ME 658 HW3

Problem 1

Do 2.5 in McLean
(Note heading angle : $\lambda = \beta + \psi$
β : side slip , ψ : yaw angle)
find the modes (e-values only)

2.5 The lateral motion of the aircraft FOXTROT-2 is to be considered. Its rudder is not used at high Mach numbers. Derive the corresponding state and output equations, if the output variables of interest are heading angle, λ, and change in roll angle, ϕ.

B.2.6 FOXTROT – A twin-engined, jet fighter/bomber aircraft

Flight Conditions

Parameter	*Flight condition* 1	2	3	4
Height (m)	S.L.	10 650	10 650	13 700
Mach no.	0.206	0.9	1.2	2.15
U_0 (m s^{-1})	70	265	350	650
$\bar{q}$ (N m^{-2})	2 997	13 550	24 090	48 070
α_0 (degrees)	11.7	2.6	1.6	1.4
γ_0 (degrees)	0	0	0	0

Lateral Motion

Stability derivative	*Flight condition* 1	2	3	4
Y_β	− 21.1	− 80.6	− 176.0	− 277.0
L'_β	− 10.4	− 18.3	− 14.1	− 8.67
L'_p	− 1.43	− 1.24	− 1.38	− 1.08
L'_r	0.929	0.395	0.318	0.22
N'_β	1.44	4.97	12.3	8.37
N'_p	− 0.026	− 0.0504	− 0.038	0.015
N'_r	− 0.215	− 0.238	− 0.4	− 0.275
$Y^*_{\delta_A}$	− 0.004	− 0.0007	− 0.0009	− 0.0005
$Y^*_{\delta_R}$	0.0053	0.0043	0.004	0.0026
L'_{δ_A}	2.74	9.0	10.9	5.35
L'_{δ_R}	0.7	1.95	3.0	2.6
N'_{δ_A}	0.42	0.2	0.67	0.36
N'_{δ_R}	− 0.67	− 2.6	− 3.2	− 1.86

For Foxtrot-2 :

A
$$Y_v = Y_\beta / U_0 = -0.304 \qquad L'_p = -1.24$$
$$U_0 = 265 \qquad L'_r = 0.395$$
$$\Gamma_0 = 0 \qquad N'_v = N'_\beta = 4.97$$
$$N'_p = -0.0504$$
$$L'_v = L'_\beta = -18.3 \qquad N'_r = -0.234$$

B
$$Y^*_{\delta_R} = 0.0043 \qquad N'_{\delta_A} = 0.2$$
$$L'_{\delta_A} = 9.0 \qquad N'_{\delta_R} = -2.6$$
$$L'_{\delta_R} = 1.95$$

$$\dot{x} = Ax + Bu$$

$$\dot{x} = \begin{pmatrix} \dot\beta \\ \dot p \\ \dot r \\ \dot\phi \\ \dot\psi \end{pmatrix} = \begin{pmatrix} Y_v & 0 & -1 & g/U_0 & 0 \\ L'_\beta & L'_p & L'_r & 0 & 0 \\ N'_\beta & N'_p & N'_r & 0 & 0 \\ 0 & 1 & \tan\Gamma_0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta \\ p \\ r \\ \phi \\ \psi \end{pmatrix} + \begin{pmatrix} 0 & Y^*_{\delta_R} \\ L'_{\delta_A} & L'_{\delta_R} \\ N'_{\delta_A} & N'_{\delta_R} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \delta_A \\ \delta_R \end{pmatrix}$$

State equation:

$$\dot{x} = \begin{pmatrix} -0.304 & 0 & -1 & 0.037 & 0 \\ -18.3 & -1.24 & 0.395 & 0 & 0 \\ 4.97 & -0.0504 & -0.238 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta \\ p \\ r \\ \phi \\ \psi \end{pmatrix} + \begin{pmatrix} 0 & 0.0043 \\ 9.0 & 1.95 \\ 0.2 & -2.6 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \delta_A \\ \delta_R \end{pmatrix}$$

Output equation for heading angle $\lambda = \beta + \psi$

$$y = Cx + Du \rightarrow D = 0, \quad y = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta \\ p \\ r \\ \phi \\ \psi \end{pmatrix} = \beta + \psi$$

Output equation for change in roll angle $\dot{\phi} = p$

$$y = Cx + Du \rightarrow D = 0, \quad y = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta \\ p \\ r \\ \phi \\ \psi \end{pmatrix} = p$$

Modes (eigenvalues):

$\lambda_1 = 0.0$ $\quad \lambda_{4,5} = -0.1473 \pm 2.293$

$\lambda_2 = -1.4761$

$\lambda_3 = -0.0114$

Problem 2

Change accordingly the C, D-matrices of Problem 1 and do 2.6 in McLean.

2.6 For exercise 2.5 derive the corresponding transfer function relating, $a_{y_{cg}}$, to aileron deflection, δ_A.

From the notes: $a_{y_{cg}} = Y_\beta U_0 \beta + Y^*_{\delta_R} U_0 \delta_r$

Therefore, the output equation should have $C = (U_0\beta, 0, 0, 0, 0)$ and $D = (0, Y^*_{\delta_R} U_0)$

$$a_{y_{cg}} = \begin{pmatrix} -21.359 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta \\ p \\ r \\ \phi \\ \psi \end{pmatrix} + \begin{pmatrix} 0 & 0.0043 \end{pmatrix} \begin{pmatrix} \delta_A \\ \delta_R \end{pmatrix}$$

From MATLAB:

$$G(s) = \frac{4272s^3 - 1150s^2 - 1755s}{s^5 + 1.782s^4 + 5.733s^3 + 7.86s^2 + 0.049s}$$

Problem 3

For CHARLIE4 find the state space description with β as output.

i) Find the e-values of the A-matrix

ii) Find the transfer function $G_{\beta\delta_R}$ from rudder δ_R to sideslip β

iii) How do poles of $G_{\beta\delta_R}(s)$ compare with e-values of A?

B.2.3 CHARLIE – A very large, four-engined, passenger jet aircraft

Flight Conditions

Parameter	Flight condition 1	2	3	4
Height (m)	S.L.	6 100	6 100	12 200
Mach no.	0.198	0.5	0.8	0.8
U_0 (m s^{-1})	67	158	250	250
$\bar{q}$ (N m^{-2})	2 810	8 667	24 420	9 911
α_0 (degrees)	8.5	6.8	0	4.6
γ_0 (degrees)	0	0	0	0

Stability derivative	Flight condition 1	2	3	4
Y_v	− 0.089	− 0.082	− 0.12	− 0.056
$Y^*_{\delta_R}$	0.015	0.014	0.014	0.012
L'_β	− 1.33	− 2.05	− 4.12	− 1.05
L'_p	− 0.98	− 0.65	− 0.98	− 0.47
L'_r	+ 0.33	+ 0.38	+ 0.29	+ 0.39
L'_{δ_A}	0.23	0.13	0.31	0.14
L'_{δ_R}	0.06	0.15	0.18	0.15
N'_β	0.17	0.42	1.62	0.6
N'_p	− 0.17	− 0.07	− 0.016	− 0.032
N'_r	− 0.217	− 0.14	− 0.232	− 0.115
N'_{δ_A}	0.026	0.018	0.013	0.008
N'_{δ_R}	− 0.15	− 0.39	− 0.92	− 0.48

State space description $\dot{x} = Ax + Bu, \quad y = Cx + Du$

$$\dot{x} = \begin{pmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} Y_v & 0 & -1 & g/U_0 \\ L'_\beta & L'_p & L'_r & 0 \\ N'_\beta & N'_p & N'_r & 0 \\ 0 & 1 & \tan\Gamma_0 & 0 \end{pmatrix} \begin{pmatrix} \beta \\ p \\ r \\ \phi \end{pmatrix} + \begin{pmatrix} 0 & Y^*_{\delta_r} \\ L'_{\delta_A} & L'_{\delta_r} \\ N'_{\delta_A} & N'_{\delta_r} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \delta_A \\ \delta_r \end{pmatrix}$$

$$\dot{x} = \begin{pmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} -0.056 & 0 & -1 & 0 \\ -1.05 & -0.47 & 0.39 & 0 \\ 0.6 & -0.032 & -0.115 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta \\ p \\ r \\ \phi \end{pmatrix} + \begin{pmatrix} 0 & 0.012 \\ 0.14 & 0.15 \\ 0.008 & -0.48 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \delta_A \\ \delta_r \end{pmatrix}$$

$$y = Cx + Du \rightarrow y = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta \\ p \\ r \\ \phi \end{pmatrix} + (0) \rightarrow y = \beta$$

3i Eigenvalues of A matrix:

From MATLAB:

$$\lambda = -0.0453 \pm 0.8088, \quad -0.5624, \quad 0.012$$

3ii From MATLAB ss2tf:

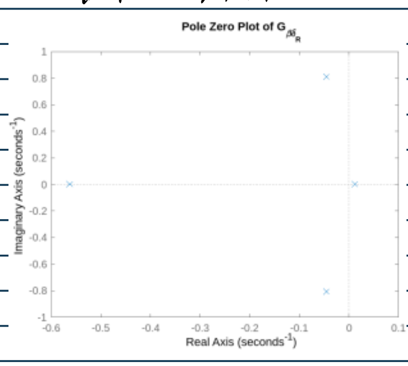
$$G_{\beta\delta_R}(s) = \frac{0.012s^3 + 0.478s^2 + 0.237s - 0.0067}{s^4 + 0.641s^3 + 0.6993s^2 + 0.361s - 0.0044}$$

3iii

The poles of $G_{\beta\delta_R}$ each correspond to one eigenvalue of the A matrix.

The plot at right shows that if you plot the real and imaginary parts of the eigenvalues on the complex plane, these locations are the same as the locations of the poles.

4x4 system



5x5 system