Quiz 9.1A AP Statistics Name:

1. For each of the following settings, define the parameter of interest and write the appropriate null and alternative hypotheses for the test that is described.

(a) The mean weight of loaves of bread produced at the bakery where you work is supposed to be one pound. You are the supervisor of quality control at the bakery, and you are concerned that new personnel are producing loaves that have a mean weight of more than one pound.

(b) According to the Humane Society, 33% of households in the United States own at least one cat. You are interested in determining whether the proportion of households of the students at your school that own at least one cat is different from the national proportion.

2. Consider the bakery problem in question 1(a). Suppose you weigh an SRS of bread loaves and find that the mean weight is 1.025 pounds, which yields a *P*-value of 0.086.

(a) Interpret the *P*-value in the context of the problem.

(b) What conclusion would you draw at the $\alpha = 0.05$ level? At the $\alpha = 0.10$ level?

3. A contract between a manufacturer and a consumer of light bulbs specifies that the mean lifetime of the bulbs must be at least 1000 hours. As part of the quality assurance program, the manufacturer will institute an inspection program for each day's production of 10,000 units. An ordinary testing procedure is difficult since 1000 hours is over 41 days! Since the lifetime of a bulb decreases as the voltage applied increases, a common procedure is to perform an accelerated lifetime test in which the bulbs are lit using 400 volts (compared to the usual 110 volts). At such a voltage, a 1000-hour bulb is expected to last only 3 hours. This is a well-known procedure, and both sides have agreed that the results from the accelerated test will be a valid indicator of lifetime of the bulb.
The manufacturer will test the hypotheses $H_0 : \mu = 3$ *versus* $H_a : \mu < 3$ at the $\alpha = 0.01$ level with an SRS of 100 bulbs.

(a) Describe what a Type I error would be in this context.

(b) What is the probability of making a Type I error when performing this test?

(c) Describe what a Type II error would be in this context.

(d) Which error—Type I or Type II—is likely to do more damage to the manufacturer's relationship with the consumer? Explain.

Quiz 9.1B AP Statistics Name: ____________________

1. For each of the following settings, define the parameter of interest and write the appropriate null and alternative hypotheses for the test that is described.

(a) The mean time needed for college students to complete a certain paper-and-pencil maze is 30 seconds. You wish to see if this is changed by vigorous exercise, so you have a randomly selected group of 25 students from a particular college exercise vigorously for 30 minutes and then complete the maze.

(b) Lumber companies dry freshly-cut wood in kilns before selling it. As a result of the drying process a certain percentage of the boards become "checked," which means that cracks develop at the ends of the boards. The current drying procedure for 1" x 4" red oak boards is known to produce cracks in 16% of the boards. The drying supervisor at a lumber company wants to test a new method to determine if fewer boards crack.

2. Consider the lumber problem in question 1(b). Suppose the drying supervisor uses the new method on an SRS of boards and finds that the sample proportion of checked boards is 0.11, which produces a *P*-value of 0.027.

(a) Interpret the *P*-value in the context of the problem.

(b) What conclusion would you draw at the α 0.05 level? At the $\alpha = 0.01$ level?

3. As a construction engineer for a city, you are responsible for ensuring that the company that is providing gravel for a new road puts as much gravel in each truckload as they claim to. It has been estimated that it will take 500 truckloads of gravel to complete this road, so you plan to measure the volume of gravel in an SRS of 25 trucks to make sure that the company isn't delivering less gravel per truckload than they claim. Each truckload is supposed to have 20 m^3 of gravel, so you will test the hypotheses $H_0 : \mu = 20$ *versus* $H_a : \mu < 20$ at the $\alpha = 0.05$ level.

(a) Describe what a Type I error would be in this context.

(b) What is the probability of making a Type I error when performing this test?

(c) Describe what a Type II error would be in this context.

(d) Which error—Type I or Type II—is a more serious problem for the city? Explain.

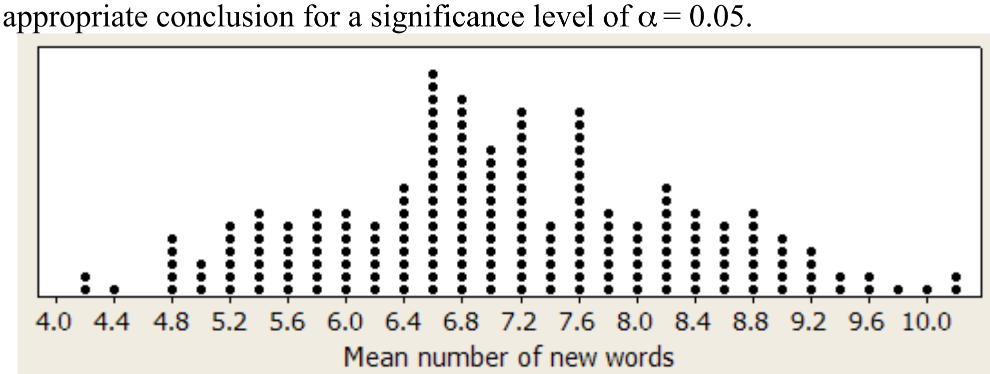
Quiz 9.1C AP Statistics Name:

1. For each of the following settings, define the parameter of interest and write the appropriate null and alternative hypotheses for the test that is described.

(a) You suspect that a certain six-sided die is not correctly balanced, so that the probability of rolling a 5 is something other than $\frac{1}{6}$. You plan to roll the die many times to test whether it's correctly balanced.

(b) Statistics can help decide the authorship of literary works. Sonnets by an Elizabethan poet are known to contain an average of $\mu = 6.9$ new words (words not used in the poet's other works) and the number of new words is approximately Normally distributed. Now a new manuscript has come to light with many new sonnets, and scholars are debating whether it is the poet's work. They take a simple random sample of five sonnets from the new manuscript and count the number of new words in each one. We expect poems by another author to contain more new words than found in the Elizabethan poet's poems.

2. Consider the test of an Elizabethan poet's sonnets from question 1(b). Scholars have determined that the number of new words in works by this poet is Normally distributed with a mean of 6.9 words and a standard deviation of $\sigma = 2.7$ words. When you examine the five new works, you find that the mean number of new words is $\bar{x} = 9.2$. Below is a dot plot showing the results of simulating 200 samples of size 5 from a Normal distribution with a mean of 6.9 and a standard deviation of 2.7, and calculating the mean for each sample. Use it to estimate the *P*-value of this test, and draw an appropriate conclusion for a significance level of $\alpha = 0.05$.



3. A certain cigarette brand advertises that the mean nicotine content of their cigarettes is 1.5 mg, but you are suspicious and plan to investigate the advertised claim by testing the hypotheses $H_0 : \mu = 1.5$ versus $H_a : \mu > 1.5$ at the $\alpha = 0.05$ significance level. You will do so by measuring the nicotine content of 30 randomly selected cigarettes of this brand.

(a) Describe what a Type I error would be in this context.

(b) Describe what a Type II error would be in this context.

(c) From the perspective of public health, which error—Type I or Type II—is more serious? Explain.

(d) Explain why you know that the probability of making a Type I error when performing this test is 0.05.

Chapter 9 Solutions

Quiz 9.1A

1. (a) μ = true mean weight of all loaves of bread produced at the bakery. $H_0: \mu = 1;\ H_a: \mu > 1$. (b) p = proportion of all of your school's students' households that own at least one cat. $H_0: p = 0.33;\ H_a: p \neq 0.33$ **2.** (a) If the true mean weight of bread loaves is 1 pound, the probability of getting a sample mean as large or larger than 1.025 pounds is 0.086. (b) Fail to reject H_0 at the $\alpha = 0.05$ level, since the *P*-value is greater than α: we do not have convincing evidence that the mean weight of bread loaves is greater than 1 pound. Reject H_0 at the $\alpha = 0.10$ level, since the *P*-value is less than α: we have convincing evidence that the mean weight of bread loaves is greater than 1 pound. **3.** (a) Type I error: concluding that the mean lifespan of bulbs is less than 3 hours when it is (at least) 3 hours. (b) $P(\text{Type I error}) = \alpha = 0.01$. (c) Type II error: not concluding that the mean lifespan of the bulbs is less than 3 hours when it is. (d) A Type II error is probably more problematic, since it means the consumer would be purchasing bulbs that don't last as long as promised.

Quiz 9.1B

1. (a) μ = The true mean time it takes a student to finish the maze after exercising vigorously for 30 minutes. $H_0: \mu = 30;\ H_a: \mu \neq 30$. (b) p = the true proportion of all 1" x 4" red oak boards that are "checked" during the new drying process. $H_0: p = 0.16;\ H_a: p < 0.16$ **2.** (a) If the true proportion cracked boards is 0.16, the probability of getting a sample proportion as low or lower than 0.11 is 0.032. (b) Reject H_0 at the $\alpha = 0.05$ level, since the *P*-value is less than α: we have convincing evidence that the true proportion of cracked boards is less than 0.016. Fail to reject H_0 at the $\alpha = 0.01$ level, since the *P*-value is greater than α: we do not have convincing evidence that the true proportion of cracked boards is less than 0.016. **3.** (a) Type I error: concluding that the mean volume of gravel per truck is below 20 m^3 when it is equal to (or greater than) 20 m^3. (b) $P(\text{Type I error}) = \alpha = 0.05$. (c) Type II error: not concluding that the mean volume of gravel per truck is less than 20 m^3 when it is. (d) A Type II error is probably more problematic for the city, since it means they would be paying full price for underweight trucks.

Quiz 9.1C

1. 1. (a) p = the true proportion of fives that would be rolled in all possible rolls of this suspicious die. $H_0: p = \frac{1}{6};\ H_a: p \neq \frac{1}{6}$. (b) μ = The mean number of new words found in all sonnets in the recently-found manuscript. $H_0: \mu = 6.9;\ H_a: \mu > 6.9$ **2.** In this simulation, 12 of 200 samples—or 6%--have a mean of 9.2 or more new words. This corresponds to a *P*-value of 0.06, which is greater than 0.05, so we fail to reject the null hypothesis at the $\alpha = 0.05$ level. Our sample does not provide convincing evidence that the mean number of new words in these works is greater than 6.9. **3.** Type I error: Concluding that the mean nicotine content per cigarette is greater than 1.5 mg when it is equal to (or less than) 1.5 mg. (b) Type II error: Not concluding that the mean nicotine level is greater than 1.5 mg per cigarette when it is. (c) A Type II error would mean that you fail to discover that the cigarettes have a higher nicotine content that the company claims, which means people will be exposed to more nicotine that they expect. A Type I error might bring unwarranted negative publicity to the tobacco company, but that is not a public health issue! (d) When we choose a significance level of $\alpha = 0.05$ we will reject the null hypothesis if we get a sample mean nicotine content that is far enough above 1.5 mg. so that it only occurs in 5% of samples *when the true mean is 1.5 mg*. In other words, if the null hypothesis is true, 5% of all possible sample means will cause us to (incorrectly) reject H_0.