UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR08012 INFORMATICS 1 - COMPUTATION AND LOGIC

Monday 8 th December 2014

09:30 to 11:30

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**
3. **CALCULATORS MAY NOT BE USED IN THIS EXAMINATION.**

Convener: D. K. Arvind
External Examiner: C. Johnson

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) For each of the following propositional expressions, use a truth table to determine whether it is tautologous, contradictory, or contingent. [*3 marks*]

   i. $A \to ((A \to B) \to B)$

   ii. $((A \to B) \to B) \to B$

   iii. $((A \to B) \to B) \to A$

   (b) This part concerns the 256 possible truth valuations of the following eight propositional letters A, B, C, D, E, F, G, H. For each of the following expressions, say how many of the 256 valuations satisfy the expression, and briefly explain your reasoning. For example, the expression D is satisfied by half of the valuations, that is 128 of the 256, since for each valuation that makes D true there is a matching valuation that make D false. [*7 marks*]

   i. $\neg A$

   ii. $E \wedge F$

   iii. $B \vee D$

   iv. $A \to H$

   v.

$$(A \to B) \wedge (B \to C) \wedge (C \to D) \wedge (D \to E) \wedge (E \to F) \wedge (F \to G) \wedge (G \to H)$$

   vi.

$$\begin{aligned}(A \to B) \wedge (B \to C) \wedge (C \to D) \wedge (D \to A)& \\ \wedge (E \to F) \wedge (F \to G) \wedge (G \to H) \wedge (H \to E)&\end{aligned}$$

   vii.

$$(A \to B \wedge C) \wedge (B \vee C \to D) \wedge (D \to E) \wedge (E \to F) \wedge (F \to G) \wedge (G \to H)$$

2. You are given the following inference rules: (Γ, Δ vary over finite sets of expressions; A, B vary over expressions):

$$\frac{}{\Gamma, A, B \vdash \Delta, A}\ (I)$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R)$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ (\wedge R)$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \to B \vdash \Delta}\ (\to L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta}\ (\to R)$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ (\neg R)$$

(Where A and B are propositional expressions, Γ, Δ are sets of expressions, and Γ, A refers to $\Gamma \cup \{A\}$.)

(a) Explain what it means to claim that these rules are: *[4 marks]*

  i. **sound**,

  ii. and **complete**.

(b) Use these rules to show that *[10 marks]*

$$P \to Q, \neg R \to \neg Q \vdash P \to R$$

(c) Use these rules to build an attempted proof of

$$P \to Q, R \to Q \vdash P \to R$$

[7 marks]

(d) Explain how you can derive a counter-example from your attempted proof. *[2 marks]*

(e) Is this a complete set of rules? Briefly justify your answer. *[2 marks]*

3. It is claimed that the proposition $\neg U$ follows from the following three assumptions:

$$\neg(T \vee Q) \qquad (P \rightarrow Q) \vee \neg(\neg S \wedge \neg T) \qquad U \rightarrow (\neg T \rightarrow (\neg S \wedge P))$$

This question concerns the resolution of this claim.

(a) Express each of the assumptions in clausal form. *[6 marks]*

i. $\neg(T \vee Q)$

ii. $(P \rightarrow Q) \vee \neg(\neg S \wedge \neg T)$

iii. $U \rightarrow (\neg T \rightarrow (\neg S \wedge P))$

(b) Explain how you would use resolution to determine whether the claim is correct. *[4 marks]*

(c) Use resolution to determine whether the claim is correct. (Show your working.) *[10 marks]*

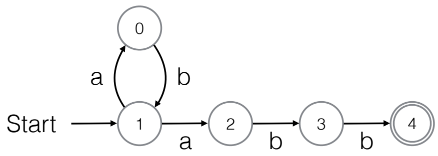
4. A vending machine takes 10p and 20p coins. Coffee costs 30p, tea costs 20p. When a user inserts coins, the machine keeps track of the amount inserted, up to a maximum of 40p.

   The user can request coffee whenever she has at least 30p credit, tea whenever she has at least 20p credit, and a refund or change whenever she has a non-zero credit — these three actions correspond to three buttons on the machine, which light up when the action is available.

   (a) Design a transducer-style finite state machine to model this system. Sketch the machine and explain clearly what the input and output alphabets mean, and what the set of states and transition function are. [*15 marks*]

   (b) Give the trace of the machine corresponding to an interaction with a user who inserts two 20p coins, then takes a coffee and collects the change. [*5 marks*]

5. (a) Which of the following strings are accepted by the NFA in the diagram? (The start state is indicated by an arrow and the accepting state by a double



border.) [*3 marks*]

i. abb

ii. abababaabb

iii. ababababab b [*3 marks*]

(b) Write a regular expression for the language accepted by this NFA.

(c) Draw a DFA that accepts the same language. Label the states of your DFA to make clear their relationship to the states of the original NFA. [*10 marks*]

(d) For each of the following regular expressions, draw a non-deterministic finite state machine that accepts the language described by the regular expression. [*9 marks*]

i. xy^*

ii. $(x|y)*$

iii. $(xy)^*$