Section 7: Memory and Caches

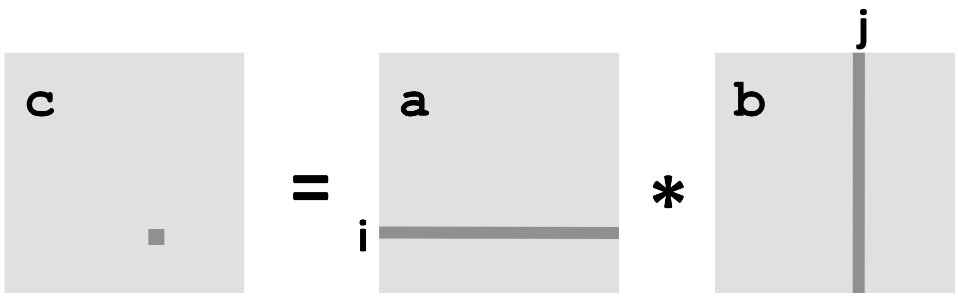
- ~~Cache basics~~
- ~~Principle of locality~~
- ~~Memory hierarchies~~
- ~~Cache organization~~
- Program optimizations that consider caches

Optimizations for the Memory Hierarchy

- **Write code that has locality**
  - Spatial: access data contiguously
  - Temporal: make sure access to the same data is not too far apart in time
- **How to achieve?**
  - Proper choice of algorithm
  - Loop transformations

Example: Matrix Multiplication

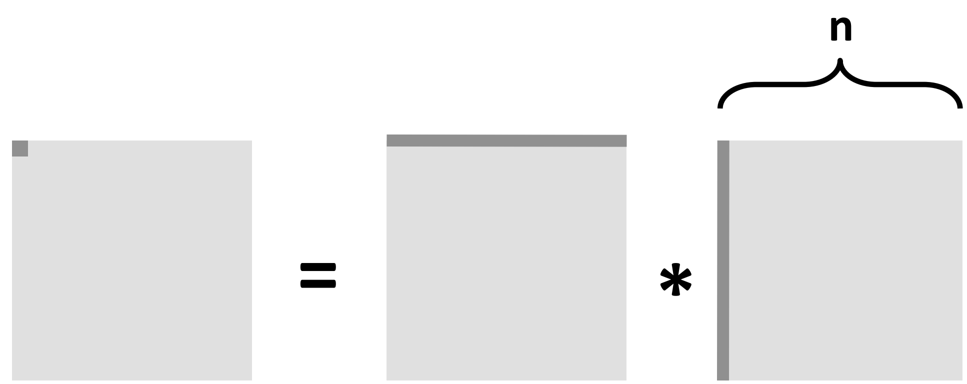
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k]*b[k*n + j];
}
```



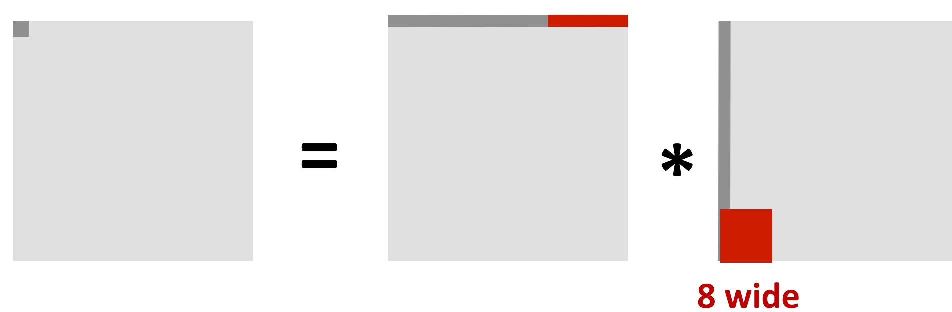
Cache Miss Analysis

- **Assume:**
  - Matrix elements are doubles
  - Cache block = 64 bytes = 8 doubles
  - Cache size C << n (much smaller than n)

- **First iteration:**
  - n/8 + n = 9n/8 misses (omitting matrix c)



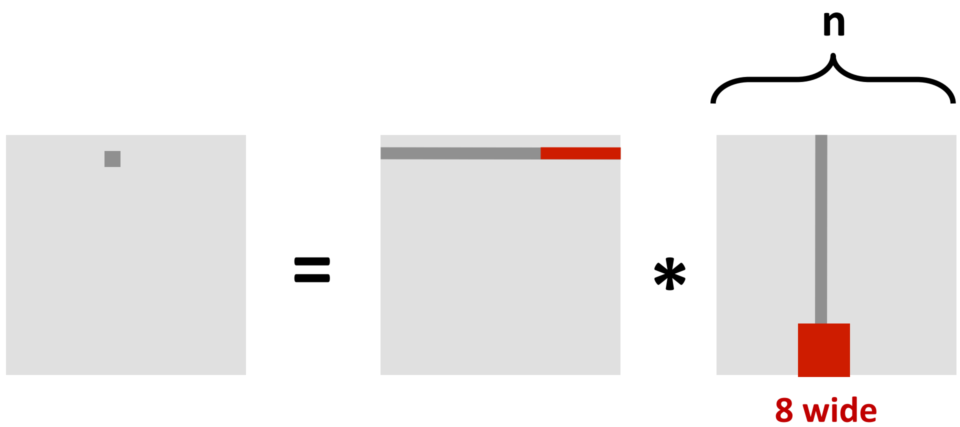
  - Afterwards in cache: (schematic)



Cache Miss Analysis

- **Assume:**
  - Matrix elements are doubles
  - Cache block = 64 bytes = 8 doubles
  - Cache size C << n (much smaller than n)

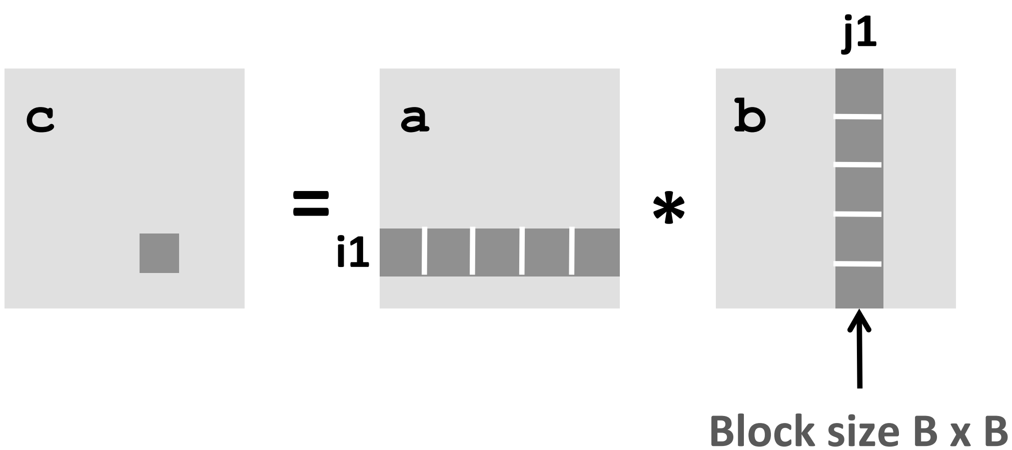
- **Other iterations:**
  - Again:
    n/8 + n = 9n/8 misses
    (omitting matrix c)



- **Total misses:**
  - $9n/8 * n^2 = (9/8) * n^3$

Blocked Matrix Multiplication

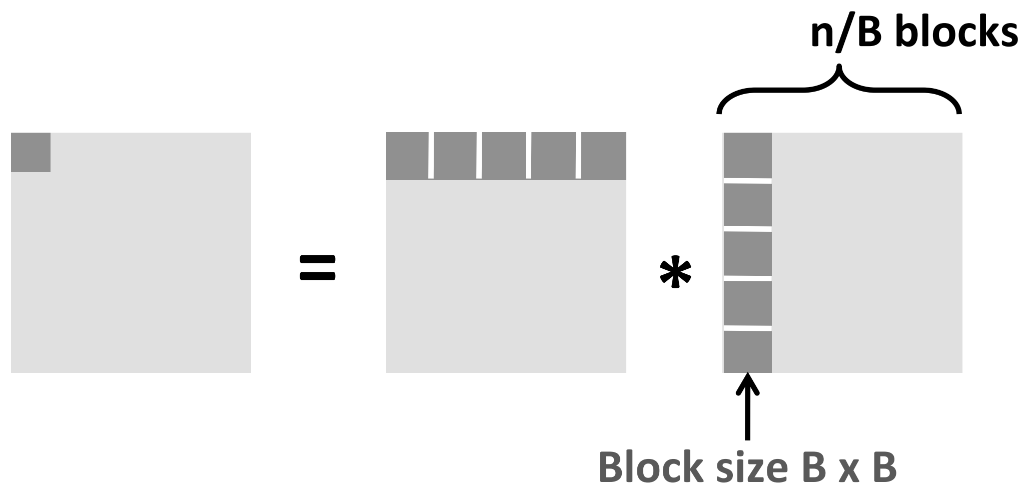
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
         for (j = 0; j < n; j+=B)
              for (k = 0; k < n; k+=B)
                   /* B x B mini matrix multiplications */
                   for (i1 = i; i1 < i+B; i1++)
                        for (j1 = j; j1 < j+B; j1++)
                            for (k1 = k; k1 < k+B; k1++)
                                 c[i1*n + j1] += a[i1*n + k1]*b[k1*n + j1];
}
```



Cache Miss Analysis

- **Assume:**
  - Cache block = 64 bytes = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$

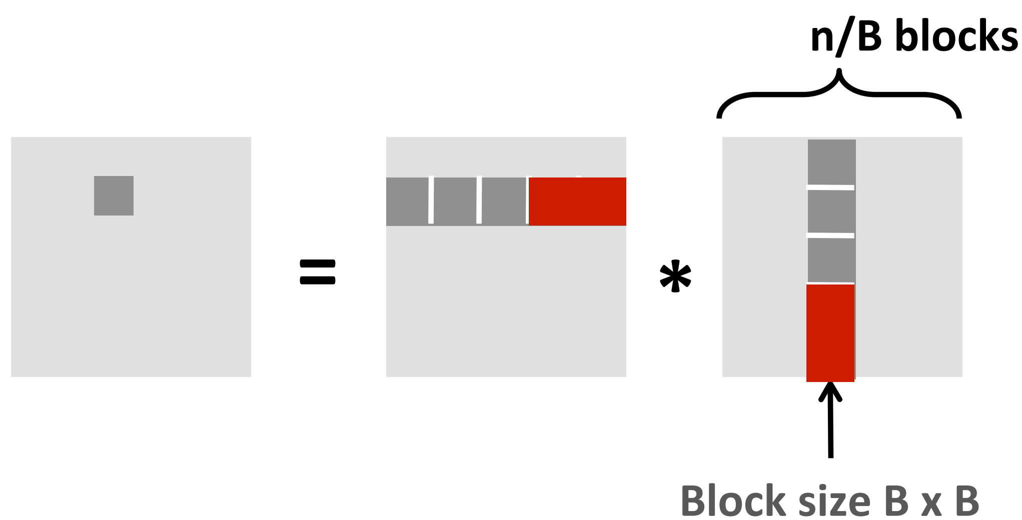
- **First (block) iteration:**
  - $B^2/8$ misses for each block
  - $2n/B * B^2/8 = nB/4$ (omitting matrix c)



  - Afterwards in cache (schematic)

Cache Miss Analysis

- **Assume:**
  - Cache block = 64 bytes = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$
- **Other (block) iterations:**
  - Same as first iteration
  - $2n/B * B^2/8 = nB/4$



- **Total misses:**
  - $nB/4 * (n/B)^2 = n^3/(4B)$

Summary

- **No blocking:** **$(9/8) * n^3$**
- **Blocking:** **$1/(4B) * n^3$**
- **If B = 8 difference is 4 * 8 * 9 / 8 = 36x**
- **If B = 16 difference is 4 * 16 * 9 / 8 = 72x**

- **Suggests largest possible block size B, but limit $3B^2 < C$!**

- **Reason for dramatic difference:**
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array element used O(n) times!
  - But program has to be written properly

Cache-Friendly Code

- **Programmer can optimize for cache performance**
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Blocking is a general technique
- **All systems favor "cache-friendly code"**
  - Getting absolute optimum performance is very platform specific
    - Cache sizes, line sizes, associativities, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)
    - Focus on inner loop code

The Memory Mountain

Intel Core i7
32 KB L1 i-cache
32 KB L1 d-cache
256 KB unified L2 cache
8M unified L3 cache

All caches on-chip