# Informed search algorithms

# Outline

- Best-first search
- Greedy best-first search
- $A^*$ search
- Heuristics
- Local search algorithms
- Hill-climbing search
- Simulated annealing search
- Local beam search
- Genetic algorithms

# Review: Tree search

- \input{\file{algorithms}{tree-search-short-algorithm}}
- A search strategy is defined by picking the order of node expansion

# Best-first search

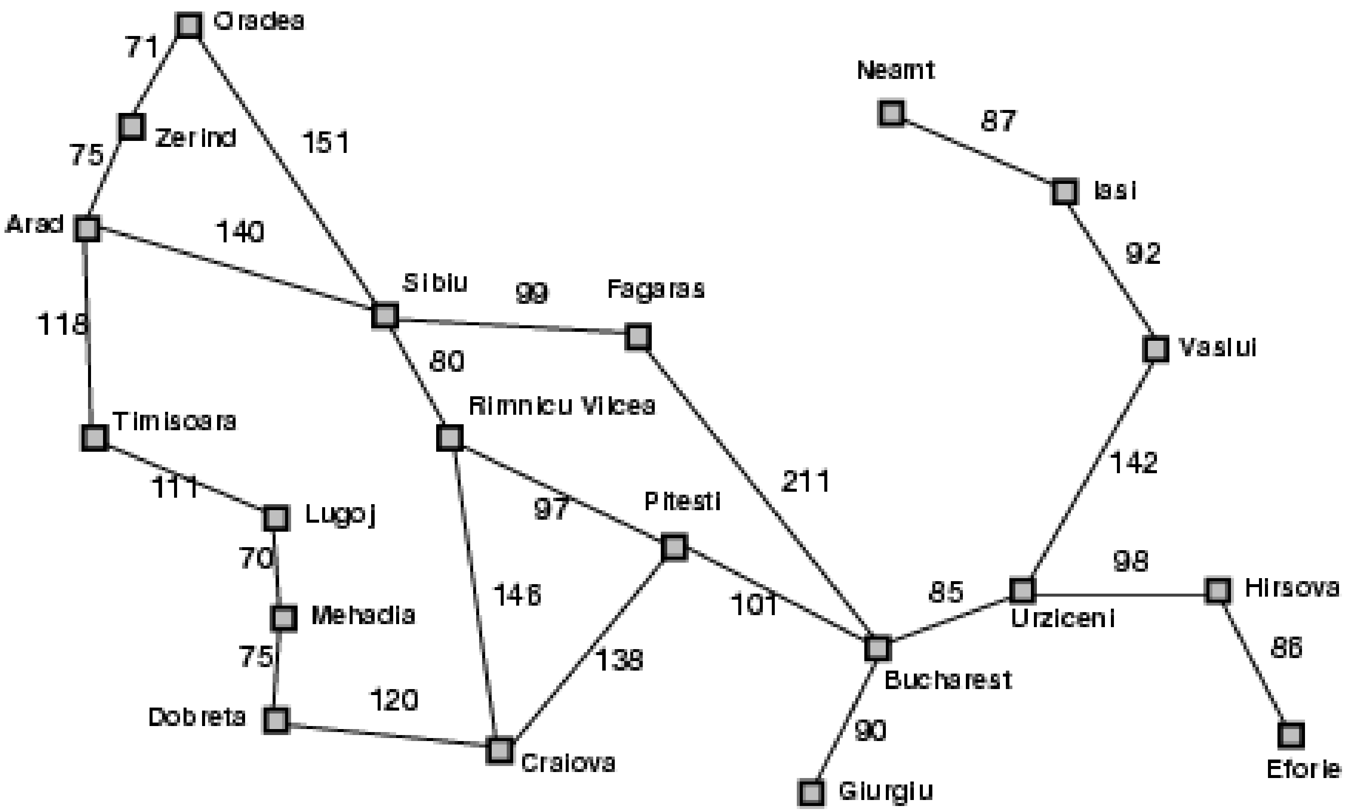
- Idea: use an evaluation function *f(n)* for each node
  - estimate of "desirability"

  → Expand most desirable unexpanded node

  →
- Implementation:

  Order the nodes in fringe in decreasing order of desirability
- Special cases:
  - greedy best-first search
  - $A^*$ search

# Romania with step costs in km




Straight–line distance to Bucharest

| | |
|---|---|
| **Arad** | 366 |
| **Bucharest** | 0 |
| **Craiova** | 160 |
| **Dobreta** | 242 |
| **Eforie** | 161 |
| **Fagaras** | 176 |
| **Giurgiu** | 77 |
| **Hirsova** | 151 |
| **Iasi** | 226 |
| **Lugoj** | 244 |
| **Mehadia** | 241 |
| **Neamt** | 234 |
| **Oradea** | 380 |
| **Pitesti** | 10 |
| **Rimnicu Vilcea** | 193 |
| **Sibiu** | 253 |
| **Timisoara** | 329 |
| **Urziceni** | 80 |
| **Vaslui** | 199 |
| **Zerind** | 374 |

# Greedy best-first search

- Evaluation function $f(n) = h(n)$ (heuristic)
- = estimate of cost from $n$ to *goal*
- e.g., $h_{SLD}(n)$ = straight-line distance from $n$ to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal

# Greedy best-first search example

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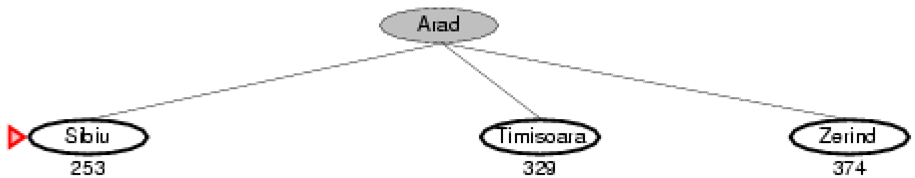
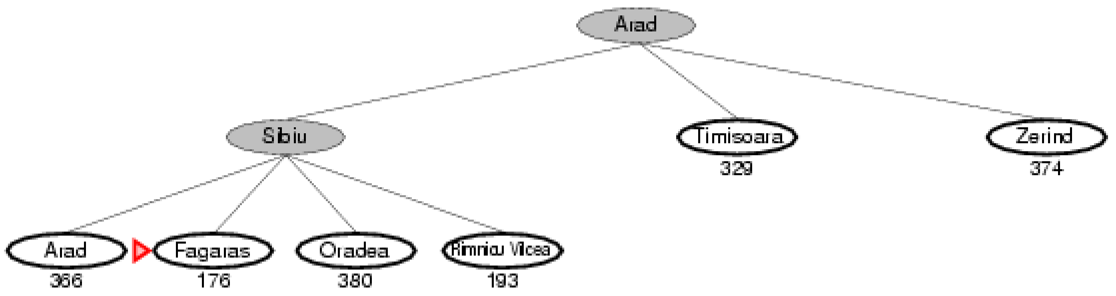
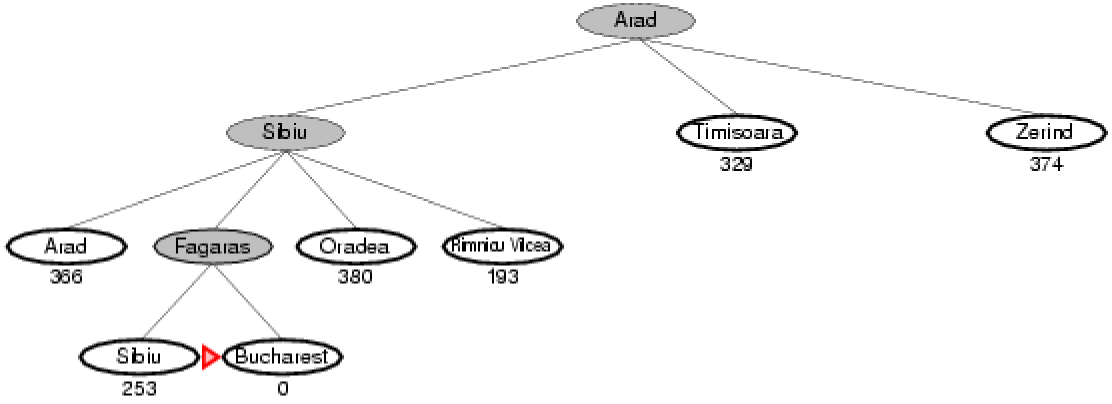


# Greedy best-first search example



# Greedy best-first search example



# Properties of greedy best-first search

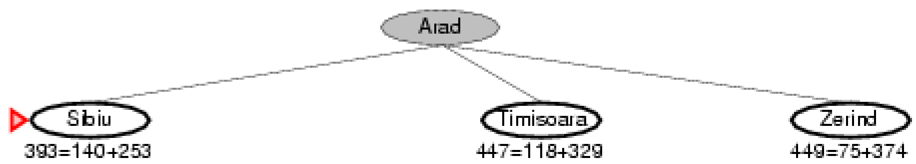
- Complete? No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ -- keeps all nodes in memory
- Optimal? No

# $A^*$ search

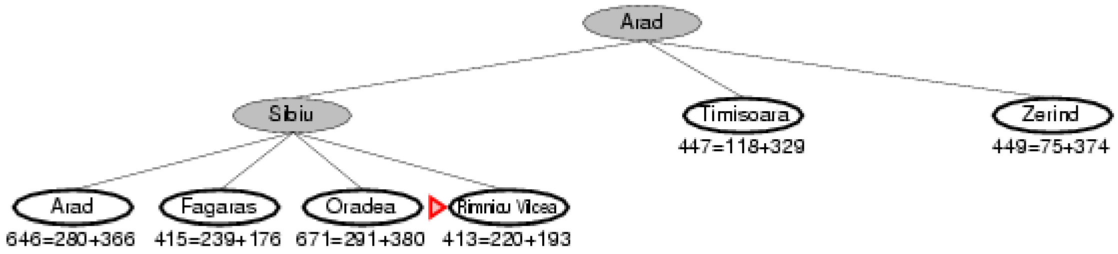
- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
- $g(n)$ = cost so far to reach $n$
- $h(n)$ = estimated cost from $n$ to goal
- $f(n)$ = estimated total cost of path through $n$ to goal

# $A^*$ search example

# A* search example



# $A^*$ search example



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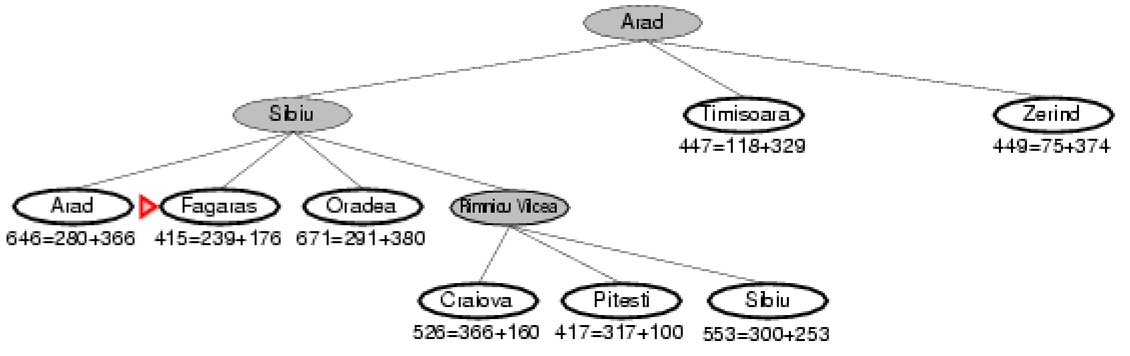
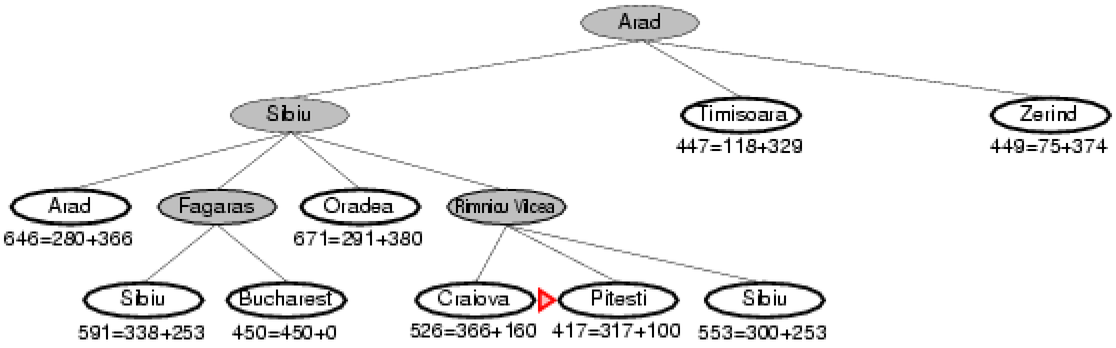
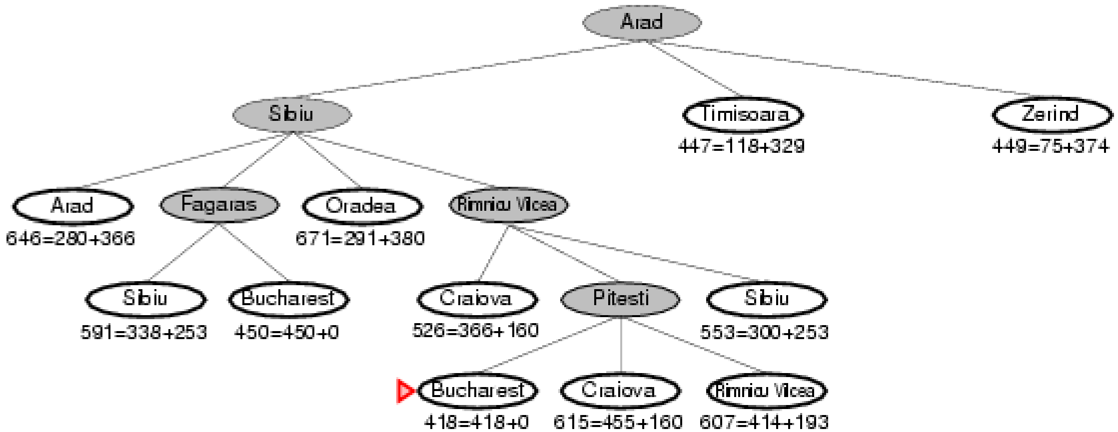


# A* search example



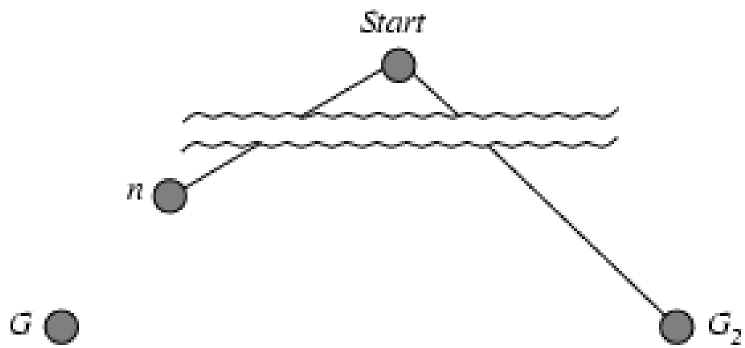
# A* search example



# Admissible heuristics

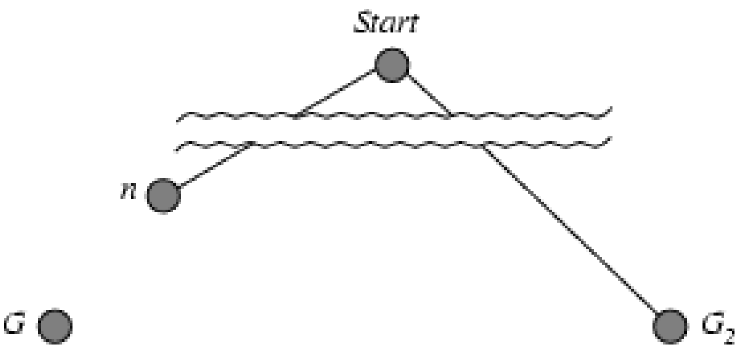
- A heuristic $h(n)$ is admissible if for every node $n$, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from $n$.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A* using `TREE-SEARCH` is optimal

# Optimality of A* (proof)

- Suppose some suboptimal goal $G_2$ has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.
- 


- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > g(G)$ since $G_2$ is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above

# Optimality of A$^*$ (proof)

- Suppose some suboptimal goal $G_2$ has been generated and is in the fringe. Let $n$ be an unexpanded node in the fringe such that $n$ is on a shortest path to an optimal goal $G$.
- 


- f(G$_2$) > f(G) from above
- h(n) ≤ h^*(n) since h is admissible
- g(n) + h(n) ≤ g(n) + h$^*$(n)
- f(n) ≤ f(G)

Hence $f(G_2) > f(n)$, and A$^*$ will never select $G_2$ for expansion

# Consistent heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,
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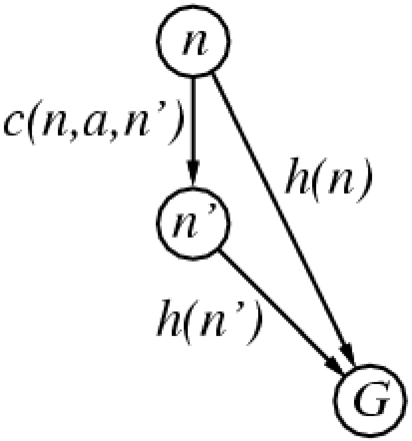


$h(n) \leq c(n,a,n') + h(n')$

- If *h* is consistent, we have
-

$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g(n) + c(n,a,n') + h(n') \\
&\geq g(n) + h(n) \\
&= f(n)
\end{aligned}
$$

- i.e., *f(n)* is non-decreasing along any path.
-
- Theorem: If *h(n)* is consistent, A* using `GRAPH-SEARCH` is optimal

# Optimality of A*

- A* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$
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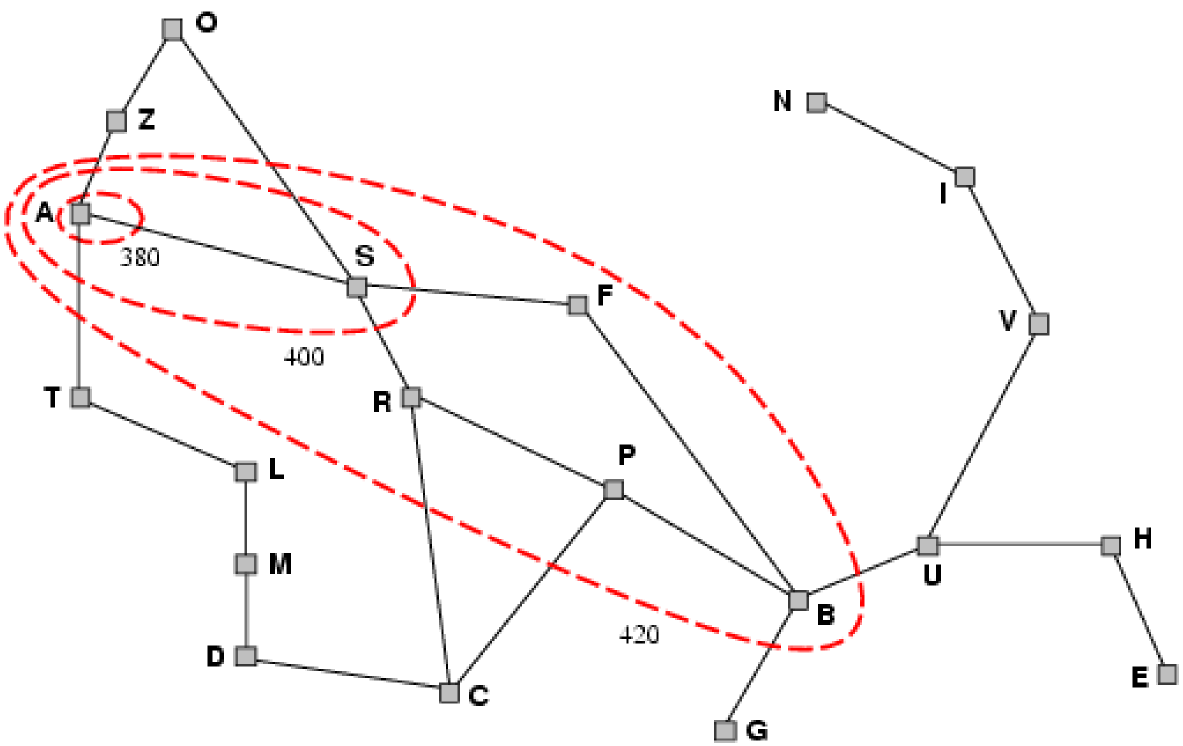


# Properties of A$^*$

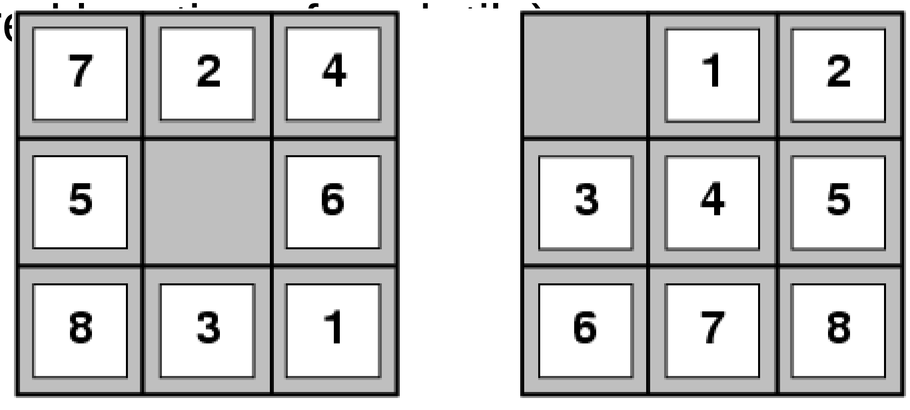
- Complete? Yes (unless there are infinitely many nodes with f ≤ *f(G)* )
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

# Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desire




Start State Goal State

- <u>$h_1(S)$ = ?</u>
- <u>$h_2(S)$ = ?</u>

# Admissible heuristics

E.g., for the 8-puzzle:

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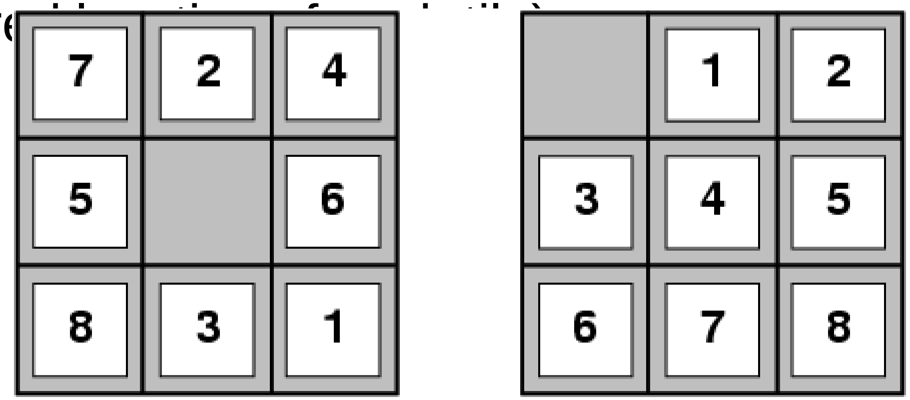


Start State Goal State

- $h_1(S) = ?$ 8
- $h_2(S) = ?$ 3+1+2+2+2+3+3+2 = 18

# Dominance

- If $h_2(n) \geq h_1(n)$ for all $n$ (both admissible)
- then $h_2$ dominates $h_1$
- $h_2$ is better for search
- 
- Typical search costs (average number of nodes expanded):
- 
- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes
- 

# Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- 
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- 
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- 
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

# Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution
- 

- State space = set of "complete" configurations
- Find configuration satisfying constraints, e.g., n-queens

- In such cases, we can use local search algorithms
- keep a single "current" state, try to improve it
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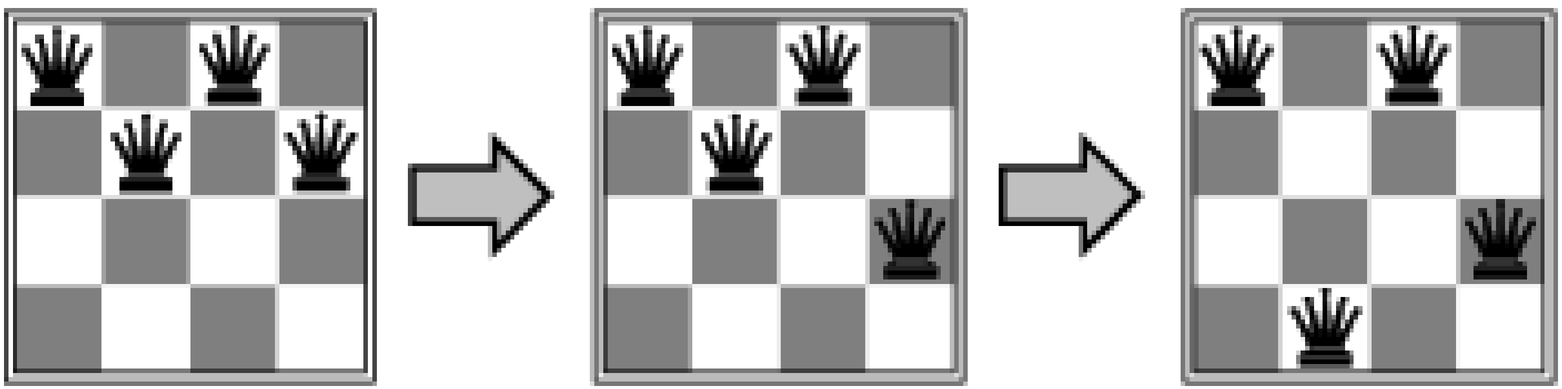
# Example: *n*-queens

- Put $n$ queens on an $n \times n$ board with no two queens on the same row, column, or diagonal
- 

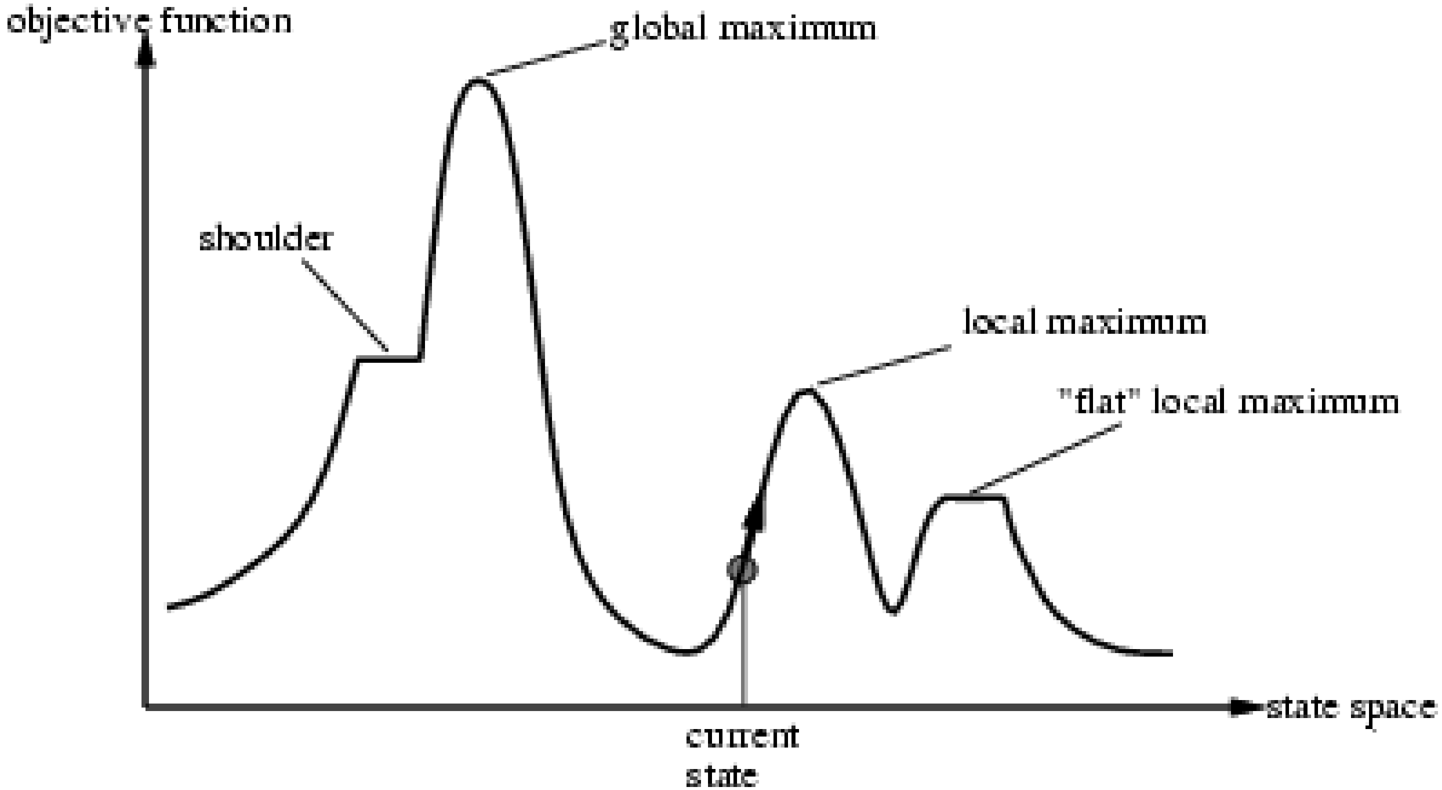
# Hill-climbing search

- "Like climbing Everest in thick fog with amnesia"
- 

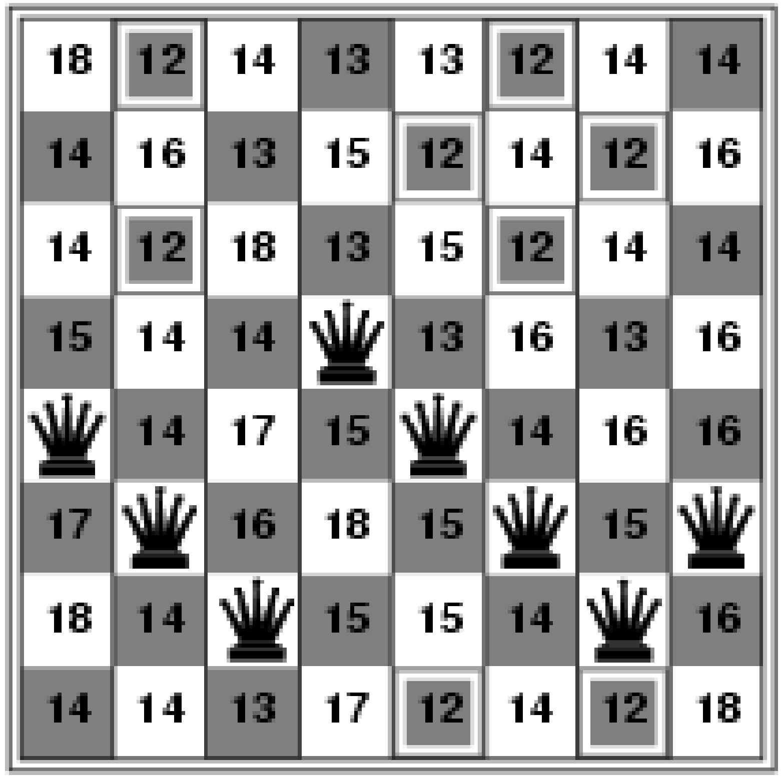
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node

    current ← MAKE-NODE(INITIAL-STATE[problem])
    loop do
        neighbor ← a highest-valued successor of current
        if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
        current ← neighbor
```

# Hill-climbing search

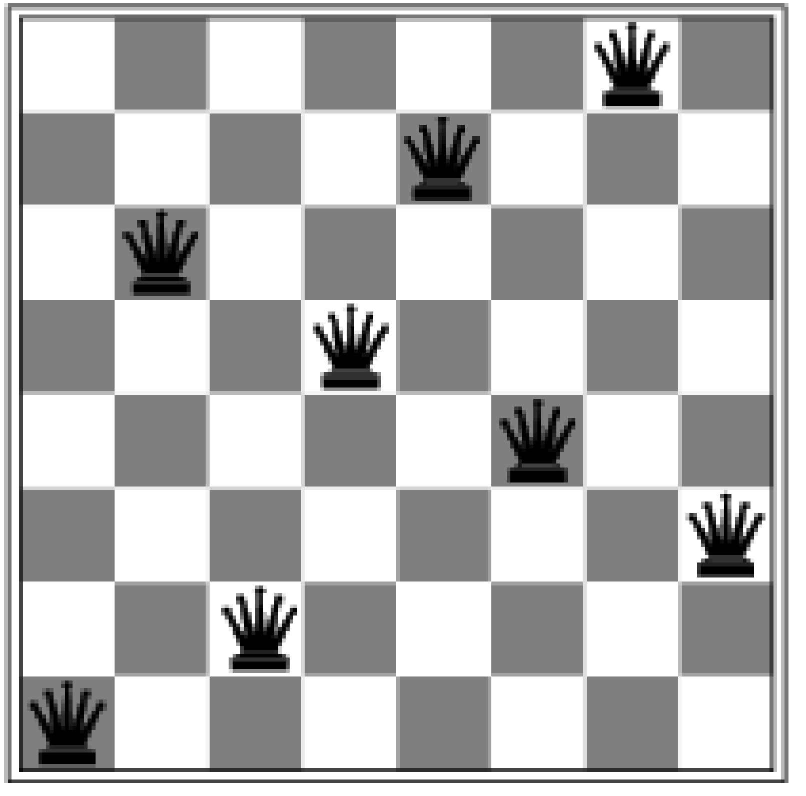
- Problem: depending on initial state, can get stuck in local maxima
- 

# Hill-climbing search: 8-queens problem




- *h* = number of pairs of queens that are attacking each other, either directly or indirectly
- *h = 17* for the above state
- 

# Hill-climbing search: 8-queens problem



- A local minimum with *h = 1*
- 

# Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency
- 

```
function SIMULATED-ANNEALING( problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(ΔE/T)
```

# Properties of simulated annealing search

- One can prove: If *T* decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1
- 
- Widely used in VLSI layout, airline scheduling, etc
- 

# Local beam search

- Keep track of *k* states rather than just one
- Start with *k* randomly generated states

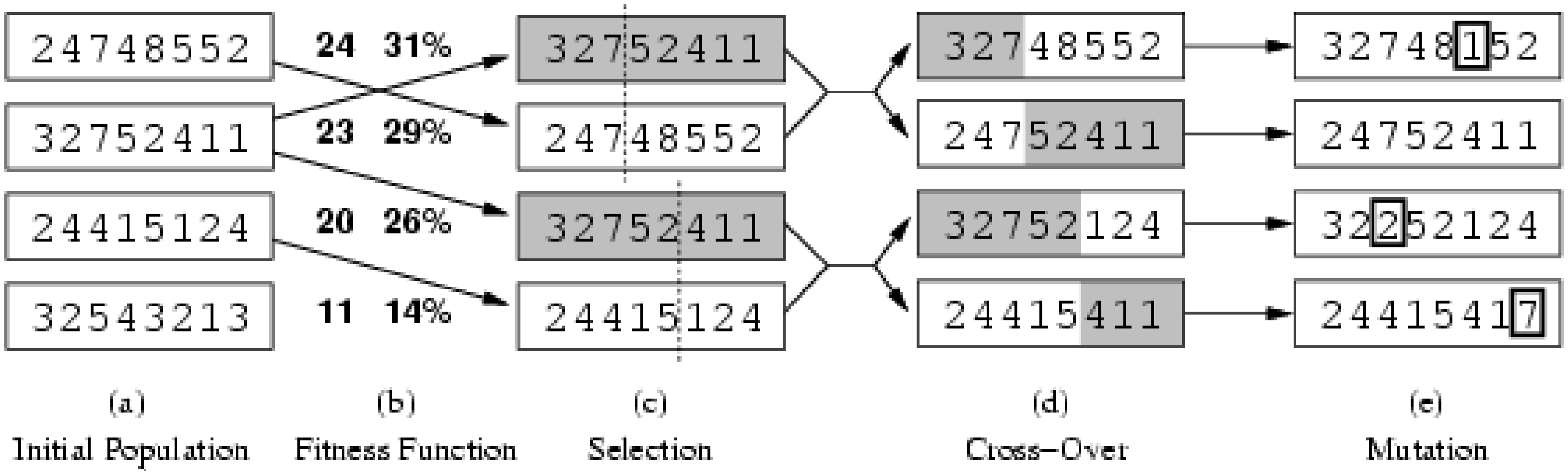
- At each iteration, all the successors of all *k* states are generated

- If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat

# Genetic algorithms

- A successor state is generated by combining two parent states
- Start with *k* randomly generated states (population)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher values for better states.
- Produce the next generation of states by selection, crossover, and mutation

# Genetic algorithms




- Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)
- 24/(24+23+20+11) = 31%
- 23/(24+23+20+11) = 29% etc

# Genetic algorithms