W271 Lab 2: CO2 1997

Ken Trinh, Lisa Wu, Ray Cao, Sophie Yeh

Contents

0.1 (3 points) Task 0a: Introduction

If you are concerned about global warming (or question whether this is true), you may have heard about the "Keeling Curve" which is named after the scientist Charles David Kneeling. Kneeling started measuring and monitoring the accumulation of carbon dioxide (CO_2) in the atmosphere in 1958. Many scientists credit the Keeling curve with first bringing our attention to the current increase of CO_2 in the atmosphere. The one key question in people's minds is whether the CO2 increase trend observed in the past will continue and at what speed, over the next few decades. The answer to this question is critical to our policy makers and environmentalists, as the forecast CO_2 results will help them evaluate how concerned they should be and what actions to take to minimize the consequences. We will conduct the study of the CO_2 data set to answer this question. We plan to explore the data set and modeling alternatives to determine whether a reliable forecast model can be developed to forecast through the year of 2022.

0.2 (3 points) Task 1a: CO2 data

The CO2 data was measured continuously at the Mauna Loa Observatory in Hawaii from 1958 to the present day (the end of 1997). Prior to this effort, measurements of CO_2 concentrations had been taken on an ad hoc basis at a variety of locations. Keeling created a frequent and consistent measurement framework of CO_2. Keeling and his collaborators measured the incoming ocean breeze above the thermal inversion layer to minimize local contamination from volcanic vents. The data were normalized to remove any influence from local contamination. His work minimized the data noises due to measurement errors or differences.

```
## [1] 0
```

```
## [1] 1959    1
```

```
## [1] 1997   12
```

This data set is a time series data that tracks CO_2 level (measured in part per million by volume (ppmv)) on a monthly frequency.There are a total of 468 monthly observations from January 1958 to December 1997 with no missing data.

As you can see in Figure 1, the CO_2 trend has steadily increased over time (close to a linear trend line), although the annual growth rate seems to be range-bound (mostly +-5%) with no clear trend. CO_2 is a greenhouse gas, so the increasing trend has significant implications for global warming. From the histogram chart, we observed that the CO_2 levels are not normally distributed, ranging from 310 to 370 ppmv. There are no extreme outlines in this data set.

Furthermore, from the CO_2 Decomposition graph, we observed trend, season and irregular components of the data set. Aside from the trend line discussed above, we observed strong seasonality, and the remaining irregular effect.

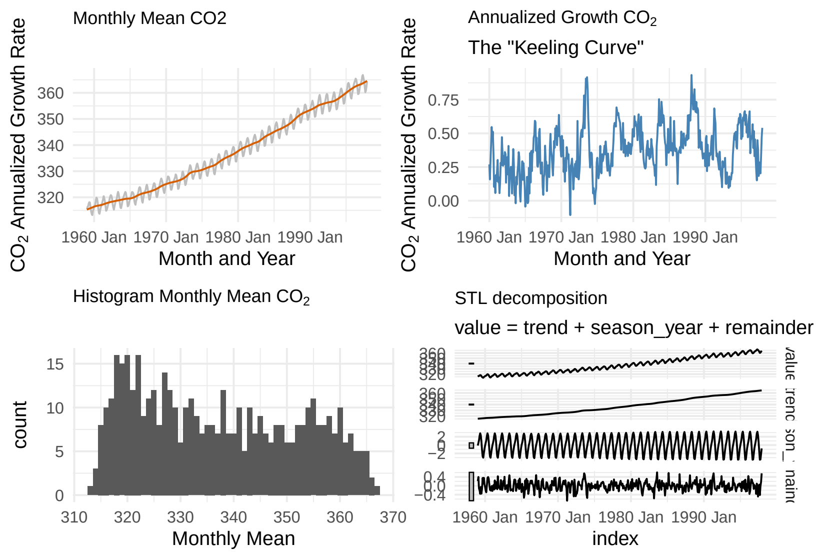


Figure 1: Atmospheric CO2 Level Time Series Overview

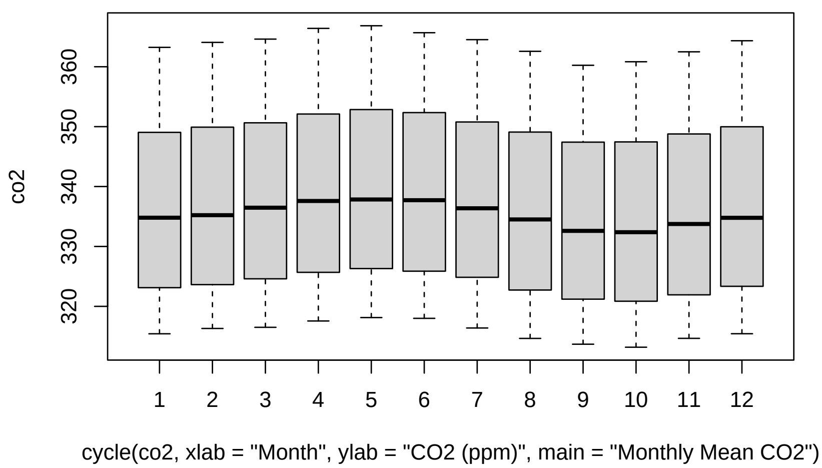


Figure 2: Seasonality CO2 Level Monthly Distribution

The boxplot in Figure 2 further validated the seasonal pattern in the Decomposition graph. We observed April-June being the peak season for CO_2, which aligns with more outdoor activities in the spring season. September-October are the low points of the year. The difference between the high and low averages is 5.5 ppmv. $(\max(\text{box}stats[3,]) - min(box\text{stats}[3,])$

0.3 (3 points) Task 2a: Linear time trend model

As discussed in the Data section above, the CO_2 time series data set follows closely to a linear trend line, so we will first explore a linear regression model for the data set. Given the annual growth rate has been range-bound (no clear sign of exponential growth) and there are extreme small or large values, we don't think it is necessary to perform a logarithmic transformation of the data. A log transformation may be more appropriate if the CO_2 level grows exponentially, which was not the case outline in both Figure 1 and Figure

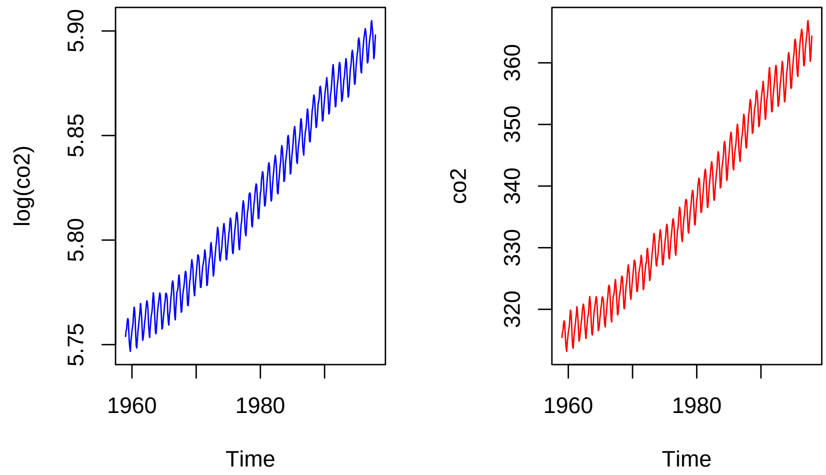


Figure 3: CO2 level regular versus logarithmic transformation

3. In these plots, the seasonal variation does not seem to change over time, and the trend does not have major curvature. Thus a logarithmic transformation is not needed for this data set.

```
# Fit a linear time trend model examine the characteristics of the residuals
mod.lm1 <- lm(co2 ~ time(co2))
plot(mod.lm1, which = c(1, 1))
```

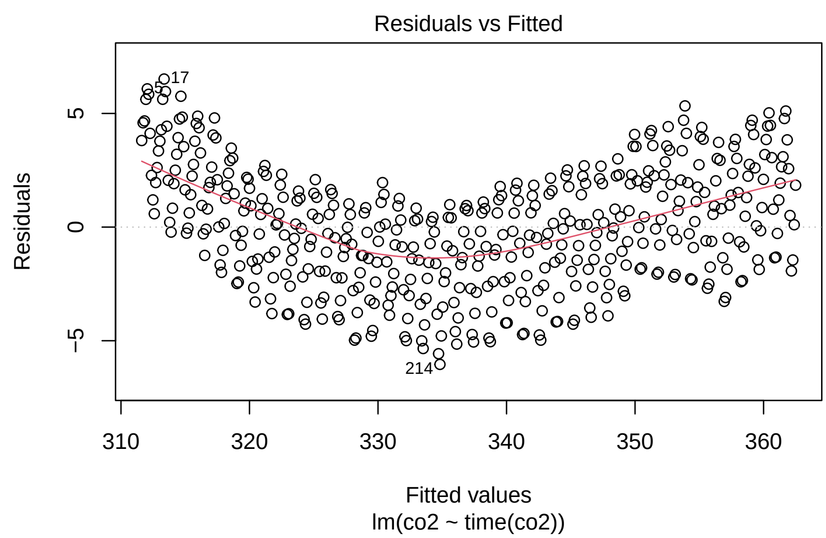


Figure 4: Linear Time Effect Model Residuals

```
# quadratic time trend model.
mod.lm2 <- lm(co2 ~ time(co2) + I(time(co2)^2))
plot(mod.lm2, which = c(1, 1))
```

First, we evaluated a linear trend model using time as the explanatory variable and CO_2 level as the response variable. Reported in Table 1 This model has an intercept of x and a positive slope of 1.3. Both coefficients are significantly different than 0 statistically, with p-value less than 0.001. In our diagnostic analysis of the model residuals Figure 4, we noted that the residuals line is curved, which violates the mean error expectation. Variance increases as the fitted values increase, which violates the homoskedasticity assumption of the classical linear model. Clearly this simple model failed to sufficiently capture the data characteristics.

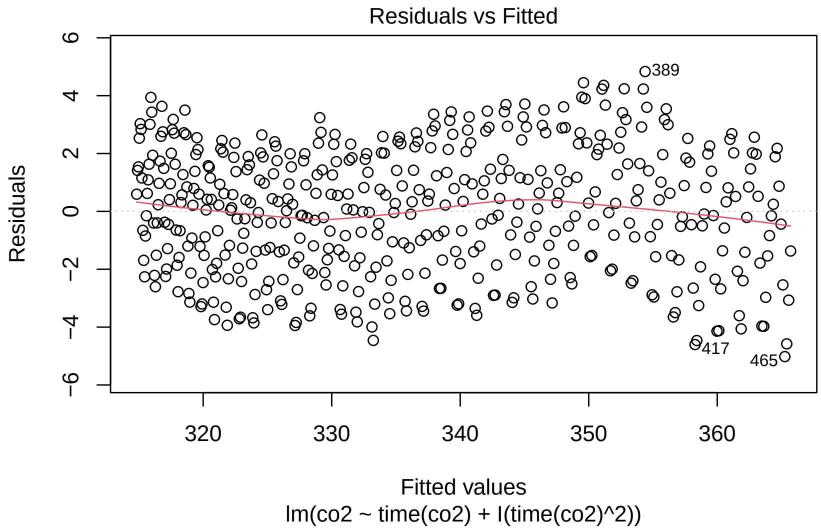


Figure 5: Quadric Time Effect Model Residuals

Table 1: Estimated Atmospheric CO2 Level

	Output Variable: CO2 Level in ppmv	
	(1)	(2)
linear time	1.307***	−49.191***
	p = 0.000	p = 0.000
quadratic time		0.013***
		p = 0.000
(Intercept)	−2,249.774***	47,702.940***
	p = 0.000	p = 0.000
Observations	468	468
R^2	0.969	0.979
Adjusted R^2	0.969	0.979
Note:		*p<0.05; **p<0.01; ***p<0.001

We then evaluated the quadratic model by adding the quadratic term of time. The coefficients are reported in Table 1 and are statistically significant. The residual plot improves with less curvature. Variance still shows some small level of heteroskedasticity in Figure 5. This model still does not adequately capture the data characteristics.

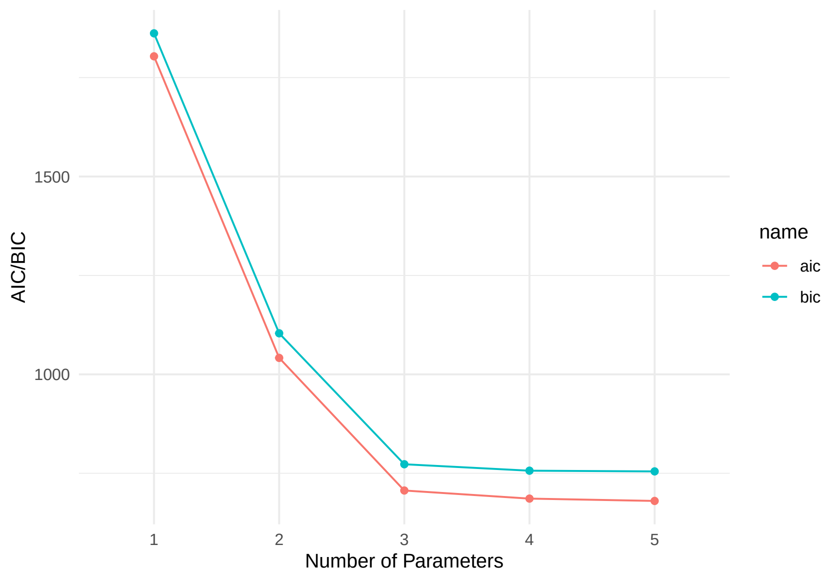


Figure 6: Polynomial trends selection using information criterions

Finally we will fit a polynomial time trend model and incorporate seasonal dummy variables. We will use the goodness-of-fit infomration criterion scores measurement to select the polynomial degree that optimizes the model fit. The three goodness-of-fit assessment measurements are AIC, AICc and BIC. Lower AIC, AICc and BIC indicates better model performance. Generally, BIC has a larger penalty for models with more parameters and therefore selects sparser models with fewer parameters compared to AIC and AICc. AICc is the preferred method and strikes the right balance of being stringent between AIC and BIC. We ran both AIC and BIC assessments and displayed the result in Figure 6.

It is noted that in our selection process we uses a range of 1 to 5 polynomial degrees for the trend variable and observed that 3 is the optimal degree with the lowest AIC and BIC score. With this, we don't recommend trying higher polynomial degrees to avoid overfitting.

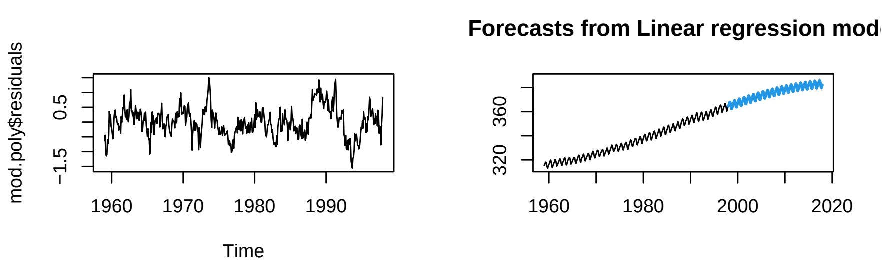


Figure 7: Polynomial Trend And Seasonality Forecast

The model results, reported in Table 2, show that all coefficients are significant, with p-value bellow 0.001. There are no variables thrown out, which validates the assumption of no perfect multicollinearity. In Figure 7, we plot both the residuals and the forecasted CO2 level for 2020. Here, the residuals in are hovering around zero, and variance does not show significant changes over time (no strong evidence of violating homoscedasticity assumption), which suggest that the estimated models is stationary.

Using fitted polynomial model to forecast the CO_2 level through 2020 (Figure 7). This polynomial time

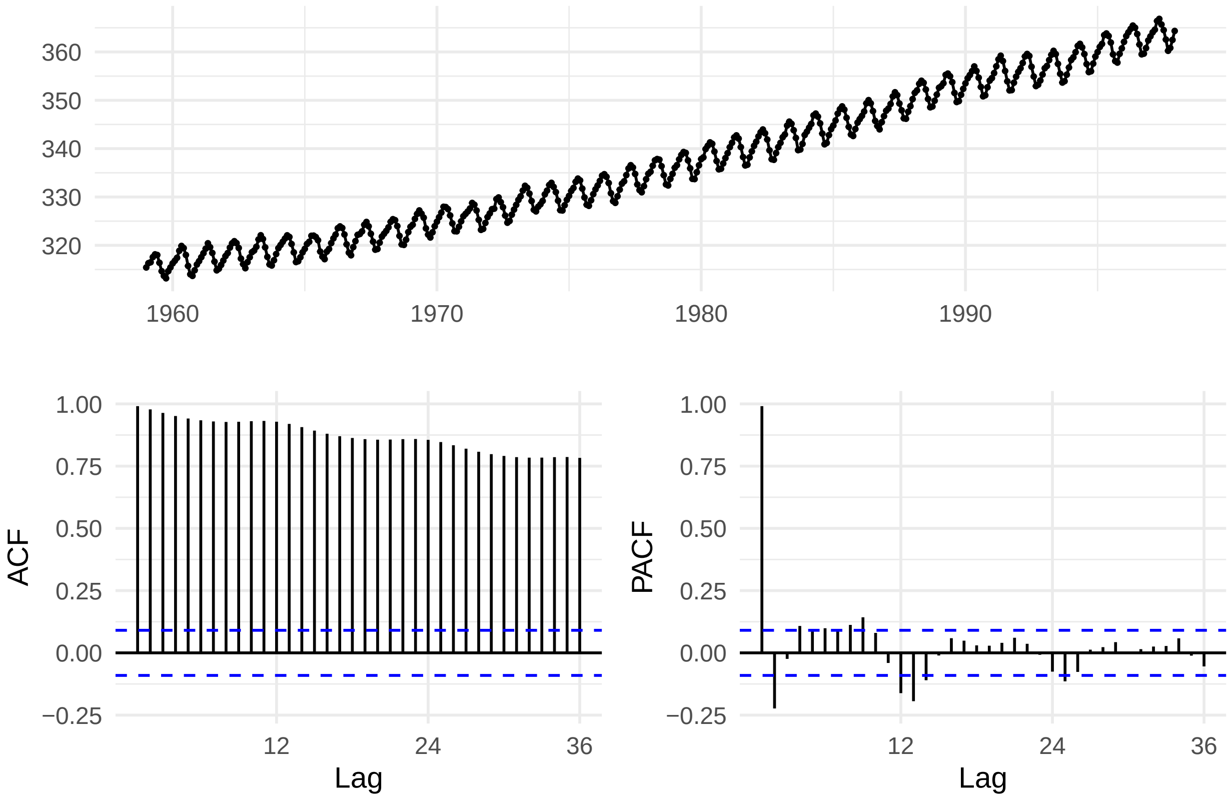
Table 2: Estimated Atmospheric CO2 Level

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	336.998	0.081	4,161.559	0
poly(trend, 3)1	319.167	0.506	631.062	0
poly(trend, 3)2	31.308	0.506	61.923	0
poly(trend, 3)3	-11.062	0.506	-21.863	0
season2	0.670	0.114	5.852	0
season3	1.419	0.114	12.390	0
season4	2.555	0.114	22.319	0
season5	3.040	0.115	26.550	0
season6	2.383	0.115	20.811	0
season7	0.868	0.115	7.578	0
season8	-1.194	0.115	-10.429	0
season9	-3.013	0.115	-26.311	0
season10	-3.191	0.115	-27.860	0
season11	-1.996	0.115	-17.428	0
season12	-0.874	0.115	-7.628	0

trend with a seasonal dummy variable was able to capture the linear and seasonal trend when forecasted to 2020. There's a slight curve at the 2020 tail of the trend due to the 3rd degree polynomial.

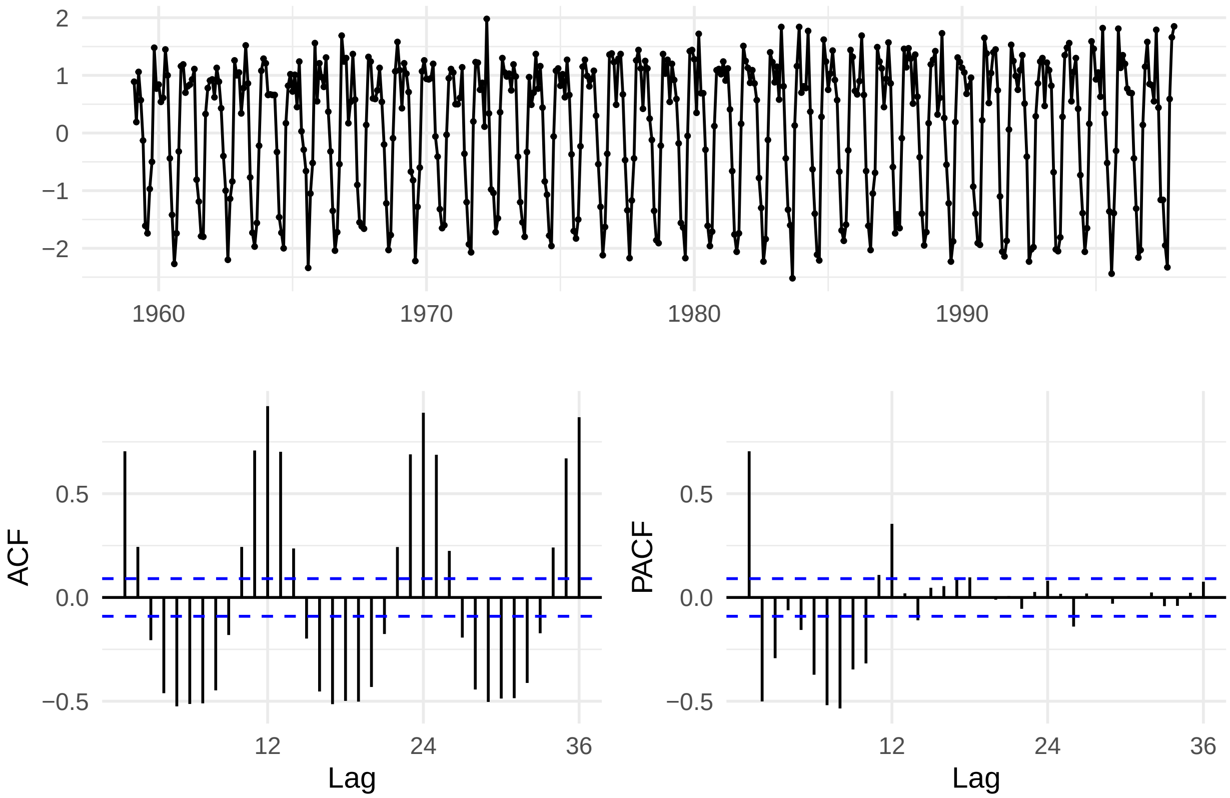
0.4 (3 points) Task 3a: ARIMA times series model

For the ARIMA model, we can conjecture the following parameters: p: the number of lag observations in the model, also known as the lag order. Inspect PACF plot. d: the number of times the raw observations are differenced; also known as the degree of differencing. q: the size of the moving average window, also known as the order of the moving average. ACF tells how many MA terms are required to remove any autocorrelation in the stationarized series.



The time series plot has a strong positive trend and is therefore non-stationary. The ACF plot has significant lags that persist with gradual decay and slight bumps due to seasonality. Applying differencing will remove this trend.

```
# 1st DIFFERENCING
diff(co2) %>% ggtsdisplay(main = "")
```




```
adf.test(diff(co2))
```

```
## Warning in adf.test(diff(co2)): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  diff(co2)
## Dickey-Fuller = -30.38, Lag order = 7, p-value = 0.01
## alternative hypothesis: stationary
```

After taking a first difference of the data, the time series plot appears to oscillate around 0 and the Augmented Dickey-Fuller test returns a significant p-value which confirms that the time series is stationary. the ACF has cyclic lags that are significant due to seasonality, but there are no signs of dampening. The PACF plot has 3 to 11 leading significant lags until the autocorrelation dampen.

1 NOT SURE IF THIS AR LOGIC IS CORRECT.

Because the ACF has persistent significant lags while the PACF has dampening oscillations, the data leans towards an AR(3) process. With the differencing component, the ARIMA(p,1,q) model will be the most appropriate. The correct MA and AR parameters will be tested.

BIC will be the model selection criteria because BIC has a larger penalty for models with more parameters and therefore selects sparser models with fewer parameters compared to AIC and AICc.

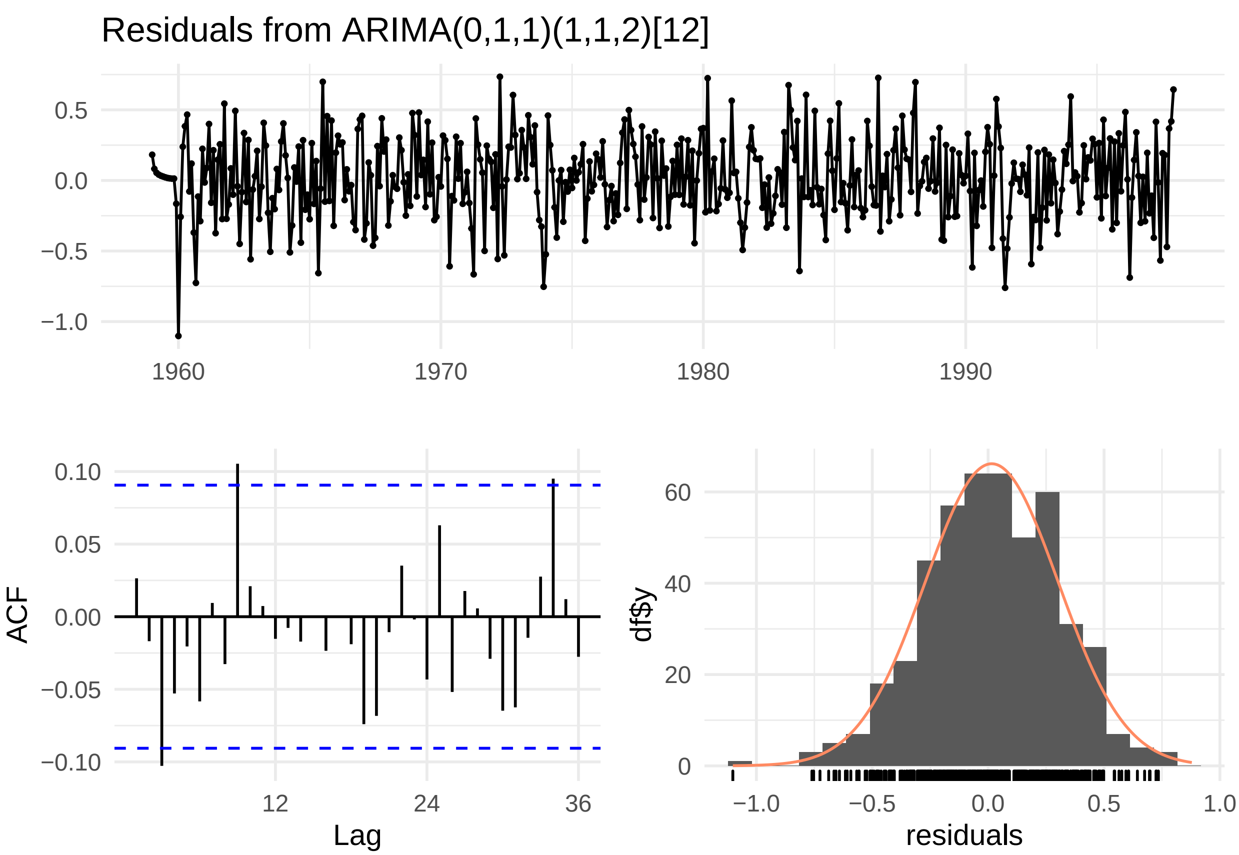
```
# TODO - check order parameter, and add other necessary parameters
# TODO - how to manually determine arima parameters? - LS7
# AUTO ARIMA
```

```r
mod.arima <- auto.arima(co2,
                        d=1,
                        ic = 'bic',
                        trace=FALSE,
                        seasonal=TRUE)

summary(mod.arima)
```

```
## Series: co2
## ARIMA(0,1,1)(1,1,2)[12]
##
## Coefficients:
##           ma1     sar1     sma1     sma2
##       -0.3482  -0.4986  -0.3155  -0.4641
## s.e.   0.0499   0.5282   0.5165   0.4367
##
## sigma^2 = 0.08603:  log likelihood = -85.59
## AIC=181.18   AICc=181.32   BIC=201.78
##
## Training set error measures:
##                      ME      RMSE       MAE         MPE       MAPE      MASE
## Training set 0.01538153 0.2879337 0.2299909 0.004479982 0.06834581 0.1816409
##                    ACF1
## Training set 0.02645309
```

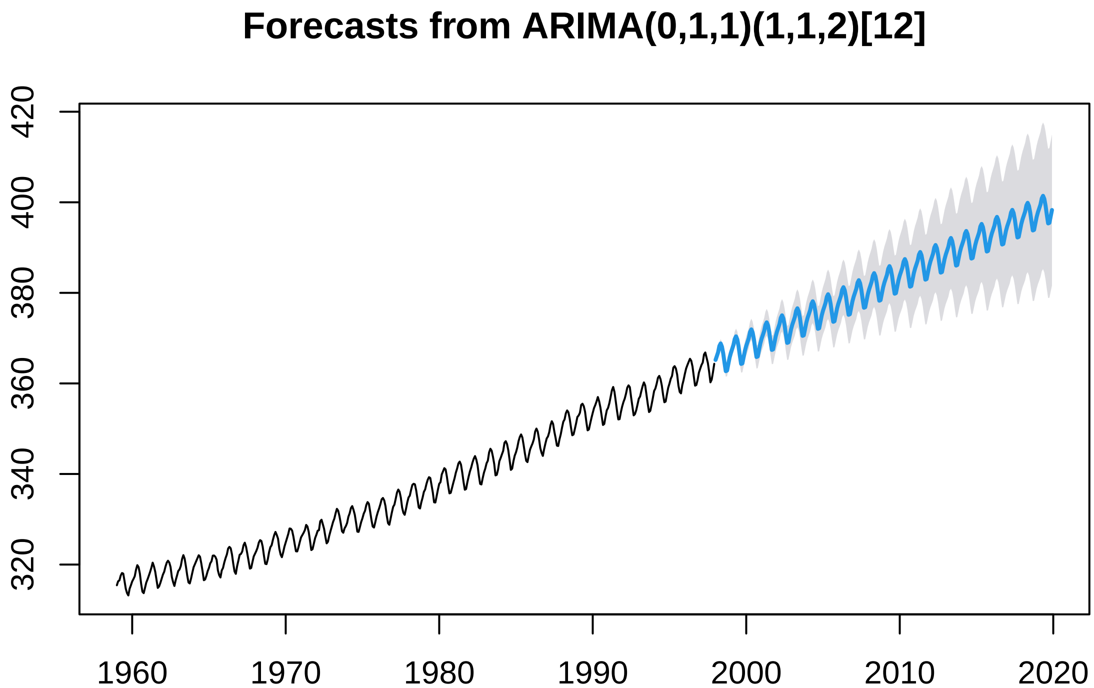
```r
checkresiduals(mod.arima)
```




```
##
```

```
##  Ljung-Box test
##
## data:  Residuals from ARIMA(0,1,1)(1,1,2)[12]
## Q* = 21.999, df = 20, p-value = 0.3406
##
## Model df: 4.   Total lags used: 24
```

```
arima_pred <- forecast::forecast(mod.arima, level = c(95), h = 22 * 12)
plot(arima_pred)
```

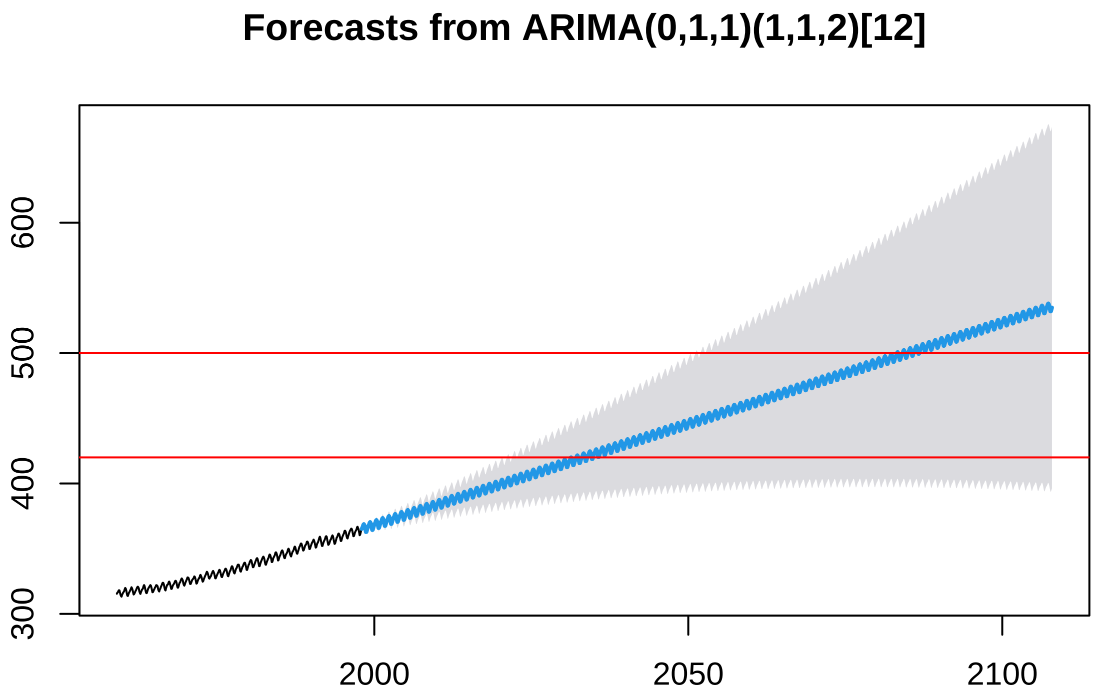



```
# SELF-DETERMINED parameters
# order <- c(1, 1, 2)
# mod.arima <- arima(co2, order = order, seasonal = list(order = order, period = NA))
# summary(mod.arima)
# arima_pred <- forecast::forecast(mod.arima, level = c(95), h = 22 * 12)
# plot(arima_pred)
```

The final model is ARIMA(1,1,1)(1,1,2) with a BIC of 205.5. Checking the residuals of the model, the residuals plot oscillates around 0, the ACF autocorrelations are all below or only slighly over the threshold value, and the distribution is Gaussian. The Ljung-Box test returns a large p-value and therefore the residuals are stationary.

Using this model, a forecast of CO2 concentrations to the year 2020 was generated. After 2010, the forecast starts to curve with a wider confidence interval.

1.1 (3 points) Task 4a: Forecast atmospheric CO2 growth




```
##          Point.Forecast    Lo.95    Hi.95
## May 2031       420.0598 392.3004 447.8192
## Oct 2086       499.4602 396.8727 602.0478
## Jan 2100       523.5351 398.8328 648.2374
```

Based on the model forecasts, CO2 concentrations is expected to reach 420 ppm by May 2031 and 500 ppm by Oct 2086. By Jan 2100, CO2 levels will be at 523.5 ppm. We are not confident about these predictions because the lower bound of the confidence interval has plateaued at approximately 390 ppm while the upper bound continues to grow higher.

```r
dcmp_multi <- co2_2 %>%
  model(stl = STL(log_value))

p_5_multi <- components(dcmp_multi) %>%
  as_tsibble() %>%
  autoplot(log_value, colour = "gray") +
  geom_line(aes(y = trend), colour = "#D55E00") +
  labs(
    y = TeX(r'(Log $CO_2$ Annualized Growth Rate)'),
    x = "Month and Year",
    title = "Monthly Mean CO2"
  ) +
  theme(plot.title = element_text(size = 10))

# plot the componentsco
p_6_multi <- components(dcmp_multi) %>%
  autoplot() + theme(plot.title = element_text(size = 10))

# plot ACF of residuals
p_7_multi <- components(dcmp_multi) %>%
  ACF(remainder) %>%
```

```r
  autoplot() + labs(title = "ACF Residuals multiplicative decomposition") +
  theme(plot.title = element_text(size = 10))

# plot PACF of residuals
p_8_multi <- components(dcmp_multi) %>%
  PACF(remainder) %>%
  autoplot() + labs(title = "PACF Residuals multiplicative decomposition") +
  theme(plot.title = element_text(size = 10))

#(p_5_multi | p_6_multi) / (p_7_multi | p_8_multi)
```

trend: strong positive linear trend across years irregular elements: The data oscilate around 0. From the ACF plot, residuals have strong autocorrelation terms, indicating trend and seasonal variation. The PACF plot ocilates with many significant lags seasonality: strong seasonality - will explore details in next plot