UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFORMATICS 1 - COMPUTATION AND LOGIC

Tuesday 20th December 2011

14:30 to 16:30

Convener: J Bradfield
External Examiner: A Preece

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**

1. (a) Using a truth table show whether the following expressions are equivalent:

   i. $((A \rightarrow C)\ and\ (B \rightarrow C)) \leftrightarrow ((A\ or\ B) \rightarrow C)$

   ii. $(A \rightarrow C) \rightarrow ((A\ or\ B) \rightarrow (B\ or\ C))$

   [*10 marks*]

   (b) For each of the two expressions above say whether it is tautologous, inconsistent or contingent? [*3 marks*]

   (c) You are given the following equivalences:

1	$not(X)\ or\ Y$	$\leftrightarrow$	$X \rightarrow Y$
2	$not(X\ or\ Y)$	$\leftrightarrow$	$not(X)\ and\ not(Y)$
3	$not(not(X))$	$\leftrightarrow$	X
4	$X\ or\ X$	$\leftrightarrow$	X
5	$(X\ and\ Y)\ or\ Z$	$\leftrightarrow$	$(X\ or\ Z)\ and\ (Y\ or\ Z)$
6	$(X\ or\ Y)\ or\ Z$	$\leftrightarrow$	$X\ or\ (Y\ or\ Z)$

   Using these equivalences, prove the following:

   i. $(A\ or\ B) \rightarrow C$ is equivalent to $(A \rightarrow C)\ and\ (B \rightarrow C)$

   ii. $(A \rightarrow C) \rightarrow (A\ or\ B)$ is equivalent to $(A\ or\ B)\ and\ ((C\ and\ not(A)) \rightarrow B)$

   [*10 marks*]

2. Assume we have the conjunctive set of premises given below:

$$[not(not(a) \rightarrow d),\ c \rightarrow d,\ b \rightarrow a,\ not(a) \rightarrow (c\ or\ not(e)),\ a \rightarrow b]$$

   (a) Convert the above expression into Conjunctive Normal Form (CNF). [*8 marks*]

   (b) Convert the CNF into clausal form. [*2 marks*]

   (c) Using Resolution show whether not(e) can be proved from the premises. [*10 marks*]

3. You are given the following proof rules:

Rule name	Sequent	Supporting proofs
immediate	$\mathcal{F} \vdash A$	$A \in \mathcal{F}$
and_intro	$\mathcal{F} \vdash A \textit{ and } B$	$\mathcal{F} \vdash A,\ \mathcal{F} \vdash B$
or_intro_left	$\mathcal{F} \vdash A \textit{ or } B$	$\mathcal{F} \vdash A$
or_intro_right	$\mathcal{F} \vdash A \textit{ or } B$	$\mathcal{F} \vdash B$
or_elim	$\mathcal{F} \vdash C$	$\mathcal{F} \vdash (A \textit{ or } B),\ [A\|\mathcal{F}] \vdash C,\ [B\|\mathcal{F}] \vdash C$
imp_elim	$\mathcal{F} \vdash B$	$A \rightarrow B \in \mathcal{F},\ \mathcal{F} \vdash A$
imp_intro	$\mathcal{F} \vdash A \rightarrow B$	$[A\|\mathcal{F}] \vdash B$

where $\mathcal{F} \vdash A$ means that expression A can be proved from set of axioms $\mathcal{F}$; $A \in \mathcal{F}$ means that A is an element of set $\mathcal{F}$; $[A|\mathcal{F}]$ is the set constructed by adding A to set $\mathcal{F}$; $A \rightarrow B$ means that A implies B; A *and* B means that A and B both are true; and A *or* B means that at least one of A or B is true.

(a) Prove the sequent $[p \rightarrow q, q \rightarrow r] \vdash (p \rightarrow s) \textit{ or } (p \rightarrow r)$ using the rules in the table. *[10 marks]*

(b) Can $[(a \textit{ and } b)] \vdash b$ be proved using only the above rules? What can be said about the soundness and completeness of these rules? *[6 marks]*

(c) How can we ensure that our rules allow proof of the sequent at (b) above? *[5 marks]*

4. (a) You are given the following languages, L_1, and L_2 over the alphabet $\{a, b\}$:

$$L_1 = \{a^n b^n \mid n > 0\}$$
$$L_2 = \{ab^n \mid n > 0\}$$

Answer TRUE or FALSE to each of the questions below: *[10 marks]*

i. L_1 is regular language.

ii. There exists a non-deterministic FSM that accepts L_2.

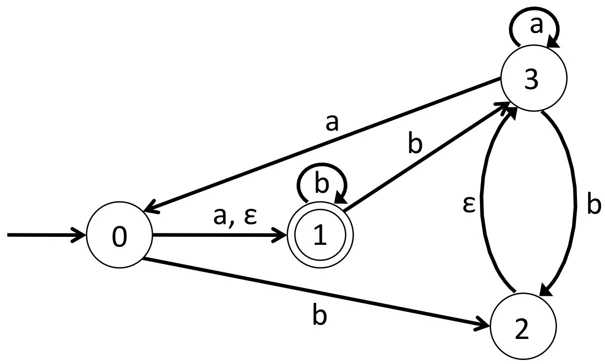
iii. $L_1 \cap L_2$ is a regular language.

iv. The context free grammar given below (where `->` is the grammar rule operator) generates palindrome strings.

$$S\texttt{->}aSb \mid S\texttt{->}bSa \mid \epsilon$$

v. The language L_3 that accepts 200 b's can be represented by a regular expression.

(b) Consider the following the non-deterministic FSM:



i. Write the transition table of the machine. [*2 marks*]

ii. Using the subset procedure give the set of transitions and the set of accepting states of the equivalent deterministic FSM. [*8 marks*]

5. (a) Prove that the following language over the alphabet $\{a, b\}$ is regular:

$$L = \{a^m b^n a^p \mid m \geq 1,\ n \geq 2,\ p \geq 3\}$$

[*4 marks*]

(b) Write a regular expression for each of the following languages over the alphabet $\{a, b\}$:

i. All strings with even number of a's followed by odd number of b's. [*4 marks*]

ii. All strings that have lengths of at least 2 and at most 4. [*4 marks*]

(c) Write a regular expression for the language accepted by the following FSM: [*4 marks*]