Module Title: Informatics 1 - Computation and Logic
Exam Diet (Dec/April/Aug): December 2012
Brief notes on answers:

1. (a) **Sensor 1 says** : $not(s2)\ and\ s3$
   Sensor 2 says : $not(s1) \rightarrow not(s3)$
   Sensor 3 says : $s3\ and\ (not(s2)\ or\ not(s1))$

   (b) Draw up a truth table with one row (where $s1$, $s2$ and $s3$ each is true) for each of the expressions above. The resulting truth values give the conclusion that Sensor 1 and Sensor 3 gave incorrect reports but what Sensor 2 says is correct.

   (c) Draw up the same truth table as for the previous question but all eight of the rows will need to be done, so as to consider all possible truth values for $s1$, $s2$ and $s3$. There is only one row in which all the expressions corresponding to peoples statements are true. This occurs when Sensor 1 and Sensor 3 are accurate and Sensor 2 is faulty.

2. Let S be the initial set of axioms $[p \rightarrow (a\ or\ b),\ a \rightarrow x,\ b \rightarrow x]$. Apply the proof rules in the following order:

$$
\begin{array}{l}
S \vdash p \rightarrow x \\
\quad Applying\ rule\ 7 \\
\quad\quad [p|S] \vdash x \\
\quad\quad\quad Applying\ rule\ 5 \\
\quad\quad\quad\quad [p|S] \vdash (aorb) \\
\quad\quad\quad\quad\quad Applying\ rule\ 6 \\
\quad\quad\quad\quad\quad\quad p \rightarrow (aorb) \in [p|S],\ [p|S] \vdash p \\
\quad\quad\quad\quad\quad\quad\quad Applying\ rule\ 1 \\
\quad\quad\quad\quad\quad\quad\quad\quad p \in [p|S] \\
\quad\quad\quad\quad [a|[p|S]] \vdash x \\
\quad\quad\quad\quad\quad Applying\ rule\ 6 \\
\quad\quad\quad\quad\quad\quad a \rightarrow x \in [a|[p|S]],\ [a|[p|S]] \vdash a \\
\quad\quad\quad\quad\quad\quad\quad Applying\ rule\ 1 \\
\quad\quad\quad\quad\quad\quad\quad\quad a \in [a|[p|S]] \\
\quad\quad\quad\quad [b|[p|S]] \vdash x \\
\quad\quad\quad\quad\quad Applying\ rule\ 6 \\
\quad\quad\quad\quad\quad\quad b \rightarrow x \in [b|[p|S]],\ [b|[p|S]] \vdash b \\
\quad\quad\quad\quad\quad\quad\quad Applying\ rule\ 1 \\
\quad\quad\quad\quad\quad\quad\quad\quad b \in [b|[p|S]]
\end{array}
$$

3. (a) An expression in clausal form $[[E1_1, \ldots, E1_n], \ldots [EN_1, \ldots, EN_m]$ corresponds to the propositional logic expression
   $(E1_1\ or\ \ldots\ or\ E1_n)\ and\ \ldots\ and\ (EN_1\ or\ \ldots\ or\ EN_m)$

   (b) The steps of conversion are:

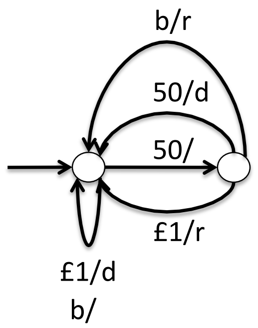
   - $[a,\quad (a\ and\ b) \rightarrow c,\quad (d\ or\ e) \rightarrow b,\quad a \rightarrow e]$
   - $[a,\quad not(a\ and\ b)\ or\ c,\quad (d\ or\ e) \rightarrow b,\quad a \rightarrow e]$
   - $[a,\quad not(a\ and\ b)\ or\ c,\quad d \rightarrow b,\quad e \rightarrow b,\quad a \rightarrow e]$

- $[a, \quad not(a \text{ } and \text{ } b) \text{ } or \text{ } c, \quad not(d) \text{ } or \text{ } b, \quad not(e) \text{ } or \text{ } b, \quad a \rightarrow e]$
- $[a, \quad not(a \text{ } and \text{ } b) \text{ } or \text{ } c, \quad not(d) \text{ } or \text{ } b, \quad not(e) \text{ } or \text{ } b, \quad not(a) \text{ } or \text{ } e]$
- $[a, \quad not(a) \text{ } or \text{ } not(b) \text{ } or \text{ } c, \quad not(d) \text{ } or \text{ } b, \quad not(e) \text{ } or \text{ } b, \quad not(a) \text{ } or \text{ } e]$
- $[[a], \ [not(a), not(b), c], \ [not(d), b], \ [not(e), b], \ [not(a), e]]$

(c) An appropriate proof is:

- Negate c as the input clause $[not(c)]$
- Resolve $[notc]$ with $[not(a), not(b), c]$ giving $[not(a), not(b)]$
- Resolve $[not(a), not(b)]$ with $[a]$ giving $[not(b)]$
- Resolve $[not(b)]$ with $[not(e), b]$ giving $[not(e)]$
- Resolve $[not(e)]$ with $[not(a), e]$ giving $[not(a)]$
- Resolve $[not(a)]$ with $[a]$ giving $[]$
- Hence $not(c)$ is contradictory
- Hence c is true.

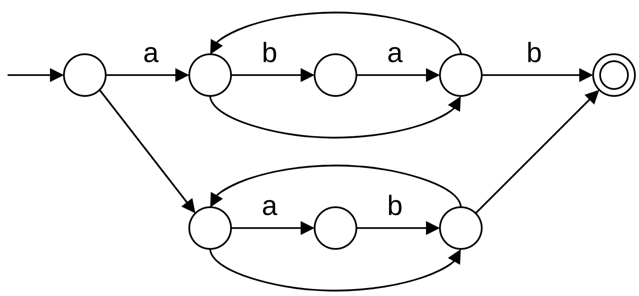
4. An appropriate transducer is:



5. The sequence of transformations is as follows:

$(a(ba)^*b)\|(ab)^*$
$\Downarrow$ $R(SR)^*$ is equivalent to $(RS)^*R$ $\Downarrow$
$(ab)^*ab\|(ab)^*$
$\Downarrow$ R^*R is equivalent to RR^* $\Downarrow$
$ab(ab)^*\|(ab)^*$
$\Downarrow$ R^* is equivalent to $RR^*\|\epsilon$ $\Downarrow$
$ab(ab)^*\|ab(ab)^*\|\theta$
$\Downarrow$ $R\|R$ is equivalent to R $\Downarrow$
$ab(ab)^*\|\theta$
$\Downarrow$ R^* is equivalent to $RR^*\|\epsilon$ $\Downarrow$
$(ab)^*$

6. The resulting FSM (built using the modular method described in lectures) is:



7. (a) $(a|b)^*a(a|b)^*$

   (b) $((a|b)^*a(a|b)^*b(a|b)^*)|((a|b)^*b(a|b)^*a(a|b)^*)$