UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR08012 INFORMATICS 1 - COMPUTATION AND LOGIC

Monday 16 th December 2013

14:30 to 16:30

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**
3. **Calculators may not be used in this examination.**

Convener: J. Bradfield
External Examiner: C. Johnson

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) For each of the following propositional expressions, state whether it is a tautology, a contradiction or a contingency. [*15 marks*]

   i. $(a \rightarrow b)$ *or* $not(a \rightarrow b)$

   ii. $(a$ *or* b *or* $c)$ *and* $(not(b)$ *or* $not(c))$

   iii. a *and* b *and* c *and* $(not(c)$ *or* $not(a))$

   iv. $(a \rightarrow b)$ *and* $(b \rightarrow c)$ *and* $(c \rightarrow not(a))$

   v. $(a \leftrightarrow a)$ *or* b

   (b) Consider the following sentences A and B. Sentence A: "Sentence B is true". Sentence B: "Sentence A is false".

   i. Explain why this is a logical paradox. [*3 marks*]

   ii. Suggest how to resolve the paradox. [*2 marks*]

2. You are given the following proof rules:

Rule number	Sequent	Supporting proofs
1	$\mathcal{F} \vdash A$	$A \in \mathcal{F}$
2	$\mathcal{F} \vdash A$ *and* B	$\mathcal{F} \vdash A,\ \mathcal{F} \vdash B$
3	$\mathcal{F} \vdash A$ *or* B	$\mathcal{F} \vdash A$
4	$\mathcal{F} \vdash A$ *or* B	$\mathcal{F} \vdash B$
5	$\mathcal{F} \vdash C$	$\mathcal{F} \vdash (A$ *or* $B),\ [A\|\mathcal{F}] \vdash C,\ [B\|\mathcal{F}] \vdash C$
6	$\mathcal{F} \vdash B$	$A \rightarrow B \in \mathcal{F},\ \mathcal{F} \vdash A$
7	$\mathcal{F} \vdash A \rightarrow B$	$[A\|\mathcal{F}] \vdash B$

   where $\mathcal{F} \vdash A$ means that expression A follow from set of axioms $\mathcal{F}$; $A \in \mathcal{F}$ means that A is an element of set $\mathcal{F}$; and $[A|\mathcal{F}]$ is the set constructed by adding A to set $\mathcal{F}$.

   Using the proof rules above, prove the following:

$$[b \rightarrow a,\ a \rightarrow d,\ not(c)] \quad \vdash \quad (b \rightarrow c)\ or\ (b \rightarrow (a\ and\ d))$$

   Show precisely how the proof rules are applied. [*15 marks*]

3. Consider the following sequent:

$$[(not(a)\ or\ b) \rightarrow c, c \rightarrow not(d)] \quad \vdash \quad (b\ and\ not(a)) \rightarrow (c\ and\ not(d))$$

   (a) Convert the sequent to a conjunction of propositional expressions so that the conjunction is a contradiction if and only if the sequent is valid. [*2 marks*]

   (b) Convert the conjunction to an equivalent CNF expression. [*7 marks*]

   (c) Give the resulting CNF in clausal form. [*1 mark*]

(d) Give a proof, using the Davis-Putnam algorithm for resolution, that the resulting CNF is a contradiction. Show each step of your proof in detail. [*10 marks*]

4. An automatic ticket vending machine at a cinema theatre operates as follows. There are two films playing at the theatre, Film 1 and Film 2. A ticket for Film 1 costs 10 pounds, and a ticket for Film 2 costs 15 pounds. The machine's start-up screen shows two options, "Buy a ticket for Film 1" and "Buy a ticket for Film 2". On the user choosing one of these options, a new screen appears with two options, "Purchase" and "Cancel". At this point, the user can choose one of these options or offer money to the machine. The machine only accepts 5 pound notes. If the user chooses the Film 1 option and attempts to purchase a ticket without paying at least 10 pounds, the machine prints a "Not enough cash" slip, and continues to show the "Purchase" and "Cancel" options. Similarly, if the user chooses the Film 2 option and attempts to purchase a ticket without paying at least 15 pounds, the machine prints a "Not enough cash" slip, and continues to show the "Purchase" and "Cancel" options. The user is free to offer as much money as she wishes to the machine. If the user has paid enough money to purchase a ticket, and chooses the Purchase option, a ticket is printed. If the user chooses the "Cancel" option at any point, the machine returns to the start-up screen. No refunds are offered.

   (a) Design a transducer-style finite state machine to model this system. Explain clearly what the input and output alphabets mean, and what the set of states and transition function are. [*15 marks*]

   (b) Give the trace of the machine corresponding to an interaction with a user who first chooses Film 1, then cancels, chooses Film 2, pays 20 pounds, and purchases a ticket. [*5 marks*]

5. Design a deterministic finite state machine over the alphabet $\{0, 1, 2\}$ which accepts only those strings which are representations of odd numbers both in base 3 *and* base 4. You may assume that the machine is fed the representation with the lowest-order digit first (i.e., it reads the number from right to left). [*10 marks*]

6. (a) Assume that your alphabet is the set of characters $\{a, b\}$. Write a regular expression for the language consisting of all strings which contain at least two occurrences of the letter a and at least one occurrence of the substring ab. [*5 marks*]

   (b) Given the following equivalences between regular expressions:

$R(SR)^*$	is equivalent to	$(RS)^*R$
R^*R	is equivalent to	RR^*
R^*	is equivalent to	$RR^*\|\epsilon$
$R\|R$	is equivalent to	R

show that the regular expression $ab(bab)^*b|abbabb(abb)^*$ is equivalent to the regular expression $abb(abb)^*$ [*10 marks*]