UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFORMATICS 1 - COMPUTATION AND LOGIC

Thursday 27 August 2009

09:30 to 11:30

Convener: M O'Boyle
External Examiner: R Irving

INSTRUCTIONS TO CANDIDATES

1. **ANSWER ALL QUESTIONS.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. The expression

$$(p \; and \; (q \; or \; r)) \; or \; (p \; and \; not(q \; and \; r))$$

can be simplified into an expression with fewer propositions and logical connectives. Simplify it as much as possible and use a truth table to show that the simplified version means the same thing as the original expression. [*10 marks*]

2. Suppose you are given a new logical operator called "xor". The expression $p \; xor \; q$ is true when ecactly one of p or q is true but is false when p and q are both false or both true. Using a truth table, explain whether or not $p \; xor \; q$ is equivalent to $(p \; or \; q) \; and \; (q \rightarrow not(p))$. [*10 marks*]

3. You are given the following set of axioms:

$$[a, \quad (a \; and \; b) \rightarrow c, \quad (d \; or \; e) \rightarrow b, \quad a \rightarrow e]$$

Convert the set of axioms above into clausal form. To help you to do this, you are given the following equivalences.

$A \rightarrow B$	is equivalent to	$not(A) \; or \; B$
$(A \; or \; B) \rightarrow C$	is equivalent to	$(A \rightarrow C) \; and \; (B \rightarrow C)$
$not(A \; and \; B)$	is equivalent to	$not(A) \; or \; not(B)$
$not(not(A))$	is equivalent to	A

[*10 marks*]

4. The following is a description, in English, of the rules of a simple grammar:

   A sentence consists of either:

   - a verbphrase followed by a nounphrase; or
   - a verbphrase followed by a nounphrase followed by the word "and" followed by a sentence.

   A verbphrase consists of a nounphrase followed by a verb. A nounphrase consists of either a noun or a determiner followed by a noun. The word "bit" is the only verb. The words "cat" and "dog" are the only nouns. The words "a" and "the" are the only determiners.

   Examples of word sequences accepted by this grammar are:

   - "the cat bit a dog and the dog bit the cat".
   - "cat bit dog".
   - "cat bit dog and the dog bit a dog and a cat bit cat".

Examples of word sequences *not* accepted by this grammar are:

- "the cat".
- "the cat and the dog bit the dog".
- "bit the cat".

Describe a finite state acceptor for this grammar. [*20 marks*]

5. Suppose that you are a safety analyst for an engine manufacturing company and have the task of relating engine faults to engine failures via a proof system. You are sure that engine failure always happens when there is low fuel pressure. There can, however, be multiple faults. A colleague in your company has defined rules for combining multiple faults and failures; one such rule is the following:

   > Assuming that low fuel pressure implies engine failure, when there is an electrical fault or low fuel pressure then we can conclude that there is an electrical fault or engine failure.

   (a) Express this rule in propositional logic and show that this rule can be proved in your proof system, which contains the following proof rules:

Rule name	Sequent	Supporting proofs
immediate	$\mathcal{F} \vdash A$	$A \in \mathcal{F}$
and_intro	$\mathcal{F} \vdash A \; and \; B$	$\mathcal{F} \vdash A,\ \mathcal{F} \vdash B$
or_intro_left	$\mathcal{F} \vdash A \; or \; B$	$\mathcal{F} \vdash A$
or_intro_right	$\mathcal{F} \vdash A \; or \; B$	$\mathcal{F} \vdash B$
or_elim	$\mathcal{F} \vdash C$	$A \; or \; B \in \mathcal{F},\ [A\|\mathcal{F}] \vdash C,\ [B\|\mathcal{F}] \vdash C$
imp_elim	$\mathcal{F} \vdash B$	$A \rightarrow B \in \mathcal{F},\ \mathcal{F} \vdash A$
imp_intro	$\mathcal{F} \vdash A \rightarrow B$	$[A\|\mathcal{F}] \vdash B$

   where $\mathcal{F} \vdash A$ means that expression A can be proved from set of axioms $\mathcal{F}$; $A \in \mathcal{F}$ means that A is an element of set $\mathcal{F}$; $[A|\mathcal{F}]$ is the set constructed by adding A to set $\mathcal{F}$; $A \rightarrow B$ means that A implies B; $A \; and \; B$ means that A and B both are true; and $A \; or \; B$ means that at least one of A or B is true.

   [*20 marks*]

   (b) Using only the proof rules above it is not possible to perform the following proof:

$$[(a \; and \; b)] \quad \vdash \quad a$$

   Briefly explain what (if anything) this observation tells us about the soundness and completeness of the proof system. [*5 marks*]

6. Suppose that you are designing a fault monitoring system for an engine. The system monitors information sent by the engine system about the presence or absence of an electrical fault and the presence or absence of low fuel pressure. In response to this information it switches on or off two separate warning lights, one for an electrical fault and the other for low fuel pressure.

   The monitoring system accepts the following input events:

   - A signal from the engine warning of an electrical fault.
   - A signal from the engine warning of low fuel pressure.
   - A signal from the engine that there is no longer an electrical fault.
   - A signal from the engine that there is no longer low fuel pressure.

   The monitoring system has the following outputs:

   - A signal to activate the electrical fault warning light.
   - A signal to activate the low fuel pressure warning light.
   - A signal to deactivate the electrical fault warning light.
   - A signal to deactivate the low fuel pressure warning light.

   It must satisfy the following functional requirements:

   - The electrical fault warning light is always activated immediately after a signal from the engine warning of an electrical fault.
   - The low fuel pressure warning light is always activated immediately after a signal from the engine warning of low fuel pressure.
   - If it is activated, the electrical fault warning light is always deactivated immediately after a signal from the engine that there is no longer an electrical fault.
   - If it is activated, the low fuel pressure warning light is always deactivated immediately after a signal from the engine that there is no longer low fuel pressure.
   - Deactivation of one of the warning lights does not deactivate the other if it is active.
   - Activation of one of the warning lights does not activate the other if it is inactive.
   - Deactivation of one of the warning lights does not activate the other if it is inactive.
   - Activation of one of the warning lights does not deactivate the other if it is active.

(a) Define a transducer FSM satisfying the engine monitoring requirements above. Notice that this must be a single FSM controlling both lights, not separate FSMs for each light. [*20 marks*]

(b) Explain, using the FSM you have defined, why any two of the eight requirements above are satisfied by your FSM. [*5 marks*]