UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR08012 INFORMATICS 1 - COMPUTATION AND LOGIC

Tuesday 12þ December 2017

14:30 to 16:30

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**
3. **CALCULATORS MAY NOT BE USED IN THIS EXAMINATION.**

Convener: I. Simpson
External Examiner: I. Gent

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. This question concerns the 64 possible truth valuations of six propositional letters, $ABCDEF$. For each of the following expressions say how many of the 64 valuations satisfy the expression:

   Use the space provided for any rough working, and to briefly explain your reasoning.

   (a) $E \vee F$

   Answer: 48

   Reason: 3/4 of the valuations

   [*3 marks*]

   (b) $(A \rightarrow B) \wedge C$

   Answer: 24

   Reason: $1/2 \times 3/4$

   [*3 marks*]

   (c) $(E \vee F) \wedge (A \rightarrow B) \wedge C$

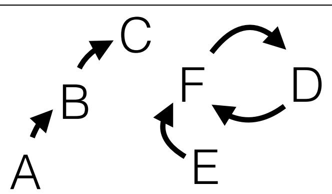
   Answer: 18

   Reason: $3(EF) \times 3(AB) \times 1(C) \times 2(D)$

   [*3 marks*]

   (d) $(A \rightarrow B) \wedge (B \rightarrow C) \wedge (D \rightarrow F) \wedge (E \rightarrow F) \wedge (F \rightarrow D)$

   Answer: 12

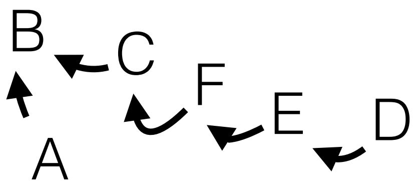


   Reason: 4×3

   [*3 marks*]

   (e) $(A \rightarrow B) \wedge (C \rightarrow B) \wedge (D \rightarrow E) \wedge (E \rightarrow F) \wedge (F \rightarrow C)$

   Answer: 11



   Reason: $(2 \times 5) + 1$

   [*3 marks*]

2. For each of the following entailments complete two Karnaugh maps, one to represent the assumption and one the conclusion, by **marking the valuations that make the expression false**.

   Place a mark in one of the check boxes provided, to indicate whether the entaiment is valid. Give a reason for your answer in the box provided.

   Use your Karnaugh maps to give a simple CNF for each assumption.

   (a) $(A \vee B) \to (C \wedge D) \models A \to C$ Valid ☒ Invalid ☐ [*1 mark*]

		CD			
assumption		00	01	11	10
AB	00				
	01	•	•		•
	11	•	•		•
	10	•	•		•

		CD			
conclusion		00	01	11	10
AB	00				
	01				
	11	•	•		
	10	•	•		

[*4 marks*]

Reason: Every state that makes the conclusion false makes the assumption false. [*5 marks*]

assumption CNF: $(\neg A \vee D) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg B \vee C)$

   (b) $(A \wedge C) \to (B \vee D) \models A \to D$ Valid ☐ Invalid ☒ [*1 mark*]

		CD			
assumption		00	01	11	10
AB	00				
	01				
	11				
	10				•

		CD			
conclusion		00	01	11	10
AB	00				
	01				
	11	•			•
	10	•			•

[*4 marks*]

Reason: There are (three) states that make the conclusion false that make the assumption true. [*5 marks*]

assumption CNF: $\neg A \vee B \vee \neg C \vee D$

3. This part concerns the satisfiability of a set of expressions.

(a) Convert each of the following expressions to CNF

- $(P \wedge Q) \rightarrow R$ — $\neg P \vee \neg Q \vee R$ [2 marks]
- $(S \ ? \ T \ : \ R \rightarrow Q)$ — $(\neg S \vee T) \wedge (S \vee \neg R \vee Q)$ [2 marks]
- $T \rightarrow (P \vee Q)$ — $\neg T \vee P \vee Q$ [2 marks]
- $(P \wedge R) \oplus (S \vee T)$ — $(\neg T \vee P) \wedge (\neg S \vee P) \wedge (\neg T \vee R) \wedge (\neg S \vee R) \wedge (\neg P \vee \neg R \vee S \vee T)$ [2 marks]

(b) Use resolution to determine whether the entailment $(A \rightarrow B) \rightarrow A \vdash A$ is valid, and produce a counterexample if it is not. [4 marks]

$(A \rightarrow B) \rightarrow A = (A \wedge \neg B) \vee A = A \wedge (\neg B \vee A)$
resolve these clauses together with $\neg A$

	A	B	C
A	$\{\}$		
$\neg B \vee A$			
$\neg A$			

Answer The entailment is valid

Counterexample? none [2 marks]

(c) Use resolution to determine whether $P \rightarrow (Q \vee R), Q \rightarrow S, R \rightarrow S \vdash P \rightarrow S$ is valid and produce a counter-example if it is not. [4 marks]

	P	Q	R	S
$^P\neg P \vee Q \vee R$	$^Q Q \vee R$	$^R S \vee R$	$^S S$	$\{\}$
$^Q\neg Q \vee S$				
$^R\neg R \vee S$				
$^P P$				
$^S\neg S$				

Answer The entailment is valid

Counterexample? none [2 marks]

Gentzen Rules

Question 4 refers to these rules.

$$\frac{}{\Gamma, A \vdash \Delta, A}\ (I)$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R)$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ (\wedge R)$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \rightarrow B \vdash \Delta}\ (\rightarrow L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \rightarrow B, \Delta}\ (\rightarrow R)$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ (\neg R)$$

A and B are propositional expressions, Γ, Δ are sets of expressions, and Γ, A refers to $\Gamma \cup \{A\}$.

4. Use the Gentzen rules, provided on the previous page, to prove the following entailment, your **goal**:

$$P \to (Q \to R),\ Q \vee \neg P \vdash P \to R \qquad \text{(goal)}$$

(a) Which of the rules have a conclusion matching this goal?
For each such rule complete a line in the table below showing the name of the rule and the bindings for Γ, Δ, A, B [*10 marks*]

Rule	Γ	Δ	A	B
$\to L$	$Q \vee \neg P$	$P \to R$	P	$Q \to R$
$\vee L$	$P \to (Q \to R)$	$P \to R$	Q	$\neg P$
$\to R$	$P \to (Q \to R),\ Q \vee \neg P$	empty	P	R

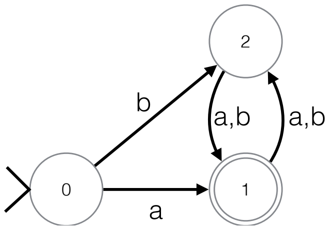
(b) Use the rules given to construct a formal proof with the goal as conclusion. Label each step in your proof with the name of the rule being applied. [*10 marks*]

$$\dfrac{\dfrac{\dfrac{}{Q, P \vdash P, R}\,(I) \qquad \dfrac{\dfrac{}{R,\ Q, P \vdash R}\,(I) \qquad \dfrac{}{Q, P \vdash R, Q}\,(I)}{Q \to R,\ Q, P \vdash R}\,(\to L)}{P \to (Q \to R),\ Q, P \vdash R}\,(\to L) \qquad \dfrac{\dfrac{}{P \to (Q \to R), P \vdash R, P}\,(I)}{P \to (Q \to R),\ \neg P, P \vdash R}\,(\neg L)}{\dfrac{P \to (Q \to R),\ Q \vee \neg P, P \vdash R}{P \to (Q \to R),\ Q \vee \neg P \vdash P \to R}\,(\to R)}\,(\vee L)$$

$$\dfrac{}{P \to (Q \to R),\ Q \vee \neg P \vdash P \to R}$$

5. Give a regular expression (`re`) for the language accepted by each FSM
Mark the check boxes to show which strings it accepts, and whether it is deterministic. Draw an equivalent DFA if it is not.

(a)

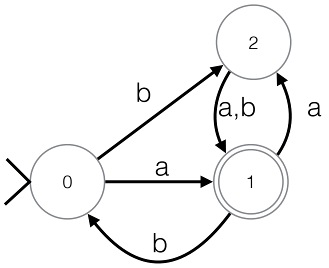


- `aba` ☒
- `abab` ☐
- `bab` ☐
- `bbba` ☒
- *DFA* ☒

`re`: `(a|b(a|b))((a|b)(a|b))*`

[*5 marks*]

(b)

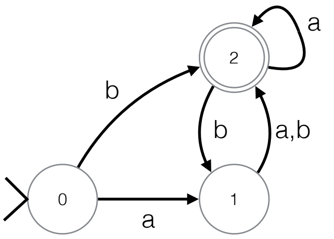


- `aba` ☒
- `bab` ☐
- `aaa` ☒
- `bbb` ☐
- *DFA* ☒

`re`: `(a|ba|bb)(ba|aa|ab|bba|bbb)*`

[*5 marks*]

(c)

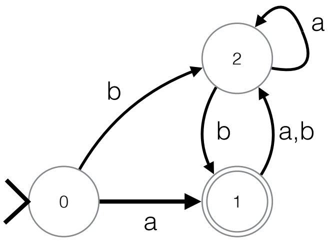


- `aba` ☒
- `abab` ☐
- `bab` ☐
- `bbba` ☒
- *DFA* ☒

`re`: `(b|aa|ab)(a|ba|bb)*`

[*5 marks*]

(d)

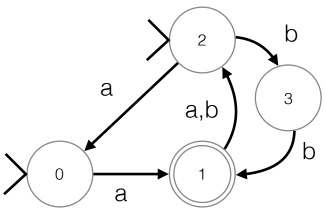


- `aba` ☐
- `bab` ☒
- `aaa` ☐
- `bbb` ☐
- *DFA* ☒

`re`: `(a|ba*b)((a|b)a*b)*`

[*5 marks*]

(e)



- `aabb` ☒
- `bbb` ☐
- `abbb` ☒
- `bbbb` ☐
- *DFA* ☐

`re`: `(a|aa|bb)((a|b)(aa|bb))*`

[*5 marks*]