UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFORMATICS 1 - COMPUTATION AND LOGIC

Wednesday 9 December 2009

09:30 to 11:30

Convener: J Bradfield
External Examiner: A Preece

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) Show that $\neg(A \rightarrow (B \rightarrow C))$ and $A \wedge B \wedge \neg C$ are equivalent using a truth table. [*10 marks*]

   (b) Are each of the two expressions above contingent, tautologous or inconsistent? [*2 marks*]

   (c) Using logical equivalences given below show that the following expressions are equivalent. Show which equivalence rules are applied.

1	$X \rightarrow Y \leftrightarrow \neg X \vee Y$
2	$\neg\neg X \leftrightarrow X$
3	$\neg(X \vee Y) \leftrightarrow \neg X \wedge \neg Y$
4	$\neg(X \wedge Y) \leftrightarrow \neg X \vee \neg Y$

   i. $\neg(A \rightarrow (B \rightarrow C))$ is equivalent to $A \wedge B \wedge \neg C$

   ii. $(A \rightarrow B) \rightarrow \neg(B \wedge \neg C)$ is equivalent to $(A \wedge \neg B) \vee (B \rightarrow C)$

   [*8 marks*]

2. Given the listed sequent rules prove the following, showing precisely which rules are applied.

Rule	Sequent	Supporting proofs
imm	$\mathcal{F} \vdash A$	$A \in \mathcal{F}$
imp_intro	$\mathcal{F} \vdash A \rightarrow B$	$[A\|\mathcal{F}] \vdash B$
and_intro	$\mathcal{F} \vdash A \wedge B$	$\mathcal{F} \vdash A, \ \mathcal{F} \vdash B$
imp_elim	$\mathcal{F} \vdash B$	$A \rightarrow B \in \mathcal{F}, \ \mathcal{F} \vdash A$
and_elim	$\mathcal{F} \vdash C$	$A \wedge B \in \mathcal{F}, \ [A, B\|\mathcal{F}] \vdash C$
or_intro_left	$\mathcal{F} \vdash A \vee B$	$\mathcal{F} \vdash A$
or_intro_right	$\mathcal{F} \vdash A \vee B$	$\mathcal{F} \vdash B$

   (a) $[p \wedge (s \rightarrow q)] \vdash s \rightarrow (p \wedge q)$ [*10 marks*]

   (b) $[(a \vee b) \rightarrow c, c \rightarrow a] \vdash b \rightarrow (a \vee c)$ [*10 marks*]

3. Given the following set of premises:

$$[a \rightarrow (d \rightarrow c), a \wedge d \wedge e, (c \wedge e) \rightarrow b]$$

   (a) Convert the premises to a single expression in Conjuctive Normal Form (CNF). [*8 marks*]

   (b) Convert the CNF expression to clausal form. [*2 marks*]

   (c) Prove b from the premises using the Resolution proof rule. Show each step in detail. [*10 marks*]

4. You are given the following requirements for a car-locking system. Note that we are only concerned with the behaviour of a single door, the driver's door, and the ignition state of the engine.

The car door can be locked or unlocked. A key is used by placing it in the door's key-slot and turning it to lock/unlock the door. Once the door is unlocked and the key has been removed from the key-slot, the door can be opened. Once opened the door can then be closed. From the inside, it is possible to lock/unlock the door using a manual switch. The key is also used to switch the car engine on and off. It is placed in the engine's key-slot and turned to switch the engine on, and turned back to turn it off. When the engine is turned on the door automatically locks, but it must be manually unlocked when the engine is switched off. The key cannot be directly removed whilst the engine is on.

The initial state is that door is closed and locked, the door's key-slot empty, the engine off and the engine's key-slot empty.

(a) List the states of the system. *[10 marks]*

(b) Compose a FSM for modelling the system. *[10 marks]*

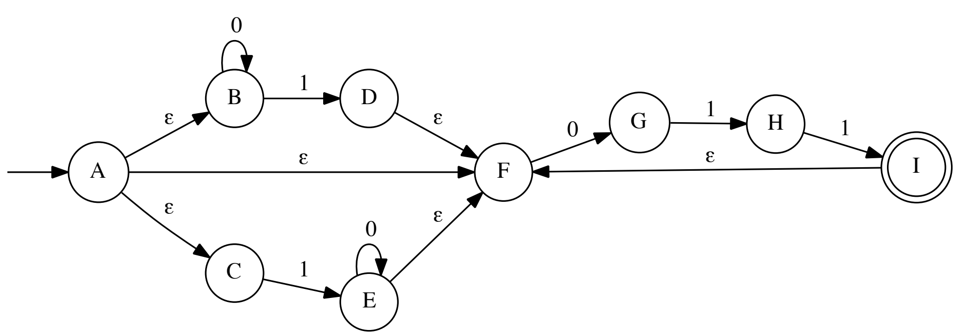
5\. (a) Draw (non-deterministic) Finite State Machines for three sub-components of the expression:

$$((aa^*(b|ca^*))|bca^*)$$

[8 marks]

(b) Compose the sub-machines using composition operators and ε-transitions where necessary to create a machine for the whole expression. *[2 marks]*

(c) Write a regular expression for the language expressed by the following Finite State Machine.



[5 marks]

(d) Show that the regular expression $((b|b)|(a^*a|\varepsilon))c$ is equivalent to $(a^*c|bc)$ using the given algebraic laws. Be explicit about which rules are applied. *[5 marks]*

1	$R\|R$	is equivalent to	R
2	R^*	is equivalent to	$RR^*\|\varepsilon$
3	R^*R	is equivalent to	RR^*
4	$R\|S$	is equivalent to	$S\|R$
5	$(R\|S)T$	is equivalent to	$RT\|RS$
6	$R\|(S\|T)$	is equivalent to	$(R\|S)\|T$