UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFORMATICS 1 - COMPUTATION AND LOGIC

Tuesday 21st August 2012

14:30 to 16:30

Convener: J Bradfield
External Examiner: A Preece

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) You are given the following equivalences:

1	$not(X)\ or\ Y$	$\leftrightarrow$	$X \rightarrow Y$
2	$not(X\ or\ Y)$	$\leftrightarrow$	$not(X)\ and\ not(Y)$
3	$not(X\ and\ Y)$	$\leftrightarrow$	$not(X)\ or\ not(Y)$
4	$(X\ and\ Y)\ or\ Z$	$\leftrightarrow$	$(X\ or\ Z)\ and\ (Y\ or\ Z)$
5	$(X\ or\ Y)\ or\ Z$	$\leftrightarrow$	$X\ or\ (Y\ or\ Z)$

Using the equivalence rules above prove the following equivalences:

i. $((A\ and\ B) \rightarrow C) \leftrightarrow (A \rightarrow (B \rightarrow C))$

ii. $((A \rightarrow C)\ and\ (B \rightarrow C)) \leftrightarrow ((A\ or\ B) \rightarrow C)$

[10 marks]

(b) For each of the following two expressions, show whether it is tautologous using a truth table.

i. $(A\ and\ (not(B) \rightarrow (B \rightarrow C))) \leftrightarrow A$

ii. $((A \rightarrow B)\ and\ (B \rightarrow C)) \rightarrow (A \rightarrow C)$

[10 marks]

2. You are given the following proof rules:

Rule name	Sequent	Supporting proofs
immediate	$\mathcal{F} \vdash A$	$A \in \mathcal{F}$
and_intro	$\mathcal{F} \vdash A\ and\ B$	$\mathcal{F} \vdash A,\ \mathcal{F} \vdash B$
or_intro_left	$\mathcal{F} \vdash A\ or\ B$	$\mathcal{F} \vdash A$
or_intro_right	$\mathcal{F} \vdash A\ or\ B$	$\mathcal{F} \vdash B$
or_elim	$\mathcal{F} \vdash C$	$(A\ or\ B) \in \mathcal{F},\ [A\vert\mathcal{F}] \vdash C,\ [B\vert\mathcal{F}] \vdash C$
and_elim	$\mathcal{F} \vdash C$	$(A\ and\ B) \in \mathcal{F},\ [A, B\vert\mathcal{F}] \vdash C$
imp_elim	$\mathcal{F} \vdash B$	$A \rightarrow B \in \mathcal{F},\ \mathcal{F} \vdash A$
imp_intro	$\mathcal{F} \vdash A \rightarrow B$	$[A\vert\mathcal{F}] \vdash B$

where $\mathcal{F} \vdash A$ means that expression A can be proved from set of axioms $\mathcal{F}$; $A \in \mathcal{F}$ means that A is an element of set $\mathcal{F}$; $[A|\mathcal{F}]$ is the set constructed by adding A to set $\mathcal{F}$; $A \rightarrow B$ means that A implies B; $A\ and\ B$ means that A and B both are true; and $A\ or\ B$ means that at least one of A or B is true.

Prove the sequent $[p\ or\ q\ or\ z \rightarrow r, r \rightarrow p] \vdash (q\ or\ z) \rightarrow r$ using the rules in the table above. *[10 marks]*

3. Assume we have the following conjunctive set of premises:

$$[not(c)\ and\ d, (b\ and\ e) \rightarrow c,\ not(a) \rightarrow e,\ not(b) \rightarrow not(e\ and\ d),\ e]$$

(a) Convert the above expression into a Conjunctive Normal Form (CNF). *[8 marks]*

(b) Convert the CNF into a clausal form. *[2 marks]*

(c) Using Resolution show whether the proposition 'a' can be proved from the premises. [*10 marks*]

4. (a) Briefly describe one advantage each of deterministic and non-deterministic FSMs. [*4 marks*]

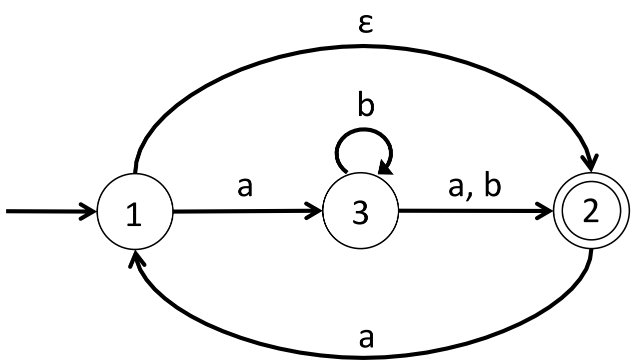
   (b) Consider the language L of strings over the alphabet $\{a, b\}$ that contain the substring "abb".

      i. Draw a deterministic FSM that accepts L. [*5 marks*]

      ii. Draw a deterministic FSM that accepts the language that is the complement of L [*5 marks*]

   (c) Draw a FSM that accepts all strings with length K such that: $K mod\, 3 \neq 0$. [*4 marks*]

   (d) Convert the following non-deterministic FSM to a deterministic FSM. [*10 marks*]



5. The following languages are defined over the alphabet $\{a, b\}$:

   (a) Is the language $L = \{a^m b^n \mid m \text{ is even}, n \text{ is odd}\}$ a regular language? [*2 marks*]

   (b) Give a regular expression for each of the following languages:

      i. L_1 = strings beginning with aa. [*2 marks*]

      ii. L_2 = strings ending with bb. [*2 marks*]

      iii. $L_1 \cap L_2$ (the intersection of L_1 and L_2). [*2 marks*]

      iv. Strings with length an odd number. [*4 marks*]

   (c) Draw the following FSMs:

      i. M_1 that accepts language L_1. [*2 marks*]

      ii. M_2 that accepts language L_2. [*2 marks*]

      iii. M_2 that accepts language $L_1 \cap L_2$, with a single final state. [*6 marks*]