
附录 截面的几何性质
Appendix Ⅰ Properties of Plane Areas

附录Ⅰ 截面的几何性质

(Appendix Ⅰ Properties of plane areas)

- §1-1 截面的静矩和形心(The first moments of the area & centroid of an area)
- §1-2 极惯性矩 惯性矩 惯性积 (Polar moment of inertia Moment of inertia Product of inertia)
- §1-3平行移轴公式 (Parallel-Axis theorem)

§1-1 截面的静矩和形心

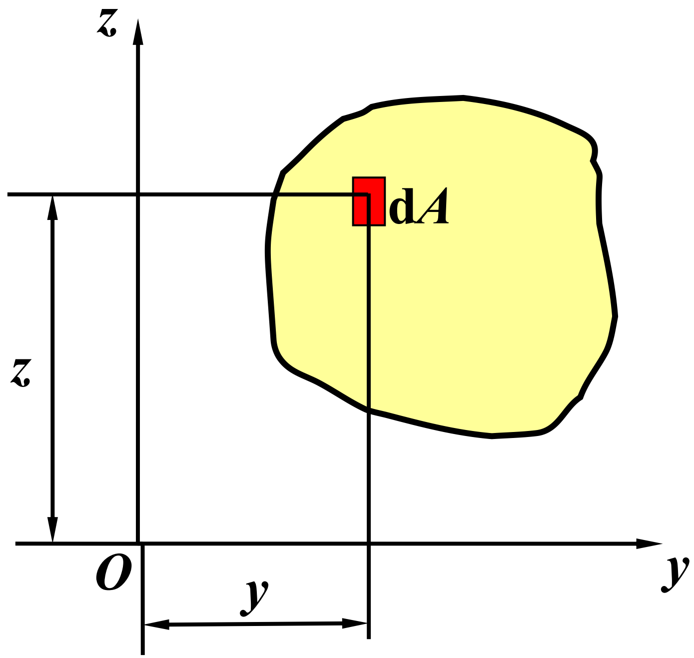
(The first moment of the area & centroid of an area)

一、静矩(The first moment of the area）

截面对 y , z 轴的静矩为

$$S_y = \int_A z\mathrm{d}A$$

$$S_z = \int_A y\mathrm{d}A$$



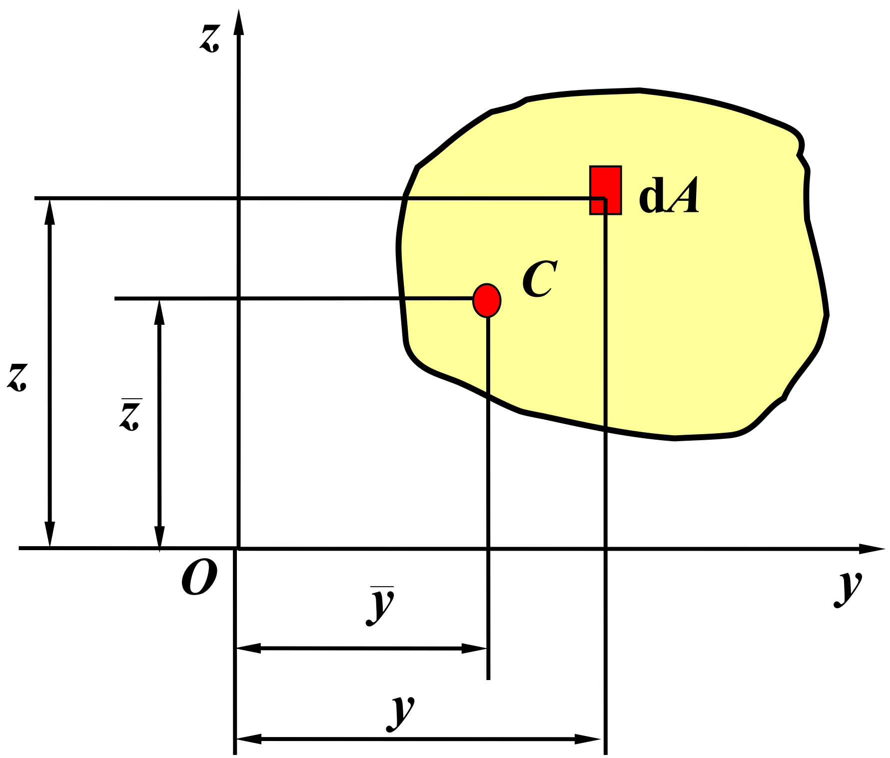
静矩可正，可负，也可能等于零.

二、截面的形心(Centroid of an area)

$$\bar{z} = \frac{\int_A z\mathrm{d}A}{A} = \frac{S_y}{A}$$

$$\bar{y} = \frac{\int_A y\mathrm{d}A}{A} = \frac{S_z}{A}$$

$$S_y = A\bar{z} \qquad S_z = A\bar{y}$$



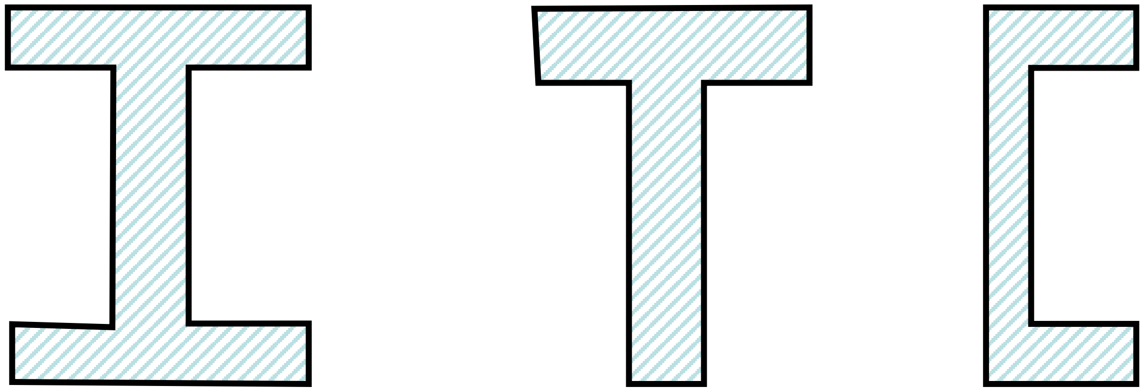
（1）若截面对某一轴的静矩等于零，则该轴必过形心.

（2）截面对形心轴的静矩等于零.

三、组合截面的静矩和形心

(The first moments &centroid of a composite area)

由几个简单图形组成的截面称为组合截面.



截面各组成部分对于某一轴的静矩之代数和，等于该截面对于同一轴的静矩.

1. 组合截面静矩(The first moments of a composite area)

$$S_y = \sum_{i=1}^{n} A_i \bar{z}_i \qquad S_z = \sum_{i=1}^{n} A_i \bar{y}_i$$

其中 A_i—第 i个简单截面面积

$(\bar{z}_i, \bar{y}_i)$ —第 i个简单截面的形心坐标

2. 组合截面形心(Centroid of a composite area)

$$\bar{z} = \frac{\sum_{i=1}^{n} A_i \bar{z}_i}{\sum_{i=1}^{n} A_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} A_i \bar{y}_i}{\sum_{i=1}^{n} A_i}$$

例题 试确定图示截面形心C的位置.

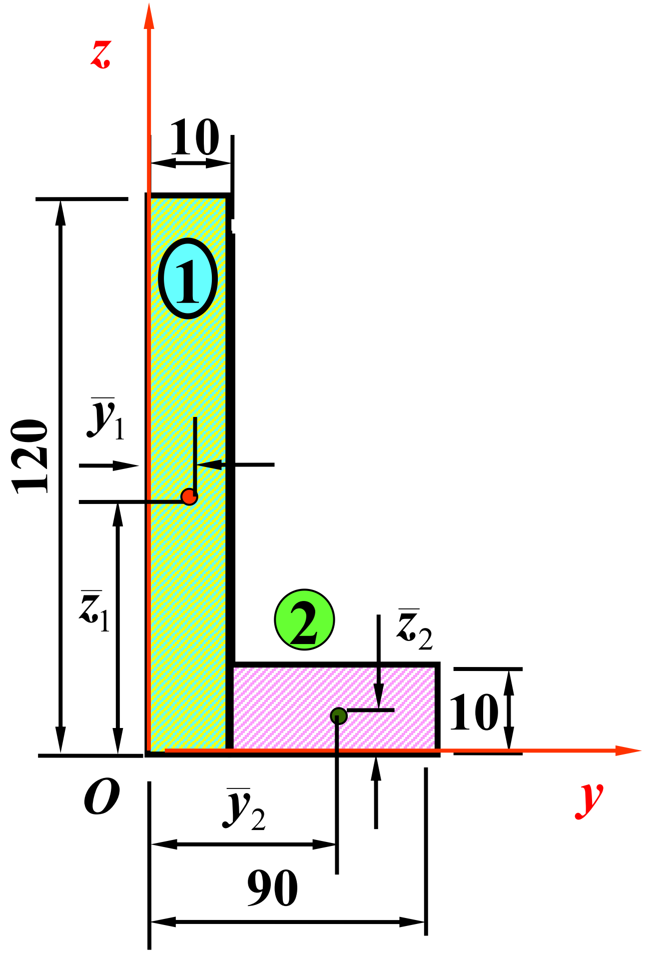
解：组合图形，用正负面积法解之.

方法1 用正面积法求解. 将截面分为1，2两个矩形.

取 z 轴和 y 轴分别与截面的底边和左边缘重合

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i \bar{y}_i}{\sum_{i=1}^{n} A_i} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{A_1 + A_2}$$

$$\bar{z} = \frac{A_1 \bar{z}_1 + A_2 \bar{z}_2}{A_1 + A_2}$$



图(a)

矩形 1 $A_1 = 10 \times 120 = 1200\text{mm}^2$

$\bar{y}_1 = 5\text{mm}$ $\bar{z}_1 = 60\text{mm}$

矩形 2 $A_2 = 10 \times 80 = 800\text{mm}^2$

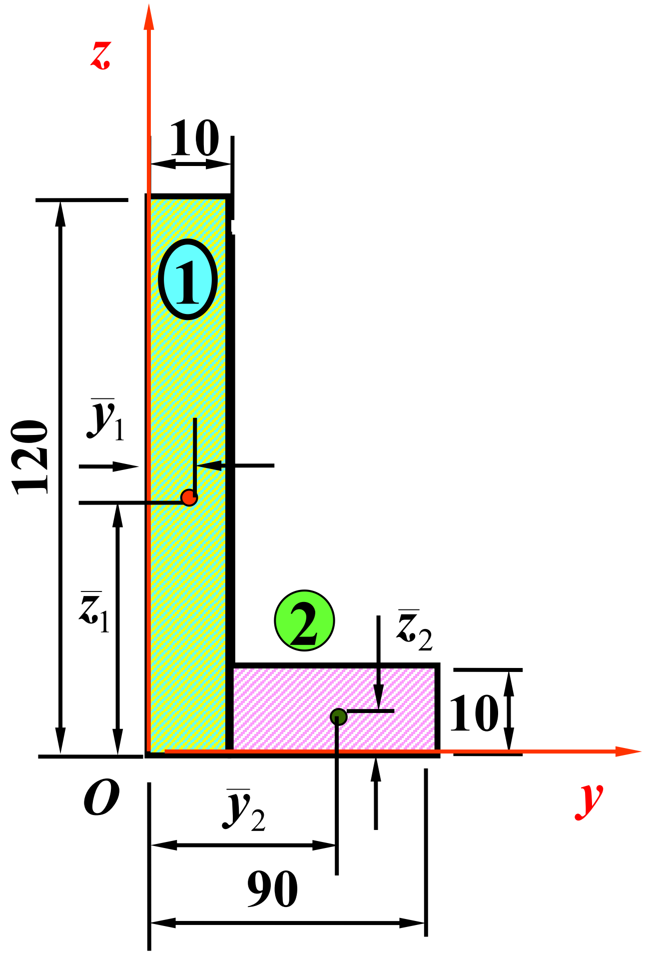
$$\bar{y}_2 = 10 + \frac{80}{2} = 50\text{mm}$$

$\bar{z}_2 = 5\text{mm}$

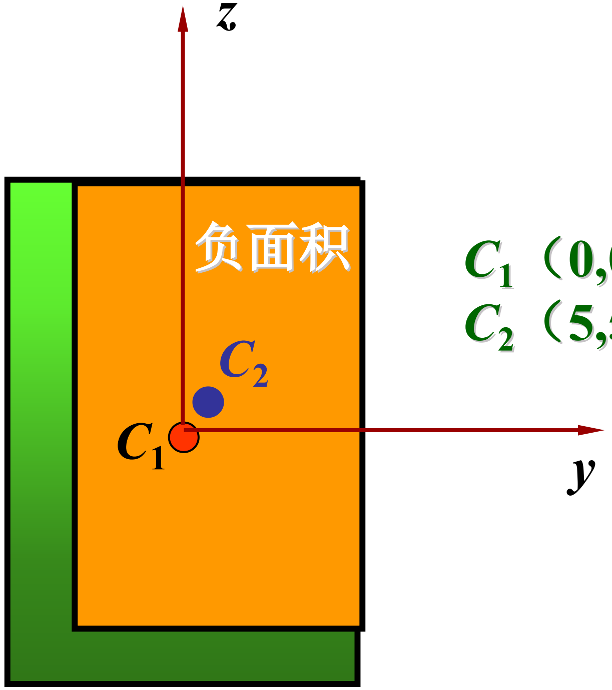
所以

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = 23\text{mm}$$

$$\bar{z} = \frac{A_1\bar{z}_1 + A_2\bar{z}_2}{A_1 + A_2} = 38\text{mm}$$



方法2　用负面积法求解，图形分割及坐标如图(b)



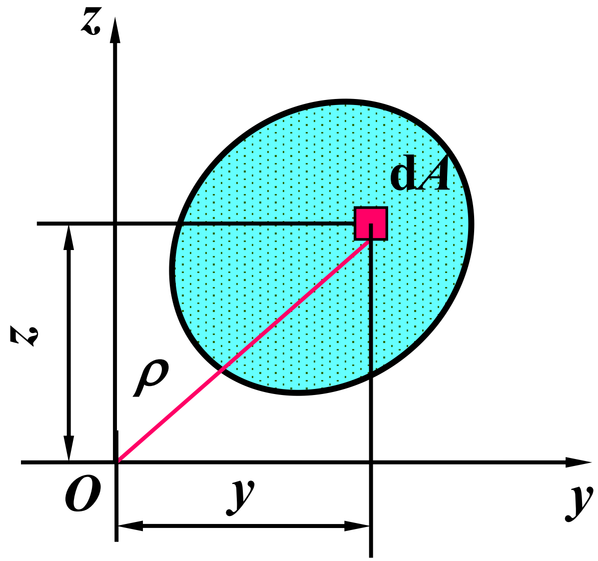
C_1（0,0）
C_2（5,5）

图(b)

$$\bar{y}=\frac{\sum \bar{y}_i A_i}{A}=\frac{\bar{y}_1 A_1+\bar{y}_2 A_2}{A_1+A_2}=$$

$$\frac{5\times(-80\times110)}{120\times90-80\times110}=-22$$

§1-2 极惯性矩、惯性矩、惯性积 (Polar moment of inertia、Moment of inertia、Product of inertia)



一、惯性矩(Moment of inertia)

$$I_y = \int_A z^2 \mathrm{d}A$$

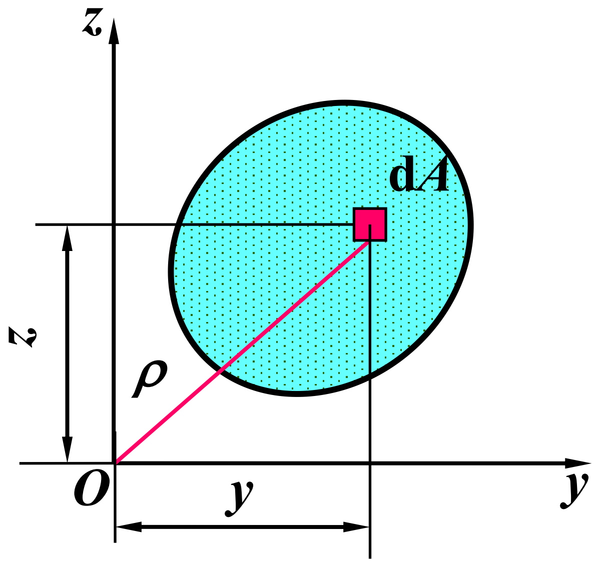
$$I_z = \int_A y^2 \mathrm{d}A$$

二、极惯性矩 (Polar moment of inertia）

$$I_P = \int_A \rho^2 \mathrm{d}A \qquad \rho^2 = z^2 + y^2 \qquad I_P = \int_A \rho^2 \mathrm{d}A$$

所以 $I_P = I_z + I_y$

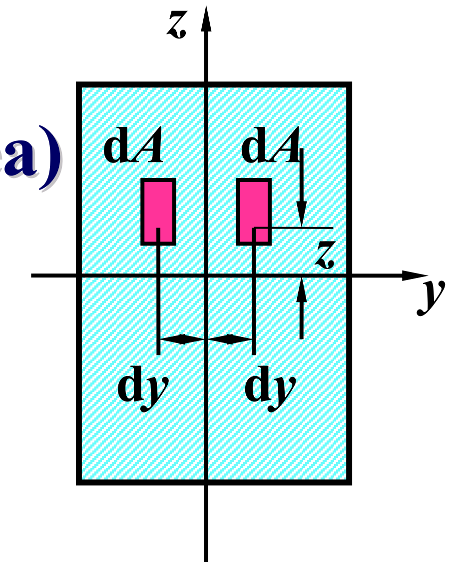
三、惯性积 (Product of inertia)



$$I_{yz} = \int_A yz\mathrm{d}A$$

（1）惯性矩的数值恒为正，惯性积则可能为正值，负值，也可能等于零；

（2）若y，z 两坐标轴中有一个为截面的对称轴，则截面对y，z轴的惯性积一定等于零.



四、惯性半径(Radius of gyration of the area)

$$i_y = \sqrt{\frac{I_y}{A}} \qquad i_z = \sqrt{\frac{I_z}{A}}$$

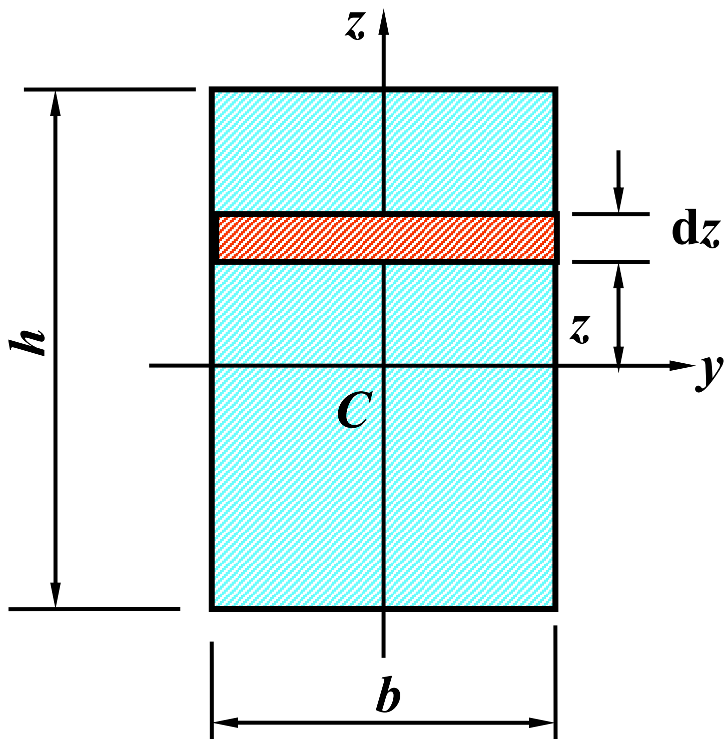
例题 求矩形截面对其对称轴y, z轴的惯性矩.

解:
$$I_y=\int_A z^2\mathrm{d}A$$

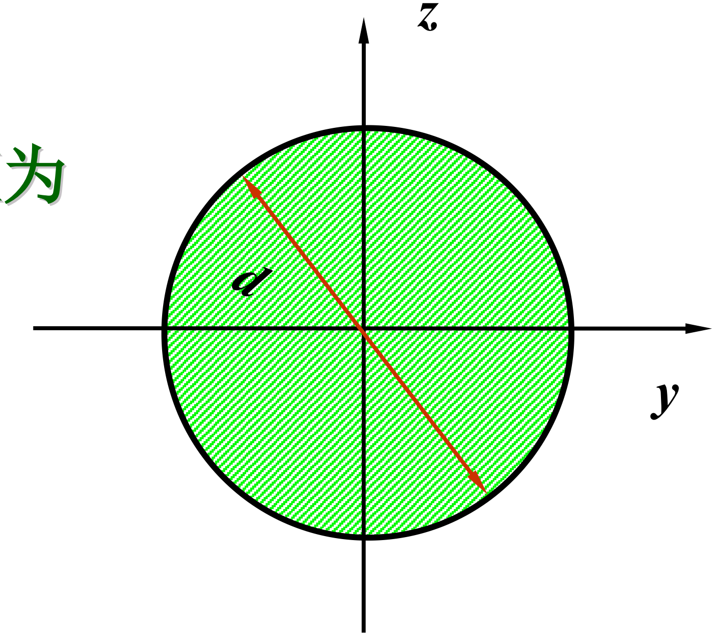
$$\mathrm{d}A=b\mathrm{d}z$$

$$I_y=\int_A z^2\mathrm{d}A=\int_{-\frac{h}{2}}^{\frac{h}{2}}bz^2\mathrm{d}z=\frac{bh^3}{12}$$

$$I_z=\frac{hb^3}{12}$$



例题 求圆形截面对其对称轴的惯性矩.



解：因为截面对其圆心 O 的极惯性矩为

$$I_{\mathrm{P}} = \frac{\pi d^4}{32}$$

$$I_y + I_z = I_{\mathrm{P}}$$

$$I_y = I_z$$

所以 $$I_y = I_z = \frac{\pi d^4}{64}$$

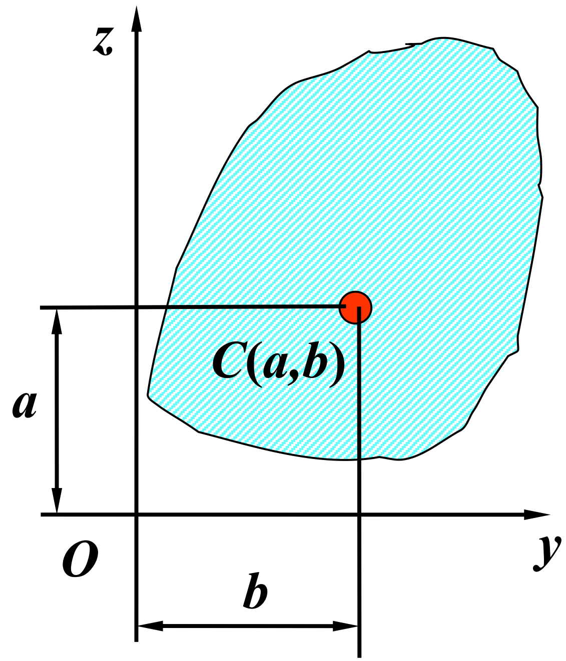
§1-3 平行移轴公式 (Parallel-axis theorem)

一、平行移轴公式(Parallel-Axis theorem for moment of inertia)

y,z — 任意一对坐标轴

C —截面形心

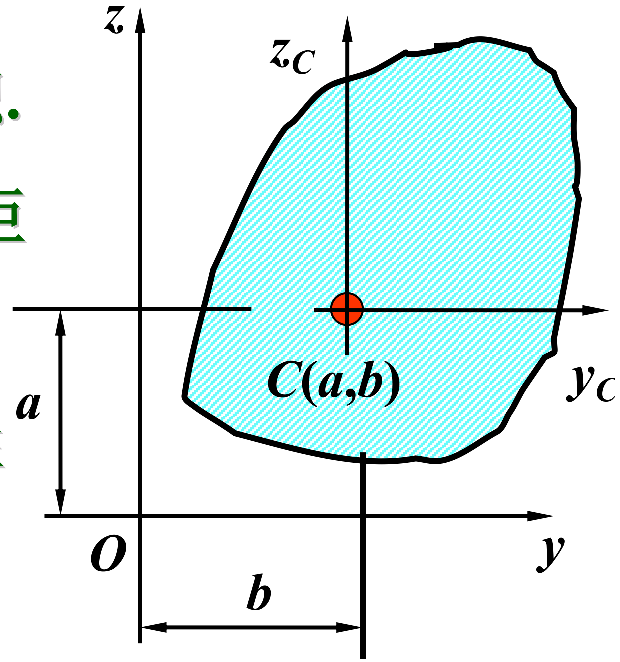
$(a\,,\,b\,)$ —形心C在 yOz坐标系下的坐标



y_C , z_C －过截面的形心 C 且与 y, z轴平行的坐标轴(形心轴）

I_y , I_z , I_{yz} －截面对 y, z 轴的惯性矩和惯性积.

I_{y_C} , I_{z_C} , $I_{y_C z_C}$ －截面对形心轴 y_C, z_C的惯性矩和惯性积.



已知截面对形心轴 y_C,z_C 的惯性矩和惯性积，求截面对与形心轴平行的 y,z轴惯性矩和惯性积，则平行移轴公式

$$I_y = I_{y_C} + a^2 A$$

$$I_z = I_{z_C} + b^2 A$$

$$I_{yz} = I_{y_C z_C} + abA$$

二、组合截面的惯性矩 、惯性积(Moment of inertia & product of inertia for composite areas ）

组合截面的惯性矩，惯性积

$$I_y = \sum_{i=1}^{n} I_{yi}$$

$$I_z = \sum_{i=1}^{n} I_{zi}$$

$$I_{yz} = \sum_{i=1}^{n} I_{yzi}$$

I_{yi}, I_{zi}, I_{yzi}—第 i个简单截面对 y, z 轴的惯性矩， 惯性积.

例题 求T形截面对其形心轴 y_C 的惯性矩.

解：将截面分成两个矩形截面.

截面的形心必在对称轴 z_C 上.

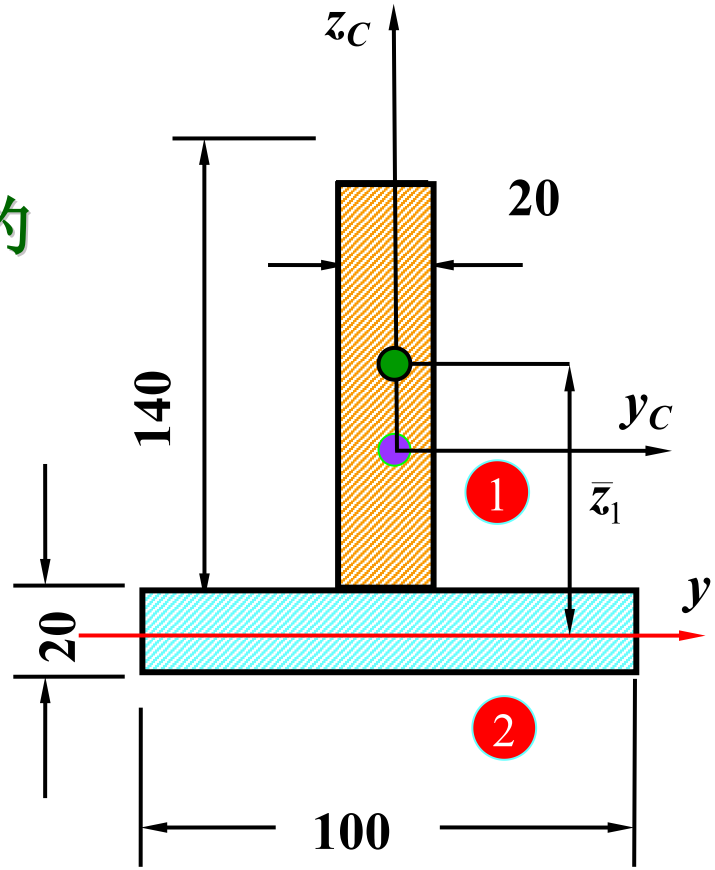
取过矩形 2 的形心且平行于底边的轴作为参考轴记作 y轴.

$$A_1 = 20 \times 140 \qquad \bar{z}_1 = 80$$

$$A_2 = 100 \times 20 \qquad \bar{z}_2 = 0$$

所以截面的形心坐标为

$$\bar{z}_C = \frac{A_1 \bar{z}_1 + A_2 \bar{z}_2}{A_1 + A_2} = 46.7\text{mm}$$

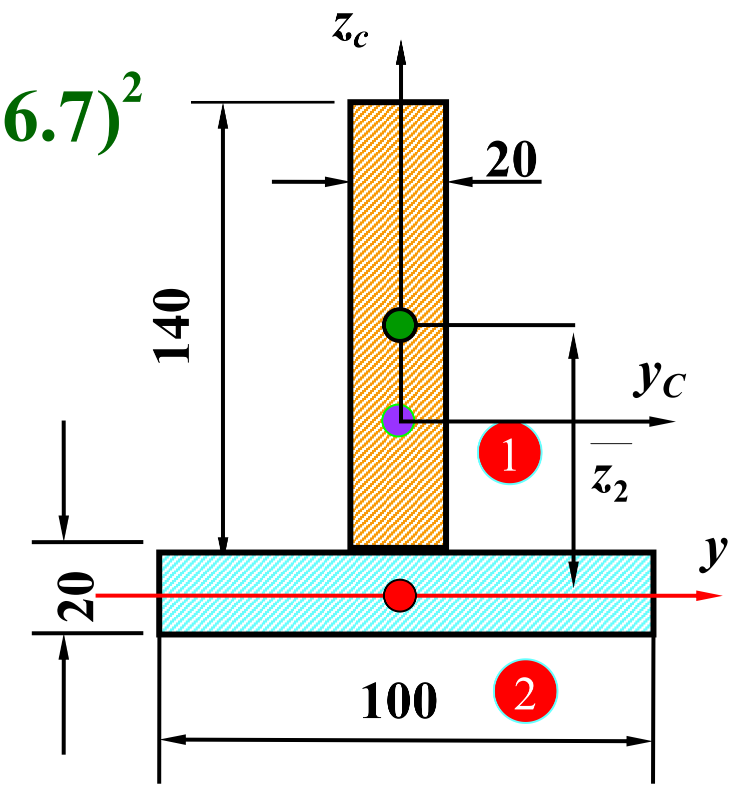


$$I_y = I_{y_C} + a^2 A$$

$$I_{y_C}^1 = \frac{1}{12} \times 20 \times 140^3 + 20 \times 140 \times (80 - 46.7)^2$$

$$I_{y_C}^2 = \frac{1}{12} \times 100 \times 20^3 + 100 \times 20 \times (46.7)^2$$

$$I_{y_C} = I_{y_C}^1 + I_{y_C}^2 = 12.12 \times 10^{-6} \mathrm{m}^4$$



附录结束

作业

I.2(b)(d)

I.6