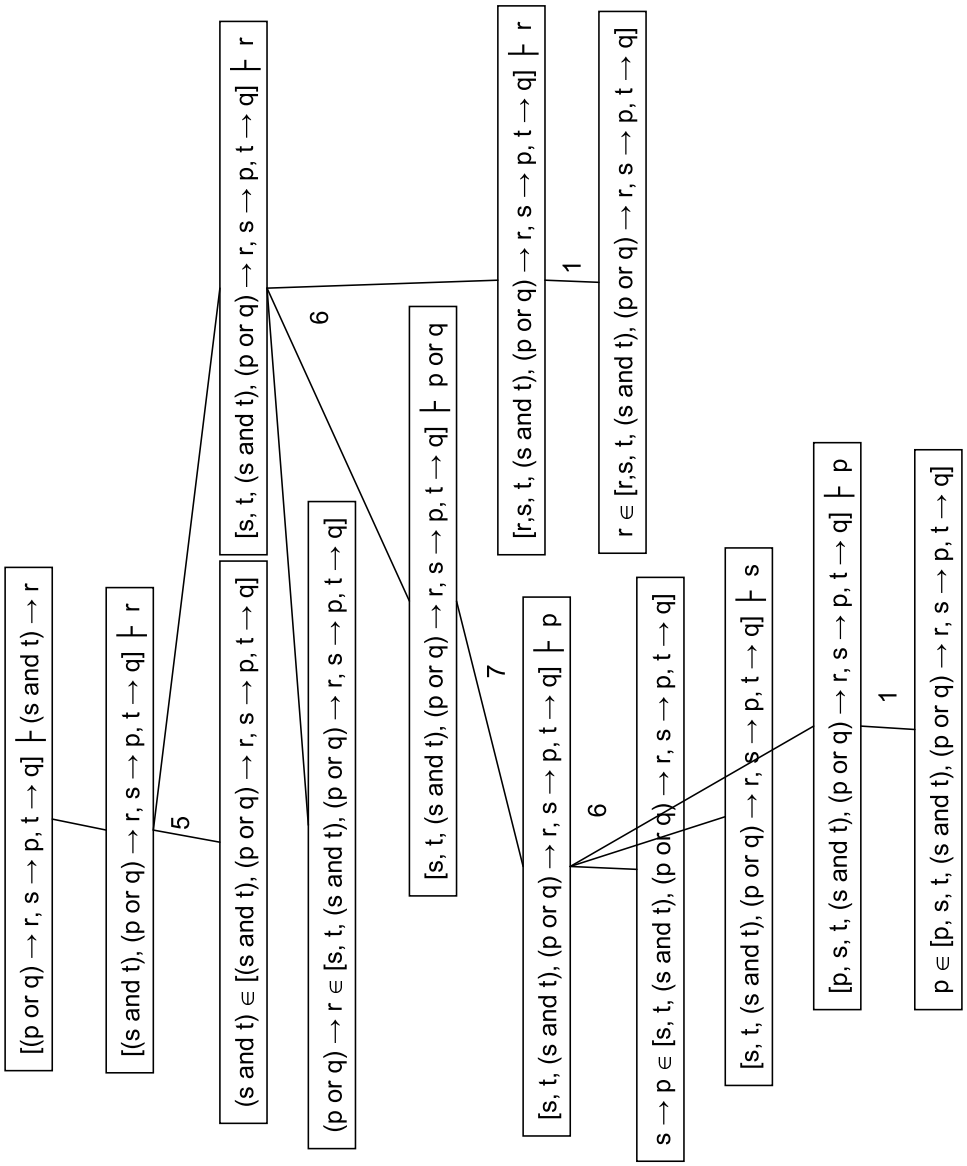
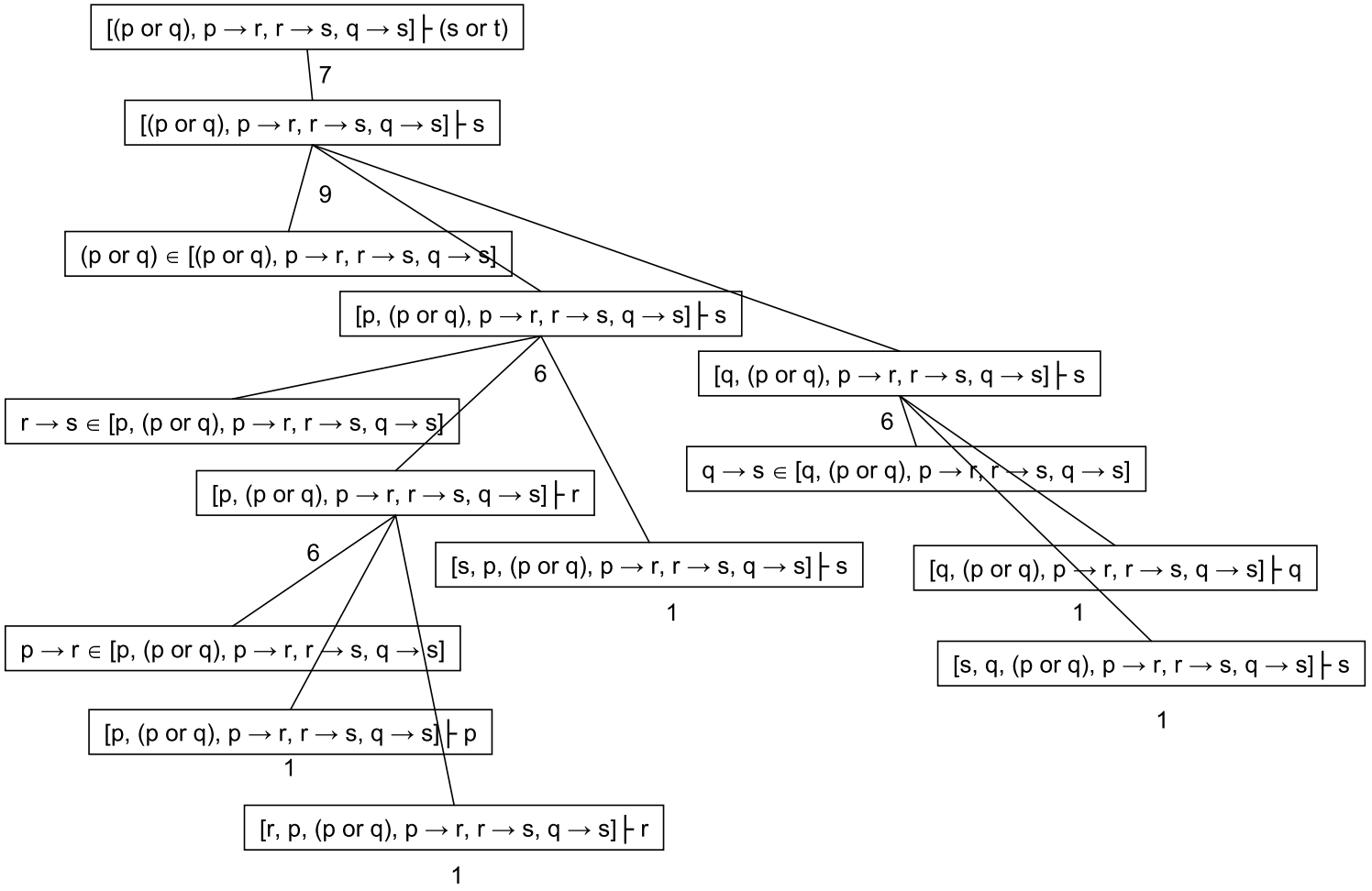
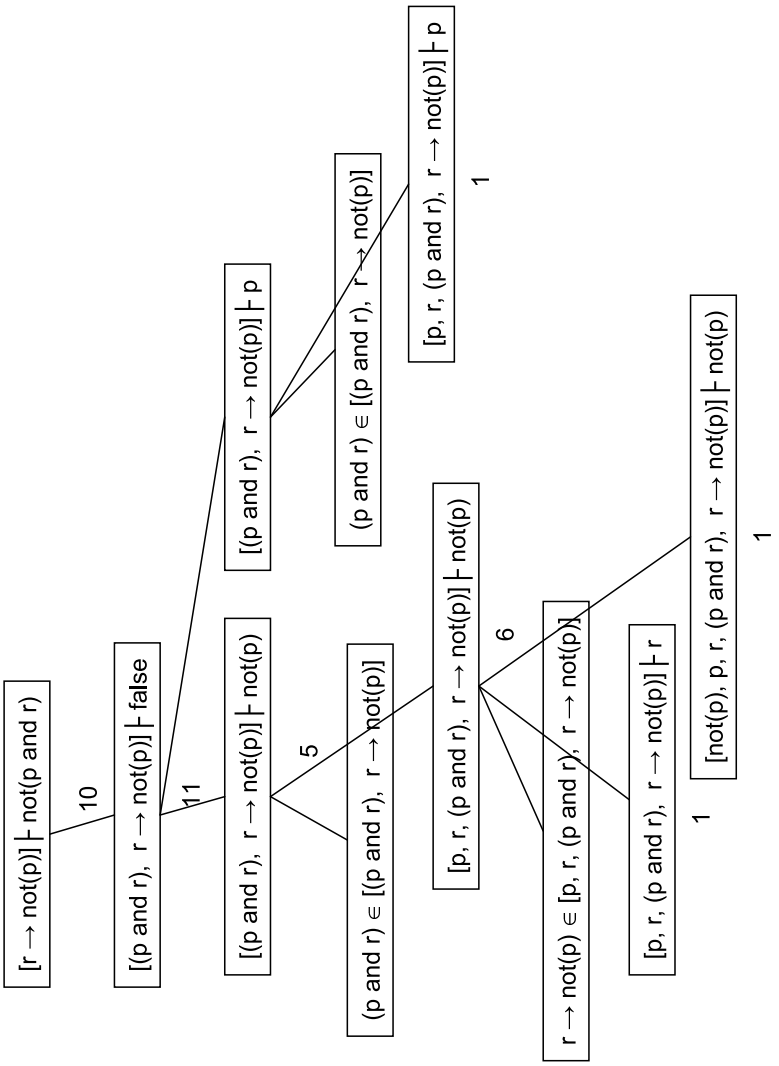
Module Title: Informatics 1A: Computation and Logic
Exam Diet: Dec 2008
Brief notes on answers:

1. The three proof trees for this question are given below:




[(p or q), p → r, r → s, q → s] ⊢ (s or t)
7
[(p or q), p → r, r → s, q → s] ⊢ s
9
(p or q) ∈ [(p or q), p → r, r → s, q → s]
[p, (p or q), p → r, r → s, q → s] ⊢ s
6
r → s ∈ [p, (p or q), p → r, r → s, q → s]
[p, (p or q), p → r, r → s, q → s] ⊢ r
6
p → r ∈ [p, (p or q), p → r, r → s, q → s]
[p, (p or q), p → r, r → s, q → s] ⊢ p
1
[r, p, (p or q), p → r, r → s, q → s] ⊢ r
1
[s, p, (p or q), p → r, r → s, q → s] ⊢ s
1
[q, (p or q), p → r, r → s, q → s] ⊢ s
6
q → s ∈ [q, (p or q), p → r, r → s, q → s]
[q, (p or q), p → r, r → s, q → s] ⊢ q
1
[s, q, (p or q), p → r, r → s, q → s] ⊢ s
1



2. (a) The required truth tables are as follows:

A	B	$not(A)$	$A \rightarrow B$	$not(A)$ *or* B
t	t	f	t	t
t	f	f	f	f
f	t	t	t	t
f	f	t	t	t

A	B	$not(B)$	A *and* $not(B)$	$A \rightarrow B$	$not(A$ *and* $not(B))$
t	t	f	f	t	t
t	f	t	t	f	f
f	t	f	f	t	t
f	f	t	f	t	t

A	B	$not(A)$	$not(B)$	$not(A)$ *and* $not(B)$	A *or* B	$not(not(A)$ *and* $not(B))$
t	t	f	f	f	t	t
t	f	f	t	f	t	t
f	t	t	f	f	t	t
f	f	t	t	t	f	f

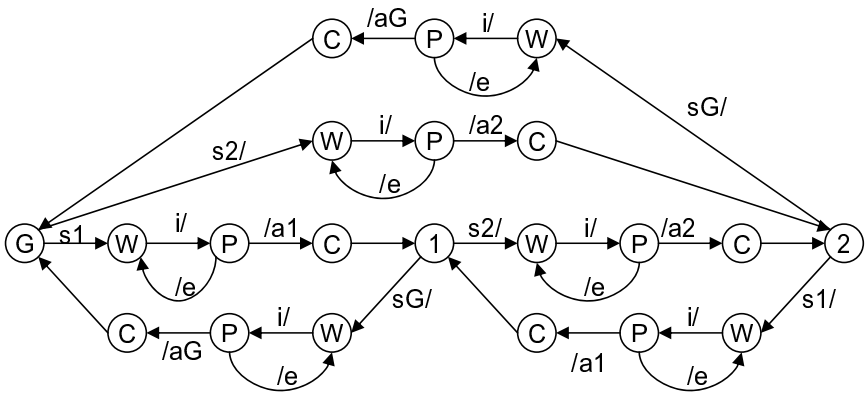
(b) The sequence of transformations is as follows:

$not(not(not(p)\ or\ q)\ and\ not(not(q\ and\ not(p))))$
$\Downarrow$
$A\ or\ B$ is equivalent to $not(not(A)\ and\ not(B))$
$\Downarrow$
$(not(p)\ or\ q)\ or\ (not(q\ and\ not(p)))$
$\Downarrow$
$A\ \rightarrow\ B$ is equivalent to $not(A)\ or\ B$
$\Downarrow$
$(p\ \rightarrow\ q)\ or\ (not(q\ and\ not(p)))$
$\Downarrow$
$A\ \rightarrow\ B$ is equivalent to $not(A\ and\ not(B))$
$\Downarrow$
$(p\ \rightarrow\ q)\ or\ (q\ \rightarrow\ p)$

3. The truth table below gives the truth value for the expression always to be true, so this is a tautology.

p	q	$p\ \rightarrow\ q$	$q\ \rightarrow\ p$	$(p\ \rightarrow\ q)\ or\ (q\ \rightarrow\ p)$
t	t	t	t	t
t	f	f	t	t
f	t	t	f	t
f	f	t	f	t

4. (a) An appropriate FSM is as follows:



G = lift at ground floor (door open)
1 = lift at 1st floor (door open)
2 = lift at 2nd floor (door open)
W = waiting for PIN
P = PIN entered
C = Correct PIN; door closed; lift moving

sF = select floor F
i = input PIN
e = error message: incorrect PIN
aF = announce move to floor F

(b) With respect to the FSM above:

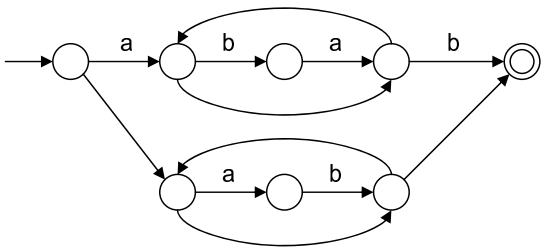
- A person with an accepted PIN is able to reach any floor from any other floor because there is a trace from each floor to each other that is uninterrupted following PIN acceptance.

- A person with a rejected PIN can not be transported to any floor because there is no trace that reaches any floor from any other without passing through a state in which the PIN is accepted.

(c) With respect to the FSM above:

- Nobody will be trapped in the lift with the doors closed because doors only close at PIN acceptance stage, after which there are no changes in state until the lift reaches its destination.
- It is not possible to revise choice of floor after a PIN is entered because there is no arc back from PIN entry to a state preceding selection of floor.

5. (a) The resulting FSM is:



(b) The sequence of transformations is as follows:

$(a(ba)^*b)\|(ab)^*$
$\Downarrow$
$R(SR)^*$ is equivalent to $(RS)^*R$
$\Downarrow$
$(ab)^*ab\|(ab)^*$
$\Downarrow$
R^*R is equivalent to RR^*
$\Downarrow$
$ab(ab)^*\|(ab)^*$
$\Downarrow$
R^* is equivalent to $RR^*\|\epsilon$
$\Downarrow$
$ab(ab)^*\|(ab(ab)^*\|\epsilon)$
$\Downarrow$
$R\|(S\|T)$ is equivalent to $(R\|S)\|T$
$\Downarrow$
$(ab(ab)^*\|ab(ab)^*)\|\epsilon$
$\Downarrow$
$R\|R$ is equivalent to R
$\Downarrow$
$ab(ab)^*\|\epsilon$
$\Downarrow$
R^* is equivalent to $RR^*\|\epsilon$
$\Downarrow$
$(ab)^*$