

# Collective Sensing

ATH and DS

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## Introduction

Recent experimental and theoretical work in the field of collective behavior has highlighted the informational and computational benefits of group living. In addition to diluting risk and increasing the availability of mates, living in social collectives allows organisms to make decisions or solve problems that are impossible at the individual level.<sup>1</sup> Examples include the ability of eusocial insects to make accurate assessments of different nest sites (though they are never compared by a given individual) and the ability of fish shoals to make consensus decision amidst conflict. The majority of these studies, however, have been concerned with the intelligence of the group. In this note I would like to flip our focus—investigating the informational benefits available to the individual.

Specifically we will take the case where an individual is attempting to measure an environmental signal  $\psi \in \mathcal{R}$ , but has limited accuracy due to noisy sensing  $y_i^{t=0} = \mathcal{N}(\psi, \sigma)$ . Galton's *wisdom of the crowds* suggests that one could beat down this noise by a factor of  $\frac{1}{\sqrt{N}}$  simply by talking with  $N$  other individuals also trying to measure  $\psi$ . This, however, requires that draw independent measurements from  $\mathcal{N}(\psi, \sigma)$  and that they can communicate their measurement perfectly. Here we focus on relaxing that second assumption. What if inter-individual communication is noisy? Or has a limited bandwidth? What happens to an individual's ability to beat down its original sensory noise?

In this note, we will introduce a simple and intuitive model and present some very basic results for a couple of (again basic) scenarios. Lastly we will present some interesting future questions and research directions.

## Toy Model

To better frame the questions above we have prescribed a toy model with simple and intuitive dynamics. In our model we have  $N$  agents which each have an internal estimate  $y_i$  of the true signal  $\psi$ . Before interacting with others, each agent arrives at an independent estimate  $y_i^0$  of the signal which is unbiased but noisy:

$$y_i^{t=0} = \mathcal{N}(\psi, \sigma) \tag{1}$$

One can think of these  $y^0$ s as a single measurement or some sort of expectation that each individual arrives at from accrued sensory information. At this point we can think of  $\sigma$  as a source of quenched noise. Agents then begin speaking with one another in order to arrive at a better estimate. In our simplest scenario we can think of a simple linear update (much like a linear filter):

$$y_i^{t+1} = (1 - k)y_i^t + k \left( \frac{1}{N} \sum_i y_i^t + \eta \right) \tag{2}$$

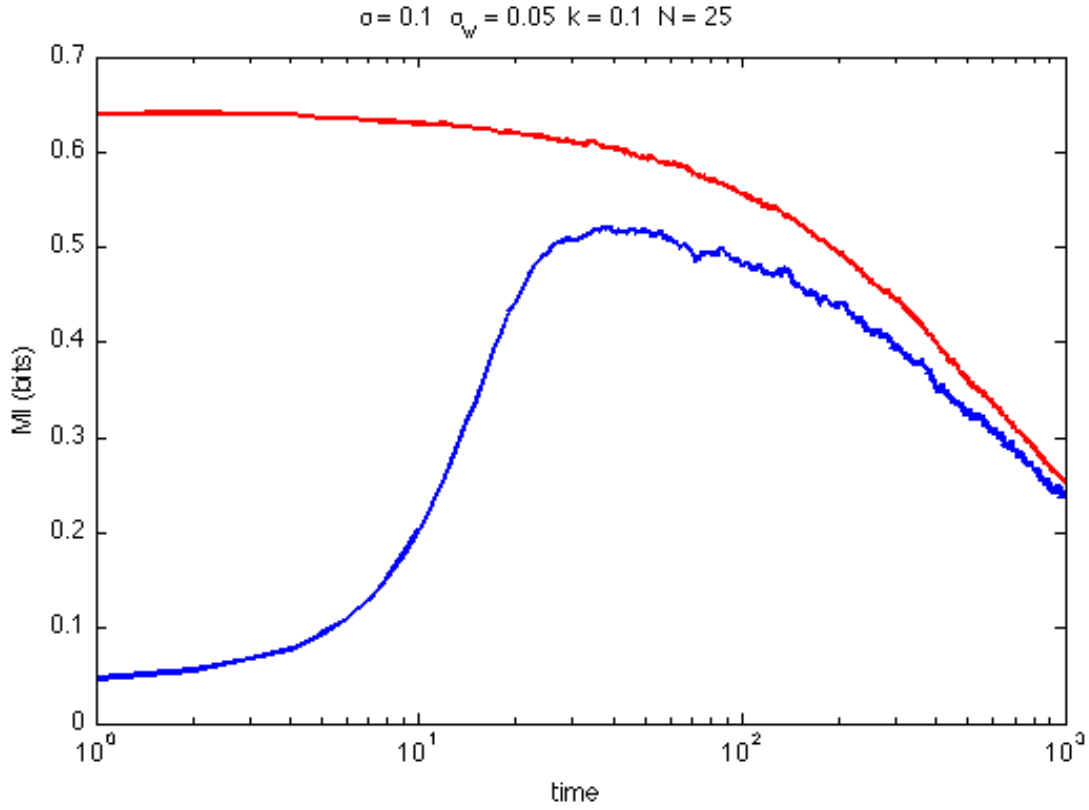
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<sup>1</sup>I would argue that this has been the most important evolutionary driver for social grouping.

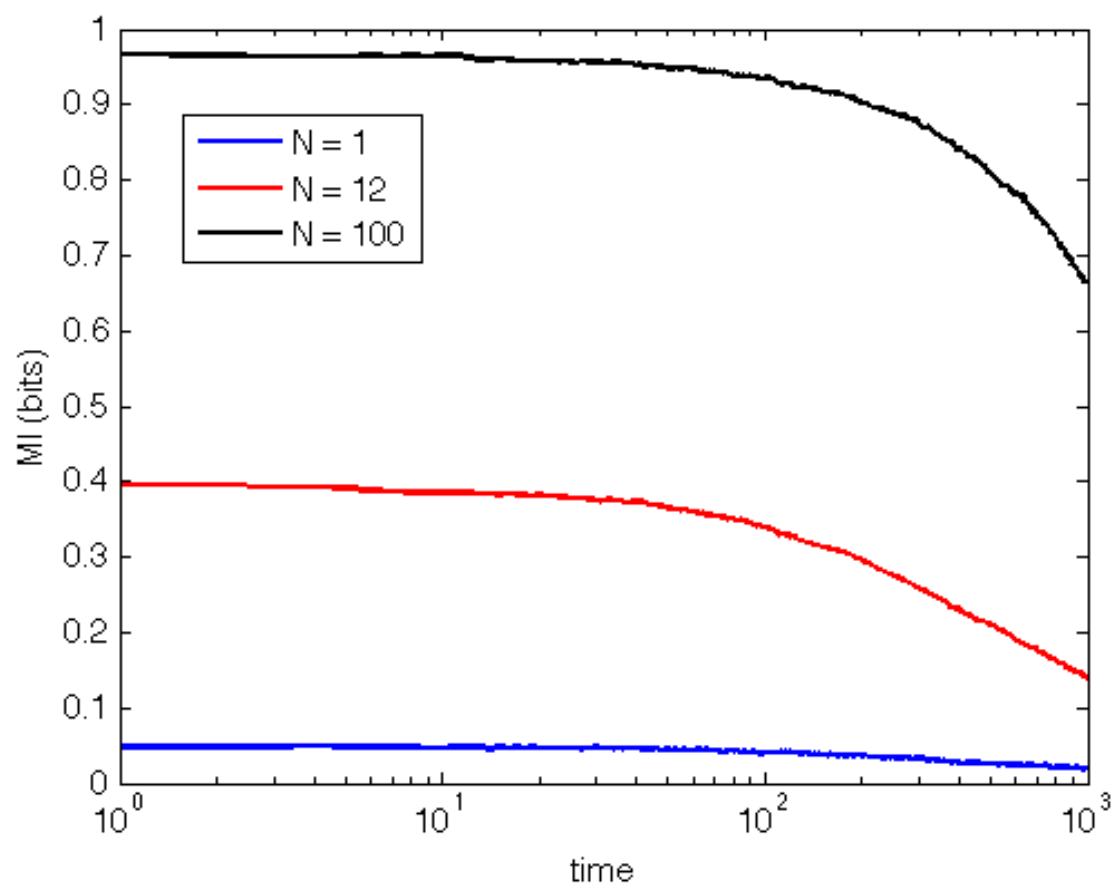
where  $\eta = \mathcal{N}(0, \sigma_w)$  is our dynamic, communication noise and  $(1 - k)$  is a behavioral inertia. Clearly, this represents an oversimplified reality as here we gather information and then communicate, however we think it is both illustrative and informative. The interesting regime is where the quenched noise is greater than the communication noise which is greater than the quenched noise beat down by  $1/\sqrt{N}$ . Several features of eq. 2 should immediately be clear. First, there is nothing anchoring agents to the original signal so for  $\sigma_w > 0$  we will eventually drift off and entirely lose  $\psi$ . This is entirely analogous to the children's game telephone. We could easily add another term to eq. 2, but for now we will use the noise to set our timescale. Second, the dynamics will take every agent to the same average value, so everyone ends up with the same answer. In other words, after sufficient time, the informational content of a given agent is the same as that of the entire population.

## Basic Results

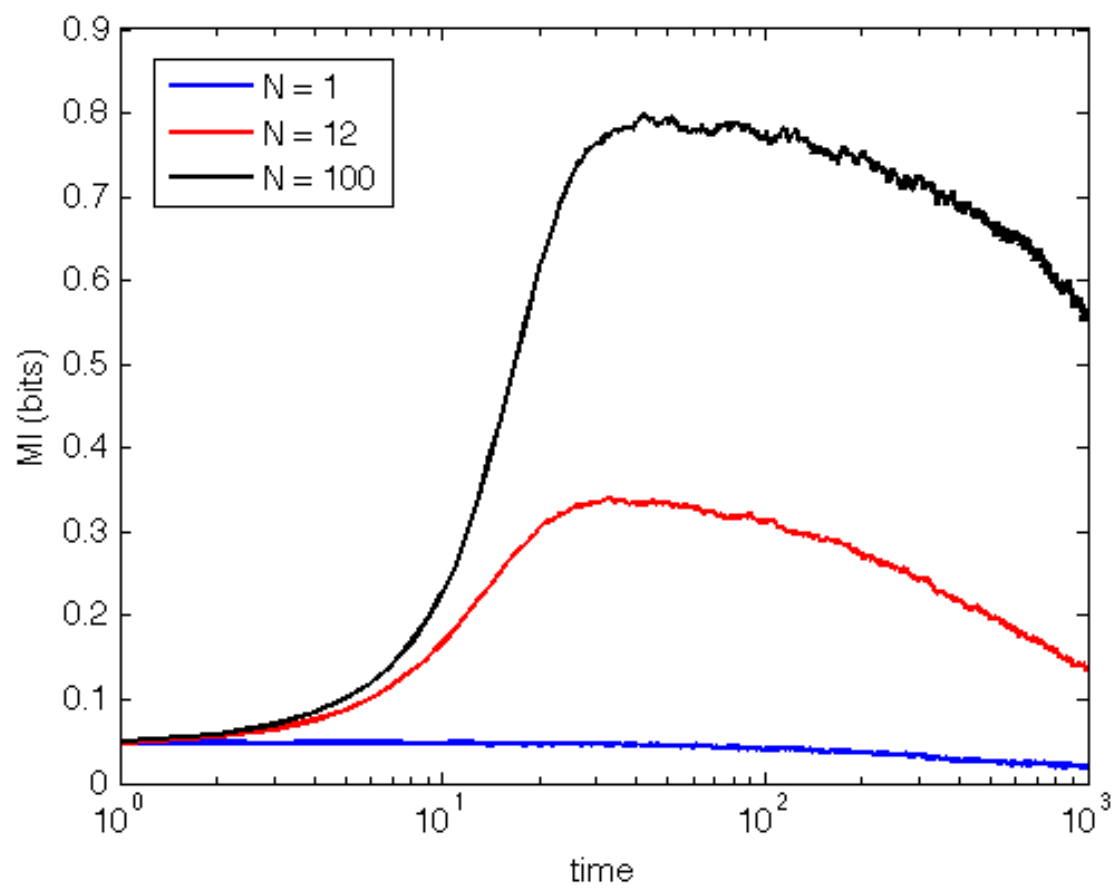
Given such a simple model, we can first ask some very basic questions. How much information does the population mean (the wisdom of the crowd) contain about the true signal? What about an individual's estimate? To investigate this we begin with a simple prior in which  $\psi = \pm\epsilon$  with equal probability. In other words, this is a one bit problem. We set  $\sigma$  such that our initial SNR  $\approx 1/8$  and  $\sigma_w \approx \sigma/4$ .



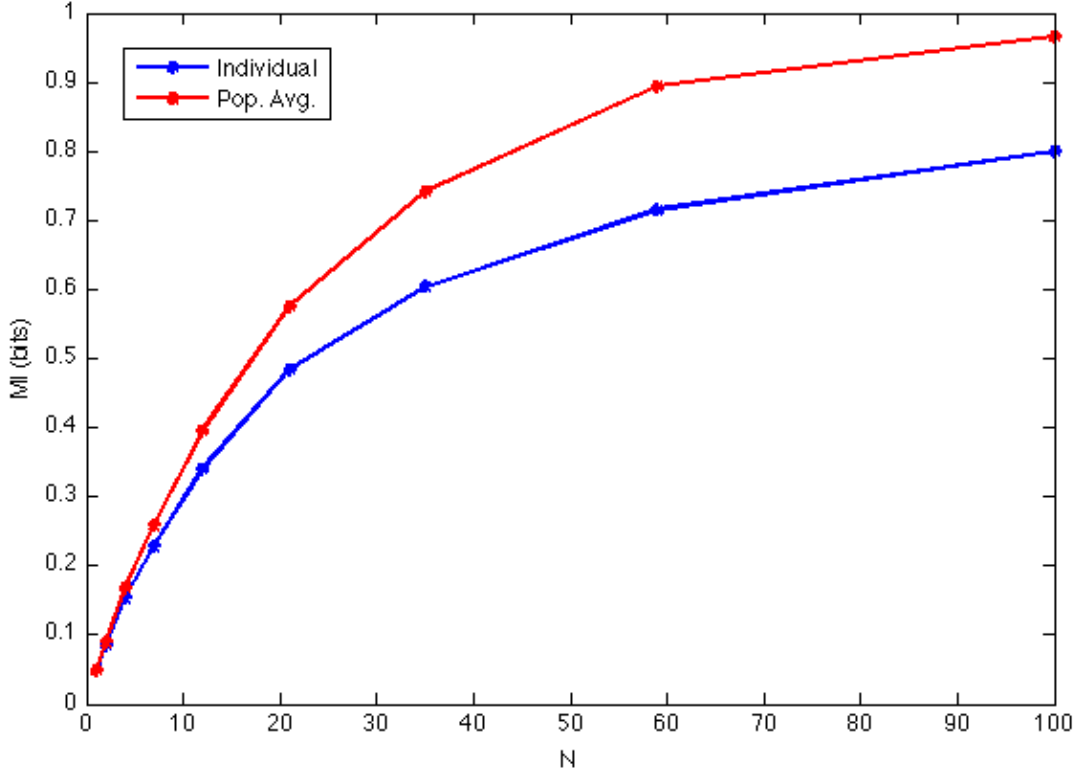
**Figure 1:** The information of the estimate of a single individual is shown in blue while the information of the mean estimate of 25 agents is shown in red. A single agent, given his measurement alone has much less than 0.1 bits. The population mean, however contains more than 0.6 bits. Over time the population information decays away due to communication noise. In contrast, individual estimates tend to get better, until they are forced down by the decaying population information. This maximum sets an ideal timescale for communication.



**Figure 2:** The information in the population mean is shown for groups of 1, 12, and 100.



**Figure 3:** The information in the a random agent's estimate is shown for groups of 1, 12, and 100.



**Figure 4:** The maximum information given by the population mean and a given individual is shown for a variety of group sizes. Both mutual informations increase with  $N$ , however the gap between individual and group becomes larger at large  $N$ s.

## Next Steps

In the framework we have presented above, a single agent arrives at a more accurate estimate of an environmental signal by being a member of a social group and consequently gains an individual benefit. As such, it seems very plausible that these computationally efficient groups could be sustained by individual selfishness. However, in the case of the all-to-all network we have described above (with asynchronous updating) it is easy to see how selfishness may cause a problem. The optimal strategy for an individual is to have  $k = 1$  or no inertia. In this way the agent can immediately adopt the group mean. However, the  $k = 1$  strategy leads to suboptimal group performance as individuals immediately throw away their independent samples. What happens in more restricted networks? Are there topologies where we get ESS?

Additionally, one can imagine complicating the model. What happens if  $t = 0$  signals are not independent and near-neighbors perceive correlated signals? What happens if communication noise is parametrized by the number of input sources or some other context dependent quantity?