CS101-Quiz9-Review

Suting Chen

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Key Points

1. Dijkstra’s algorithm
2. Bellman-Ford algorithm
3. A* Search Algorithm

Dijkstra’s algorithm

1. Solves single-source shortest path problem.
2. Choose the vertex “nearest” to the source each time.
3. Does not work for graphs with negative weight edges.
4. Time complexity of Dijkstra's algorithm is $O\left((V+E)\log V\right)$, optimized with binary heap. $O((V+E)\log V) = O(E\log V)$ if $V = O(E)$ (in common cases).

Dijkstra's algorithm

Algorithm analysis

1. The outer loop is executed V (or $V-1$) times. In each iteration, the shortest path to a new vertex is determined.
2. Each **edge** is processed once when relaxing the distances.
3. **Binary heap** is used to find the vertex with the minimum distance.

Dijkstra's algorithm

Algorithm analysis — Time

1. Every edge visit could update distances in binary heap: $O\left(E \log V\right)$
2. Popping the vertex (whose shortest path has not been determined) with minimum distance from the heap takes $O(\log V)$ time. This is done $O(V)$ times.
3. Total time complexity: $O(V \log V + E \log V) = O(E \log V)$ if $V = O(E)$.

Dijkstra's algorithm

Algorithm analysis — Better **time** complexity with Fibonacci heap

1. Every edge visit could update distances in Fibonacci heap: $O(E)$, because the Fibonacci heap supports $O(1)$ decrease-key operation.
2. Popping the vertex (whose shortest path has not been determined) with minimum distance from the heap takes $O(\log V)$ time. This is done $O(V)$ times.
3. Total time complexity: $O\left(V \log V + E\right)$

但fib确实难写，而且因为常数巨大，所以我们也不会让你写。

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Key Points

1. Dijkstra's algorithm
2. **Bellman-Ford algorithm**
3. A* Search Algorithm

Bellman-Ford algorithm

1. Solves single-source shortest path problem.
2. Update cost of all vertices in each iteration.
3. Work for graphs with negative weight edges.
4. Time complexity is $O(VE)$

Bellman-Ford algorithm

Algorithm analysis — Time

1. In each iteration, the algorithm goes through all the edges in the graph: $O(E)$

2. Relaxation process is run for $O(V)$ times.

3. Total time complexity: $O(VE)$

Comparison

	Dijkstra's	Bellman-Ford
Time complexity	~~$O\left((V+E)\log V\right)$~~ $O\left(V\log V + E\right)$	$O(VE)$
Negative weights	✗	✓
Negative cycle	✗	**Detect**

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3. **A* Search Algorithm**

A* Search Algorithm

1. Heuristic-based Pathfinding algorithm
2. Admissible and consistent
3. We don’t care the time complexity

A* Search Algorithm

Admissible and consistent

1. Admissible: never overestimates.
2. Consistent: triangle inequality.

3. Consistency implies admissibility (See post @173)

两边之和大于第三边

A* Search Algorithm

Tree search and Graph search

1. Tree search is the algorithm that **believes** it is running on a tree, so it will not check whether a vertex is visited
2. By using tree search:
   2.1. You could fall into an infinite loop
   2.2. "Scores" or "Costs" of vertices can be updated multiple times
3. By using graph search:
   3.1. Your algorithm will terminate
   3.2. Avoid visiting the same vertices a second time

A* Search Algorithm

Tree search and Graph search

	None	Admissible	Consistent
Graph	✗	✗	✓
Tree	✗	✓	✓

Tree的搜索比Graph“充分”；

Consistent的结论比Admissible强；

总之要想optimal，要么搜的多，要么猜的准。

红色的叉只是不保证optimal，不代表一定很差。