Module Title: Informatics 1
Exam Diet (Dec/April/Aug): December 2014
Brief notes on answers:

1. (a) Answer to first part of question 1

A	B	$A \to B$	$A \to ((A \to B) \to B)$	$((A \to B) \to B) \to B$	$((A \to B) \to B) \to A$
$\bot$	$\bot$	$\top$	$\top$	$\top$	$\top$
$\bot$	$\top$	$\top$	$\top$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\top$	$\bot$	$\top$
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$

   (i). tautologous

   (ii). contingent

   (iii). contingent

   (b) Answer to second part of question 1

   (i). $\neg A$ 128 (1/2)

   (ii). $E \wedge F$ 64 (1/4)

   (iii). $B \vee D$ 192 (3/4)

   (iv). $A \to H$ 192 (3/4)

   (v). 9 (the valuation must repect the ordering)

$$(A \to B) \wedge (B \to C) \wedge (C \to D) \wedge (D \to E) \wedge (E \to F) \wedge (F \to G) \wedge (G \to H)$$

   (vi). 4 (the two cycles place the letters in two groups each group must be all true or all false)

$$(A \to B) \wedge (B \to C) \wedge (C \to D) \wedge (D \to A) \\ \wedge (E \to F) \wedge (F \to G) \wedge (G \to H) \wedge (H \to E)$$

   (vii). 10 (count cuts of the partial ordering)

$$(A \to B \wedge C) \wedge (B \vee C \to D) \wedge (D \to E) \wedge (E \to F) \wedge (F \to G) \wedge (G \to H)$$

2. Answers to question 2

   (a) Sound: any provable entailment is valid. Complete any valid entailment is provable. Where an entailment is valid if any valuation that makes all of its premises true will make at least one of its conclusions true.

   (b)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P \to Q, P, R \vdash R}\,(I)}{P \to Q, P \vdash \neg R, R}\,(\neg R) \quad \dfrac{\dfrac{}{P, \neg Q \vdash P, R}\,(I) \quad \dfrac{\dfrac{}{P, Q \vdash Q, R}\,(I)}{P, Q, \neg Q \vdash R}\,(\neg L)}{P \to Q, P, \neg Q \vdash R}\,(\to L)}{P \to Q, \neg R \to \neg Q, P \vdash R}\,(\to L)}{P \to Q, \neg R \to \neg Q \vdash P \to R}\,(\to R)$$

   (c)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P \vdash P, R}\,(I) \quad P, Q \vdash R}{P \to Q, P \vdash R}\,(\to L) \quad \dfrac{\dfrac{}{P, Q \vdash P, R}\,(I) \quad P, Q \vdash R}{P \to Q, P, Q \vdash R}\,(\to L)}{P \to Q, R \to Q, P \vdash R}\,(\to L)}{P \to Q, R \to Q \vdash P \to R}\,(\to R)$$

   (d) We can falsify the conclusion by falsifying any undischarged assumption. So taking $P, Q, \bar{R}$ suffices to falsify the conclusion.

   (e) Since we can trivially falsify any entailment containing only atoms, unles some atom occurs on each side — in which case we could discharge the assumtion using (I) — this set of rules is complete.

3. (a) Answer to first part of question 3

   (i). $\neg(T \vee Q)$: $\{\{\neg T\}, \{\neg Q\}\}$

   (ii). $(P \to Q) \vee \neg(\neg S \wedge \neg T)$: $\{\{\neg P, Q, S, T\}\}$

   (iii). $U \to (\neg T \to (\neg S \wedge P))$: $\{\{\neg U, T, \neg S\}, \{\neg U, T, P\}\}$

   (b) Answer to second part of question 3
   Add $\{U\}$ — the negation of the conclusion — to our set of clauses and then apply resolution. The claim is true iff the empty clause is derivable.

   (c) Answer to third part of question 3
   Start from the clauses

$$\Big\{\{\neg T\}, \{\neg Q\}, \{\neg P, Q, S, T\}, \{\neg U, T, \neg S\}, \{\neg U, T, P\}, \{U\}\Big\}$$

   Any order for resolution is OK. E.g. Starting with T then U we get

$$\Big\{\{\neg Q\}, \{\neg P, Q, S\}, \{\neg U, \neg S\}, \{\neg U, P\}, \{U\}\Big\}$$

$$\Big\{\{\neg Q\}, \{\neg P, Q, S\}, \{\neg S\}, \{P\}\Big\}$$

   and hence, taking P, Q, S in any order,

$$\Big\{\{\}\Big\}$$

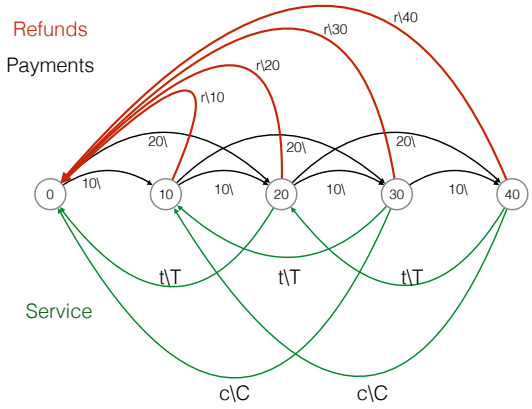
4. Answer to question 4

(a) Alphabets:

inputs 10, 20 — coins
t, c, r — requests for tea/coffee/refund

outputs T, C — tea and coffee
10, 20, 30, 40 — change/refund

States correspond to user credit. Coffee button enabled when credit ≥ 30; tea button enabled when credit ≥ 20; change/refund button enabled when credit ≥ 10.



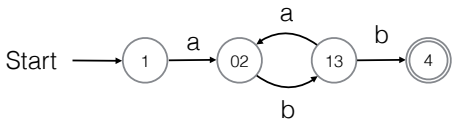
(b) trace is 20/, /20, c/C, r/10

5. Answers to question 5

   (a) (i) and (iii)

   (b) (ab)*abb

   (c) The states of the DFA correspond to sets of states of the NFA



   (d)