Graphics Note

Microfacet model

这里假设macrosurface的面积为dA，法向量为$\mathbf{n}$, microsurface的面积为dA^m，法向量为$\mathbf{m}$。入射方向为$\mathbf{i}$，出射方向为$\mathbf{o}$。定义Microfacet distribution function$D(\mathbf{m}):\mathbb{R}^3\rightarrow\mathbb{R}$，这是一个密度函数；定义Bidirectional shadowing-masking function $G(\mathbf{i},\mathbf{o},\mathbf{m}):\mathbb{R}^3\times\mathbb{R}^3\times\mathbb{R}^3\rightarrow\{0,1\}$

$$dA^m=D(\mathbf{m})G(\mathbf{i},\mathbf{o},\mathbf{m})d\omega_m dA$$

这里列出$D(\mathbf{m})$\$的几条性质：

1. $D(\mathbf{m})$是一个大于等于0的密度函数

$$0\le D(\mathbf{m})\le\infty$$

2. microsurface的面积和至与macrosurface一样大

$$dA\le\int D(\mathbf{m})d\omega_w dA\Rightarrow 1\le\int D(\mathbf{m})d\omega_m$$

3. microsurface和macroface在某个方向上$\mathbf{v}$的投影面积相同

$$(\mathbf{v}\cdot\mathbf{n})dA=\int D(\mathbf{m})(\mathbf{v}\cdot\mathbf{m})d\omega_m dA\Rightarrow(\mathbf{v}\cdot\mathbf{n})=\int D(\mathbf{m})(\mathbf{v}\cdot\mathbf{m})d\omega_m$$

BSDF

辐射照度定义：

$$E=\frac{d\Phi}{A}$$

辐射度定义:

$$L=\frac{dE}{d\omega\cos\theta}=\frac{d^2\Phi}{dAd\omega\cos\theta}$$

对于Macrosurface，在入射方向$\mathbf{i}$和出射方向$\mathbf{o}$分别有：

$$L_i = \frac{d^2\Phi_i}{dA \cdot d\omega_i \cdot |\mathbf{i} \cdot \mathbf{n}|}$$

$$L_o = \frac{d^2\Phi_o}{dA \cdot d\omega_o \cdot |\mathbf{o} \cdot \mathbf{n}|} = \frac{\rho(\mathbf{i}, \mathbf{o}) d^2\Phi_o}{dA d\omega_o \cdot |\mathbf{o} \cdot \mathbf{n}|}$$

$$E_i = L_i \cdot d\omega_i \cdot |\mathbf{i} \cdot \mathbf{n}|$$

其中$\rho(\mathbf{i}, \mathbf{o})$是Fresnell函数

根据BSDF的定义：

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{dL_o}{dE_i}$$

$$= \frac{\rho(\mathbf{i}, \mathbf{o}) \cdot d\Phi_i}{L_i \cdot dA \cdot d\omega_i \cdot d\omega_o \cdot |\mathbf{i} \cdot \mathbf{n}| \cdot |\mathbf{o} \cdot \mathbf{n}|}$$

对于L_i，从microfacesurface上分析（面积为dA^m，法向量为$\mathbf{m}$）可以得到

$$L_i = \frac{d^2\Phi_i}{d\omega_i \cdot dA^m \cdot |\mathbf{i} \cdot \mathbf{m}|} = \frac{d^2\Phi_i}{d\omega_i \cdot D(\mathbf{m}) \cdot G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \cdot d\omega_m \cdot dA \cdot |\mathbf{i} \cdot \mathbf{m}|}$$

带入上式可以得到

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| \cdot |\mathbf{o} \cdot \mathbf{n}|} \cdot \rho(\mathbf{i}, \mathbf{o}) \cdot D(\mathbf{m}) \cdot G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \cdot \frac{d\omega_m}{d\omega_o}$$