UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR08012 INFORMATICS 1 - COMPUTATION AND LOGIC

Friday 16th August 2013

14:30 to 16:30

INSTRUCTIONS TO CANDIDATES

1. **Note that ALL QUESTIONS ARE COMPULSORY.**
2. **DIFFERENT QUESTIONS MAY HAVE DIFFERENT NUMBERS OF TOTAL MARKS. Take note of this in allocating time to questions.**

CALCULATORS MAY NOT BE USED IN THIS EXAMINATION.

Convener: J Bradfield
External Examiner: A Preece

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. Using truth tables, answer each of the following three questions:

   (a) Is $(not(b)\ and\ (a \rightarrow b)) \rightarrow not(a)$ a tautology?

   (b) Is $(not(a)\ and\ (a \rightarrow b)) \rightarrow not(b)$ a contradiction?

   (c) Is $(a \rightarrow b)\ and\ c$ equivalent to $(not(a)\ and\ c)\ or\ (b\ and\ c)$?

   [*15 marks*]

2. You are given the following proof rules:

Rule number	Sequent	Supporting proofs
1	$\mathcal{F} \vdash A$	$A \in \mathcal{F}$
2	$\mathcal{F} \vdash A \leftrightarrow B$	$\mathcal{F} \vdash A \rightarrow B,\ \mathcal{F} \vdash B \rightarrow A$
3	$\mathcal{F} \vdash A \rightarrow B$	$[A\|\mathcal{F}] \vdash B$
4	$\mathcal{F} \vdash A\ and\ B$	$\mathcal{F} \vdash A,\ \mathcal{F} \vdash B$
5	$\mathcal{F} \vdash C$	$(A\ and\ B) \in \mathcal{F},\ [A, B\|\mathcal{F}] \vdash C$
6	$\mathcal{F} \vdash C$	$A \rightarrow B \in \mathcal{F},\ \mathcal{F} \vdash A,\ [B\|\mathcal{F}] \vdash C$
7	$\mathcal{F} \vdash A\ orB$	$\mathcal{F} \vdash A$
8	$\mathcal{F} \vdash A\ orB$	$\mathcal{F} \vdash B$
9	$\mathcal{F} \vdash C$	$(A\ or\ B) \in \mathcal{F},\ [A\|\mathcal{F}] \vdash C,\ [B\|\mathcal{F}] \vdash C$
10	$\mathcal{F} \vdash not(A)$	$[A\|\mathcal{F}] \vdash false$
11	$\mathcal{F} \vdash B$	$\mathcal{F} \vdash not(A),\ \mathcal{F} \vdash A$

   where $\mathcal{F} \vdash A$ means that expression A can be proved from set of axioms $\mathcal{F}$; $A \in \mathcal{F}$ means that A is an element of set $\mathcal{F}$; $[A|\mathcal{F}]$ is the set constructed by adding A to set $\mathcal{F}$; $A \rightarrow B$ means that A implies B; $A\ and\ B$ means that A and B both are true; $A\ or\ B$ means that at least one of A or B is true; and $not(A)$ means that A is not true.

   Using the proof rules above, prove the following:

   (a) $[(p\ or\ q)\ \rightarrow\ r,\ s\ \rightarrow\ p,\ t\ \rightarrow\ q]\ \ \vdash\ \ (s\ and\ t)\ \rightarrow\ r$

   (b) $[p\ or\ q,\ p\ \rightarrow\ r,\ r\ \rightarrow\ s,\ q\ \rightarrow\ s]\ \ \vdash\ \ s\ or\ t$

   Show precisely how the proof rules are applied. [*20 marks*]

3. The proposition d can be proved from the following set of axioms in clausal form:

$$[[a],\ [b],\ [not(a), not(b), c],\ [not(c), d]]]$$

   (a) Explain what clausal form notation means, in terms of the conjunction and disjunction of propositional expressions. [*5 marks*]

   (b) Give a proof, using resolution, of d from the axioms above in clausal form above. Show each step of your proof in detail. [*10 marks*]

4. Suppose that you are designing a fault monitoring system for an engine. The system monitors information sent by the engine system about the presence or absence of an electrical fault and the presence or absence of low fuel pressure. In response to this information it switches on or off two separate warning lights, one for an electrical fault and the other for low fuel pressure.

   The monitoring system accepts the following input events:
   - A signal from the engine warning of an electrical fault.
   - A signal from the engine warning of low fuel pressure.
   - A signal from the engine that there is no longer an electrical fault.
   - A signal from the engine that there is no longer low fuel pressure.

   The monitoring system has the following outputs:
   - A signal to activate the electrical fault warning light.
   - A signal to activate the low fuel pressure warning light.
   - A signal to deactivate the electrical fault warning light.
   - A signal to deactivate the low fuel pressure warning light.

   It must satisfy the following functional requirements:
   - The electrical fault warning light is always activated immediately after a signal from the engine warning of an electrical fault.
   - The low fuel pressure warning light is always activated immediately after a signal from the engine warning of low fuel pressure.
   - If it is activated, the electrical fault warning light is always deactivated immediately after a signal from the engine that there is no longer an electrical fault.
   - If it is activated, the low fuel pressure warning light is always deactivated immediately after a signal from the engine that there is no longer low fuel pressure.
   - Deactivation of one of the warning lights does not deactivate the other if it is active.
   - Activation of one of the warning lights does not activate the other if it is inactive.
   - Deactivation of one of the warning lights does not activate the other if it is inactive.
   - Activation of one of the warning lights does not deactivate the other if it is active.

   Define a transducer FSM satisfying the engine monitoring requirements above. [*25 marks*]

5. Draw a non-deterministic finite state machine that accepts the language described by the regular expression:

$$(a|bb)^*(a^*|c)$$

[*10 marks*]

6. The following is a description, in English, of the rules of a simple grammar:

> A sentence consists of a conjunct followed by a verbphrase. A conjunct consists of either a nounphrase or, alternatively, a nounphrase followed by the word "and" followed by a conjunct. A nounphrase consists of a determiner followed by a noun. A verbphrase consists of either a verb or, alternatively, a verb followed by a conjunct. The word "bit" is the only verb. The words "cat" and "dog" are the only nouns. The word "the" is the only determiner.

Examples of word sequences accepted by this grammar are:

- "the cat bit".
- "the cat and the cat bit".
- "the cat bit the dog".
- "the cat and the dog bit the dog".
- "the dog and the dog bit the dog and the cat".

Examples of word sequences *not* accepted by this grammar are:

- "the cat".
- "the rabbit and the dog bit the dog".
- "bit the cat".

Write a finite state acceptor for this grammar. [*15 marks*]