comprehensive optimization model that incorporate LTL consolidation will need to establish a precise cost function for merged shipments. In the context of synthesized data, incorporating LTL costs could skew the comparison between the Naïve and Consolidation methods, since synthesized data may not accurately reflect the variability and complexity of real-world LTL cost structures.

For third limitation stems from the stochastic nature of random variables: LTL cost and order volume. Future research will need to include these variables as random with a 95% confidence level, potentially using stochastic gradient approximation within the objective functions for both the Naïve and Consolidation methods. This approach would allow for

the management of uncertainty and variability, leading to solutions that are robust against the unpredictable elements inherent in shipping logistics. To handle randomness, at each iteration of SGD, randomly select a subset of orders to calculate the gradient of the objective function with respect to the decision variables. Then, update the decision variables in the direction that minimally decrease the total shipping cost. Repeat the process of random sampling and gradient updates for many iterations. Over time, SGD converges to a solution that minimizes the expected total shipping cost across the distribution of possible scenarios, making the solution inherently more robust to randomness. This approach is suggested for future research to address randomness.

References

1. Opex Analytics. (2018). Why Use Optimization Techniques? A Transportation Case. Medium.
<https://medium.com/@OpexAnalytics/why-use-optimization-techniques-a-transportation-case-2951718dac32>

Appendix

1. Consolidation method when including LTL shipping model – suggestion for future research

Objective function:

Minimize the total shipping cost by considering both TL and LTL shipments.

$$\text{Minimize } \sum_{i=1}^n \sum_{j=1, j \neq i}^n (\text{Cost}_{\text{TL}} \cdot z_{ij}^{\text{TL}} + \text{Cost}_{\text{LTL}, ij} \cdot z_{ij}^{\text{LTL}}) \quad (1)$$

where Cost_{TL} is the fixed cost of shipping a TL, $\text{Cost}_{\text{LTL}, ij}$ is the variable cost of shipping order i with order j via LTL, and z_{ij}^{TL} and z_{ij}^{LTL} are binary decision variables that equal 1 if order i is shipped with order j in the same TL or LTL shipment, respectively, and 0 otherwise.

Constraints:

Truckload Capacity Constraint (TL)

The total weight of a TL shipment must not exceed the truck capacity.

$$\sum_{i=1}^n W_i \cdot z_{ij}^{\text{TL}} \leq \text{Capacity}_{\text{TL}} \quad \forall j \quad (2)$$

Less than truckload capacity constraint (LTL)

The total weight of any group of orders consolidated into an LTL shipment must not exceed the LTL capacity.

$$\sum_{i=1}^n W_i \cdot z_{ij}^{\text{LTL}} \leq \text{Capacity}_{\text{LTL}} \quad \forall j \quad (3)$$

Mixed shipment constraint

An order cannot be assigned to both TL and LTL in the same model solution.

$$z_{ij}^{\text{TL}} + z_{ij}^{\text{LTL}} \leq 1 \quad \forall i, j \quad (4)$$

Fulfilment constraint

Each order must be shipped, taking both TL and LTL into account.

$$\sum_{j=1, j \neq i}^n (z_{ij}^{\text{TL}} + z_{ij}^{\text{LTL}}) \geq 1 \quad \forall i \quad (5)$$

Non-negativity and binary constraints

The decision variables must be non-negative and binary.

$$z_{ij}^{\text{TL}}, z_{ij}^{\text{LTL}} \in \{0, 1\} \quad \forall i, j \quad (6)$$

2. Constraints when handling randomness

To incorporate the randomness of LTL costs, we define a constraint such that the expected cost does not exceed a certain limit, based on a confidence level:

$$P(\text{Cost}_{\text{LTL}} \cdot y_i \leq \text{Cost}_{\text{LTL}, \text{limit}}) \geq 0.95 \quad \forall i \quad (1)$$

where P represents the probability, and $\text{Cost}_{\text{LTL}, \text{limit}}$ is the upper cost limit for LTL that we are 95% confident the actual costs will not exceed.

Similarly, to manage the uncertainty in order volume, a constraint is included to handle the variability:

$$P(W_i \cdot y_i \leq W_{i, \text{limit}}) \geq 0.95 \quad \forall i \quad (2)$$

where $W_{i, \text{limit}}$ is the weight limit for an order that we are 95% confident the actual weight will not exceed.