UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR08012 INFORMATICS 1 - COMPUTATION AND LOGIC

Friday 18th December 2015

14:30 to 16:30

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**
3. **CALCULATORS MAY NOT BE USED IN THIS EXAMINATION.**

Convener: D. K. Arvind
External Examiner: C. Johnson

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) For each of the following propositional expressions, use a truth table to determine whether it is tautologous, contradictory, or contingent. [*2 marks*]

   i. $((\neg B \to \neg A) \to B) \to B$

   ii. $((A \vee B) \to A) \to (B \to A)$

   (b) This part concerns the 256 possible truth valuations of the following eight propositional letters A, B, C, D, E, F, G, H. For each of the following expressions, say how many of the 256 valuations satisfy the expression, and briefly explain your reasoning. For example, the expression $\neg D$ is satisfied by half of the valuations, that is 128 of the 256, since for each valuation that makes D true there is a matching valuation that makes D false.

   i. A

   ii. $E \oplus F$

   iii. $B \vee \neg D$

   iv. $\neg A \to H$

   v.
$$(A \to B) \wedge (B \to C) \wedge (C \to D)$$

   vi.
$$(A \to C) \wedge (B \to C) \wedge (C \to E) \wedge (D \to E)$$

   vii.
$$\begin{aligned}(A \to B) \wedge (B \to C) \wedge (C \to D) \wedge (D \to E)&\\ \wedge (E \to G) \wedge (F \to G) \wedge (G \to H) \wedge (H \to E)&\end{aligned}$$

   viii.
$$(A \to (B \wedge C)) \wedge (B \to D) \wedge (D \to E) \wedge (E \to F) \wedge (F \to G) \wedge (G \to H)$$

[*8 marks*]

2. You are given the following inference rules, due to Gentzen:

$$\frac{}{\Gamma, A \vdash \Delta, A}\ (I)$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R)$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ (\wedge R)$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \to B \vdash \Delta}\ (\to L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta}\ (\to R)$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ (\neg R)$$

Here, A and B are propositional expressions, Γ, Δ are finite sets of expressions, and Γ, A is shorthand for $\Gamma \cup \{A\}$.

An entailment $\Gamma \vdash \Delta$ is *valid* iff every valuation that makes each expression in Γ true makes some expression in Δ true.

(a) i. Explain what it means to say that a valuation V is a **counterexample** for an entailment, $\Gamma \vdash \Delta$. *[2 marks]*

ii. Show that the rule $(\to L)$ has the property that a valuation V is a counterexample for the rule's conclusion iff it is a counterexample to at least one of its assumptions. *[4 marks]*

(b) Use the Gentzen rules to show that *[4 marks]*

$$(Q \to P) \to Q \vdash Q$$

(c) Use the Gentzen rules to build an attempted proof of *[4 marks]*

$$(P \wedge Q) \to R, P \vee Q \vdash R$$

(d) Derive a counter-example from your attempted proof. (Show your working, and briefly justify your answer in terms of the particular properties of this set of rules.) *[2 marks]*

(e) i. What does it mean to say a set of rules is **complete**? *[2 marks]*

ii. Is this a complete set of rules? Briefly justify your answer. *[2 marks]*

3. (a) Express each of the following expressions in clausal form. Write your answers in the table provided for Q3c. (Please order the literals in each clause alphabetically.) *[4 marks]*

   i. $P \rightarrow (Q \vee R)$

   ii. $(P \wedge Q) \rightarrow \neg R$

   iii. $\neg P \rightarrow (R \rightarrow Q)$

   iv. $(Q \wedge \neg P) \rightarrow R$

(b) Use resolution to determine whether there is a valuation satisfying the conjunction of these four expressions. *[6 marks]*

(c) Using the table provided on the final page of this paper, record the search for a satisfying valuation for these clauses, using the one watched literal algorithm.

You may use the table on this page for rough working, but you *must* use the final page to submit your answer to Q3(c).

Start with the empty valuation, watching the first literal in each clause. At each step of the search use one column of the table provided to record in the first 3 squares the truth values assigned to the atoms, P, Q, R, and in the remaining four, any changed position (1,2,3) of the watched literal for each of the four clauses. When you need to find a new literal to watch always choose the first one available (reading each expression left-to-right). Place an $\times$ in the watched literal square when no suitable literal is available.

P											
Q											
R											
(i)											
(ii)											
(iii)											
(iv)											

Satisfying valuation: *[10 marks]*

This question continues on the next page ...

(d) What is the invariant property of the watched literals that has to be maintained? *[2 marks]*

(e) What must you do when it is not possible to maintain the invariant?

(f) Sketch the search tree explored by this search. *[4 marks]*

(g) What invariant must be maintained for the 2-watched literals algorithm? *[2 marks]*

(h) What advantage is gained by watching two literals? *[2 marks]*

4. A cruise control system for a car communicates with the engine to maintain a set speed.

   The engine periodically provides **inputs**

   c: correct

   s: slow

   f: fast

   to the controller and the controller matches these with appropriate **outputs**

   a: acc

   d: dec

   to indicate that the engine should accelerate or decelerate.

   Requirements In addition to this function, the controller must interact with the driver.

   - The driver should always be able to turn the system off.
   - The driver should be able to request the system to maintain the current speed.
   - If the driver brakes the system should go into standby mode.
   - The system should allow the driver to travel faster than the set speed by using the accelerator.

   The system has the following **additional inputs** to sense the user's actions and commands

onoff: A button that toggles the system on and off.

set: set the cruise speed to the current speed

brake: the brake has been pressed

accP: the accelerator has been pressed

accR: the accelerator has been released

resume: resume travelling at the set speed

States The system has 5 states:

Off: The system is not operational.

Ready: The system is switched on but so far no speed has been set to cruise at.

Steady: A cruise speed has been set and the system is maintaining it.

Standby: The system has a set cruise speed but at some time the driver used the brake and caused the system to wait until the resume button is pressed to bring the car back into cruise control.

Acc: The system has a set cruise speed but the accelerator has been pressed to override cruise control until the accelerator is released.

(a) Design a transducer-style finite state machine to model this system. [*8 marks*]

(b) Justify your design by explaining how it meets each of the requirements. If any of the requirements is ambiguous or unclear, explain how you have resolved the ambiguity. [*12 marks*]

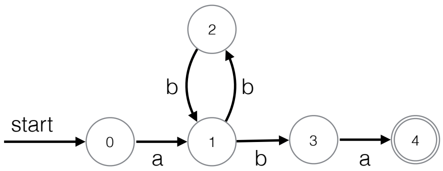
i. The driver should always be able to turn the system off.

ii. The driver should be able to request the system to maintain the current speed.

iii. If the driver brakes the system should go into standby mode.

iv. The system should allow the driver to travel faster than the set speed by using the accelerator.

5. (a) Which of the following strings are accepted by the NFA in the diagram? (The start state is indicated by an arrow and the accepting state by a double border.)



i. abb
ii. aba
iii. abbbbba
iv. abbbbbba

[*4 marks*]

(b) Write a regular expression for the language accepted by this NFA. [*2 marks*]

(c) Draw a DFA that accepts the same language. Label the states of your DFA to make clear their relationship to the states of the original NFA. [*5 marks*]

(d) For each of the following regular expressions, draw a non-deterministic finite state machine that accepts the language described by the regular expression.

i. x^*y

ii. $((x|y)z)*$

iii. $(x^*|y^*)$

[*9 marks*]

Answer to Q 3(c) Fill in your answer to this part in the table provided below. **FILL IN YOUR EXAMINATION NUMBER** and place this sheet inside your answer book.

Examination number..

Start with the empty valuation, watching the first literal in each clause. At each step of the search use one column of the table provided to record in the first 3 squares the truth values assigned to the atoms, P, Q, R, and in the remaining four, any changed position (1,2,3) of the watched literal for each of the four clauses. When you need to find a new literal to watch always choose the first one available (reading each expression left-to-right). Place an × in the watched literal square when no suitable literal is available.

P											
Q											
R											
(i)											
(ii)											
(iii)											
(iv)											

Satisfying valuation: [*10 marks*]