

## Quiz 9.2A      AP Statistics      Name: \_\_\_\_\_

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1. Eleven percent of the products produced by an industrial process over the past several months have failed to conform to specifications. The company modifies the process in an attempt to reduce the rate of nonconformities. In a random sample of 300 items from a trial run, the modified process produces 16 nonconforming items.

(a) Do these results provide convincing evidence that the modification is effective? Support your conclusion with a test of significance.

(b) Explain what the  $P$ -value of your test means in the context of this problem.

2. The germination rate of seeds is defined as the proportion of seeds that, when properly planted and watered, sprout and grow. A certain variety of grass seed usually has a germination rate of 0.80, and a company wants to see if spraying the seeds with a chemical that is known to change germination rates in other species will change the germination rate of this grass species.
- (a) Suppose the company plans to spray a random sample of 400 seeds and conduct a two-sided test of  $H_0 : p = 0.8$ , using  $\alpha = 0.05$ . They determine that the power of this test against the alternative  $p = 0.75$  is 0.69. Explain what this means in the context of the problem.
- (b) Describe two ways the company can increase the power of the test.
- (c) The company researchers spray 400 seeds with the chemical, and 307 of the seeds germinate. This produces a 95% confidence interval for the proportion of seeds that germinate of (0.726, 0.809). Use this confidence interval to determine whether this test would reject or fail to reject the null hypothesis. Explain your reasoning.

1. Nationally, the proportion of red cars on the road is 0.12. A statistically-minded fan of the Philadelphia Phillies (whose team color is red) wonders if fans who park at Citizens Bank Park (the Phillies home field) are more likely to drive red cars. One day during a home game, he takes an SRS of 210 cars parked in the lot while a game is being played, and counts 35 red cars. (There are 21,000 parking spaces.)

- (a) Is this convincing evidence that Phillies fans prefer red cars more than the general population? Support your conclusion with a test of significance.

- (b) Explain what the  $P$ -value of your test means in the context of this problem.

2. Do political “attack ads” work? A congressional candidate who currently has the support of only 44% of the voters runs a television spot that aggressively attacks the character of his opponent. He wants to know if the television spot has changed his support level among voters.
- (a) Suppose his pollsters plan to survey a random sample of 450 voters and conduct a two-sided test of  $H_0 : p = 0.44$ , using  $\alpha = 0.05$ . They determine that the power of this test against the alternative  $H_0 : p = 0.40$  is 0.36. Explain what this means in the context of the problem.
- (b) Describe two ways the pollsters can increase the power of the test.
- (c) The pollsters survey an SRS of 450 voters and find that 186 support the candidate. This produces a 95% confidence interval for the proportion of voters who support him of (0.368, 0.459). Use the confidence interval to determine whether this test would reject or fail to reject the null hypothesis. Explain your reasoning.

## Quiz 9.2C      AP Statistics    Name: \_\_\_\_\_

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1. LeRoy, a starting player for a major college basketball team, made only 40% of his free throws last season. During the summer, he worked on developing a softer shot in hopes of improving his free throw accuracy. In the first eight games of this season, LeRoy made 25 free throws in 40 attempts. You want to investigate whether LeRoy's work over the summer will result in a higher proportion of free-throw successes this season. What conclusion would you draw at the  $\alpha = 0.01$  level about LeRoy's free throw shooting? Justify your answer with a complete significance test.

2. Is the ratio of male births to female births even? A simple random sample of births in a major metropolitan area found 1345 boys among 2546 firstborn children. A 99% confidence interval for  $p$  = the proportion of male births in this population is given by (0.5028, 0.5538).

(a) Use the confidence interval to draw a conclusion about the hypothesis  $H_0 : p = 0.5$  against  $H_a : p \neq 0.5$ . Be sure to indicate the appropriate significance level.

(b) What information is provided by the confidence interval that would not be provided by a test of significance alone?

(c) If the actual proportion of male births in this population is 0.51, the power of the test in part (a) is only 0.06. Explain what this means and suggest a way to increase it.

### Quiz 9.2A

1. (a) State: We wish to test  $H_0 : p = 0.11$  versus  $H_a : p < 0.11$ , where  $p$  = the true proportion of nonconforming items. We will use a significance level of  $\alpha = 0.05$ . Plan: The procedure is a one-sample  $z$ -test for a proportion. Conditions: *Random*: A random sample of items was taken. *10%*: We must assume that the trial run consisted of at least  $(10)(300) = 3000$  items. *Large counts*: Assuming

$H_0 : p = 0.11$  is true,  $np = (300) \cdot (0.11) = 33 \geq 10$ , and  $n(1 - p) = (300) \cdot (0.89) = 267 \geq 10$ .

Do:  $\hat{p} = \frac{16}{300} = 0.0533$ , so  $z = \frac{0.0533 - 0.11}{\sqrt{\frac{(0.11)(0.89)}{300}}} = -3.14$ ;  $P$ -value = 0.0008. Conclude: A  $P$ -value of

0.0008 is less than  $\alpha = 0.05$ , so we reject  $H_0$  and conclude that there is convincing evidence that the true proportion of nonconforming items is less than 0.11. (b) If  $H_0 : p = 0.11$  is true, there is a probability of 0.0008 of getting a sample proportion of nonconforming items as far or farther below 0.11 as 0.05333 is. 2. (a) Power = 0.69 measures the probability of correctly rejecting the null hypothesis and concluding that the true proportion of seeds that germinate is different from 0.8 when it is actually 0.75. (b) Increase power by increasing the sample size or increasing the significance level. (c) Since the 95% confidence interval contains the null value of 0.8, we cannot reject  $H_0$  at the  $\alpha = 0.05$  level. We do not have convincing evidence that the germination rate of the seeds was changed by the chemical spray.

### Quiz 9.2B

1. (a) State: We wish to test  $H_0 : p = 0.12$  versus  $H_a : p > 0.12$ , where  $p$  = the true proportion of red cars in the parking lot. We will use a significance level of  $\alpha = 0.05$ . Plan: The procedure is a one-sample  $z$ -test for a proportion. Conditions: *Random*: The fan took an SRS of 210 cars. *10%*: 210 is clearly less than 10% of all the cars in the lot. *Large counts*: Assuming  $H_0 : p = 0.12$  is true,

$np = (210) \cdot (0.12) = 25.2 \geq 10$ , and  $n(1 - p) = (210) \cdot (0.88) = 184.8 \geq 10$ . Do:  $\hat{p} = \frac{35}{210} = 0.1667$ , so

$z = \frac{0.1667 - 0.12}{\sqrt{\frac{(0.12)(0.88)}{210}}} = 2.081$ ;  $P$ -value = 0.0187. Conclude: A  $P$ -value of 0.0187 is less than  $\alpha = 0.05$ , so

we reject  $H_0$  and conclude that there is convincing evidence that the true proportion of red cars at Citizens Bank Park is greater than 0.12. (b) If  $H_0 : p = 0.12$  is true, there is a probability of 0.0187 of getting a sample proportion of red cars as far or farther above 0.12 as 0.1667 is. 2. (a) Power = 0.36 measures the probability of correctly rejecting the null hypothesis and concluding that the true proportion of voters that support the candidate is different from 0.44 when it is actually 0.40. (b) Increase power by increasing the sample size or increasing the significance level. (c) Since the 95% confidence interval contains the null value of 0.44, we cannot reject  $H_0$  at the  $\alpha = 0.05$  level. We do not have convincing evidence that the attack ads change the level of support for the candidate.

### Quiz 9.2C

1. State: We wish to test  $H_0 : p = 0.4$  versus  $H_a : p > 0.4$ , where  $p$  = Leroy's new free throw accuracy, expressed as the true proportion of all shots he will take this season that he'll make. We will use a significance level of  $\alpha = 0.01$ . Plan: The procedure is a one-sample  $z$ -test for a proportion. Conditions: Random: We will have to assume that LeRoy's first 40 attempts this season are an SRS of all shots he might over the remainder of his career (and that he will take more than 400 shots). Large counts:

Assuming  $H_0 : p = 0.4$  is true,  $np = (40) \cdot (0.4) = 16 \geq 10$ , and  $n(1 - p) = (40) \cdot (0.6) = 24 \geq 10$ . Do:

$$\hat{p} = \frac{25}{40} = 0.625, \text{ so } z = \frac{0.625 - 0.4}{\sqrt{\frac{(0.4)(0.6)}{40}}} = 2.90; P\text{-value} = 0.0018. \text{ Conclude: A } P\text{-value of } 0.0018 \text{ is less}$$

than  $\alpha = 0.01$ , so we reject  $H_0$  and conclude that there is convincing evidence that LeRoy's free-throw shooting percentage has improved.

2. (a) Since the 99% confidence interval does not contain the null value of 0.5, we can reject  $H_0$  at the  $\alpha = 0.01$  level. We have convincing evidence that the proportion of male births is different from the proportion of female births. (b) The confidence interval gives a range of plausible values for the proportion of male births; the test of significance does not. (c) This means that there is only a 0.06 probability that we will reject the (false) null hypothesis that  $p = 0.5$ . Power can be increased by increasing the sample size or by increasing the level of significance (which was 0.01 for this test, since we used a 99% confidence interval).