UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFORMATICS 1 - COMPUTATION AND LOGIC

Tuesday 17 August 2010

14:30 to 16:30

Convener: J Bradfield
External Examiner: A Preece

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) Using logical equivalences below, show that the following expressions are equivalent. Show precisely which rules are applied.

   i. $\neg(\neg A \to B) \vee \neg(\neg C)$ is equivalent to $(A \vee B) \to C$ [*5 marks*]

   ii. $\neg(A \wedge B) \wedge (\neg A \vee C)$ is equivalent to $A \to (\neg B \wedge C)$ [*5 marks*]

Rule	Equivalence
1	$X \to Y \leftrightarrow \neg X \vee Y$
2	$\neg\neg X \leftrightarrow X$
3	$\neg(X \vee Y) \leftrightarrow \neg X \wedge \neg Y$
4	$\neg(X \wedge Y) \leftrightarrow \neg X \vee \neg Y$
5	$X \vee (Y \wedge Z) \leftrightarrow (X \vee Y) \wedge (X \vee Z)$

   (b) Using a truth table show that

$$\neg(\neg A \to B) \vee \neg(\neg C) \leftrightarrow (A \vee B) \to C$$

   is a tautology. [*10 marks*]

2. Using the rules below prove the following, showing precisely which rules are applied.

   (a) $[(p \to q) \wedge (q \to s)] \vdash p \to (s \vee q)$ [*10 marks*]

   (b) $[] \vdash ((p \wedge q) \to s) \to (p \to (q \to s))$ [*10 marks*]

Rule	Sequent	Supporting proofs
1	$\mathcal{F} \vdash A$	$A \in \mathcal{F}$
2	$\mathcal{F} \vdash A \to B$	$[A \vert \mathcal{F}] \vdash B$
3	$\mathcal{F} \vdash A \wedge B$	$\mathcal{F} \vdash A, \mathcal{F} \vdash B$
4	$\mathcal{F} \vdash A \vee B$	$\mathcal{F} \vdash A$
5	$\mathcal{F} \vdash A \vee B$	$\mathcal{F} \vdash B$
6	$\mathcal{F} \vdash B$	$A \to B \in \mathcal{F}, \mathcal{F} \vdash A$
7	$\mathcal{F} \vdash C$	$A \wedge B \in \mathcal{F}, [A, B \vert \mathcal{F}] \vdash C$

3. Given the following premises:

$$[x \vee w, (z \to y) \wedge (x \to z), \neg(z \wedge y)]$$

   (a) Convert them to a single expression in:

   i. Conjuctive Normal Form using logical equivalences [*6 marks*]

   ii. Clausal Form (CF) [*2 marks*]

   (b) Prove w from the premises using the Resolution proof rule. Show each step in detail. [*12 marks*]

4. (a) Draw a FSM that accepts the language of strings over the alphabet $\{a, b\}$ with at least two a's followed by either two or more b's, or a b and then an a. *[2 marks]*

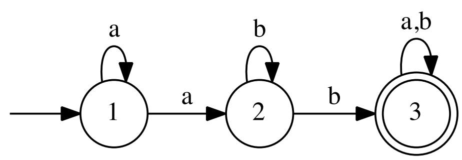
   (b) Draw separate FSMs that accept the language of strings over the alphabet $\{a, b\}$ that have the following properties:

      i. an even number of b's *[2 marks]*

      ii. an odd number of a's *[2 marks]*

   Then compose the two machines together to create a new machine that accepts strings which have an even number of b's followed by an odd number of a's. *[2 marks]*

   (c) Use the subset procedure to convert the following non-deterministic FSM to a deterministic FSM. Either draw or give the formal definition of the resulting deterministic FSM. *[12 marks]*

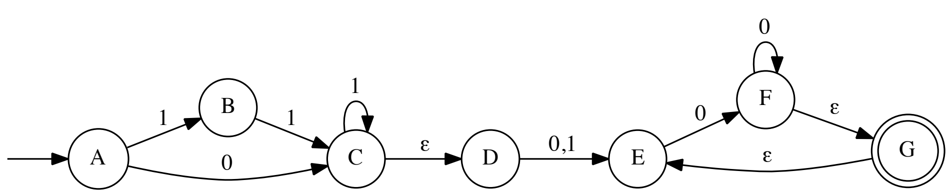


5. (a) Give a regular expression for the language that describes all the strings over the alphabet $\{a, b\}$ that have the following properties:

      i. end in three a's *[2 marks]*

      ii. contain at least two a's and two b's. *[6 marks]*

   (b) Write a regular expression for the language accepted by the following FSM. *[4 marks]*



(c) Show that the following regular expression:

$$(b(ab)^*ac)|((ba)^*c)$$

is equivalent to:

$$(ba)^*c$$

using the given algebraic laws below. Indicate which rules are used. *[8 marks]*

Rule	Equivalence		
1	$R\|R$	$=$	R
2	R^*	$=$	$RR^*\|\varepsilon$
3	R^*R	$=$	RR^*
4	$R\|S$	$=$	$S\|R$
5	$(R\|S)T$	$=$	$RT\|ST$
6	$R\|(S\|T)$	$=$	$(R\|S)\|T$
7	$(RS)^*R$	$=$	$R(SR)^*$