Module Title: INFORMATICS 1 - COMPUTATION AND LOGIC
Exam Diet (Dec/April/Aug): August 2010
Brief notes on answers:

1. (a) (i). $\neg(\neg A \rightarrow B) \vee \neg(\neg C)$ is equivalent to $(A \vee B) \rightarrow C$

$$\neg(\neg A \rightarrow B) \vee \neg(\neg C) \qquad (2)$$
$$\neg(\neg A \rightarrow B) \vee C \qquad (1)$$
$$(\neg A \rightarrow B) \rightarrow C \qquad (1)$$
$$(\neg(\neg A) \vee B) \rightarrow C \qquad (2)$$
$$(A \vee B) \rightarrow C$$

(ii). $\neg(A \wedge B) \wedge (\neg A \vee C)$ is equivalent to $A \rightarrow (\neg B \wedge C)$

$$\neg(A \wedge B) \wedge (\neg A \vee C) \qquad (4)$$
$$(\neg A \vee \neg B) \wedge (\neg A \vee C) \qquad (5)$$
$$\neg A \vee (\neg B \wedge C) \qquad (1)$$
$$A \rightarrow (\neg B \wedge C)$$

(b) $(\neg(\neg A \rightarrow B) \vee \neg(\neg C)) \leftrightarrow ((A \wedge B) \rightarrow C)$

A	B	C	$\neg$ A	$\neg(\neg A \rightarrow B)$	$\vee C$	$\leftrightarrow$	$(A \vee B) \rightarrow C$
t	t	t	f	f	t	t	t
t	t	f	f	f	f	t	f
t	f	t	f	f	t	t	t
t	f	f	f	f	f	t	f
f	t	t	t	f	t	t	t
f	t	f	t	f	f	t	f
f	f	t	t	t	t	t	t
f	f	f	t	t	t	t	t

2. (a) $[(p \rightarrow q) \wedge (q \rightarrow s)] \vdash p \rightarrow (s \vee q)$
One proof tree is:

$$[(p \rightarrow q) \wedge (q \rightarrow s)] \vdash p \rightarrow (s \vee q) \qquad (2)$$
$$[p, (p \rightarrow q) \wedge (q \rightarrow s)] \vdash s \vee q \qquad (7)$$
$$[p, p \rightarrow q, q \rightarrow s] \vdash s \vee q \qquad (5)$$
$$[p, p \rightarrow q, q \rightarrow s] \vdash q \qquad (6)$$
$$[p, q \rightarrow s] \vdash p \qquad \text{(1 to finish)}$$

Alternatively,

$$[(p \rightarrow q) \wedge (q \rightarrow s)] \vdash p \rightarrow (s \vee q) \qquad (2)$$
$$[p, (p \rightarrow q) \wedge (q \rightarrow s)] \vdash s \vee q \qquad (7)$$
$$[p, p \rightarrow q, q \rightarrow s] \vdash s \vee q \qquad (4)$$
$$[p, p \rightarrow q, q \rightarrow s] \vdash s \qquad (6)$$
$$[p, p \rightarrow q] \vdash q \qquad (6)$$
$$[p] \vdash p \qquad \text{(1 to finish)}$$

(b) $[] \vdash ((p \wedge q) \to s) \to (p \to (q \to s))$

$$
\begin{array}{rr}
[] \vdash ((p \wedge q) \to s) \to (p \to (q \to s)) & (2) \\
[(p \wedge q) \to s] \vdash p \to (q \to s) & (2) \\
[(p \wedge q) \to s, p] \vdash q \to s & (2) \\
[(p \wedge q) \to s, p, q] \vdash s & (6) \\
[p, q] \vdash p \wedge q & (3) \\
[p, q] \vdash p & \text{(1 to finish left branch)} \\
[p, q] \vdash q & \text{(1 to finish right branch)}
\end{array}
$$

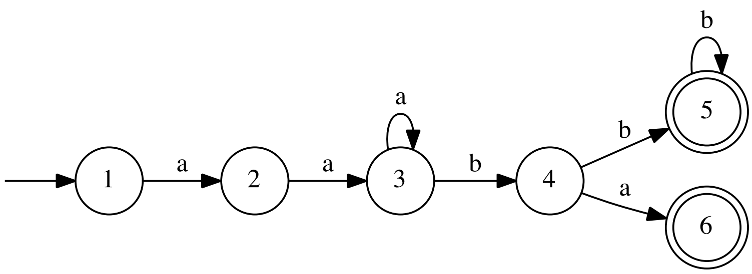
3. Given the following premises:
   $[x \vee w, (z \to y) \wedge (x \to z), \neg(z \wedge y)]$

   (a) (i). $[(x \vee w) \wedge (\neg z \vee y) \wedge (\neg x \vee z) \wedge (\neg z \vee \neg y)]$
   (ii). $[[x, w], [\neg z, y], [\neg x, z], [\neg z, \neg y]]$

   (b) Add $\neg w$ to axioms: $[[x, w], [\neg z, y], [\neg x, z], [\neg z, \neg y], [\neg w]]$
   An example proof is:

$$
\begin{array}{rr}
[[x, w], [\neg z, y], [\neg x, z], [\neg z, \neg y], [\neg w]] & ([\neg z, y], [\neg z, \neg y]) \\
[[x, w], [\neg z], [\neg x, z], [\neg w]] & ([x, w], [\neg x, z]) \\
[[w, z], [\neg z], [\neg w]] & ([w, z], [\neg w]) \\
[[z], [\neg z]] & ([z], [\neg z]) \\
[[]] &
\end{array}
$$

4. (a) See figure.

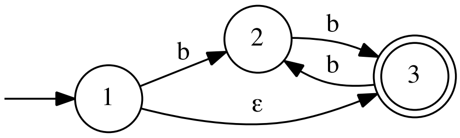


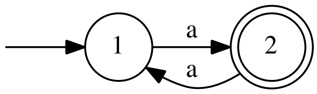
   (b) (i). See figure
   (ii). See figure
   See figure for composition of i) and ii)

   (c) New machine:

$$M = \langle \{\{\}, \{1\}, \{1, 2\}, \{2, 3\}, \{3\}\}, \{a, b\}, \{1\}, \{\{2, 3\}, \{3\}\}, \delta \rangle$$

   $\delta = \{\langle \{1\}, a, \{1, 2\} \rangle, \langle \{1\}, b, \{\} \rangle, \langle \{\}, a, \{\} \rangle, \langle \{\}, b, \{\} \rangle,$
   $\langle \{1, 2\}, a, \{1, 2\} \rangle,\ \langle \{1, 2\}, b, \{2, 3\} \rangle, \langle \{2, 3\}, b, \{2, 3\} \rangle, \langle \{2, 3\}, a, \{3\} \rangle,$
   $\langle \{3\}, a, \{3\} \rangle, \langle \{3\}, b, \{3\} \rangle\}$
   See figure for FSM.





5. (a) (i). $(a^*|b^*)^*aaa$

(ii).

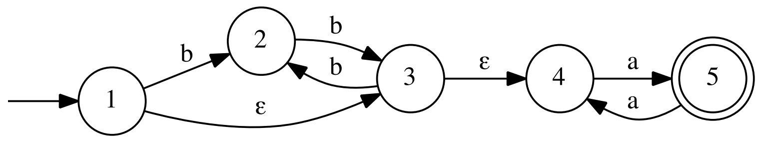
$$aaa^*ba^*b(a^*|b^*)^*|abaa^*b(a^*|b^*)^*|abbb^*a(a^*|b^*)^*|$$
$$bbb^*ab^*a(a^*|b^*)^*|babb^*a(a^*|b^*)^*|baaa^*b(a^*|b^*)^*$$

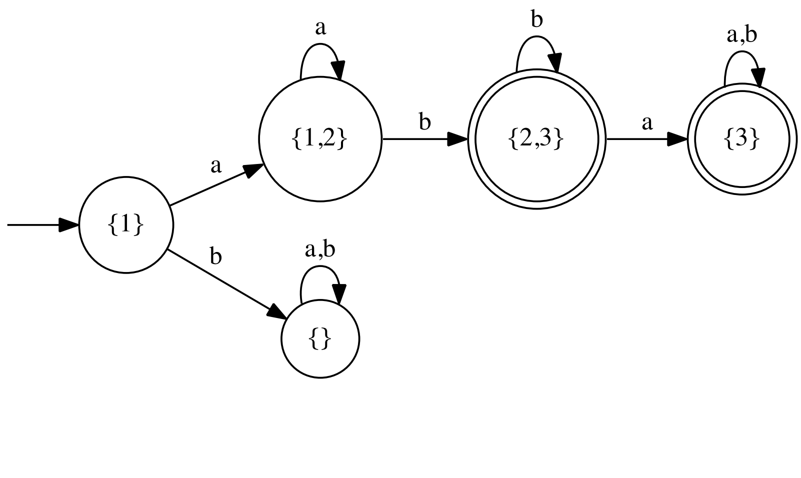
which factors to:
$((a(aa^*ba^*b|b(aa^*b|bb^*a))|(b(bb^*ab^*a|a(bb^*a|aa^*b))))(a^*|b^*)^*$

(b) $(11|0)1^*(1|0)00^*(00^*)^*$

(c) $(b(ab)^*ac)|((ba)^*c) = (ba)^*c$

$$(b(ab)^*ac)|((ba)^*c) \tag{5}$$
$$(b(ab)^*a|(ba)^*)c \tag{7}$$
$$((ba)^*ba|(ba)^*)c \tag{2}$$
$$((ba)^*ba|(ba(ba)^*|\varepsilon))c \tag{6}$$
$$((ba)^*ba|(ba(ba)^*)|\varepsilon)c \tag{3}$$
$$(ba(ba)^*|(ba(ba)^*)|\varepsilon)c \tag{1}$$
$$(ba(ba)^*|\varepsilon)c \tag{2}$$
$$(ba)^*c$$


1
2
3
4
5
b
b
b
ε
ε
a
a


{1}
{1,2}
{2,3}
{3}
{}
a
b
a
b
b
a
a,b
a,b