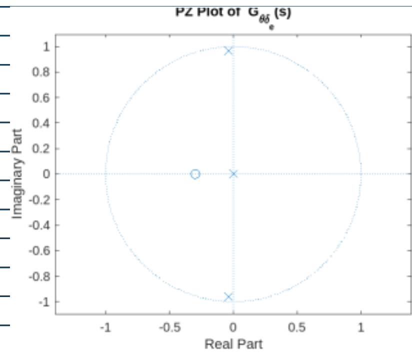
ME 658 HW4

Tuesday, April 25, 2023 8:59 AM

P1 Consider pitch dynamics of Notes

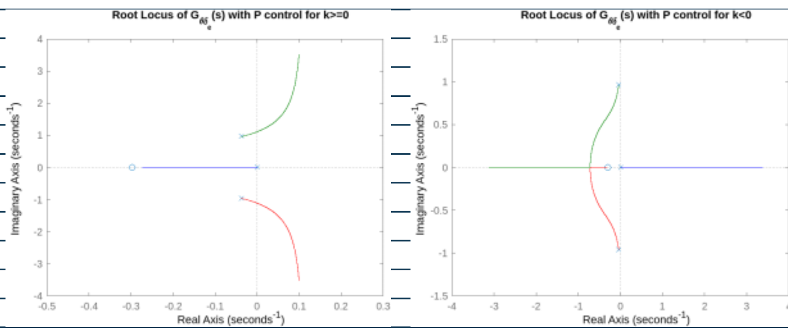
$$G_{\theta\delta_e}(s) = -\frac{1.1569\,s + 0.3435}{s^3 + .0741\,s^2 + 0.9272\,s}$$

a) Find poles and zeros of $G_{\theta\delta_e}(s)$
(Identify the short period mode)



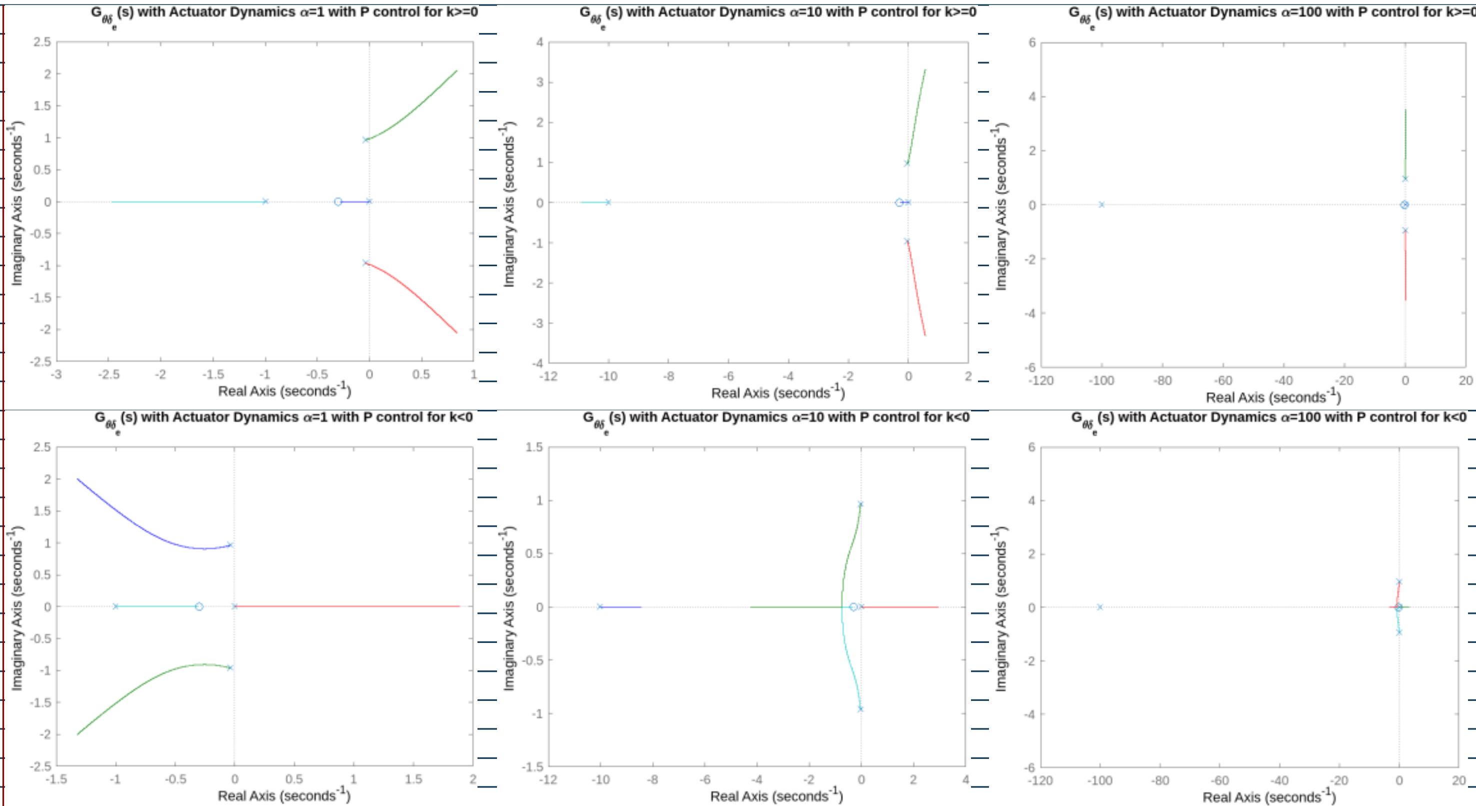
Zero at $s = -0.2969$
Poles at $s = 0.0 + 0.0j, -0.0371 \pm 0.9622j$
The short period mode corresponds to the pole at $s = -0.0371 \pm 0.9622j$

b) Consider a proportional controller $G_c(s) = k$ & draw a Root Locus $(H(s) = 1)$ for i) $k \geq 0$, ii) $k < 0$ (can use Matlab) and comment if you can obtain a stable CL system



Since the locus plots show that we can move the poles of the system into the LHP we can obtain a stable closed loop system

c) Repeat b) if there are actuator dynamics $H(s) = \frac{\alpha}{s+\alpha}$ for various values of $\alpha > 0$:
$\alpha = 1, 10, 100$
and comment how Root Locus changes relatively to the "speed" of the actuator



As alpha increases, the system becomes harder to control. For values of k that are greater than or equal to 0, as alpha increases, the asymptotes of the conjugate pair of poles moves closer and closer to the imaginary axis, implying that there will be more oscillations in the response to the corresponding mode. Additionally, the root locus plots show that the final pole on the real axis moves further into the LHP as alpha increases, this increases the damping for this mode.

For values of k that are less than 0, a similar pattern is seen: The paths of the pair of complex poles move closer to the imaginary axis, and the path of the single pole moves further into the LHP.

d) Consider P-D controller:

$$G_c(s) = -k(s+z)$$

$$k \geq 0, \quad z \geq 0$$

and identify the proportional & the derivative gains in terms of k, z.

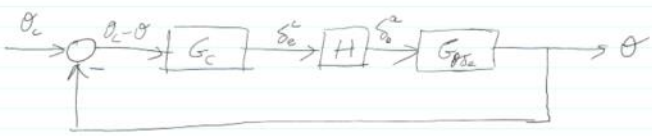
$G_c(s) = -k(s+z) = -ks - kz$

Proportional Gain: $K_p = -kz$

Derivative Gain: $K_d = -k$

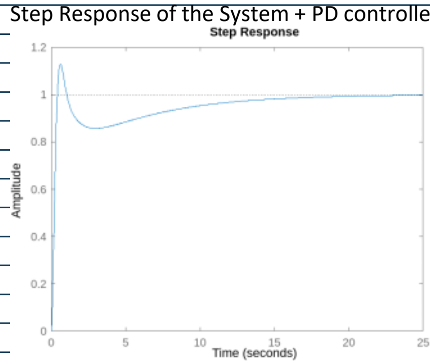
e) For $H(s) = \frac{10}{s+10}$ find a set of k, z that provides CL stability and settling time to step commands $t_s \le 15$ sec and overshoot $\le 20\%$

Note: There are many solutions to choose from. No need to do (or, show) analytical work. You can choose your z & do Root Locus with Matlab.



Using MATLAB:
$K_p = 2.124, K_d = 4.0475, t_s = 14.6s, Overshoot = 13.2\%$
So $k = 4.0475$ and $z = 0.523$

Step Response of the System + PD controller:



2) Consider longitudinal eqns:

$$\dot{x} = Ax + B\delta$$

with

$$A = \begin{pmatrix} -.01 & .1 & 0 & -32.2 \\ -.40 & -.8 & 180 & 0 \\ 0 & -.003 & -.5 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ -10 \\ -2.8 \\ 0 \end{pmatrix}$$

and x is as usual $x = \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix}$ and δ is δ_e: elevator deflection

a) Find open loop poles & identify the short period & phugoid modes in terms of their damping ratio & natural frequency

Open loop poles at $s = -0.6667 \pm 0.7351j$ and $s = 0.0116 \pm 0.1977j$

	Short Period	Phugoid
Poles	$-0.6667 \pm 0.7351j$	$0.0116 \pm 0.1977j$
Damping Ratio	0.6718	-0.0587
Natural Frequency	0.9924 rad/s	0.1981 rad/s

b) Suppose state feedback

$$\delta(t) = -K x(t)$$

is available.
Using formula in notes find the gain matrix K to create CL poles at locations corresponding to modes

$$\zeta_1 = 0.6, \quad \omega_1 = 3 \text{ rad/sec}$$

and

$$\zeta_2 = .05 \quad \omega_2 = 0.1 \text{ rad/sec}$$

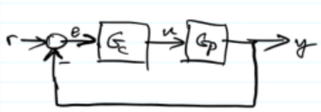
where ζ_i, ω_i are damping ratios & natural frequencies respectively of the CL modes $i = 1, 2$.

For mode $i = 1$:
$Re\{s\} = \zeta_1 * \omega_1 = 0.6 * 3 = 1.8$
$Im\{s\} = \sqrt{(\omega_1^2 - Re\{s\}^2)} = \sqrt{3^2 - 1.8^2} = 2.4$
For $\zeta_1 = 0.6, \omega_1 = 3\, rad/s$, the pole is located at $s = -1.8 \pm 2.4$

For mode $i = 2$:
$Re\{s\} = \zeta_2 * \omega_2 = 0.05 * 0.1 = 0.005$
$Im\{s\} = \sqrt{(\omega_2^2 - Re\{s\}^2)} = \sqrt{0.1^2 - 0.005^2} = 0.0999$
For $\zeta_2 = 0.05, \omega_2 = 0.1\, rad/s$, the pole is located at $s = -0.005 \pm 0.0999$

Using MATLAB 'place' function: K = [-0.0055, -0.0120, -0.7785, -0.0656]

P3 Bonus for UG's, required for G's



$L =$ "loop" t.f. $= G_p G_c$
OL zeros = zeros of $L(s)$
OL poles = poles of $L(s)$
$G_{CL} =$ "CL" t.f. $= \frac{y}{r} = \frac{L}{1+L}$

a) If CL zeros are the zeros of $G_{CL}(s)$, how do OL zeros relate to CL zeros?

$$G_{OL} = G_p G_c, \quad G_{CL} = \frac{G_p G_c}{1 + G_p G_c}$$

The open loop zeros are in the same location as the closed loop zeros because the numerator of G_{CL} is equal to G_{OL}

b) How do zeros of $G_c(s)$ and $G_p(s)$ relate to CL zeros?

$$\{\text{zeros of } G_c(s)\} \cup \{\text{zeros of } G_p(s)\} = \{\text{zeros of } G_{CL}\}$$

The union of the sets containing the zeros of $G_c(s)$ and $G_p(s)$ is equal to the set containing the zeros of G_{CL}

c) Consider tracking error $e = r - y$ and show that $\frac{e}{r} = H_{er}(s) = \frac{1}{1 + L(s)}$

$e = r - y = r - eG_c G_p = r - eL(s)$

$e(1 + L(s)) = r \quad \therefore \quad \frac{e}{r} = \frac{1}{1 + L(s)}$

d) Show that $H_{er}(s) + G_{CL}(s) = 1$

$H_{er}(s) + G_{CL}(s) = \frac{1}{1+L} + \frac{L}{1+L} = \frac{1+L}{1+L} = 1$ ✓