Module Title: INFORMATICS 1 - COMPUTATION AND LOGIC
Exam Diet (Dec/April/Aug): Dec 2009
Brief notes on answers:

1. (a) $\neg(A \rightarrow (B \rightarrow C)) \leftrightarrow A \wedge B \wedge \neg C$

A	B	C	¬ C	$B \rightarrow C$	$A \rightarrow (B \rightarrow C)$	$\neg(A \rightarrow (B \rightarrow C))$	$A \wedge B \wedge \neg C$
t	t	t	f	t	t	f	f
t	t	f	t	f	f	t	t
t	f	t	f	t	t	f	f
t	f	f	t	t	t	f	f
f	t	t	f	t	t	f	f
f	t	f	t	f	t	f	f
f	f	t	f	t	t	f	f
f	f	f	t	t	t	f	f

(b) Both contingent.

(c) Four marks for each sub-problem.

(i). $\neg(A \rightarrow (B \rightarrow C)) \leftrightarrow A \wedge B \wedge \neg C$

$$\begin{array}{rr}
\neg(A \rightarrow (B \rightarrow C)) & (X \rightarrow Y \leftrightarrow \neg X \vee Y) \\
\neg(A \rightarrow (\neg B \vee C)) & (X \rightarrow Y \leftrightarrow \neg X \vee Y) \\
\neg(\neg A \vee (\neg B \vee C)) & (\neg(W \vee Z) \leftrightarrow \neg W \wedge \neg Z) \\
\neg\neg A \wedge \neg(\neg B \vee C) & (\neg\neg X \leftrightarrow X) \\
A \wedge \neg(\neg B \vee C) & (\neg(W \vee Z) \leftrightarrow \neg W \wedge \neg Z) \\
A \wedge \neg\neg B \wedge \neg C) & (\neg\neg X \leftrightarrow X) \\
A \wedge B \wedge \neg C &
\end{array}$$

(ii). $(A \rightarrow B) \rightarrow \neg(B \wedge \neg C) \leftrightarrow (A \wedge \neg B) \vee (B \rightarrow C)$

$$\begin{array}{rr}
(A \rightarrow B) \rightarrow \neg(B \wedge \neg C) & (X \rightarrow Y \leftrightarrow \neg X \vee Y) \\
(\neg A \vee B) \rightarrow \neg(B \wedge \neg C) & (\neg(W \wedge Z) \leftrightarrow \neg W \vee \neg Z) \\
(\neg A \vee B) \rightarrow (\neg B \vee \neg\neg C) & (\neg\neg X \leftrightarrow X) \\
(\neg A \vee B) \rightarrow (\neg B \vee C) & (X \rightarrow Y \leftrightarrow \neg X \vee Y) \\
\neg(\neg A \vee B) \vee (\neg B \vee C) & (\neg(W \vee Z) \leftrightarrow \neg W \wedge \neg Z) \\
(\neg\neg A \wedge \neg B) \vee (\neg B \vee C) & (\neg\neg X \leftrightarrow X) \\
(A \wedge \neg B) \vee (\neg B \vee C) & (\neg X \vee Y \leftrightarrow X \rightarrow Y) \\
(A \wedge \neg B) \vee (B \rightarrow C) &
\end{array}$$

2. (a) $[p \wedge (s \rightarrow q)] \vdash s \rightarrow (p \wedge q)$

$$\begin{array}{rr}
[p \wedge (s \rightarrow q)] \vdash s \rightarrow (p \wedge q) & (\textit{imp_intro}) \\
[p \wedge (s \rightarrow q), s] \vdash p \wedge q & (\textit{and_intro}) \\
[p \wedge (s \rightarrow q), s] \vdash p, [p \wedge (s \rightarrow q), s] \vdash q & (\textit{and_elim} \text{ on both}) \\
[p, s \rightarrow q, s] \vdash p, [p, s \rightarrow q, s] \vdash q & (\textit{imm} \text{ to finish first}) \\
[p, s \rightarrow q, s] \vdash q & (\textit{imp_elim}) \\
[p, s] \vdash s & (\textit{imm} \text{ to finish})
\end{array}$$

(b) $[(a \vee b) \rightarrow c, c \rightarrow a] \vdash b \rightarrow (a \vee c)$

$$
\begin{array}{rr}
[(a \vee b) \rightarrow c, c \rightarrow a] \vdash b \rightarrow (a \vee c) & (imp\_intro) \\
[b, (a \vee b) \rightarrow c, c \rightarrow a] \vdash a \vee c & (or\_intro\_left) \\
[b, (a \vee b) \rightarrow c, c \rightarrow a] \vdash a & (imp\_elim) \\
[b, (a \vee b) \rightarrow c] \vdash c & (imp\_elim) \\
[b] \vdash a \vee b & (or\_intro\_right) \\
[b] \vdash b & (imm \text{ to finish})
\end{array}
$$

An alternative proof from step 2 is given here:

$$
\begin{array}{rr}
[b, (a \vee b) \rightarrow c, c \rightarrow a] \vdash a \vee c & (or\_intro\_right) \\
[b, (a \vee b) \rightarrow c, c \rightarrow a] \vdash c & (imp\_elim) \\
[b, c \rightarrow a] \vdash a \vee b & (or\_intro\_right) \\
[b, c \rightarrow a] \vdash b & (imm \text{ to finish})
\end{array}
$$

3. (a) $[(\neg a \vee \neg d \vee c) \wedge a \wedge d \wedge e \wedge (\neg c \vee \neg e \vee b)]$
   (b) $[[\neg a, \neg d, c], [a], [d], [e], [\neg c, \neg e, b]]$
   (c) Add $\neg b$ to axioms: $[[\neg a, \neg d, c], [a], [d], [e], [\neg c, \neg e, b], [\neg b]]$
   Then apply Resolution proof rule:

$$
\begin{array}{rr}
[[\neg a, \neg d, c], [a], [d], [e], [\neg c, \neg e, b], [\neg b]] & ([\neg a, \neg d, c], [a]) \\
[[\neg d, c], [d], [e], [\neg c, \neg e, b], [\neg b]] & ([\neg d, c], [d]) \\
[[c], [e], [\neg c, \neg e, b], [\neg b]] & ([e], [\neg c, \neg e, b]) \\
[[c], [\neg c, b], [\neg b]] & ([c], [\neg c, b]) \\
[[b], [\neg b]] & \\
[] &
\end{array}
$$

   N.B. There are various way to derive the empty clause.

4. There are various potential ways of describing and modelling such a system.

   (a) List states of the system.

   Objects of system: driver's door, door key-slot, car's engine, engine's key-slot, key, door handle, door manual lock switch.
   Object states:

Object	States
Door	open/ closed, locked/ unlocked
Door's key-slot	empty/contains key
Engine	on/off
Engine's key-slot	empty/contains key

   Joint states:

State ID	Description
1	initial state: door closed and locked, engine off, key-slots empty
2	same as 1, but door key-slot contains key
3	same as 2, but door unlocked
4	same as 3, but key-slot empty
5	same as 4, but door open
6	same as 4, but key in engine slot
7	same as 6, but engine on, door locked
8	same as 7, but engine off
9	same as 8, but engine key-slot empty

(b) Goal of marker is to check for key behaviours in system:

- Door won't open when locked - but can handle action (loop at state)
- Unlocked door won't open when key in either key-slot (loop at state)
- Key cannot be removed whilst engine is on, must be turned off first.

An example FSM for modelling the system.
Transitions:

From	To	Action
1	1	open door
1	2	key placed in door key slot
2	1	key removed from door key slot
2	3	key turned
3	2	key turned
3	4	remove key
4	3	replace key
4	5	open door
4	6	put key in engine slot
5	4	close door
6	4	remove key from engine slot
6	7	turn key
6	6	open door
7	8	turn key
7	7	open door
8	7	turn key
8	9	remove key from engine slot
8	8	open door
9	4	flip manual switch
9	9	open door

5. (a) There are several ways to partition: $((aa^*(b|ca^*))|bca^*)$. An example solution is as follows were we partition the expression into three machines: aa^*, $b|ca^*$, bca^*. Or even to split $b|ca^*$.

(b) Compose the machines together using Sequence and Choice operators to create the complete machine. (See FSM diagram)

(c) $(0^*1|10^*|\varepsilon)011(011)^*$

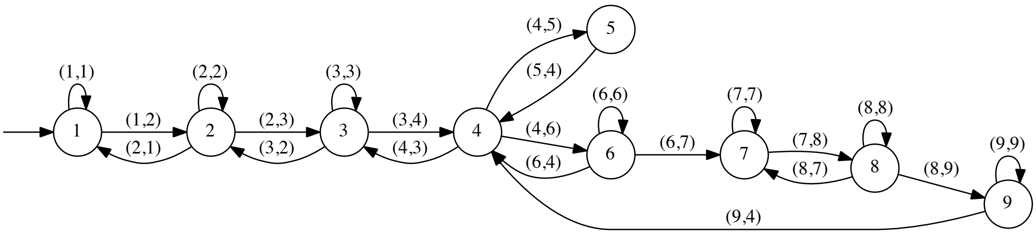

Figure 1: Q4. b)

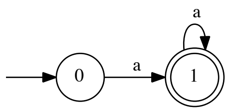

Figure 2: Q5. a) sub-component 1

(d)

$$
\begin{aligned}
((b|b)|(a^*a|\varepsilon))c & \qquad \text{(rule 1)}\\
(b|(a^*a|\varepsilon))c & \qquad \text{(rule 5)}\\
(b|(aa^*|\varepsilon))c & \qquad \text{(rule 2)}\\
(b|a^*)c & \qquad \text{(rule 3)}\\
(a^*|b)c & \qquad \text{(rule 4)}\\
(a^*c|bc) &
\end{aligned}
$$

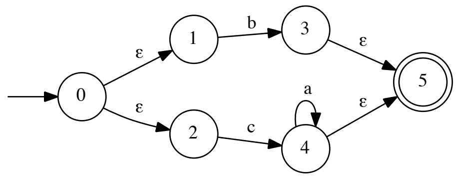

Figure 3: Q5. a) sub-component 2

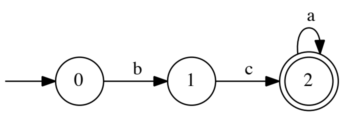

Figure 4: Q5. a) sub-component 3

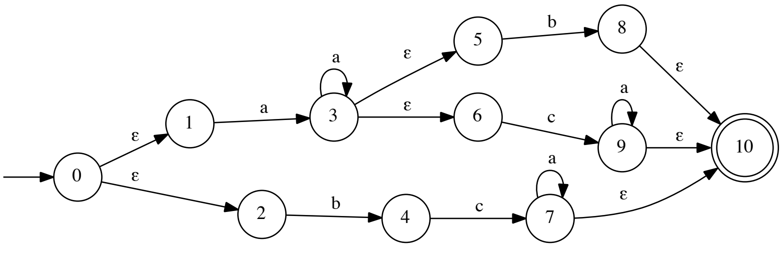

Figure 5: Q5. b)