UM–SJTU JOINT INSTITUTE
PHYSICS LABORATORY
(VP241)

LABORATORY REPORT

EXERCISE 2

THE HALL PROBE: CHARACTERISTICS AND APPLICATIONS

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Contents

1 OBJECTIVE

- Use Hall probe to understand the Hall effect.
- Discover the practical applications of the principle of the Hall effect.
- Verify the fact that the Hall voltage is proportional to the magnetic field.
- Study the sensitivity of an integrated Hall probe.
- Measure the magnetic field distribution along the axis of the solenoid and compare it with the theoretical value.

2 THEORETICAL BACKGROUND

2.1 HALL EFFECT

Hall effect refers to the situation where a conducting sheet is placed in a magnetic field so that the plane of the sheet is perpendicular to the direction of the magnetic field B, and then if an electric current passes through the sheet in a particular direction, an electric potential difference will be generated between side a and b, which is shown in Fig.1. Such potential difference will result in a electric field and we define the corresponding potential difference as the Hall voltage, denoted as U_H.

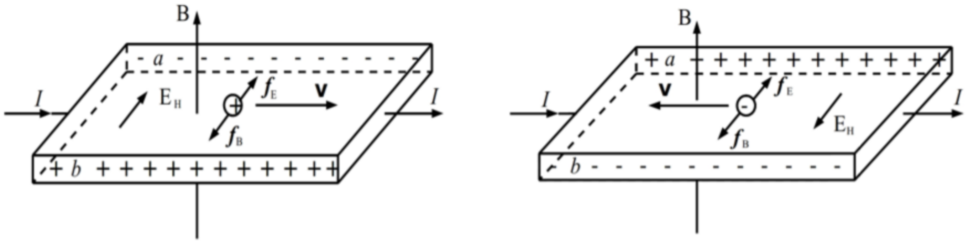

Figure 1: The principle of the Hall effect.[1]

If we view the Hall effect microscopically, we will find that it is actually caused by the Lorentz force F_B, which leads the deflection of moving charges. As a result, the accumulation of the charges on one side of the sheet increases the transverse electric field's magnitude (denoted by the Hall field E_H). However, because of the effect of the field, an electron force force F_E in the charges will act upon the opposite direction of F_B. Eventually, a balance will be reached such that the net force of F_E and F_B is zero. At that time, U_H stabilizes. Hence, we are able to determine the type of charge carriers in semiconductors by the analysis of the sign of U_H.

Further more, we find that when the external magnetic field is not strong to a certain extent, we have the following relationship between the magnitude of the magnetic field B, current I and the thickness of the sheet d shown in Eq.(1):

$$U_{\mathrm{H}} = R_{\mathrm{H}} \frac{IB}{d} = KIB \tag{1}$$

where we call R_H as the Hall coefficient and K_H as the sensitivity of the Hall element. Here $K = R_H/d = K_H/I$. From the equation, we find that the Hall voltage is proportional to I and B, and is inverse proportional to d.

2.2 Integrated Hall Probe

If we fix the sensitivity K_H and the current I, the magnitude of the magnetic field B can be calculated accordingly by measuring the Hall voltage U_H. However, for the reason that the Hall voltage is small, we need to amplify the Hall voltage before measuring it.

The integrated Hall probe is a device which includes the Hall probe and the electric circuits as a single device. The probe we use in this lab is SS495A, which consists of a Hall sensor, an amplifier and a voltage compensator used to make the voltage stable and compensate for the differences (Figure 2).

As shown in Fig.2, the working voltage U_S = 5V with output voltage U_0 approximately equals to 2.5V when the magnetic field is zero (Figure 2). The relation between the output voltage U and the magnitude of the magnetic field B can be written as follows:

$$B = \frac{U - U_0}{K_H} \tag{2}$$

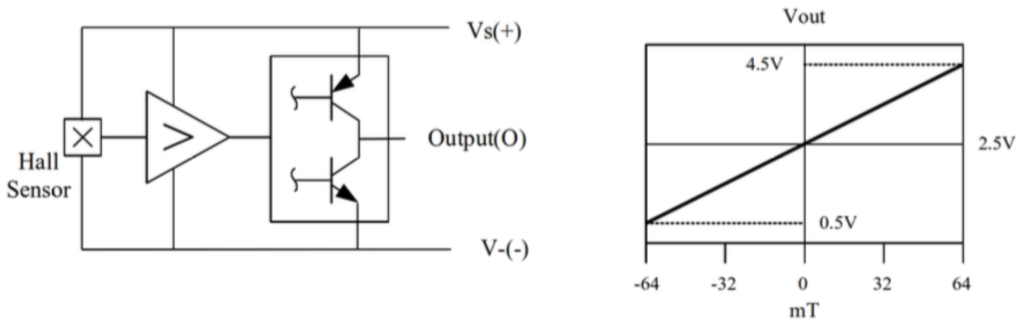

Figure 2: The integrated Hall probe SS495A (left) and the relationship between U and B (right).[1]

2.3 Magnetic Field Distribution Inside a Solenoid

The magnetic field distribution on the axis of a single layer solenoid is given by the following formula:

$$B(x) = \mu_0 \frac{N}{L} I_{\mathrm{M}} \left\{ \frac{L + 2x}{2\left[D^2 + (L + 2x)^2\right]^{\frac{1}{2}}} + \frac{L - 2x}{2\left[D^2 + (L - 2x)^2\right]^{\frac{1}{2}}} \right\} = C(x) I_{\mathrm{M}} \tag{3}$$

where I_M is the current through the solenoid wire, L is the length, N is the number of turns of the solenoid, and D is the diameter of the solenoid. Moreover, $\mu_0 = 4\pi \times 10^{-7} H/m$ in the equation is the magnetic permeability of vacuum.

The solenoid we use in the lab has ten layers. According to Eq.(3), we obtained theoretical value of the magnetic field inside the solenoid when $I_M = 0.1A$. The details are shown in Table 1.

x [cm]	B[mT]	x [cm]	B [mT]
$\pm$ 0.0	1.4366	$\pm$ 8.0	1.4057
$\pm$ 1.0	1.4364	$\pm$ 9.0	1.3856
$\pm$ 2.0	1.4356	$\pm$ 10.0	1.3478
$\pm$ 3.0	1.4343	$\pm$ 11.0	1.2685
$\pm$ 4.0	1.4323	$\pm$ 11.5	1.1963
$\pm$ 5.0	1,4292	$\pm$ 12.0	1.0863
$\pm$ 6.0	1.4245	$\pm$ 12.5	0.9261
$\pm$ 7.0	1.4173	$\pm$ 13.0	0.7233

Table 1: Theoretical value of the magnetic field inside the solenoid.[1]

2.4 Study of the Geomagnetic Field with a Hall Probe

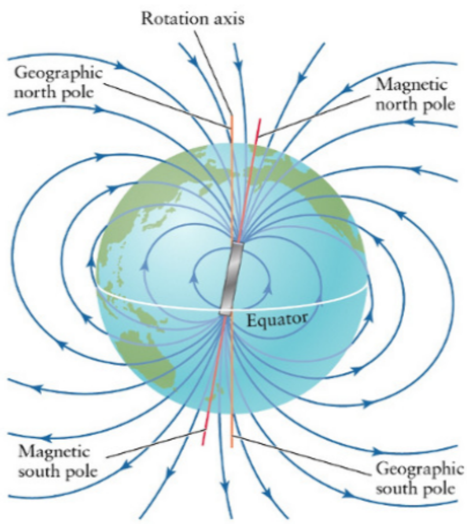

Figure 3: Magnetic field of the Earth.[1]

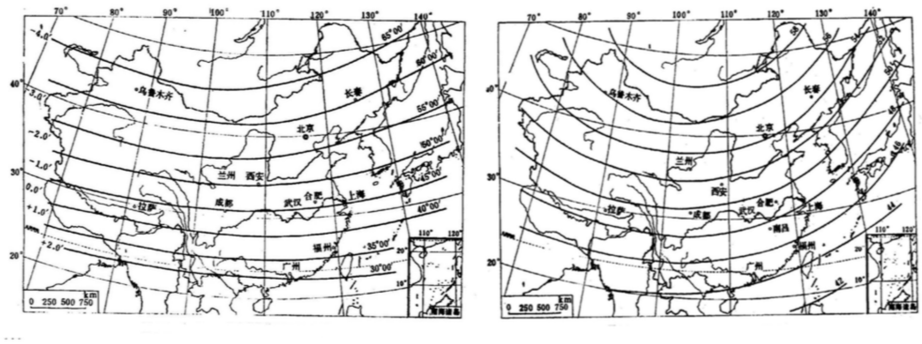

Figure 4: Geomagnetic inclination (1970) (left). The magnitude of geomagnetic field (right).[1]

As shown in Fig.3, we can see that the geomagnetic field as an analogue to that of a bar magnet tilted about 11.5° from the spin axis of the Earth. Consequently, the ”bar magnet” forms the geomagnetic field.

In Fig.4, we see the geomagnetic field distribution of China in 1970. The magnetic inclination is 44.5° with the corresponding magnetic field in Shanghai 48000nT.

3 Apparatus and Experimental Setup

We set up the experiment as shown in Fig.5. Fig.6 shows the integrated Hall probe SS495A we use. In all, the apparatuses we use in this lab are listed below:

- An integrated Hall probe SS495A with $K_H = 31.25 \pm 1.25 V/T$ at the working voltage 5 V.
- A solenoid.
- A power supply.
- A voltmeter.
- A DC voltage divider.
- A set of connecting wires.

Figure 5: Measurement setup.[1]

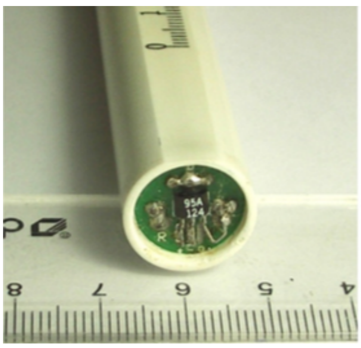

Figure 6: Integrated Hall probe SS495A.[1]

Then, the uncertainties for the quantities we measured in this lab are listed in Table 2:

Quantity	Uncertainty
U_S (voltage source)	0.5% [V]
U_0 & U (voltimeter)	$0.05\% + 6 \times 10^{-3}$ or 6×10^{-4} [V]
I_M (current source)	2% [mA]
x (distance)	0.05 [cm]

Table 2: Uncertainties of quantities measured in this lab.

Note that for U_0 & U, the uncertainty depends, and we choose the correct uncertainty accordingly.

4 Procedures [1]

4.1 Relation Between Sensitivity K_H and Working Voltage U_S

1. First, we place the integrated Hall probe at the center of the solenoid.
2. Second, we set the working voltage at 5 V and measure the output voltage U_0 (I_M = 0) and U (I_M = 250 mA).
3. Third, we take the theoretical value of B(x = 0) from Table 1 and calculate the sensitivity of the probe K_H by using Eq. (2).
4. Eventually, we measure K_H for different values of U_S (from 2.8 V to 10 V). Calculate K_H/U_S and plot the curve K_H/U_S vs. U_S.

4.2 Relation Between Output Voltage U and Magnetic Field B

1. First, we connect the 2.4 - 2.6 V output terminal of the DC voltage divider and the negative port of the voltmeter with B=0 and U_S=5 V. Then adjust the voltage until U_0 = 0.
2. Second, we place the integrated Hall probe at the center of the solenoid and measure the output voltage U for different values of I_M ranging from 0 to 500 mA, with intervals of 50 mA.
3. Third, we explain the relation between B(x = 0) and the Hall voltage U_H. Notice that the output voltage has been amplified and the theoretical value of B(x = 0) can be found from introduction part.
4. Finally, we plot the curve U vs. B and find the sensitivity K_H through linear fit. Compare the value with (1) the theoretical one (2) the value in first part.

4.3 Magnetic Field Distribution Inside the Solenoid

1. First, we measure the magnetic field distribution along the axis of the solenoid under the condition that I_M = 250mA. Then we record the output voltage U and position x and find out B = B(x). by using K_H we obtain in first part.

2. Then, we plot the theoretical and experimental curve to show the magnetic field distribution inside the solenoid. Note that for experimental curve we use dot points and for the theoretical one we use solid line.

4.4 Measurement of the Geomagnetic Field

1. We use an integrated Hall probe to measure both the magnitude and the direction of the geomagnetic field.

5 Results

5.1 Relation Between Sensitivity K_H and Working Voltage U_S

In this part, we find out the relation between sensitivity K_H and working voltage U_S. The data is recorded in Table 3.

U_S [V] $\pm$ 0.5% [V]	U_0 (I_M = 0) [V] $\pm$ 0.05% + 6×10^{-3}/6×10^{-4} [V]	U (I_M = 250 mA) [V] $\pm$ 0.05% + 6×10^{-3}/6×10^{-4} [V]
5.00	2.482	2.602

Table 3: Data for U_0 and U with U_S = 5 V.

To proceed, we need to first find out the value of K_H based on Eq.(2) and Table 1. From Table 1, we know that when $I_{M,t}$ = 0.1 A, the magnetic field in the middle of the solenoid is B = 1.4366 mT.

However, in this section, when measuring, the real current we choose is $I_{M,r}$ = 250 mA. Hence, we ought to find out the theoretical value of B at I_M = 250 mA = 0.25 A. Accordingly to Eq.(2), we have:

$$B_M = B \times \frac{I_{M,r}}{I_{M,t}} = 1.4366 \times \frac{0.25}{0.1} = 3.5915[mT]$$

Note that this is the calculation based on the theoretical values. Hence no uncertainty is generated here.

Then, based on our results, we are able to find out the sensitivity of the probe K_H as follows (Detailed calculations of uncertainties are shown in A.1):

$$K_H = \frac{U - U_0}{B_M} = \frac{2.602 - 2.482}{3.5915} = 0.033 \pm 0.003[kV/T]$$

with a relative uncertainty r_{K_H} = 9%.

Then, we are going to find out K_H for different values of U_S varying from 2.8 V to 10 V through Eq.(2) and then find out $\frac{K_H}{U_S}$.

Take U_S = 2.80 V as an example, and we will obtain $\frac{K_H}{U_S}$ at this voltage is (Detailed calculations of uncertainties are shown in A.1):

$$\frac{K_H}{U_S} = \frac{U - U_0}{B_M U_S} = \frac{1.4694 - 1.4019}{3.5915 \times 2.80} = 0.00671 \pm 0.00019 [mT^{-1}]$$

with a relative uncertainty r_{K_H/U_S} = 3%.

Now, we do the same calculations for all the 18 pairs of data we collected, and we will obtain Table 4:

	U_S [V] $\pm$ 0.5% [V]	u_{U_S}	U_0 [V] $\pm$ 0.05% + $6 \times 10^{-4}/6 \times 10^{-3}$ [V]	U [V] $\pm$ 0.05% + $6 \times 10^{-4}/6 \times 10^{-3}$ [V]	$\frac{K_H}{U_S}$	$u_{\frac{K_H}{U_S}}$
1	2.800	0.014	1.4019	1.4694	0.00671	0.00019
2	3.200	0.016	1.5977	1.6767	0.00687	0.00017
3	3.600	0.018	1.7970	1.8848	0.00679	0.00017
4	4.00	0.02	1.9922	2.0893	0.00676	0.00016
5	4.40	0.02	2.186	2.292	0.0067	0.0006
6	4.80	0.02	2.382	2.495	0.0066	0.0006
7	5.20	0.03	2.577	2.700	0.0066	0.0006
8	5.60	0.03	2.772	2.905	0.0066	0.0005
9	6.00	0.03	2.966	3.107	0.0065	0.0005
10	6.40	0.03	3.160	3.306	0.0064	0.0005
11	6.80	0.03	3.351	3.504	0.0063	0.0004
12	7.20	0.04	3.541	3.700	0.0061	0.0004
13	7.60	0.04	3.735	3.901	0.0061	0.0004
14	8.00	0.04	3.929	4.097	0.0058	0.0004
15	8.40	0.04	4.120	4.292	0.0057	0.0004
16	8.80	0.04	4.309	4.480	0.0054	0.0004
17	9.40	0.04	4.593	4.767	0.0052	0.0003
18	10.00	0.05	4.880	5.061	0.0050	0.0003

Table 4: Data for U_0 and U with different U_S.

Now, we are able to use *Origin* to plot the curve $\frac{K_H}{U_S}$ vs. U_S based on the data in Table 4. The result is shown in Fig.7. From the curve, we can observe that with the **increase** of U_S, $\frac{K_H}{U_S}$ has a general trend to **decrease**.

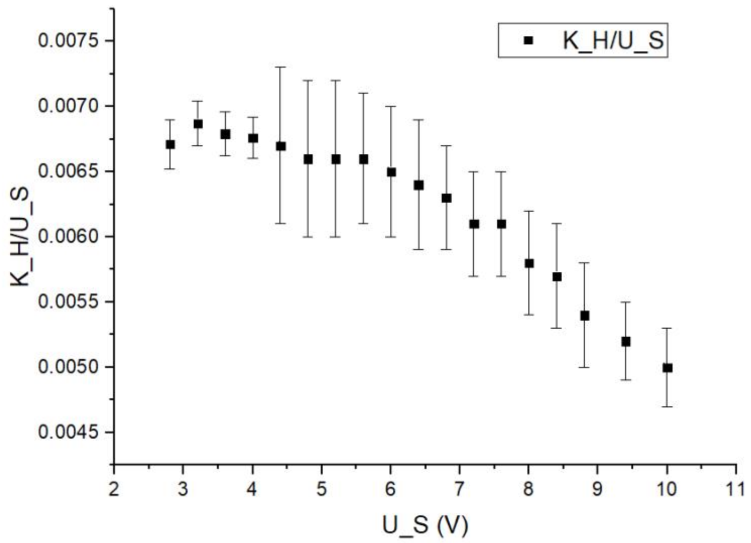

Figure 7: $\frac{K_H}{U_S}$ vs. U_S.

5.2 Relation Between Output Voltage U and Magnetic Field B

According to Eq.(3), we have the following formula:

$$C(x) = \frac{B(x)}{I_M}$$

Based on the data in Table 1, we are able to obtain the value of C(x) when x = 0 as follows:

$$C(x) = \frac{1.4366}{0.1} = 14.366[mT/A]$$

Now, we are able to find out B through C(x) and I_M, where I_M is the value we measured in the lab. Then, we are able to obtain the data in Table 5.

	I_M [mA] $\pm$ 2% [mA]	U [V] $\pm$ 0.05% + 6×10^{-4} [V]	u_U	B [mT]	u_B [mT]
1	0	0.0000	0.0006	0	0
2	50	0.0256	0.0006	0.718	0.014
3	100	0.0510	0.0006	1.44	0.03
4	150	0.0731	0.0006	2.15	0.04
5	200	0.0974	0.0006	2.87	0.06
6	250	0.1205	0.0007	3.59	0.07
7	300	0.1419	0.0007	4.31	0.09
8	350	0.1664	0.0007	5.03	0.10
9	400	0.1856	0.0007	5.75	0.12
10	450	0.2102	0.0007	6.46	0.13
11	500	0.2324	0.0007	7.18	0.14

Table 5: Measurement data for the I_M vs. U relation.

As sample calculation, we choose the data when I_M = 50 mA, and we have (Detailed calculations of uncertainties are shown in A.2):

$$B = C(x) \times I_M = 14.366 \times 50 \times 10^{-3} = 0.718 \pm 0.014[mT]$$

with a relative uncertainty 2%.

Now, we are able to use *Origin* to plot a linear fit for U vs. B. The result is shown in Fig.8.

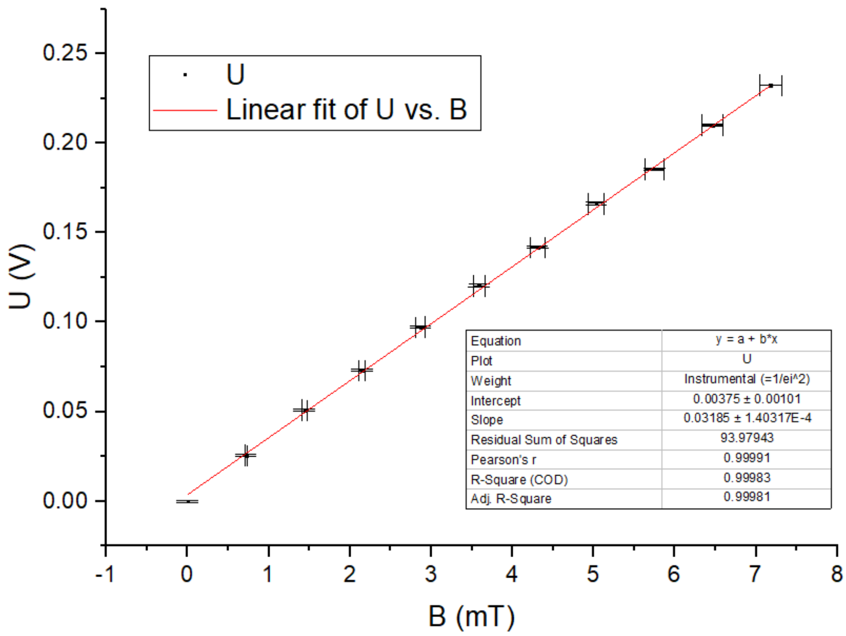

Figure 8: $\frac{K_H}{U_S}$ vs. U_S.

Since U is the output signal of U_H, we can find out the relationship between B and the Hall voltage U_H through the figure. Since the Pearson's R is 0.99991, which is very close to 1, we can conclude that they do have a linear relationship.

Also, from the linear fit, we can obtain K_H as:

$$K_H = 0.0319 \pm 0.0003[kV/T]$$

with a relative uncertainty $r_{K_H} = 1.0\%$.

In the first part, the K_H we obtain is $K_{H,1} = 0.033 \pm 0.003[kV/T]$. We can find out their relative error is 3%, which is acceptable, indicating that the two answers we obtain are very close to each other.

5.3 Magnetic Field Distribution Inside the Solenoid

In order to find out B(x), we can use the following formula:

$$B = \frac{U}{K_H}$$

where K_H is what we have find out before.

Then, we collected 52 pairs of data, which is recorded in Table 6. For the calculation of B, we take the case when x = 0 [cm] as a sample calculation (Detailed calculations of uncertainties are shown in A.3):

$$B = \frac{U}{K_H} = \frac{0.0097}{0.0319 \times 10^3} = 0.00030 \pm 0.00002[mT]$$

	x [cm] $\pm$ 0.05 [cm]	U [V] $\pm$ 0.05% + 6×10^{-4} [V]	B [mT]	u_B		x [cm] $\pm$ 0.05 [cm]	U [V] $\pm$ 0.05% + 6×10^{-4} [V]	B [mT]	u_B
1	0.0	0.0097	0.304	0.019	27	18.0	0.1201	3.76	0.03
2	0.2	0.0119	0.373	0.019	28	19.0	0.1200	3.76	0.03
3	0.4	0.0136	0.426	0.019	29	20.0	0.1200	3.76	0.03
4	0.6	0.0150	0.470	0.019	30	21.0	0.1200	3.76	0.03
5	0.8	0.0173	0.542	0.019	31	22.0	0.1195	3.75	0.03
6	1.0	0.0194	0.608	0.019	32	23.0	0.1187	3.72	0.03
7	1.2	0.0227	0.712	0.019	33	24.0	0.1170	3.67	0.03
8	1.4	0.0248	0.777	0.019	34	25.0	0.1158	3.63	0.03
9	1.6	0.0286	0.897	0.019	35	26.0	0.1123	3.52	0.03
10	1.8	0.0335	1.050	0.019	36	26.2	0.1112	3.49	0.03
11	2.0	0.0383	1.201	0.019	37	26.4	0.1097	3.44	0.03
12	3.0	0.0707	2.22	0.02	38	26.6	0.1064	3.34	0.02
13	4.0	0.0965	3.03	0.02	39	26.8	0.1064	3.34	0.02
14	5.0	0.1098	3.44	0.03	40	27.2	0.1013	3.18	0.02
15	6.0	0.1149	3.60	0.03	41	27.6	0.0945	2.96	0.02
16	7.0	0.1176	3.69	0.03	42	28.0	0.0854	2.68	0.02
17	8.0	0.1181	3.70	0.03	43	28.2	0.0781	2.45	0.02
18	9.0	0.1185	3.71	0.03	44	28.4	0.0736	2.31	0.02
19	10.0	0.1202	3.77	0.03	45	28.6	0.0656	2.06	0.02
20	11.0	0.1205	3.78	0.03	46	28.8	0.0591	1.85	0.02
21	12.0	0.1204	3.77	0.03	47	29.0	0.0515	1.61	0.02
22	13.0	0.1206	3.78	0.03	48	29.2	0.0463	1.45	0.02
23	14.0	0.1203	3.77	0.03	49	29.4	0.0384	1.204	0.019
24	15.0	0.1205	3.78	0.03	50	29.6	0.0345	1.082	0.019
25	16.0	0.1205	3.78	0.03	51	29.8	0.0298	0.934	0.019
26	17.0	0.1204	3.77	0.03	51	30.0	0.0252	0.790	0.019

Table 6: Measurement data for the I_M vs. U relation.

Then, we derive the theoretical value based on Table 1. As what we have done in the first part, since B is in proportional to I_M, we are able to find out the value of B when I_M = 0.25 A. The result is shown in Table 7.

Hence, with the data in Table 6 and Table 7, we can use *Origin* to plot the theoretical and experimental value of B vs. x. The curve is shown in Fig.9.

x [cm]	B[mT]	B_M[mT]	x [cm]	B [mT]	B_M[mT]
$\pm$ 0.0	1.4366	3.5915	$\pm$ 8.0	1.4057	3.5143
$\pm$ 1.0	1.4364	3.5908	$\pm$ 9.0	1.3856	3.4640
$\pm$ 2.0	1.4356	3.5890	$\pm$ 10.0	1.3478	3.3695
$\pm$ 3.0	1.4343	3.5858	$\pm$ 11.0	1.2685	3.1713
$\pm$ 4.0	1.4323	3.5808	$\pm$ 11.5	1.1963	2.9908
$\pm$ 5.0	1,4292	3.5730	$\pm$ 12.0	1.0863	2.7158
$\pm$ 6.0	1.4245	3.5613	$\pm$ 12.5	0.9261	2.3153
$\pm$ 7.0	1.4173	3.5433	$\pm$ 13.0	0.7233	1.8083

Table 7: Theoretical value of the magnetic field when I_M = 250 mA.

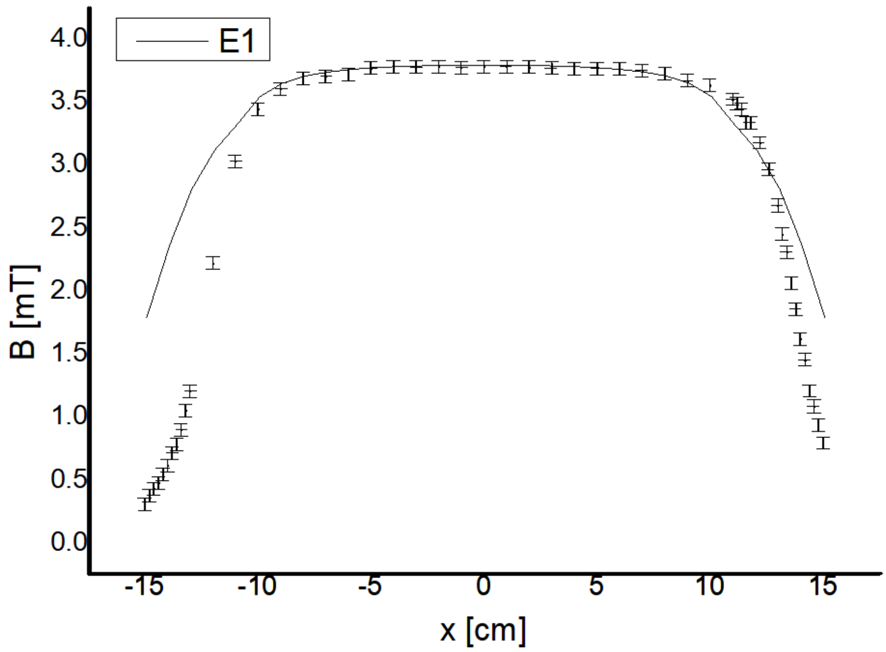

Figure 9: Magnetic field distribution inside the solenoid.

6 Conclusion and Discussion

6.1 Discussion

In this lab, generally speaking we have done a successful job. However, there are still some sources of error. The sources of error and their corresponding solutions are listed below:

1. In the part "Relation Between Sensitivity K_H and Working Voltage U_S":
   - When measuring, we measure the voltage only once, which may lead to a relatively large error while applying Eq.(2).

     [Solution]: We can measure the voltage for multiple times and calculate the answer. Then by taking the average we can reduce the error.
   - In Fig.7, though our result is satisfactory overall, it still exist some error. It may come from the time when we are adjusting the apparatuses.

     [Solution]: Under the process of the experiment, we should adjust the apparatuses as less as possible to reduce the error.
2. In the part "Relation Between Output Voltage U and Magnetic Field B":
   - In this part, we have some little errors. This is because when calculating the magnetic field B, we should use C(x). Hence if C(x) has error, our result will be affected accordingly.

     [Solution]: We can record more groups of data and find out C(x) by taking the average. In this way the error can be reduced.
3. In the part "Magnetic Field Distribution Inside the Solenoid":

   In this part, plot the magnetic field distribution inside the solenoid. The general shape of our result is similar to that of the theoretical one. However, some error still exists. It may be caused by:
   - When the Hall probe are in the two sides, the value of magnetic field changes rapidly, which may cause the error.

     [Solution]: We can record more groups of data when the Hall probe is near the two sides.
   - The theoretical value we use in this lab is measured when I_M = 0.1 A. However, in this lab our I_M = 0.25 A. Although we have changed the scale, some errors may still occur.

     [Solution]: We can either find a theoretical with I_M = 0.25 A, or in the experiment we simply choose I_M = 0.1 A.
4. When measuring the values near the two sides, the Hall probe will have a small inclination. In other words, the Hall probe is not horizontal to the ground. This may leads to error. The case is shown in FIg.10.

   [Solution]: We can improve the stability if the apparatus. In other words, we can build the probe stronger.

Figure 10: The Hall probe is not perfectly horizontal, and is inclined for a small angle.

5. When connecting the circuit, the wire, and even the solenoid has a small resistance. However, in this lab we just ignore them.

   [Solution]: We can take into consideration the resistance of the wires.

6. When we read the values on the screen, the readings is not stable and is always oscillating. The phenomenon is particularly obvious when the values are small.

   [Solution]: We can wait for a longer time until the reading is stable.

6.2 Conclusion

In this lab, I have really learned a lot. We successfully fulfil the objectives, and find out the following phenomenon:

- With the increase of U_S, $\dfrac{K_H}{U_S}$ has a general trend to decrease.
- The Hall voltage is proportional to the magnetic field.
- Inside the solenoid, we find that the magnetic field is the largest in the middle and decrease when going to the sides. The decrease is rapid near the two sides.
- The quantities of Hall probe and their relationships.

Also, I learned the following after the lab:

- The principle of the Hall effect.
- The applications of how to use a Hall probe.
- How to discover the magnetic field of a solenoid.

Furthermore, we can use a Hall probe to detect the magnetic field on the Earth.

A Measurement uncertainty analysis

A.1 Relation Between Sensitivity K_H and Working Voltage U_S

Since K_H is calculated through $K_H = (U - U_0)/B_M$, the uncertainty u_{K_H}can be find out by using uncertainty propagation formula:

$$u_{K_H} = \sqrt{(\frac{\partial K_H}{\partial U})^2(u_U)^2 + (\frac{\partial K_H}{\partial U_0})^2(u_{U_0})^2} = \frac{\sqrt{u_U^2 + u_{U_0}^2}}{B_M}$$

Hence, by plugging in the values, we obtain:

$$U_{K_H} = \frac{\sqrt{(2.482 \times 0.05\% + 6 \times 10^{-3})^2 + (2.602 \times 0.05\% + 6 \times 10^{-3})^2}}{3.5915} = 0.003[kV/T]$$

Hence, K_H can be written as:

$$K_H = 0.033 \pm 0.003[kV/T]$$

Then, we need to calculate the uncertainty of $\frac{K_H}{U_S}$. Since $\frac{K_H}{U_S}$ is calculated through $\frac{K_H}{U_S} = \frac{U-U_0}{B_M U_S}$, the uncertainty u_{K_H/U_S}can be find out by using uncertainty propagation formula similarly. Take U_S = 2.80 V as an example, and we will obtain:

$$\begin{aligned}
u_{K_H/U_S} &= \sqrt{(\frac{\partial f}{\partial U})^2(u_U)^2 + (\frac{\partial f}{\partial U_0})^2(u_{U_0})^2 + (\frac{\partial f}{\partial U_S})^2(u_{U_S})^2} \\
&= \sqrt{\left(\frac{1}{B_M U_S} u_U\right)^2 + \left(\frac{1}{B_M U_S} u_{U_0}\right)^2 + \left(\frac{U - U_0}{B_M U_S^2} \times u_{U_S}\right)^2} \\
&= \sqrt{\left(\frac{1}{3.5915 \times 2.8} \times u_U\right)^2 + \left(\frac{1}{3.5915 \times 2.8} \times u_{U_0}\right)^2 + \left(\frac{1.4694 - 1.4019}{3.5915 \times 2.8^2} \times 2.8 \times 0.5\%\right)^2} \\
&= 0.00019[mT^{-1}]
\end{aligned}$$

Hence, the value of K_H/U_S when U_S = 2.80 V can be written as:

$$\frac{K_H}{U_S} = 0.00671 \pm 0.00019[mT^{-1}]$$

A.2 Relation Between Output Voltage U and Magnetic Field B

To calculate the uncertainty of B, we note that B is given by $B = C(x) \times I_M$, where C(x) is a constant and we do not need to take into consideration its uncertainty. Hence, the uncertainty of B is simply the type-B uncertainty, which is shown as follows:

$$u_B = C(x) \times u_B$$

Take the case when $I_M = 50mA$ as a sample calculation, we will obtain:

$$u_B = 14.366 \times 2\% \times 0.05 = 0.014[mT]$$

And the other values can be calculated in exactly the same way.

To find out the uncertainty of the slope, we can calculate through the following formula:

$$u = t_{0.95} \times Std = 2.26 \times 1.40317 \times 10^{-4} = 0.0003[kV/T]$$

A.3 Magnetic Field Distribution Inside the Solenoid

In this part, we calculate B through the following formula: $B = \frac{U}{K_H}$. Based on uncertainty propagation, we can find out the uncertainty of B as follows:

$$\begin{aligned} u_B &= \sqrt{(\frac{\partial B}{\partial U})^2 (u_U)^2 + (\frac{\partial B}{\partial K_H})^2 (u_{K_H})^2} \\ &= \sqrt{\frac{1}{K_H^2}(u_U)^2 + \frac{U^2}{K_H^4}(u_{K_H})^2} \end{aligned}$$

Take the case when x = 0 [cm] as a sample calculation, and we will have:

$$u_B = \sqrt{\frac{1}{31.9^2}(0.0097 \times 0.05\% + 6 \times 10^{-4})^2 + \frac{0.0097^2}{31.9^4} \times 0.0003^2} = 0.000019[T] = 0.019[mT]$$

B Data sheet

See the attached data sheet.

References

[1] Krzyzosiak, M. & VP241 TA Groups. *Exercise 2 - lab manual [rev 4.3].pdf*. 2019.