無線通訊 HW2

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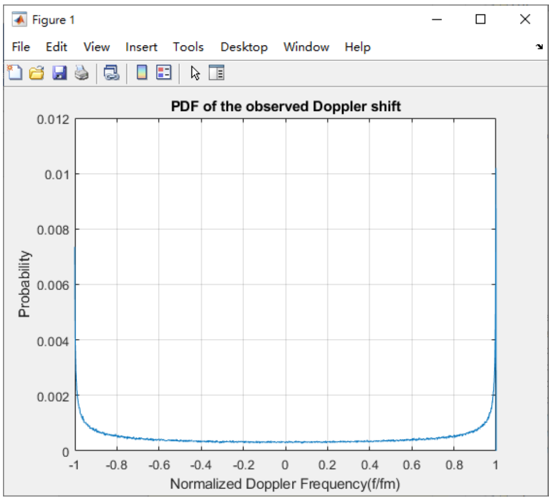
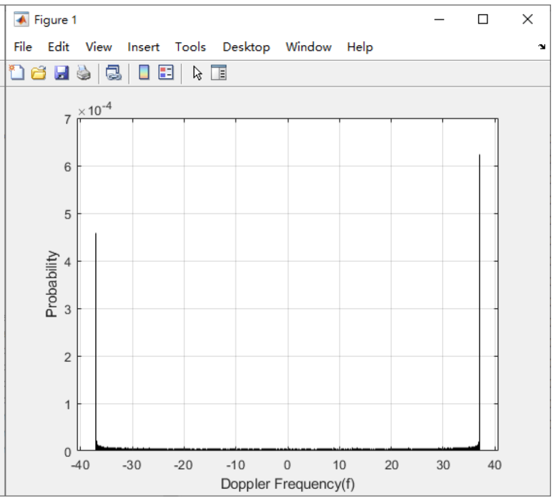
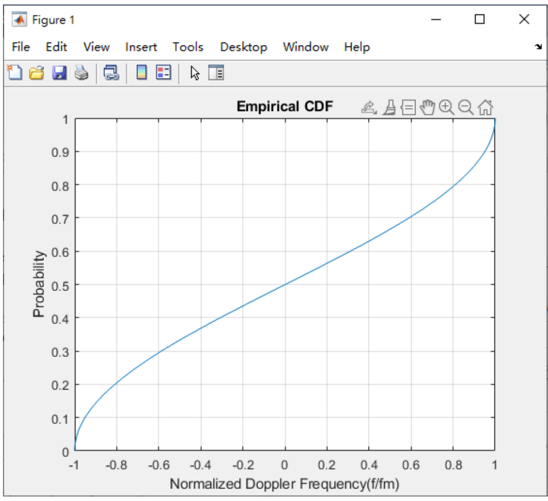
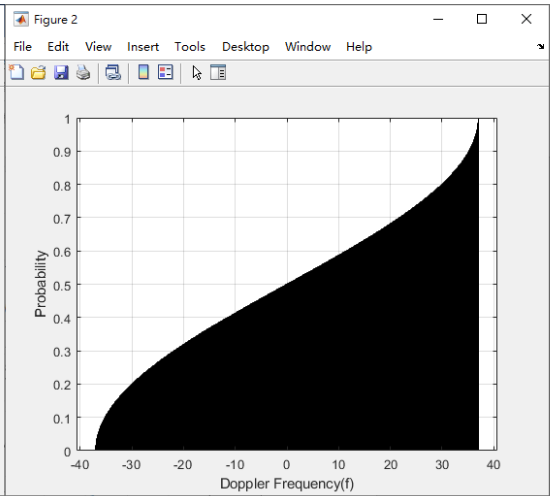
1. (40%) Consider that an MS with a velocity v receives an unmodulated carrier with a frequency f_c. The incidence angle $\theta(t)$ of the incoming wave is assumed to be uniformly distributed between $-\pi$ and π.
   a) If $v = 20$ km/hr and $f_c = 2$ GHz, find the distribution function (cdf) and the probability density function (pdf) of the observed Doppler shift via simulation.
   b) If $v = 90$ km/hr and $f_c = 26$ GHz, find the cdf and the pdf of the observed Doppler shift via simulation.
   c) If $f_c = 2$ GHz and v is uniformly distributed between 20 km/hr and 90 km/hr, find the cdf and the pdf of the observed Doppler shift via simulation.
   d) Derive the cdf and the pdf of the observed Doppler shift for fixed v and f_c. Compare the simulation results with the theoretical results.

a)









b)

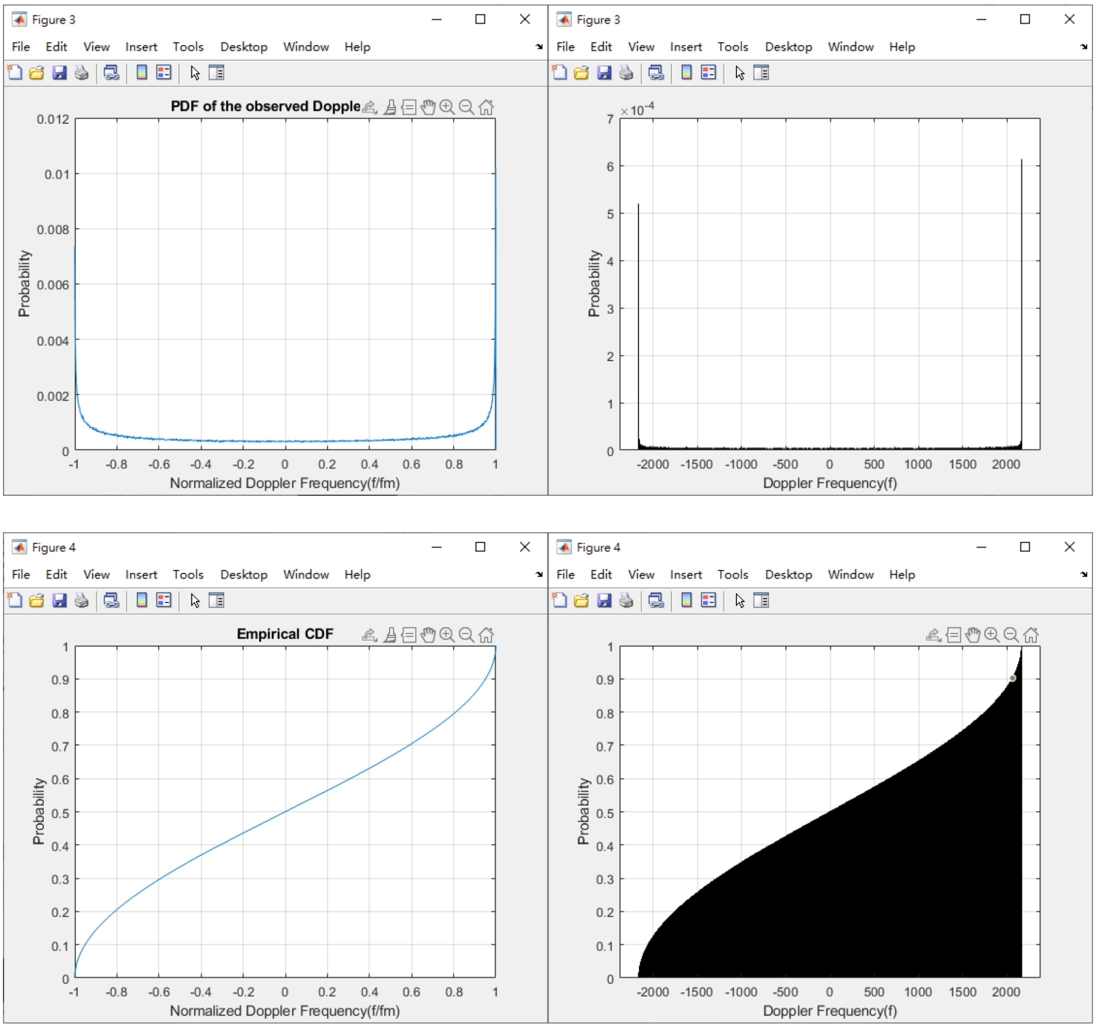


Figure 3
PDF of the observed Dopple
Probability
Normalized Doppler Frequency(f/fm)
Figure 3
Probability
Doppler Frequency(f)
Figure 4
Empirical CDF
Probability
Normalized Doppler Frequency(f/fm)
Figure 4
Probability
Doppler Frequency(f)

c)

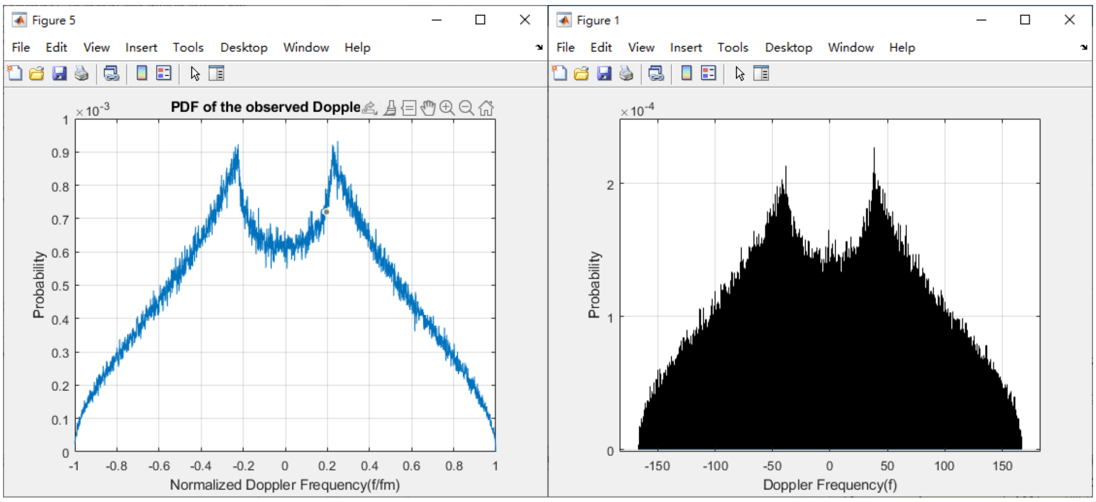
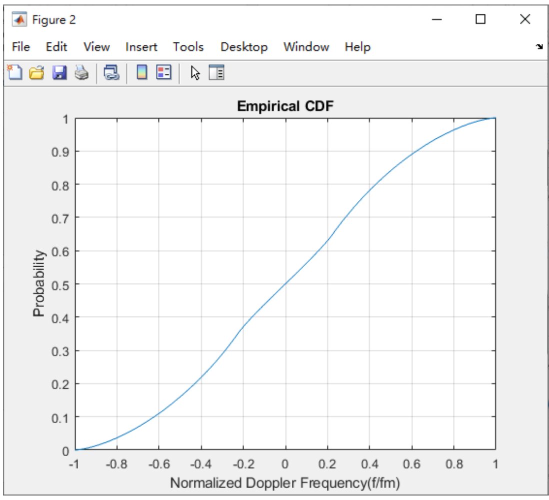
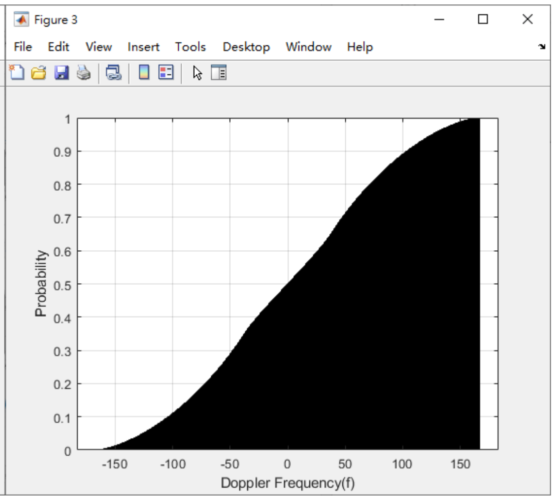
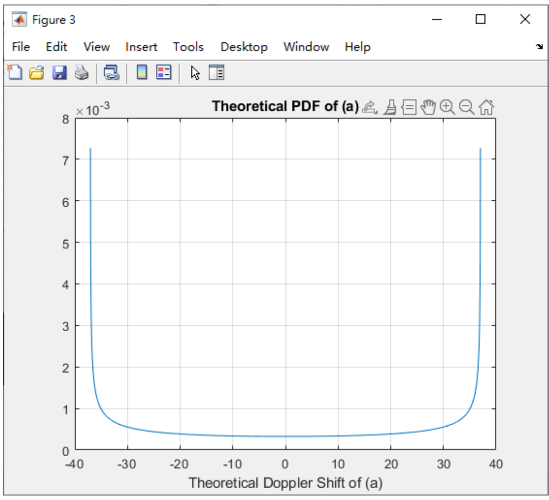


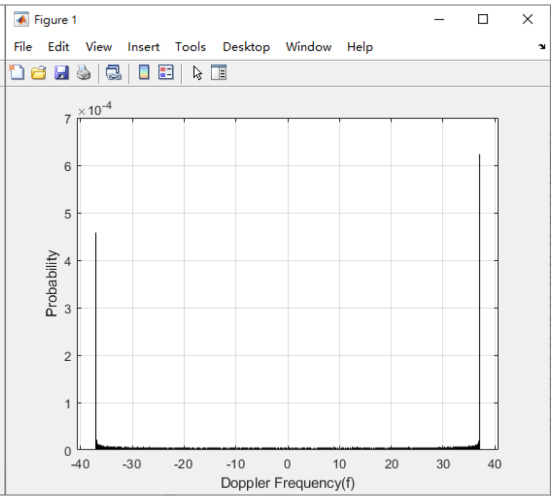
Figure 5
PDF of the observed Dopple
Probability
Normalized Doppler Frequency(f/fm)
Figure 1
Probability
Doppler Frequency(f)

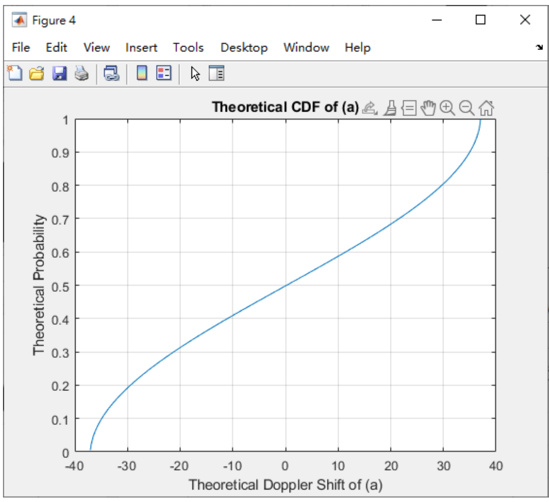


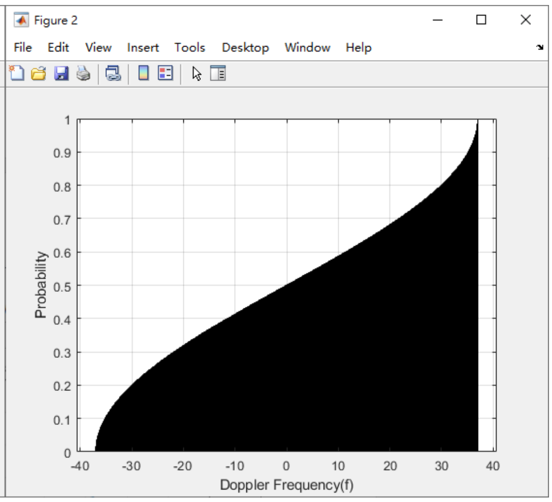


d)









討論：

pdf of $\boldsymbol{\theta}$: $f(\theta) = \frac{1}{2\pi}$, $-\pi \le \theta \le \pi$　　令r.v $Y = f_D = f_m * \cos(\theta)$

$$f_Y(\mathrm{y}) = f\left(\theta = cos^{-1}\left(\frac{y}{f_m}\right)\right)\left|\frac{d\left(-\pi - cos^{-1}\left(\frac{y}{f_m}\right)\right)}{dx}\right| \text{ for } -\pi \le \theta \le 0$$

$$= \frac{1}{2\pi} * \frac{1}{f_m * \sqrt{1-\cos(\theta)^2}} \text{ , } -f_m \le y \le f_m$$

$$f_Y(\mathrm{y}) = f\left(\theta = cos^{-1}\left(\frac{y}{f_m}\right)\right)\left|\frac{d\left(cos^{-1}\left(\frac{y}{f_m}\right)\right)}{dx}\right| \qquad \text{for } 0 \le \theta \le \pi$$

$$= \frac{1}{2\pi} * \frac{1}{f_m * \sqrt{1-\cos(\theta)^2}} \text{ , } -f_m \le y \le f_m$$

所以 **pdf of** f_D $f_Y(\mathrm{y}) = \frac{1}{\pi} * \frac{1}{f_m * \sqrt{1-\cos(\theta)^2}}$, $-f_m \le y \le f_m$,

上述為利用機率的變數變換所推導 Doppler Shift 的分布。
由 d) 左側為理論結果和右側模擬結果來比較，發現單點機率有差別，原因是理論值只有 2000 個而模擬產生的亂數卻有 1000000 個之多，也因而降低了單點機率(為了力求曲線平滑的結果)。然而雖然如此，可由兩邊 pdf 總合為一表示符合機率定理。