Module Title: Informatics 1A: Logic and Computation
Exam Diet (Dec/April/Aug): August 2004
Brief notes on answers:

1. (a) **p1 reports** : $not(p2)\ and\ p3$
   p2 reports : $not(p1)\ \rightarrow\ not(p3)$
   p3 reports : $p3\ and\ (not(p2)\ or\ not(p1))$

   (b) Draw up a truth table with one row (where $p1$, $p2$ and $p3$ each is true) for each of the expressions above. This is the first row of the truth table below. The resulting truth values give the conclusion that p1 and p3 lied but what p2 says is true.

   (c) Draw up the same truth table as for the previous question but (unless the student is very insightful) all eight of the rows will need to be done, so as to consider all possible truth values for $p1$, $p2$ and $p3$. The full truth table is below. There is only one row (the third one) in which all the expressions corresponding to reports are true. This occurs when p1 and p3 are non-faulty and p2 is faulty.

$p1$	$p2$	$p3$	$not(p2)$	$not(p2)\ and\ p3$	$not(p1)$	$not(p3)$	$not(p1)\ and\ not(p3)$	$not(p2)\ or\ not(p1)$	$p3\ and\ (not(p2)\ or\ not(p1))$
t	t	t	f	f	f	f	t	f	f
t	t	f	f	f	f	t	t	f	f
t	f	t	t	t	f	f	t	t	t
t	f	f	t	f	f	t	t	t	f
f	t	t	f	f	t	f	f	t	t
f	t	f	f	f	t	t	t	t	f
f	f	t	t	t	t	f	f	t	t
f	f	f	t	f	t	t	t	t	f

2. Let S be the initial set of axioms $[p \rightarrow (a\ or\ b),\ a \rightarrow x,\ b \rightarrow x]$. Apply the proof rules in the following order:

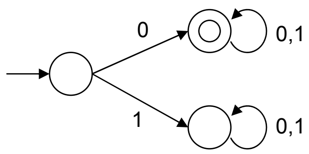
$$
\begin{array}{l}
S\ \vdash\ p \rightarrow x \\
\quad Applying\ rule\ 7 \\
\quad\quad [p|S]\ \vdash\ x \\
\quad\quad\quad Applying\ rule\ 5 \\
\quad\quad\quad\quad [p|S]\ \vdash\ (aorb) \\
\quad\quad\quad\quad\quad Applying\ rule\ 6 \\
\quad\quad\quad\quad\quad\quad p \rightarrow (aorb)\ \in\ [p|S],\ [p|S]\ \vdash\ p \\
\quad\quad\quad\quad\quad\quad\quad Applying\ rule\ 1 \\
\quad\quad\quad\quad\quad\quad\quad\quad p\ \in\ [p|S] \\
\quad\quad\quad\quad [a|[p|S]]\ \vdash\ x \\
\quad\quad\quad\quad\quad Applying\ rule\ 6 \\
\quad\quad\quad\quad\quad\quad a \rightarrow x\ \in\ [a|[p|S]],\ [a|[p|S]]\ \vdash\ a \\
\quad\quad\quad\quad\quad\quad\quad Applying\ rule\ 1 \\
\quad\quad\quad\quad\quad\quad\quad\quad a\ \in\ [a|[p|S]] \\
\quad\quad\quad\quad [b|[p|S]]\ \vdash\ x \\
\quad\quad\quad\quad\quad Applying\ rule\ 6 \\
\quad\quad\quad\quad\quad\quad b \rightarrow x\ \in\ [b|[p|S]],\ [b|[p|S]]\ \vdash\ b \\
\quad\quad\quad\quad\quad\quad\quad Applying\ rule\ 1 \\
\quad\quad\quad\quad\quad\quad\quad\quad b\ \in\ [b|[p|S]]
\end{array}
$$

3. (a) A set of sets, where the outer set represents a conjunction and the elemental sets represent disjunctions. Each elemental set consists of propositions or their negations.

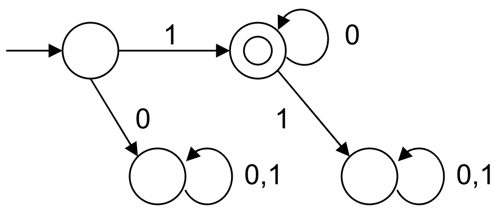
(b) An appropriate proof is:

- Negate d as the input clause $[not(d)]$
- Resolve $[notd]$ with $[not(c), d]$ giving $[not(c)]$
- Resolve $[not(c)]$ with $[not(a), not(b), c]$ giving $[not(a), not(b)]$
- Resolve $[not(a), not(b)]$ with $[a]$ giving $[not(b)]$
- Resolve $[not(b)]$ with $[b]$ giving $[]$
- Hence $not(d)$ is contradictory
- Hence d is true.

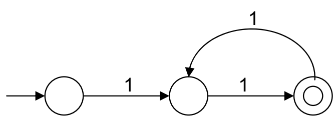
4. (a) (i). One answer is as follows (others possible):



(ii). One answer is as follows (others possible):

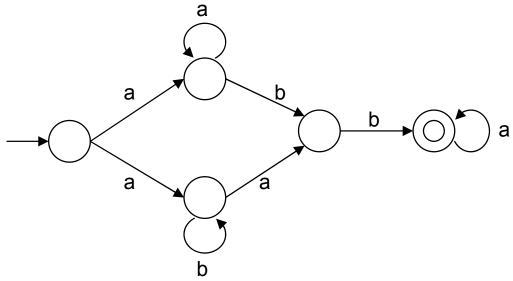


(b) (i). One answer is as follows (others possible):

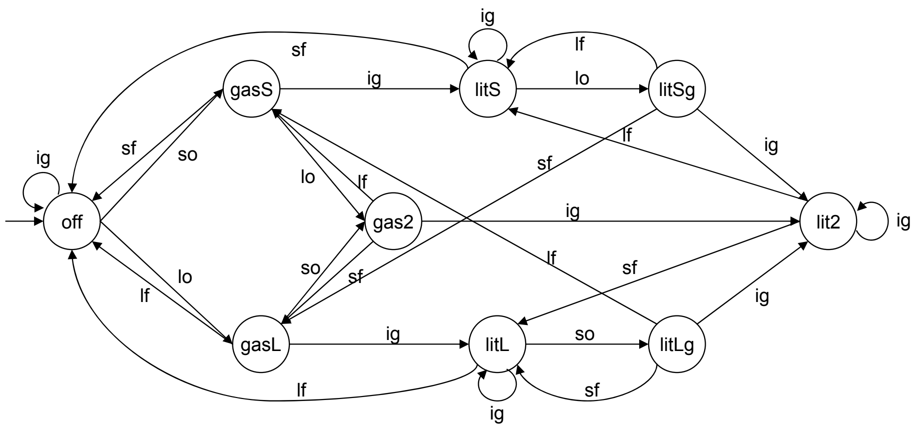


(ii). Not possible to construct this machine because we can't use the "trick" we had in binary to determine a power via local state; we need to count to an arbitrarily high number.

5. One answer is as follows (others possible):



6. (a) One answer is as follows (others possible):



(b) Answers to this question involve demonstrating that all possible traces through the FSM give the desired property:

- Show that in every trace containing a so or lo followed by a ig we reach an appropriate *lit* state.
- Show that for all traces containing a sf or lf there follows a transition to a state that is not one of the *lit* ones.
- Show that for all states that are not *lit* states there is no *lit* state that can be reached without passing through a so or lo followed by an ig.
- Show that there is no trace to the off state for which so has occurred without a succeeding sf or lo has occurred without a succeeding lf.

7. Sum the probabilities for all valid traces for "aab", where the probability for a trace is the product of the probabilities for its transitions. This gives the probability for the example as:

$$(0.2 * 1 * 0.6) + (0.8 * 0.5 * 1) + (0.8 * 0.5 * 0.6) = 0.76$$