FOR EXTERNAL EXAMINER (date of this version: 3/11/2016)

UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR08012 INFORMATICS 1 - COMPUTATION AND LOGIC

Wednesday 0 December 2016

00:00 to 00:00

INSTRUCTIONS TO CANDIDATES

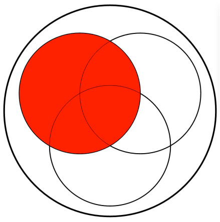
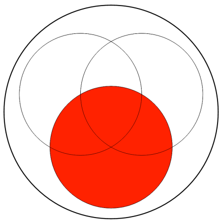
1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**
3. **CALCULATORS MAY NOT BE USED IN THIS EXAMINATION.**

Convener: I. Simpson
External Examiner: I. Gent

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) The entailment $A \vdash C$, is invalid.

   - $A \vdash C$

A	C	$A \vdash C$
		Valid ☐ Invalid ☒

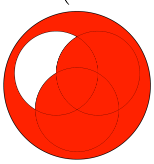
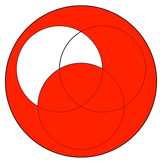
How is this invalidity shown by comparing the two Venn diagrams above? [*4 marks*]

The entailment is invalid if there is some region coloured for the assumption that is not coloured for the conclusion. Here there are two such.

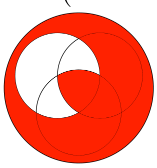
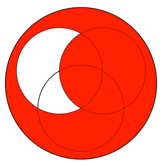
(b) For each of the following entailments, complete the two Venn diagrams to represent the assumption and conclusion, and place a mark in one of the check boxes provided to indicate whether the entailment is valid.

(You should use the same encoding as in the example above, where each circle represents one of the propositions A, B, C.)

i. $A \to (B \vee C) \vdash A \to C$ [*8 marks*]

$A \to (B \vee C)$	$A \to C$	$A \to (B \vee C) \vdash A \to C$
		Valid ☐ Invalid ☒

ii. $A \to (B \wedge C) \vdash A \to B$ [*8 marks*]

$A \to (B \wedge C)$	$A \to B$	$A \to (B \wedge C) \vdash A \to B$
		Valid ☒ Invalid ☐

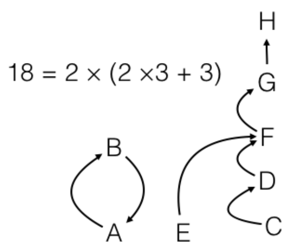
2. This question concerns the 256 possible truth valuations of the following eight propositional letters A, B, C, D, E, F, G, H. For each of the following expressions, say how many of the 256 valuations satisfy the expression, and briefly explain your reasoning. For example, the expression D is satisfied by half of the valuations, that is 128 of the 256, since for each valuation that makes D true there is a matching valuation that make D false.

(a) $A \vee B$ — Answer $192 = 3/4 * 256$ *[1 mark]*

(b) $(A \wedge B) \vee C$ — Answer $160 = 5/8 * 256$ *[2 marks]*

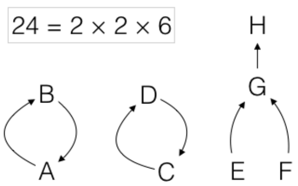
(c) $A \rightarrow (B \rightarrow C)$ — Answer $224 = 7/8 * 256$ *[2 marks]*

(d) $(A \rightarrow B) \wedge (B \rightarrow A) \wedge (C \rightarrow D) \wedge (D \rightarrow F) \wedge (E \rightarrow F) \wedge (F \rightarrow G) \wedge (G \rightarrow H)$ *[5 marks]*



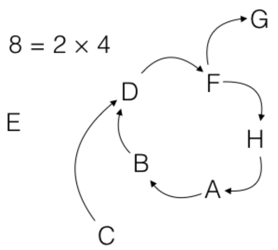
Answer 18

(e) $(A \rightarrow B) \wedge (B \rightarrow A) \wedge (C \rightarrow D) \wedge (D \rightarrow C) \wedge (E \rightarrow G) \wedge (F \rightarrow G) \wedge (G \rightarrow H)$ *[5 marks]*



Answer 24

(f) $(H \rightarrow A) \wedge (A \rightarrow B) \wedge (B \vee C \rightarrow D) \wedge (D \rightarrow F) \wedge (F \rightarrow H) \wedge (F \rightarrow G)$ *[5 marks]*



Answer 8

3. You are given the following inference rules (Γ, Δ vary over finite sets of expressions; A, B vary over expressions):

$$\frac{}{\Gamma, A \vdash \Delta, A}\ (I)$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R)$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ (\wedge R)$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \to B \vdash \Delta}\ (\to L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta}\ (\to R)$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ (\neg R)$$

(Where A and B are propositional expressions, Γ, Δ are sets of expressions, and Γ, A refers to $\Gamma \cup \{A\}$.) This question concerns the use of these rules to prove the following entailment. This is your **goal**.

$$P \to (Q \to R),\ Q \vee \neg P \vdash P \to R \tag{1}$$

(a) Which of these rules have a conclusion matching the goal (1)?
For each such rule complete a line in the table below showing the name of the rule and the bindings for Γ, Δ, A, B *[10 marks]*

Rule	Γ	Δ	A	B
$\to L$	$Q \vee \neg P$	$P \to R$	P	$Q \to R$
$\vee L$	$P \to (Q \to R)$	$P \to R$	A	$\neg P$
$\to R$	$P \to (Q \to R),\ Q \vee \neg P$	empty	P	R

(b) Use the rules given to construct a formal proof with the goal as conclusion, making any remaining assumptions as simple as possible.
Label each step in your proof with the name of the rule being applied. [*10 marks*]

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{(I)}
\UnaryInfC{$Q, P \vdash P, R$}
\AxiomC{}
\RightLabel{(I)}
\UnaryInfC{$R,\ Q, P \vdash R$}
\AxiomC{}
\RightLabel{(I)}
\UnaryInfC{$Q, P \vdash R, Q$}
\RightLabel{$(\rightarrow L)$}
\BinaryInfC{$Q \rightarrow R,\ Q, P \vdash R$}
\RightLabel{$(\rightarrow L)$}
\BinaryInfC{$P \rightarrow (Q \rightarrow R),\ Q, P \vdash R$}
\AxiomC{}
\RightLabel{(I)}
\UnaryInfC{$P \rightarrow (Q \rightarrow R), P \vdash R, P$}
\RightLabel{$(\neg L)$}
\UnaryInfC{$P \rightarrow (Q \rightarrow R),\ \neg P, P \vdash R$}
\RightLabel{$(\vee L)$}
\BinaryInfC{$P \rightarrow (Q \rightarrow R),\ Q \vee \neg P, P \vdash R$}
\RightLabel{$(\rightarrow R)$}
\UnaryInfC{$P \rightarrow (Q \rightarrow R),\ Q \vee \neg P \vdash P \rightarrow R$}
\end{prooftree}
$$

$$
\frac{}{P \rightarrow (Q \rightarrow R),\ Q \vee \neg P \vdash P \rightarrow R}
$$

4. (a) What does it mean for an entailment to be valid? *[2 marks]*

Answer Every valuation making the premises true makes the conclusion true

(b) How can resolution be used to determine whether an entailment is valid? *[2 marks]*

Answer We use resolution to determine whether the premises are inconsistent with the negation of the conclusion, by attempting to derive the empty clause.

(c) Use resolution to determine whether the entailment $(A \to B) \to A \vdash A$ is valid, and produce a counterexample if it is not. *[4 marks]*

A	B	C	
$(A \wedge \neg B) \vee A$ A $\neg B \vee A$ $\neg A$	$\{\}$		

Answer The entailment is valid

Counterexample? none *[2 marks]*

(d) Use resolution to determine whether $P \to (Q \vee R), (Q \wedge R) \to S \vdash P \to S$ is valid and produce a counter-example if it is not. *[4 marks]*

P	Q	R	S	
$^{P}\neg P \vee Q \vee R$ $^{Q}\neg Q \vee S$ $^{R}\neg R \vee S$ ^{P}P $^{S}\neg S$	$^{Q}Q \vee R$	$^{R}S \vee R$	^{S}S	$\{\}$

AnswerThe entailment is valid

Counterexample?none *[2 marks]*

(e) Explain what it means to claim that the resolution procedure is:

i. **sound** *[2 marks]*

Answer If we derive the empty clause then the entailment is valid.

ii. **complete**. *[2 marks]*

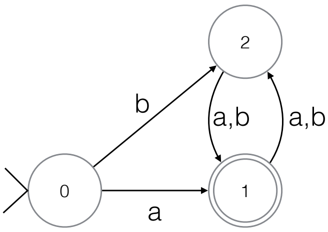
Answer If the entailment is valid then we will derive the empty clause.

5. Each diagram shows an FSM. In each case give a regular expression for the language accepted by the FSM, make a mark in the check box against each string that it accepts (and no mark against those strings it does not accept), make a mark in the DFA check box if it is deterministic, and draw an equivalent DFA if it is not.

[*4 marks*]

(a)

(a|b(a|b))((a|b)(a|b))*

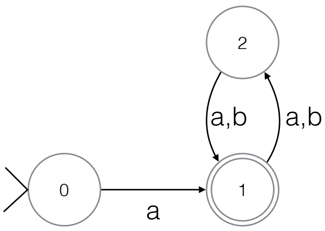


aab ☒
aba ☒
bab ☐
aaa ☒
bbb ☐
DFA ☒

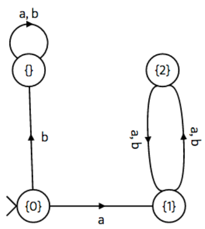
[*4 marks*]

(b)

a((a|b)(a|b))*



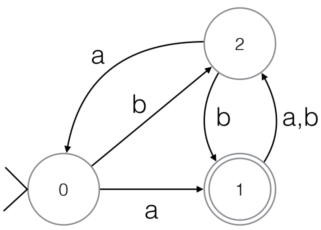
aab ☒
aba ☒
bab ☐
aaa ☒
bbb ☐
DFA ☐



[*4 marks*]

(c)

(b|a(a|b))(bb|(aa|b)(a|b))*

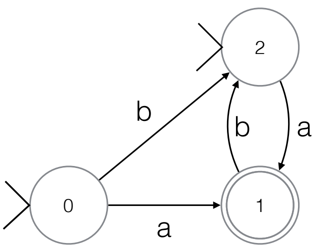


aab ☒
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☒

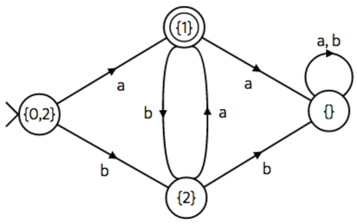
[*4 marks*]

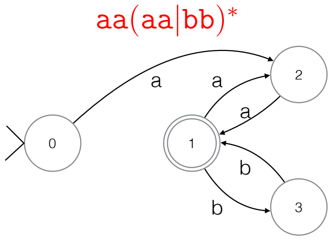
(d)

(a|ba)(ba)*

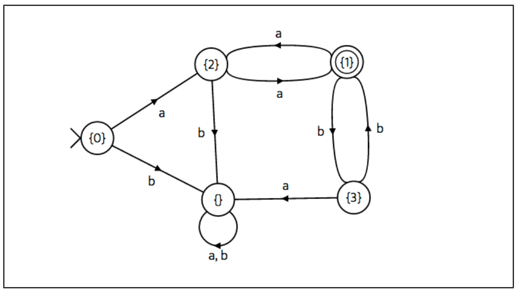


aab ☐
aba ☒
bab ☐
aaa ☐
bbb ☐
DFA ☐



(e) 

aab ☐
aba ☐
bab ☐
aaa ☐
bbb ☐
DFA ☐



[4 marks]