Chapter 3: Convolution Neural Network

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Convolutional Neural Networks

Computer vision

Computer Vision Problems

Image Classification

64x64

Cat? (0/1)

Neural Style Transfer

Object detection

Deep Learning on large images

64x64 × 3

→ Cat? (0/1)

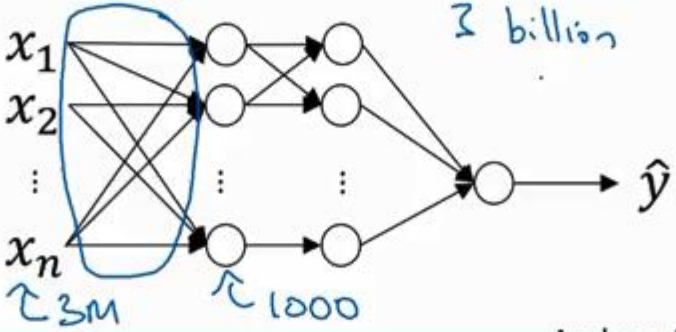
12288

1000 × 1000 × 3
= 3 million

$x \in \mathbb{R}^{3M}$

$W^{[1]}$ (1000, 3M)

3 billion



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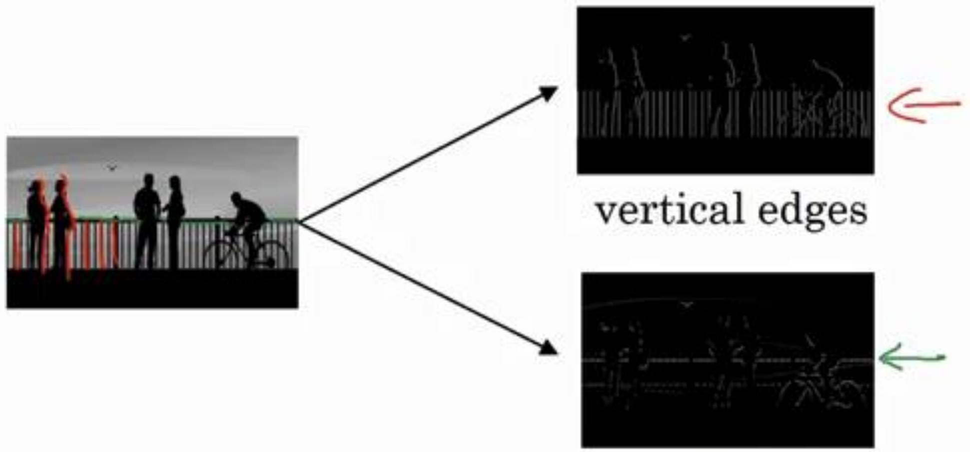
Convolutional Neural Networks

Edge detection example

Computer Vision Problem



vertical edges

horizontal edges

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Vertical edge detection

$$3\times1 + 1\times1 + 2\times1 + 0\times0 + 5\times0 + 7\times0 + 1\times-1 + 8\times-1 + 2\times-1 = -5$$

"convolution"

$$\underbrace{\begin{bmatrix} 3 & 0 & 1 & 2 & 7 & 4 \\ 1 & 5 & 8 & 9 & 3 & 1 \\ 2 & 7 & 2 & 5 & 1 & 3 \\ 0 & 1 & 3 & 1 & 7 & 8 \\ 4 & 2 & 1 & 6 & 2 & 8 \\ 2 & 4 & 5 & 2 & 3 & 9 \end{bmatrix}}_{\boxed{6\times6}} * \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}}_{\substack{3\times3 \\ \text{filter}}} = \underbrace{\begin{bmatrix} -5 & -4 & 0 & 8 \\ -10 & -2 & 2 & 3 \\ 0 & -2 & -4 & -7 \\ -3 & -2 & -3 & -16 \end{bmatrix}}_{4\times4}$$

python: conv-forward

tensorflow: tf.nn.conv2d

keras: Conv2D

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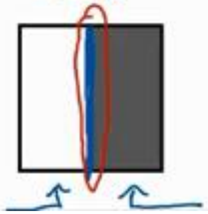
Vertical edge detection

$$\underset{6\times6}{\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix}} * \underset{3\times3}{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}} = \underset{4\times4}{\begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}}$$



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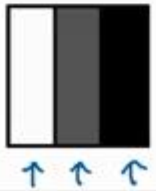

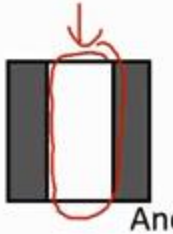

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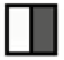
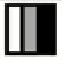
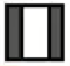
Convolutional Neural Networks

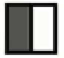
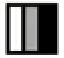
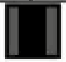
More edge detection

Vertical edge detection examples

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \end{bmatrix}$$

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Vertical and Horizontal Edge Detection

1	0	-1
1	0	-1
1	0	-1

Vertical

1	1	1
0	0	0
-1	-1	-1

Horizontal

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10

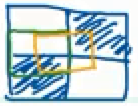
6 x 6

*

1	1	1
0	0	0
-1	-1	-1

=

0	0	0	0
30	10	-10	-30
30	10	-10	-30
0	0	0	0



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Learning to detect edges

1	0	-1
1	0	-1
1	0	-1

1	0	-1
2	0	-2
1	0	-1

Sobel filter

3	0	-3
10	0	-10
3	0	-3

Scharr filter

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

convolution

$*$

$$\underbrace{\begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix}}_{3\times 3}$$

$=$

45°
70°
73°

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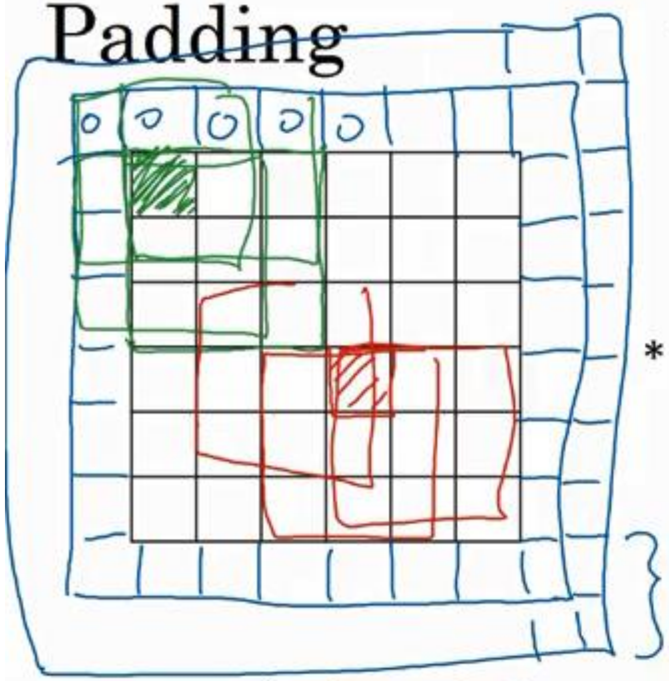
Convolutional Neural Networks

Padding

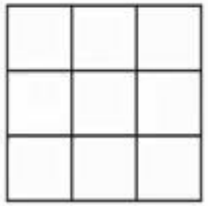
Padding

- shrinking output
- throw away info from edge



p=2

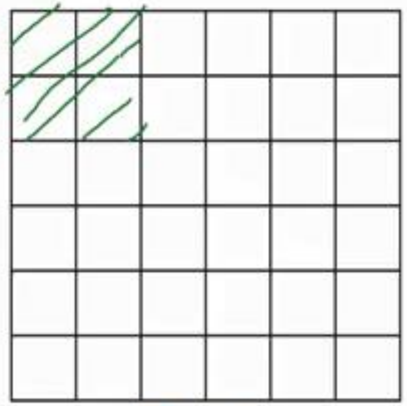
*



3×3

f×f

=



6×6

~~4×4~~

6×6 → 8×8

n×n

$n-f+1 \times n-f+1$

$6-3+1=4$

p = padding = 1

$n+2p-f+1 \times n+2p-f+1$

$6+2-3+1 \times \_\_\_ = 6\times 6$

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Valid and Same convolutions

no padding

"Valid": $n \times n \ * \ f \times f \rightarrow n-f+1 \times n-f+1$

$6 \times 6 \ * \ 3 \times 3 \rightarrow 4 \times 4$

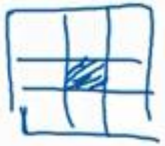
"Same": Pad so that output size is the same as the input size.

$n+2p-f+1 \times n+2p-f+1$

$\cancel{n}+2p-f+1 = \cancel{n} \Rightarrow \boxed{p = \frac{f-1}{2}}$

$3 \times 3 \quad p = \frac{3-1}{2} = 1 \quad \Big| \quad \underset{f=5}{5 \times 5} \quad p = 2$

f is usually odd



1×1
3×3
5×5
7×7

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Convolutional Neural Networks

Strided convolutions

Strided convolution

2	3	7	4	6	2	9
6	6	9	8	7	4	3
3	4	8	3	8	9	7
7	8	3	6	6	3	4
4	2	1	8	3 ^{3}	4 ^{4}	6 ^{4}
3	2	4	1	9 ^{1}	8 ^{0}	3 ^{2}
0	1	3	9	2 $^{-1}$	1 ^{0}	4 ^{3}

$\underline{7 \times 7}$

*

3	4	4
1	0	2
-1	0	3

$\underline{3 \times 3}$

=

91	100	83
69	91	127
44	72	74

$\underline{3 \times 3}$

stride = 2

$\lfloor z \rfloor = \text{floor}(z)$

$n \times n$ * $f \times f$

padding p stride s

$s = 2$

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

$$\frac{7+0-3}{2} + 1 = \frac{4}{2} + 1 = 3$$

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Summary of convolutions

$n \times n$ image $\quad f \times f$ filter

padding $p \quad$ stride s

Output size:

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

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Technical note on cross-correlation vs. convolution

Convolution in math textbook:

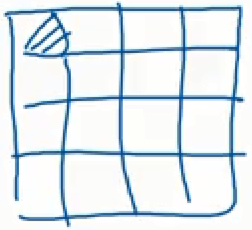
2^{7}	3^{2}	7^{5}	4	6	2
6^{9}	6^{0}	9^{4}	8	7	4
3^{-1}	4^{1}	8^{3}	3	8	9
7	8	3	6	6	3
4	2	1	8	3	4
3	2	4	1	9	8

*

3	4	5
1	0	2
-1	9	7

7	2	5
9	0	4
-1	1	3

=



$$(A * B) * C = A * (B * C)$$

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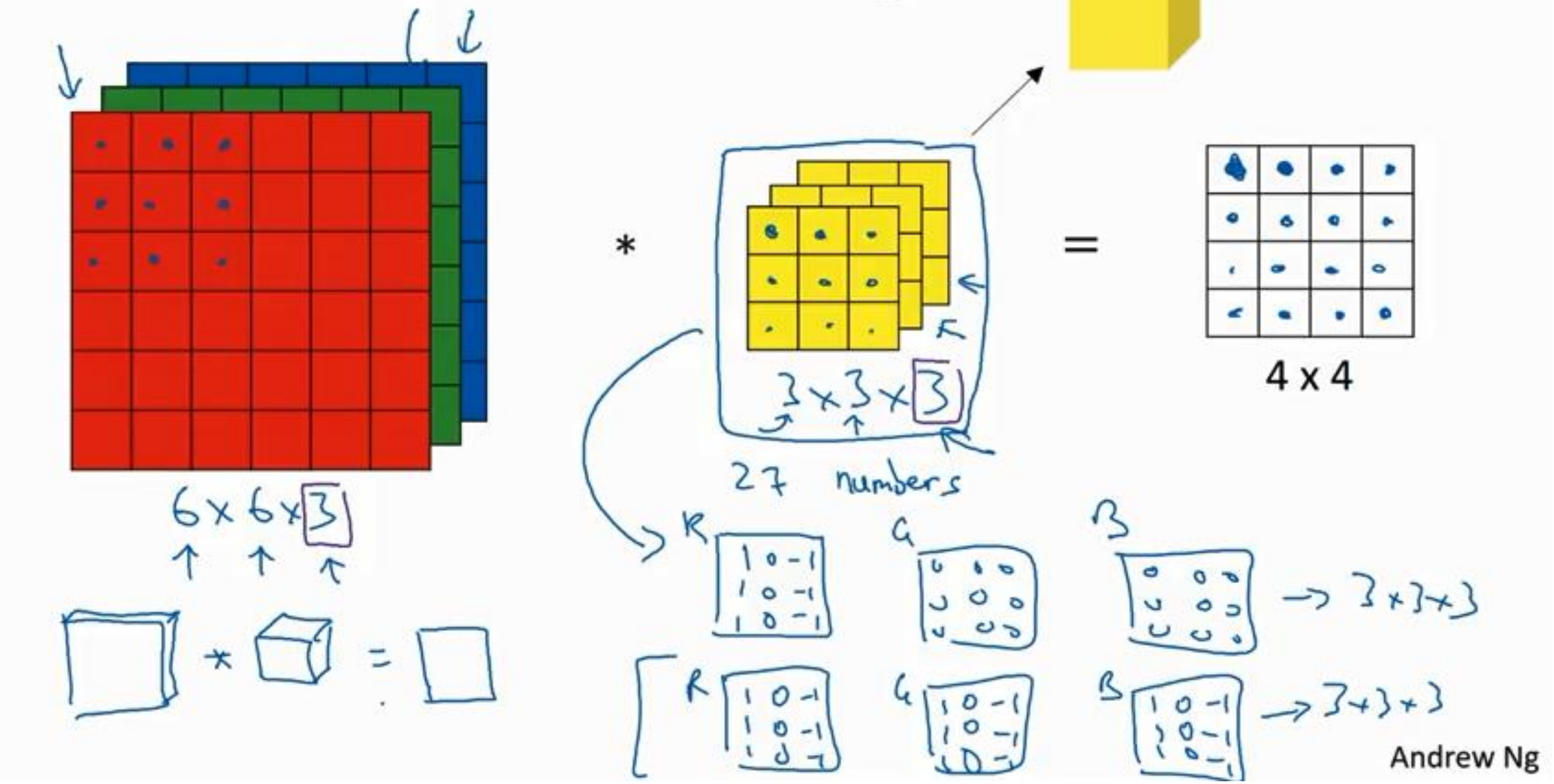
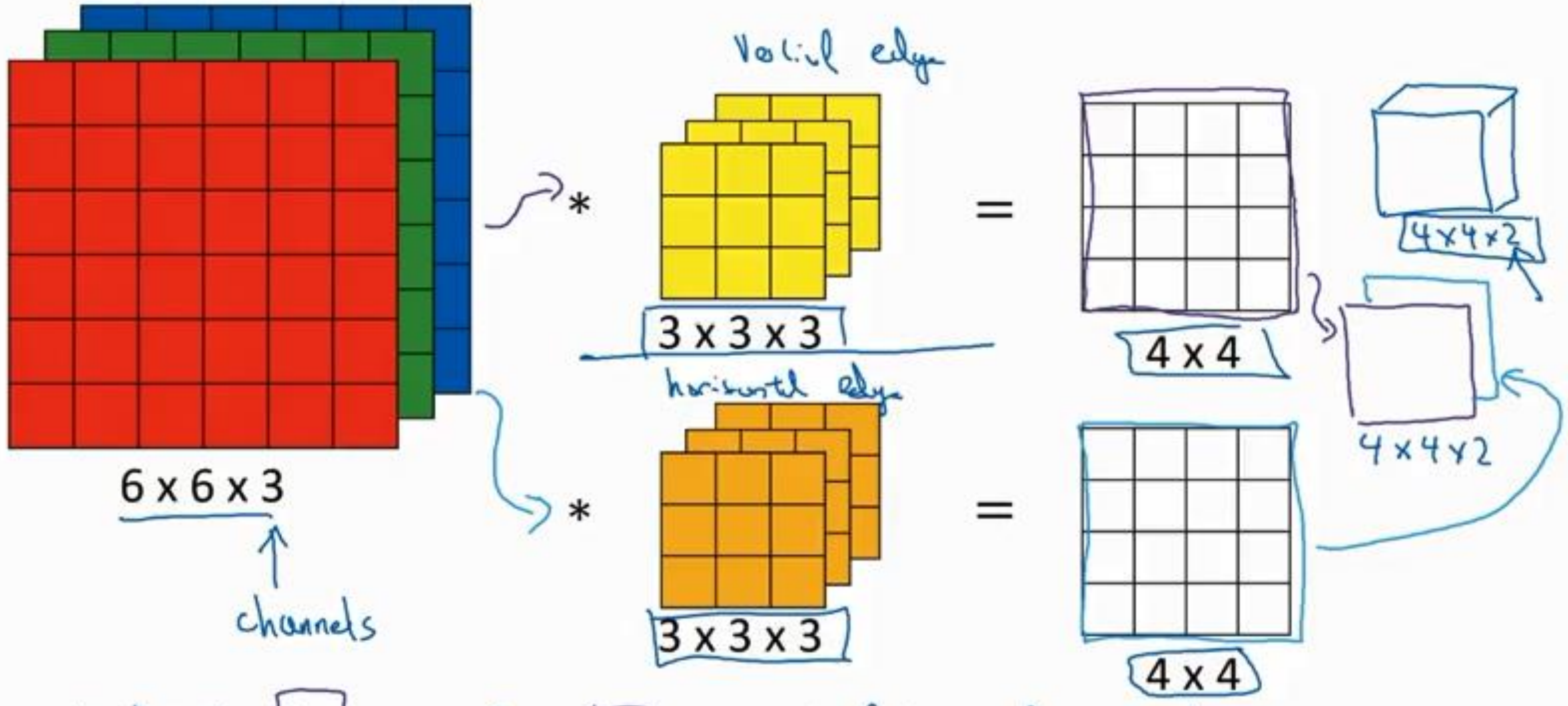
Convolutional Neural Networks

Convolutions over volumes

Convolutions on RGB image



Multiple filters



Summary: $n \times n \times \boxed{n_c} \ * \ f \times f \times \boxed{n_c} \longrightarrow n-f+1 \times n-f+1 \times n_c'$

$6 \times 6 \times 3 \qquad 3 \times 3 \times 3 \qquad 4 \times 4 \times 2$ ← #filters

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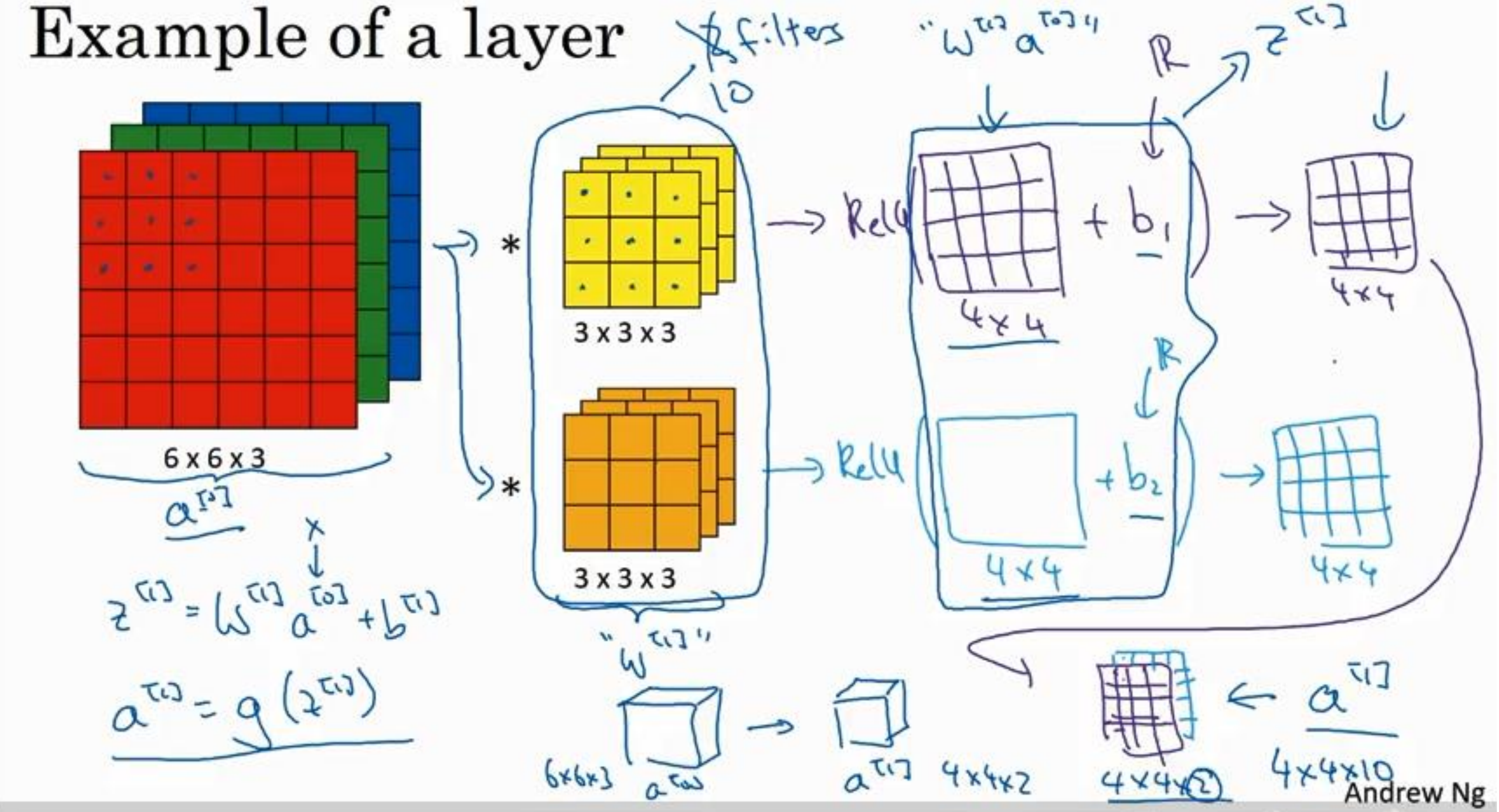
Convolutional Neural Networks

One layer of a convolutional network

Example of a layer



Number of parameters in one layer

If you have 10 filters that are 3 x 3 x 3 in one layer of a neural network, how many parameters does that layer have?

3x3x3

27 parameters.

+bias

→ 28 parameters.

280 parameters.

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Summary of notation

If layer l is a convolution layer:

$f^{[l]}$ = filter size

$p^{[l]}$ = padding

$s^{[l]}$ = stride

$\boxed{n_c^{[l]}}$ = number of filters

→ Each filter is: $f^{[l]} \times f^{[l]} \times \boxed{n_c^{[l-1]}}$

Activations: $a^{[l]} \rightarrow n_H^{[l]} \times n_W^{[l]} \times n_c^{[l]}$

Weights: $f^{[l]} \times f^{[l]} \times n_c^{[l-1]} \times n_c^{[l]}$ ← #filters in layer l.

bias: $n_c^{[l]}$ — $(1,1,1,n_c^{[l]})$

Input: $n_H^{[l-1]} \times n_W^{[l-1]} \times \boxed{n_c^{[l-1]}}$ ←

Output: $n_H^{[l]} \times n_W^{[l]} \times \boxed{n_c^{[l]}}$ ←

$$n_{H/W}^{[l]} = \left\lfloor \frac{n_{H/W}^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

$A^{[l]} \rightarrow m \times \underbrace{n_H^{[l]} \times n_W^{[l]} \times n_c^{[l]}}$

$n_c \times n_H \times n_W$

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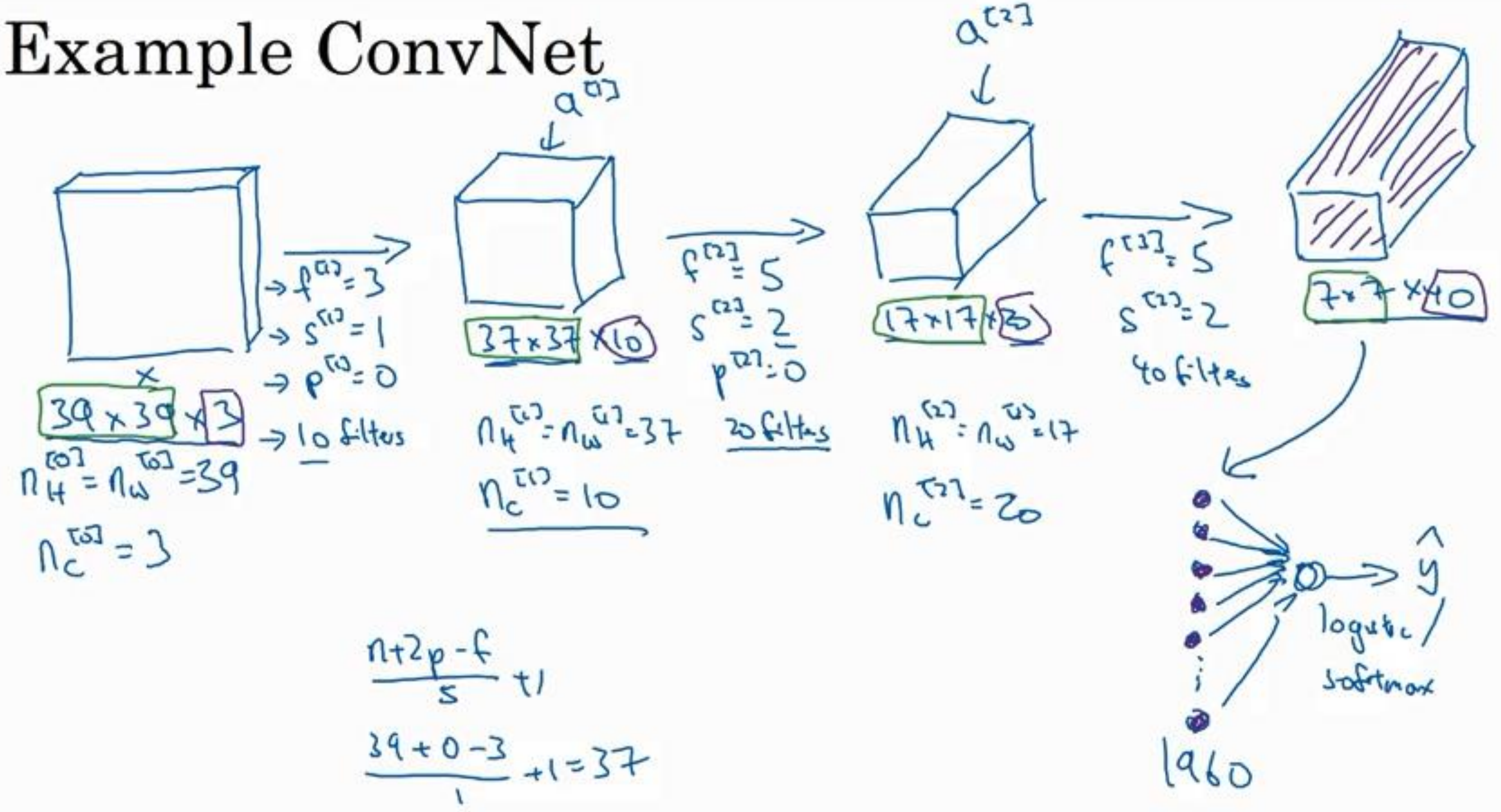
Convolutional Neural Networks

A simple convolution network example

Example ConvNet



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Types of layer in a convolutional network:

- Convolution (CONV) ←
- Pooling (POOL) ←
- Fully connected (FC) ←

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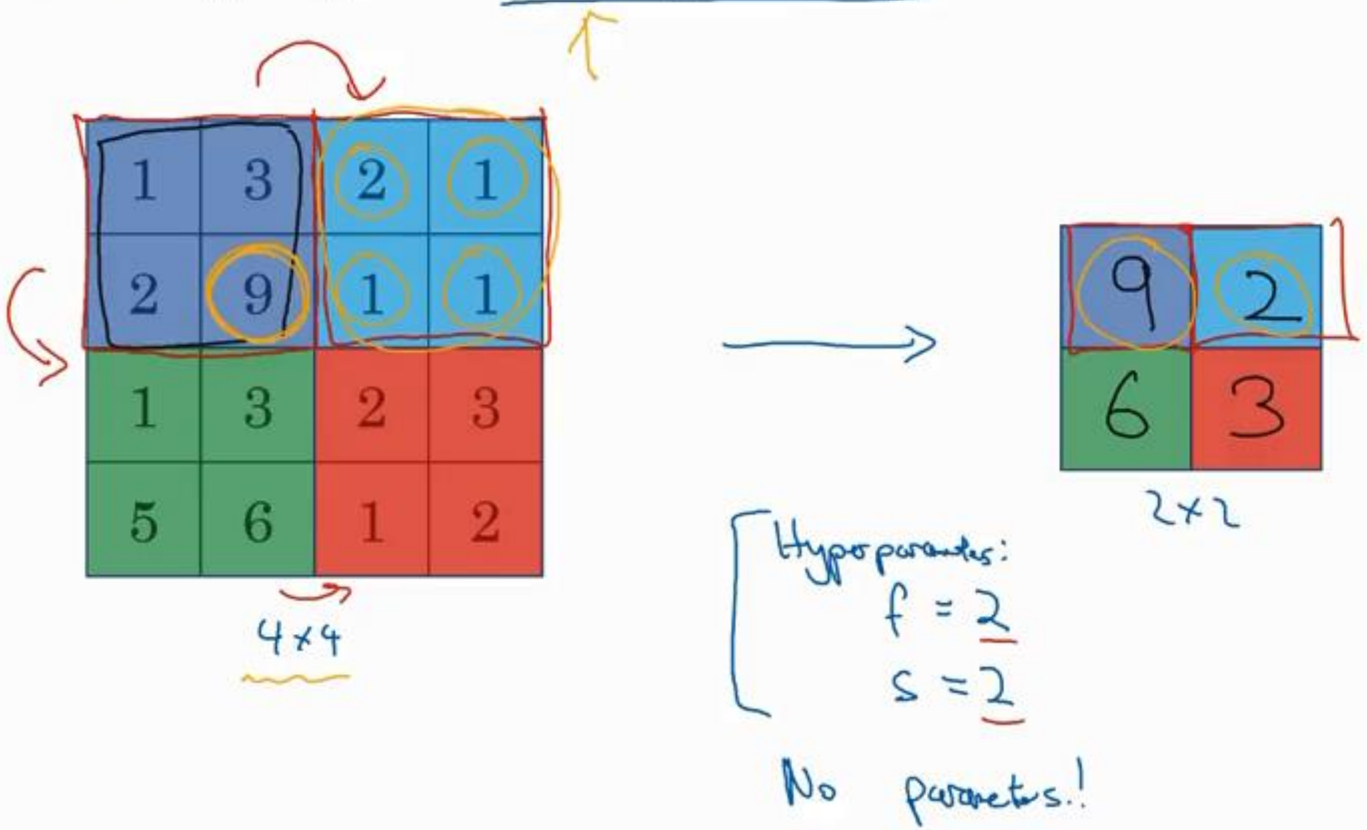
Convolutional Neural Networks

Pooling layers

Pooling layer: Max pooling



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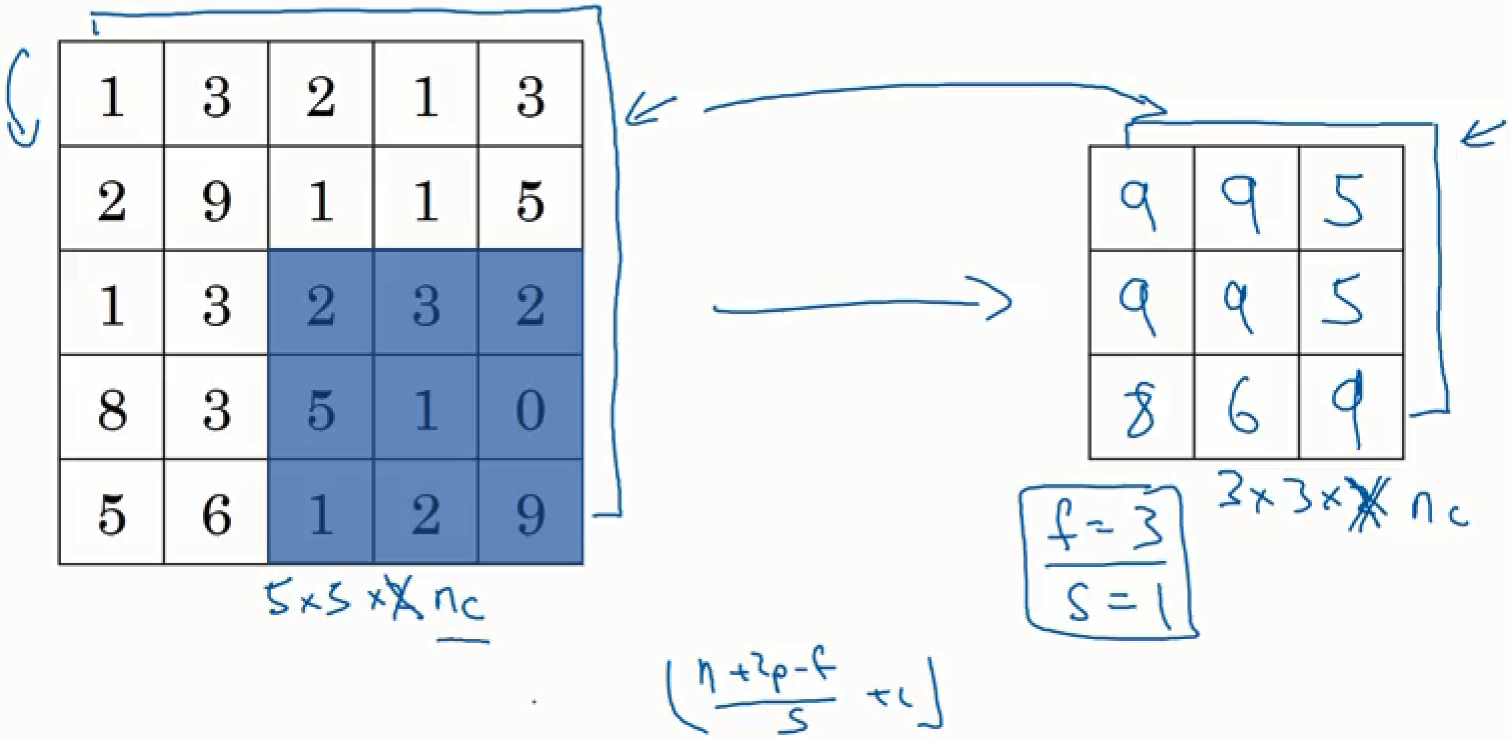
Pooling layer: Max pooling



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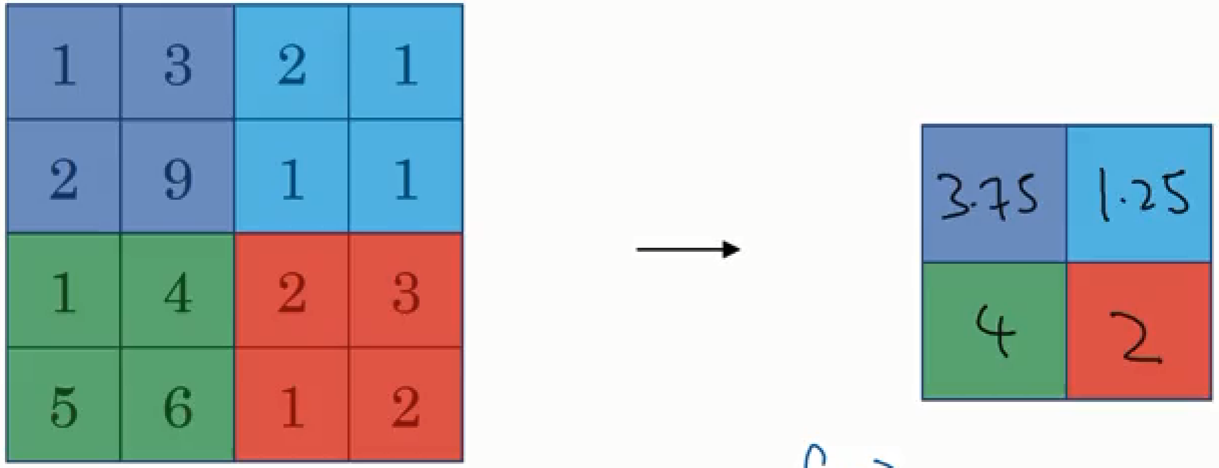
Pooling layer: Average pooling



$f=2$

$s=2$

$\underline{7\times 7}\times 1000 \rightarrow 1\times 1\times 1000$

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Summary of pooling

Hyperparameters:

f : filter size $\quad f=2, s=2$

s : stride $\quad f=3, s=2$

Max or average pooling

→ ~~p: padding.~~

No parameters to learn!

$$n_H \times n_W \times n_C$$

$$\downarrow$$

$$\left\lfloor \frac{n_H - f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n_W - f}{s} + 1 \right\rfloor \times n_C$$

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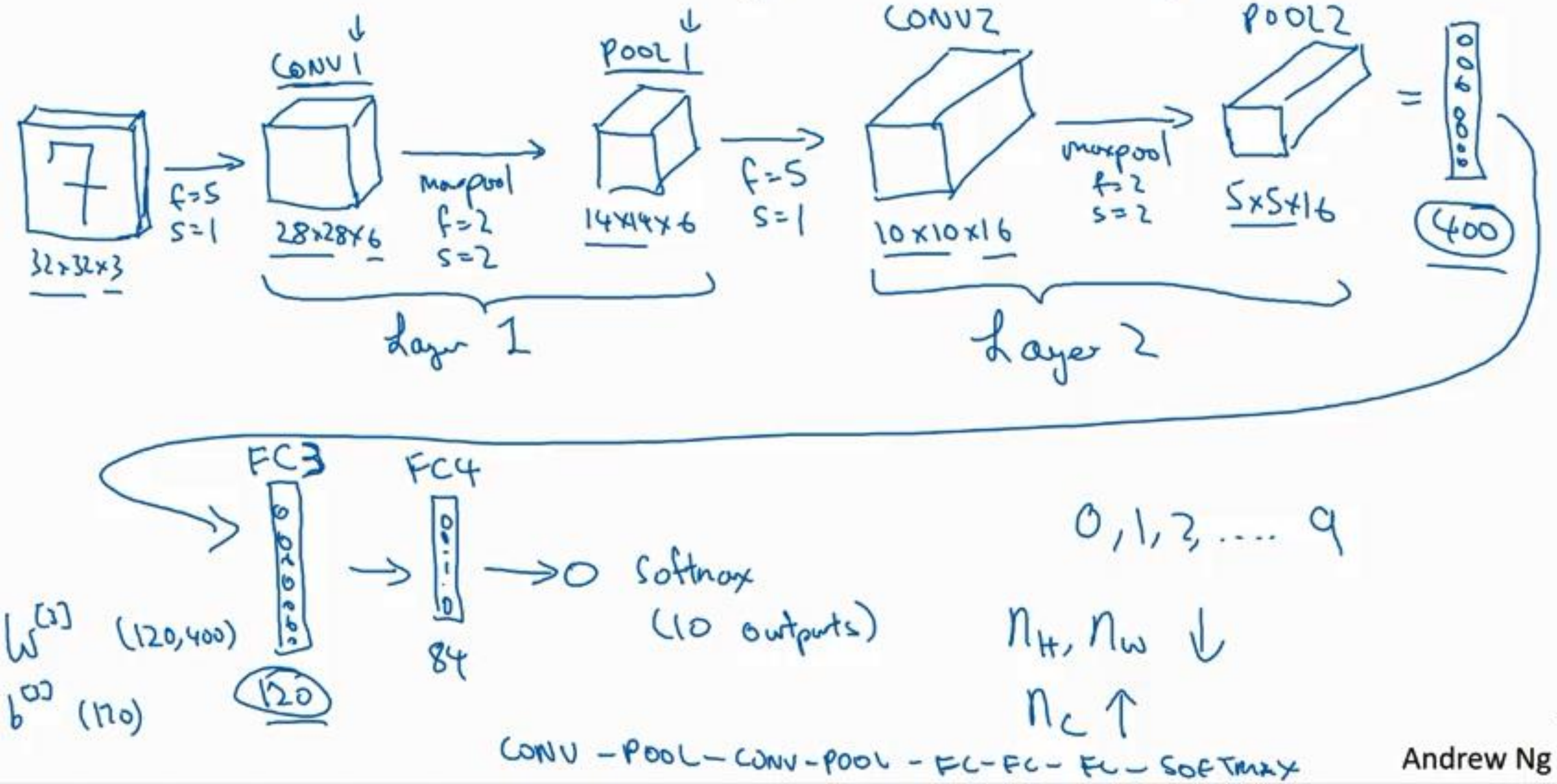
Convolutional neural network example

Neural network example (LeNet-5)



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Neural network example

	Activation shape	Activation Size	# parameters
Input:	(32,32,3)	3,072 $a^{[0]}$	0
CONV1 (f=5, s=1)	(28,28,8)	6,272	208
POOL1	(14,14,8)	1,568	0
CONV2 (f=5, s=1)	(10,10,16)	1,600	416
POOL2	(5,5,16)	400	0
FC3	(120,1)	120	48,001
FC4	(84,1)	84	10,081
Softmax	(10,1)	10	841

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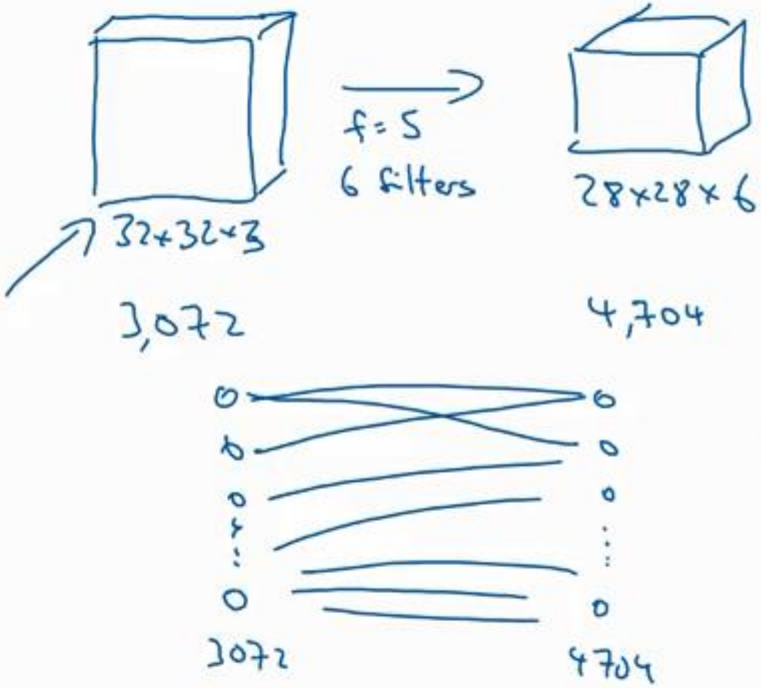
Why convolutions?

Why convolutions



5×5 – 25

26

6 × 26 = 156 parameters

3,072 × 4,704 ≈ 14M

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Why convolutions

translation invariance

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}}_{3\times 3} = \begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}$$

Parameter sharing: A feature detector (such as a vertical edge detector) that's useful in one part of the image is probably useful in another part of the image.

Sparsity of connections: In each layer, each output value depends only on a small number of inputs.

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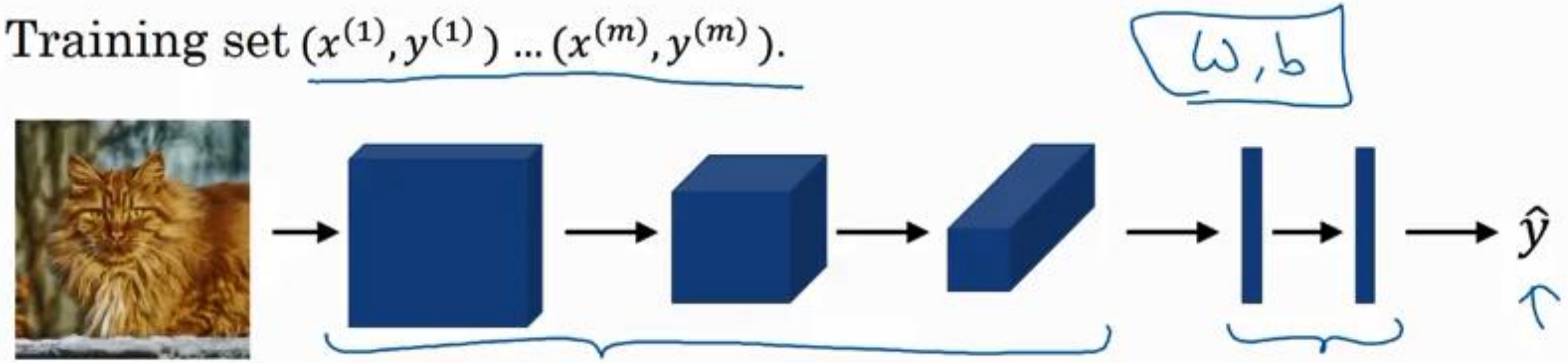
Putting it together

Training set $(x^{(1)}, y^{(1)}) \dots (x^{(m)}, y^{(m)})$.



Cost $J = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

Use gradient descent to optimize parameters to reduce J