UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFORMATICS 1 - COMPUTATION AND LOGIC

Tuesday 9 December 2008

09:30 to 11:30

Convener: M O'Boyle
External Examiner: R Irving

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. You are given the following proof rules:

Rule number	Sequent	Supporting proofs
1	$\mathcal{F} \vdash A$	$A \in \mathcal{F}$
2	$\mathcal{F} \vdash A \leftrightarrow B$	$\mathcal{F} \vdash A \rightarrow B,\ \mathcal{F} \vdash B \rightarrow A$
3	$\mathcal{F} \vdash A \rightarrow B$	$[A\vert\mathcal{F}] \vdash B$
4	$\mathcal{F} \vdash A \textit{ and } B$	$\mathcal{F} \vdash A,\ \mathcal{F} \vdash B$
5	$\mathcal{F} \vdash C$	$(A \textit{ and } B) \in \mathcal{F},\ [A, B\vert\mathcal{F}] \vdash C$
6	$\mathcal{F} \vdash C$	$A \rightarrow B \in \mathcal{F},\ \mathcal{F} \vdash A,\ [B\vert\mathcal{F}] \vdash C$
7	$\mathcal{F} \vdash A \textit{ or} B$	$\mathcal{F} \vdash A$
8	$\mathcal{F} \vdash A \textit{ or} B$	$\mathcal{F} \vdash B$
9	$\mathcal{F} \vdash C$	$(A \textit{ or } B) \in \mathcal{F},\ [A\vert\mathcal{F}] \vdash C,\ [B\vert\mathcal{F}] \vdash C$
10	$\mathcal{F} \vdash \textit{not}(A)$	$[A\vert\mathcal{F}] \vdash \textit{false}$
11	$\mathcal{F} \vdash B$	$\mathcal{F} \vdash \textit{not}(A),\ \mathcal{F} \vdash A$

where $\mathcal{F} \vdash A$ means that expression A can be proved from set of axioms $\mathcal{F}$; $A \in \mathcal{F}$ means that A is an element of set $\mathcal{F}$; $[A|\mathcal{F}]$ is the set constructed by adding A to set $\mathcal{F}$; $A \rightarrow B$ means that A implies B; $A \textit{ and } B$ means that A and B both are true; $A \textit{ or } B$ means that at least one of A or B is true; $\textit{not}(A)$ means that A is not true; and $\textit{false}$ means that you have a contradiction.

Using the proof rules above, prove the following:

(a) $[(p \textit{ or } q) \rightarrow r,\ s \rightarrow p,\ t \rightarrow q] \vdash (s \textit{ and } t) \rightarrow r$

(b) $[p \textit{ or } q,\ p \rightarrow r,\ r \rightarrow s,\ q \rightarrow s] \vdash s \textit{ or } t$

(c) $[r \rightarrow \textit{not}(p)] \vdash \textit{not}(p \textit{ and } r)$

Show precisely how the proof rules are applied. [*20 marks*]

2. You are given the following equivalences between expressions:

$A \rightarrow B$	is equivalent to	$\textit{not}(A) \textit{ or } B$
$A \rightarrow B$	is equivalent to	$\textit{not}(A \textit{ and } \textit{not}(B))$
$A \textit{ or } B$	is equivalent to	$\textit{not}(\textit{not}(A) \textit{ and } \textit{not}(B))$

(a) Prove, using truth tables, that each of these three equivalences is correct. [*15 marks*]

(b) Apply the equivalences correctly to show that the expression:

$$\textit{not}(\textit{not}(\textit{not}(p) \textit{ or } q) \textit{ and } \textit{not}(\textit{not}(q \textit{ and } \textit{not}(p))))$$

can be transformed into the equivalent expression

$$(p \rightarrow q) \textit{ or } (q \rightarrow p)$$

[*10 marks*]

3. Is $(p \rightarrow q)$ *or* $(q \rightarrow p)$ a tautology? Justify your answer in detail using a truth table. [*5 marks*]

4. You are given the following requirements specification of a system to control part of the operation of a lift:

   The lift moves between three floors: the ground floor, first floor and second floor. The lift waits on whichever floor it has last visited with its door open. When a person enters the lift he must first select the required floor and then enter a personal identification number (PIN). If the PIN is accepted then the system announces that it is moving to the chosen floor and signals the lift to move to that floor (where it is assumed the lift then waits with the doors open). If the PIN is rejected then an error message is given and the system waits for the PIN to be re-entered. A person with an accepted PIN should be able to reach any floor from any other floor. A person with a rejected PIN should not be transported to any floor.

   (a) Define a transducer FSM satisfying the lift controller description above. It should contain states in which the following six conditions are true (although you may have more than one instance of each of these sorts of state in your FSM).

State	Conditions true at state
G	Lift at ground floor with door open
1	Lift at first floor with door open
2	Lift at second floor with door open
W	System is waiting for PIN to be entered, with door open
P	PIN has been entered
C	Lift door closed and lift is moving to new floor

   It also should show the relevant input and output when moving between states. [*20 marks*]

   (b) Demonstrate, using the FSM you have defined, that the following requirements from the lift specification are satisfied by your FSM:

   - A person with an accepted PIN should be able to reach any floor from any other floor.
   - A person with a rejected PIN should not be transported to any floor. [*5 marks*]

   (c) Demonstrate, using the FSM you have defined, whether or not your FSM satisfies the following optional requirements:

   - Nobody will be trapped in the lift by your system with the doors closed (assuming the mechanical movement system works properly).
   - It is possible to revise choice of floor after a PIN is entered. [*5 marks*]

5. (a) FSMs can be constructed in a modular way by combining smaller FSM fragments into larger designs. Show how this approach can be used to produce a finite state machine corresponding to the regular expression $(a(ba)^*b)|(ab)^*$ and give the resulting FSM. *[10 marks]*

(b) Given the following equivalences between regular expressions:

$R(SR)^*$	is equivalent to	$(RS)^*R$
R^*R	is equivalent to	RR^*
R^*	is equivalent to	$RR^*\|\epsilon$
$R\|R$	is equivalent to	R
$R\|(S\|T)$	is equivalent to	$(R\|S)\|T$

show that the regular expression $(a(ba)^*b)|(ab)^*$ is equivalent to the regular expression $(ab)^*$ *[10 marks]*