UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR08012 INFORMATICS 1 - COMPUTATION AND LOGIC

Thursday 20th December 2012

14:30 to 16:30

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**
3. **Calculators may not be used in this examination.**

Convener: J Bradfield
External Examiner: A Preece

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. Imagine that you have a system with three sensor units: Sensor 1, Sensor 2 and Sensor 3. Each sensor is monitoring some aspect of the environment. Sensors can become faulty so there is a system of self-monitoring between the sensors, allowing each one to provide a report on which of the sensors it claims to be operating correctly (*i.e.* it believes these to be non-faulty) and which it believes to be faulty.

   Each sensor has made the following report:

   Sensor 1 reports : "Sensor 2 is faulty and Sensor 3 is non-faulty."

   Sensor 2 reports : "If Sensor 1 is faulty then so is Sensor 3."

   Sensor 3 reports : "I am non-faulty but at least one of the other two sensors is faulty."

   Let $s1$, $s2$ and $s3$ mean "Sensor 1 is non-faulty", "Sensor 2 is non-faulty" and "Sensor 3 is non-faulty", respectively.

   (a) Rewrite the reports made by Sensor 1, Sensor 2 and Sensor 3, each as an expression in propositional logic. [*3 marks*]

   (b) If all the sensors are non-faulty (so $s1$, $s2$ and $s3$ are true), which sensors gave incorrect reports? Explain your answer using a truth table. [*3 marks*]

   (c) If the statements made by all the sensors are true, which sensors are faulty? Explain your answer using a truth table. [*6 marks*]

2. You are given the following proof rules:

Rule number	Sequent	Supporting proofs
1	$\mathcal{F} \vdash A$	$A \in \mathcal{F}$
2	$\mathcal{F} \vdash A \textit{ and } B$	$\mathcal{F} \vdash A,\ \mathcal{F} \vdash B$
3	$\mathcal{F} \vdash A \textit{ or } B$	$\mathcal{F} \vdash A$
4	$\mathcal{F} \vdash A \textit{ or } B$	$\mathcal{F} \vdash B$
5	$\mathcal{F} \vdash C$	$\mathcal{F} \vdash (A \textit{ or } B),\ [A\vert\mathcal{F}] \vdash C,\ [B\vert\mathcal{F}] \vdash C$
6	$\mathcal{F} \vdash B$	$A \rightarrow B \in \mathcal{F},\ \mathcal{F} \vdash A$
7	$\mathcal{F} \vdash A \rightarrow B$	$[A\vert\mathcal{F}] \vdash B$

where $\mathcal{F} \vdash A$ means that expression A can be proved from set of axioms $\mathcal{F}$; $A \in \mathcal{F}$ means that A is an element of set $\mathcal{F}$; $[A|\mathcal{F}]$ is the set constructed by adding A to set $\mathcal{F}$; $A \rightarrow B$ means that A implies B; $A \textit{ and } B$ means that A and B both are true; and $A \textit{ or } B$ means that at least one of A or B is true.

Using the proof rules above, prove the following:

$$[p \rightarrow (a \textit{ or } b),\ a \rightarrow x,\ b \rightarrow x] \quad \vdash \quad p \rightarrow x$$

Show precisely how the proof rules are applied. *[20 marks]*

3. The proposition c can be proved from the following set of axioms:

$$[a, \quad (a \textit{ and } b) \rightarrow c, \quad (d \textit{ or } e) \rightarrow b, \quad a \rightarrow e]$$

(a) Explain what clausal form notation means, in terms of the conjunction and disjunction of propositional expressions. *[5 marks]*

(b) Explain how to convert the set of axioms above into clausal form using the following equivalences.

$A \rightarrow B$	is equivalent to	$not(A) \textit{ or } B$
$(A \textit{ or } B) \rightarrow C$	is equivalent to	$(A \rightarrow C) \textit{ and } (B \rightarrow C)$
$not(not(A))$	is equivalent to	A

Show, in detail, each step in your conversion. *[10 marks]*

(c) Give a proof, using resolution, of c from the axioms in clausal form above. Show each step of your proof in detail. *[15 marks]*

4. A simple ticket dispenser accepts one pound or fifty pence coins and its only output is to dispense a ticket. Tickets cost one pound (so a ticket can be bought for either two fifty pence pieces or a pound coin). If too much money is entered then the machine returns the coins entered. If the user presses the "return" button at any stage the machine also returns any coins entered. Draw a transducer-style finite state machine to model this system. *[5 marks]*

5. Given the following equivalences between regular expressions:

$R(SR)^*$	is equivalent to	$(RS)^*R$
R^*R	is equivalent to	RR^*
R^*	is equivalent to	$RR^*\|\epsilon$
$R\|R$	is equivalent to	R

show that the regular expression $(a(ba)^*b)|(ab)^*$ is equivalent to the regular expression $(ab)^*$ *[14 marks]*

6. Finite State Machines (FSMs) can be constructed in a modular way by combining smaller FSM fragments into larger designs. Show how this approach can be used to produce a FSM corresponding to the regular expression $(a(ba)^*b)|(ab)^*$ and give the resulting FSM. *[14 marks]*

7. Assuming that your alphabet is the set of characters $\{a, b\}$, write regular expressions for the following languages:

   (a) All strings containing at least one a.

   (b) All strings containing at least one occurrence of a and at least one occurrence of b.

[5 marks]