Module Title: Informatics 1A: Computation and Logic
Exam Diet (Dec/April/Aug): Dec 2007
Brief notes on answers:

1. (a) A tautologous expression is true whatever the truth values of the atomic propositions from which it is composed.

   (b) An appropriate truth table is:

a	b	c	a *and* b	1 $(a$ *and* $b) \rightarrow c$	$b \rightarrow c$	2 $a \rightarrow (b \rightarrow c)$	$1 \leftrightarrow 2$
t	t	t	t	t	t	t	t
t	t	f	t	f	f	f	t
t	f	t	f	t	t	t	t
t	f	f	f	t	t	t	t
f	t	t	f	t	t	t	t
f	t	f	f	t	f	t	t
f	f	t	f	t	t	t	t
f	f	f	f	t	t	t	t

   (c) An appropriate proof is:

$$
\begin{array}{l}
[] \quad \vdash \quad ((a \textit{ and } b) \rightarrow c) \leftrightarrow (a \rightarrow (b \rightarrow c)) \\
\quad 2 : [] \quad \vdash \quad ((a \textit{ and } b) \rightarrow c) \rightarrow (a \rightarrow (b \rightarrow c)) \\
\quad\quad 3 : [((a \textit{ and } b) \rightarrow c)] \quad \vdash \quad (a \rightarrow (b \rightarrow c)) \\
\quad\quad\quad 3 : [a, ((a \textit{ and } b) \rightarrow c)] \quad \vdash \quad (b \rightarrow c) \\
\quad\quad\quad\quad 3 : [b, a, ((a \textit{ and } b) \rightarrow c)] \quad \vdash \quad c \\
\quad\quad\quad\quad\quad 6 : [b, a, ((a \textit{ and } b) \rightarrow c)] \quad \vdash \quad (a \textit{ and } b) \\
\quad\quad\quad\quad\quad\quad 4 : [b, a, ((a \textit{ and } b) \rightarrow c)] \quad \vdash \quad a \\
\quad\quad\quad\quad\quad\quad\quad 1 \\
\quad\quad\quad\quad\quad\quad 4 : [b, a, ((a \textit{ and } b) \rightarrow c)] \quad \vdash \quad b \\
\quad\quad\quad\quad\quad\quad\quad 1 \\
\quad\quad\quad\quad\quad 6 : [c, b, a, ((a \textit{ and } b) \rightarrow c)] \quad \vdash \quad c \\
\quad\quad\quad\quad\quad\quad 1 \\
\quad 2 : [] \quad \vdash \quad (a \rightarrow (b \rightarrow c)) \rightarrow ((a \textit{ and } b) \rightarrow c) \\
\quad\quad 3 : [(a \rightarrow (b \rightarrow c))] \quad \vdash \quad ((a \textit{ and } b) \rightarrow c) \\
\quad\quad\quad 3 : [(a \textit{ and } b), (a \rightarrow (b \rightarrow c))] \quad \vdash \quad c \\
\quad\quad\quad\quad 5 : [a, b, (a \textit{ and } b), (a \rightarrow (b \rightarrow c))] \quad \vdash \quad c \\
\quad\quad\quad\quad\quad 6 : [a, b, (a \textit{ and } b), (a \rightarrow (b \rightarrow c))] \quad \vdash \quad a \\
\quad\quad\quad\quad\quad\quad 1 \\
\quad\quad\quad\quad 5 : [(b \rightarrow c), a, b, (a \textit{ and } b), (a \rightarrow (b \rightarrow c))] \quad \vdash \quad c \\
\quad\quad\quad\quad\quad 6 : [(b \rightarrow c), a, b, (a \textit{ and } b), (a \rightarrow (b \rightarrow c))] \quad \vdash \quad b \\
\quad\quad\quad\quad\quad\quad 1 \\
\quad\quad\quad\quad\quad 6 : [c, (b \rightarrow c), a, b, (a \textit{ and } b), (a \rightarrow (b \rightarrow c))] \quad \vdash \quad c \\
\quad\quad\quad\quad\quad\quad 1
\end{array}
$$

2. (a) An expression in clausal form

$$[[E1_1, \ldots, E1_n], \ldots [EN_1, \ldots, EN_n]$$

corresponds to the propositional logic expression

$$(E1_1 \text{ or } \ldots \text{ or } E1_n) \text{ and } \ldots \text{ and } (EN_1 \text{ or } \ldots \text{ or } EN_n)$$

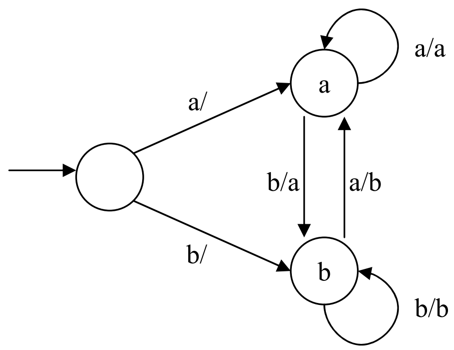
(b) The steps of conversion are:

- $[a, \quad (a \text{ and } b) \rightarrow c, \quad (d \text{ or } e) \rightarrow b, \quad a \rightarrow e]$
- $[a, \quad not(a \text{ and } b) \text{ or } c, \quad not(d \text{ or } e) \text{ or } b, \quad not(a) \text{ or } e]$
- $[a, \quad not(a) \text{ or } not(b) \text{ or } c, \quad not(d \text{ or } e) \text{ or } b, \quad not(a) \text{ or } e]$
- $[a, \quad not(a) \text{ or } not(b) \text{ or } c, \quad (not(d) \text{ and } not(e)) \text{ or } b, \quad not(a) \text{ or } e]$
- $[a, \quad not(a) \text{ or } not(b) \text{ or } c, \quad (not(d) \text{ or } b) \text{ and } (not(e) \text{ or } b), \quad not(a) \text{ or } e]$
- $[a, \quad not(a) \text{ or } not(b) \text{ or } c, \quad not(d) \text{ or } b, \quad not(e) \text{ or } b, \quad not(a) \text{ or } e]$
- $[[a], \ [not(a), not(b), c], \ [not(d), b], \ [not(e), b], \ [not(a), e]]$

(c) An appropriate proof is:

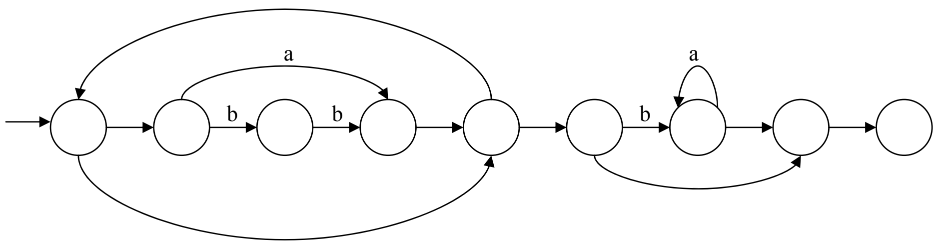
- Negate c as the input clause $[notc]$
- Resolve $[notc]$ with $[not(a), not(b), c]$ giving $[not(a), not(b)]$
- Resolve $[not(a), not(b)]$ with $[a]$ giving $[not(b)]$
- Resolve $[not(b)]$ with $[not(e), b]$ giving $[not(e)]$
- Resolve $[not(e)]$ with $[not(a), e]$ giving $[not(a)]$
- Resolve $[not(a)]$ with $[a]$ giving $[]$
- Hence $not(c)$ is contradictory
- Hence c is true.

3. (a) An appropriate FSM is:



(b) Depending on the FSM used in the previous answer, the number of states is likely to be $|A|^D$ where A is the alphabet set and D is the number of units of delay. A precise answer is not essential here - only evidence that the student understand that we have a significant scaling issue.

4. An appropriate FSM is:



5. (a) $(a|b)^*a(a|b)^*$
   (b) $((ba^*b)|a^*)^*$
   (c) $((a|b)^*a(a|b)^*b(a|b)^*)|((a|b)^*b(a|b)^*a(a|b)^*)$

6. The steps of proof are:

$$\begin{aligned}((aa^*|\epsilon)c)|((b|b)c) &= ((a^*c)|((b|b)c)\\ ((a^*c)|((b|b)c) &= ((a^*c)|((bc)\\ ((a^*c)|((bc) &= (a^*|b)c\end{aligned}$$