

Description of the forecasting exercise performed in Matlab.

1. We downloaded the necessary data from the FRED database. The variables we use are the following:
 - GDP (quarterly)
 - GDP deflator (quarterly)
 - Unemployment (quarterly)
2. We generated the variable for $\pi_t = 400 \ln(P_t/P_{t-1})$
3. An AR(2) benchmark was established instead of the AR(AIC) proposed in the paper. The process of construction is as follows:
 - (a) Find $\Delta\pi_t$ for $h = 1$, that means $\Delta\pi_t = \pi_t - \pi_{t-1}$,
 - (b) with this vector we calculated 4 lags of the variable,
 - (c) we took the variable $\Delta\pi_t$ as dependent variable and as independents we took a constant and the first to lags of the process.
 - (d) after the variable specification we ran OLS ($\Delta\pi_t = \alpha + \phi_1\Delta\pi_{t-1} + \phi_2\Delta\pi_{t-2} + \epsilon_t$) as follow:
 - take the information between 1960-I and 1970-I and estimate the coefficients, obtain $\hat{\Delta\pi}_t$ to obtain $\epsilon_t^2 = (\Delta\pi_t - \hat{\Delta\pi}_t)^2$
 - take the information between 1960-I and 1970-II and estimate the coefficients, obtain $\hat{\Delta\pi}_{t+1}$ to obtain $\epsilon_{t+1}^2 = (\Delta\pi_{t+1} - \hat{\Delta\pi}_{t+1})^2$

The process is performed from the period 1970-I until period 1983-IV. This exercise gave us a vector of ϵ^2 s, one for each OLS performed.

- (e) We average the vector of ϵ^2 s, such that we have the MSE for the AR(2) with $h = 1$

This exercise is performed also for $h = 2$ and $h = 4$, keeping in mind that h gives us the distance in periods.

4. The AO was computed for $h = 1, 2, 4$ as follows:
 - (a) For $h = 1$, $AO_t^1 = \pi_t$,
 - (b) for $h = 2$, $AO_t^2 = \frac{1}{2}(\pi_t + \pi_{t-1})$
 - (c) and for $h = 4$, $AO_t^3 = \frac{1}{4}(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})$

5. With this information we computed the square of errors for AO as:

- (a) for $h = 1, \epsilon_t = (\pi_t - AO_{t-1}^1)^2$
- (b) for $h = 2, \epsilon_t = (\pi_t - AO_{t-1}^2)^2$
- (c) for $h = 4, \epsilon_t = (\pi_t - AO_{t-1}^3)^2$

With this vector we obtained the MSE for the AO process. To compare the benchmark $AR(2)$ and the AO we created the scalar $AOh_i = \frac{MSE_{AO_t^i}}{MSE_{AR(2)}^1}$ for all the values of $i \in h$. The result of these are:

- $AOh_1^{70-83} = 0.4365$
- $AOh_2^{70-83} = 0.4554$
- $AOh_4^{70-83} = 0.3582$
- $AOh_1^{84-04} = 0.3640$
- $AOh_2^{84-04} = 0.2931$
- $AOh_4^{84-04} = 0.2106$

6. Next step is the Phillips curve

$$\Delta\pi_t = \alpha + \phi_1\Delta\pi_{t-1} + \phi_2\Delta\pi_{t-2} + \beta u_t + \delta_1\Delta u_{t-1} + \delta_2\Delta u_{t-2} + \epsilon_t.$$

In this exercise we need the variable (and the lags) for unemployment, u . These variables are calculated as $\Delta u_t = u_t - u_{t-1}$, and use the later to calculate the phillips curve equation using as dependent variable $\Delta\pi_t$ that we create in the initial step of the entire exercise.

The OLS exercise is performed for each value of h and in the exec same way we calculate the $AR(2)$, i.e. take period 1960-I to 1970-I, calculate coefficients and error term for that period, enlarge the period until 1970-II and obtain errors, and repeat the process until 1983-IV and find MSE of tis process.

The results for this exercise are:

- $(PC - u_{h=1})^{70-83} = 0.8080$
- $(PC - u_{h=2})^{70-83} = 0.7888$
- $(PC - u_{h=4})^{70-83} = 0.9420$
- $(PC - u_{h=1})^{84-04} = 1.2991$
- $(PC - u_{h=2})^{84-04} = 1.3225$
- $(PC - u_{h=4})^{84-04} = 1.1283$

7. To calculate $PC_{\Delta y}$

$$\Delta\pi_t = \alpha + \phi_1\Delta\pi_{t-1} + \phi_2\Delta\pi_{t-2} + \delta_1\Delta y_{t-1} + \delta_2\Delta y_{t-2} + \epsilon_t,$$

we did the same process to the variable GDP as for unemployment to find $\Delta y_t = y_t - y_{t-1}$, and perform the estimation in the same way as the Phillips curve. The obtained results are:

- $(PC - \Delta y_t^{h=1})^{70-83} = 0.9225$
- $(PC - \Delta y_t^{h=2})^{70-83} = 0.9094$
- $(PC - \Delta y_t^{h=4})^{70-83} = 0.9729$
- $(PC - \Delta y_t^{h=1})^{84-04} = 1.0021$
- $(PC - \Delta y_t^{h=2})^{84-04} = 0.9945$
- $(PC - \Delta y_t^{h=4})^{84-04} = 0.9760$

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