MC System (system model)

→ Transmitter assumed to be a point transmit particles to a spherical receiver located at a distance d.

→ Particles follow Brownian motion (diffuse randomly & independently)

→ particles reaches receiver at diff time slots → causes ISI.

$$f_{hit}^{3D}(t) = \frac{r(d-r)}{d\sqrt{4\pi D t^3}} e^{\frac{-(d-r)^2}{4Dt}}$$

↑ hitting rate of each particle

d = distance b/w Tx & Rx

r – $\frac{\text{diameter}}{2}$ of spherical Rx

D – diffusion constant

→ $s_i = 1$ → Tx sends N_{Tx} particles

$s_i = 0$ → Tx " no " .

– Hitting probability of an absorbing receiver → (to absorb one particle after t seconds)

$$P_{hit}(t) = \int_0^t f_{hit}(t)\,dt = \frac{r}{d}\,\text{erfc}\left(\frac{d-r}{\sqrt{4Dt}}\right)$$

so

$$P_{i-1} = \frac{r}{d}\,\text{erfc}\left(\frac{d-r}{\sqrt{4DiT}}\right) - \frac{r}{d}\,\text{erfc}\left(\frac{d-r}{\sqrt{4D(i-1)T}}\right)$$

↑ Prob. one particle hits Rx at (i-1)th time slot.

$\boxed{c_j = N_{TX}\, P_j}$ → avg no of particles (received) at jth time slot if N_{TX} particles are released.

$r_i \sim \text{Poisson}(I_i + s_i c_0)$; $I_i = \underbrace{\lambda_0 T + \sum_{j=1}^{L} c_j s_{i-j}}_{\text{Backgrnd noise + ISI}}$

↑ no of received particles

Probability of receiving r_i particles –

$$P(r_i | I_i + s_i c_0) = \frac{e^{-(I_i + s_i c_0)} (I_i + s_i c_0)^{r_i}}{r_i!}$$

$$SNR = 10\log_{10}\left(\frac{C_0}{2\lambda_0 T}\right)$$

$$\frac{SNR}{10} = \log_{10}\left(\frac{C_0}{2\lambda_0 T}\right) \rightarrow \quad 10^{\frac{SNR}{10}}(2\lambda_0 T) = C_0 \longrightarrow$$

$$N_{TX} P_0 = \left(10^{SNR/10}\right)(2\lambda_0 T)$$

$$\boxed{N_{TX} = \frac{2\lambda_0 T 10^{SNR/10}}{P_0}} \rightarrow \text{no of released particles from Tx}$$

Optimal zero Bit Memory Receiver

$\bar{s_i}$ = estimate of symbol s_i at time slot i.

$$\tilde{s_i} = \begin{cases} 0 & r_i \le \tau \\ 1 & r_i > \tau \end{cases}$$

τ can be found by

$$P(r_i = \tau \mid s_i = 0) = P(r_i = \tau \mid s_i = 1)$$

Prob. of receiving r_i particles condn upon $s_i \rightarrow$

$$P(r_i \mid s_i) = \frac{e^{-\lambda|s_i} (\lambda|s_i)^{r_i}}{r_i!}$$

$$\lambda|s_i = \lambda_0 T + C_0 s_i + \frac{\sum_{j=1}^{L} C_j'}{2}$$

finding threshold: $\rightarrow$ $P(r_i = \tau \mid s_i = 0) = P(r_i = \tau \mid s_i = 1)$

$$\frac{e^{-\lambda|s_i=0} (\lambda|s_i=0)^{r_i}}{\cancel{r_i!}} = \frac{e^{-\lambda|s_i=1} (\lambda|s_i=1)^{r_i}}{\cancel{r_i!}}$$

$$e^{-(C_0 \cdot 0) + \sum C_j/2 + \lambda_0 T)} \cdot \left(\frac{\sum C_j}{2} + \lambda_0 T\right)^{r_i} = e^{-(C_0 + \frac{\sum C_j}{2} + \lambda_0 T)} \left(C_0 + \frac{\sum C_j}{2} + \lambda_0 T\right)^{r_i}$$

$$\frac{e^{-(\sum C_j/2 + \lambda_0 T)}}{e^{-(\sum C_j/2 + \lambda_0 T + C_0)}} = \left(\frac{\left(C_0 + \frac{\sum C_j}{2} + \lambda_0 T\right)}{\left(\frac{\sum C_j}{2} + \lambda_0 T\right)}\right)^{r_i} \quad r_i = \tau$$

$$e^{C_0} = (\;\;''\;\;)^{\tau} \rightarrow \text{taking ln Both sides}$$

$$C_0 = T \ln\left(\frac{C_0}{\sum g_j/2 + \lambda_0 T} + 1\right)$$

$$\boxed{T = \frac{C_0}{\ln\left(1 + \left(\frac{C_0}{\sum_{j=1}^{L} \frac{g_j}{2} + \lambda_0 T}\right)\right)}} \rightarrow \text{sub optimal threshold.}$$

<u>Optimal threshold calculation</u> —

optimal threshold that minimizes BER of the zero-bit memory receiver.

$$(T^*, P_e^*) = \arg\min_T \underbrace{P_e(T)}_{} \rightarrow \text{Ber as funcn of } T.$$

$$P_e(T) = \frac{1}{2^L} \sum_{s_{i-1}} P_e(s_{i-1}, T)$$

2^L → Total possibilities (each bit has 2 poss., total bits $= L$).

$\sum_{s_{i-1}}$ → means summing over all permutations of L bit sequence

$$P_e(s_{i-1}, T) = \frac{1}{2}\Big[Q(\lambda_0 T + \sum_{j=1}^{L} s_{i-j} g_j, \lceil T \rceil) + 1 - Q(\lambda_0 T + \sum_{j=1}^{L} s_{i-j} g_j + C_0, \lceil T \rceil)\Big\} \quad \text{— (1)}$$

Proof $P_e(s_{i-1}, T) = \frac{1}{2}\left[P(r_i \geq T \mid s_i = 0, s_{i-1}) + P(r_i < T \mid s_i = 1, s_{i-1})\right]$

now, $P(r_i \geq T \mid s_i = 0, s_{i-1}) = P(r_i \geq T \mid \lambda_0 T + \sum_{j=1}^{L} s_{i-j} g_j)$

$$= \sum_{k=\lceil T \rceil}^{\infty} \frac{e^{-(\lambda_0 T + \sum s_{i-j} g_j)} (\lambda_0 T + \sum s_{i-1} g_j)^k}{k!} \quad \text{— (A)}$$

$$= Q(\lambda_0 T + \sum s_{i-j} g_j, \lceil T \rceil) \rightarrow \text{where } Q(\lambda, n) = \sum_{k=n}^{\infty} \frac{e^{-\lambda} \lambda^k}{k!}$$

similarly,

$$P(r_i < T \mid s_i = 1, s_{i-1}) = \sum_{k=0}^{\lceil T \rceil - 1} (\;\text{"}\;) = 1 - \sum_{k=\lceil T \rceil}^{\infty} (\;\text{"}\;)$$

$$= 1 - Q\left(\lambda_0 T + \sum_{j=1}^{L} s_{i-j}\, c_j + c_0, \lceil \tau \rceil\right) \quad \text{— (B)}$$

(A) + (B) = gives — (1) Hence proved.

Optimal One Bit Memory Receiver —

more prior information than the zero bit memory receiver.

$$\tilde{s}_i = \begin{cases} 0, & r_i \le \tau|_{s_{i-1}} \\ 1, & r_i > \tau|_{s_{i-1}} \end{cases}$$

$\tau|_{s_{i-1}}$ = denotes threshold for ith symbol when prev. transmitted symbol s_{i-1}

Hence τ changes from time slot to time slot.

OPT. Thres $\leftarrow \tau^{*}|_{s_{i-1}} = \underset{\tau}{\operatorname{argmin}}\ P_e(\tau, s_{i-1})$

where $$P_e(\tau, s_{i-1}) = \frac{1}{2^{L-1}} \underbrace{\sum_{s_{i-2}, s_{i-3}, \dots, s_{i-L}}}_{\text{summing over rest } L-1 \text{ bits as } s_{i-1} = \text{known.}} P_e(s_{i-1}, \tau)$$

$L-1$ bits, each has 2 possibilities

$$BER = P_e = \frac{m+n}{2} \rightarrow$$

$$m = \frac{1}{2^L} \sum_{s_{i-1}} \sum_{\tilde{s}_{i-1}} Q\left(\lambda|_{s_{i-1}, s_i = 0}, \lceil \tau|_{\tilde{s}_i} \rceil\right) \psi(s_{i-1}, \tilde{s}_{i-1}, m, n)$$

$$n = \frac{1}{2^L} \sum \sum \left(1 - Q(\lambda|_{s_{i-1}, s_i = 1}, \lceil \tau|_{\tilde{s}_i} \rceil\right) \psi(s_{i-1}, \tilde{s}_{i-1}, m, n))$$

$\tau|_{\tilde{s}_{i-1}}$ = optimal threshold to prev. detected bit $\tilde{s}_{i-1}$.

$\lambda|_{s_{i-1}, s_i = 0}$ = avg no of particles condn on current symbol being 0 and the prev. L symbols s_{i-1}.

$$\lambda|_{s_{i-1}, s_i = 0} = \sum_{j=1}^{L} s_{i-j}\, c_j + \lambda_0 T$$

where $\psi(s_{i-1}, \tilde{s}_{i-1}, m, n) = \begin{cases} m, & s_{i-1}=0,\ \tilde{s}_{i-1}=\phi \\ 1-m, & s_{i-1}=0,\ \tilde{s}_{i-1}=0 \\ n, & s_{i-1}=1 \quad \tilde{s}_{i-1}=0 \\ 1-n, & s_{i-1}=1 \quad \tilde{s}_{i-1}=1 \end{cases}$

Proof $\underset{P_e}{BER} = \frac{1}{2}\left[P(\hat{s}_i=1|s_i=0) + P(\hat{s}_i=0|s_i=1)\right]$

$$P(\hat{s}_i=1|s_i=0) = \sum_{s_{i-1},\tilde{s}_{i-1}} P(\hat{s}_i=1|s_i=0, \underbrace{s_{i-1}, \tilde{s}_{i-1}}) \times P(\underbrace{\tilde{s}_{i-1}, s_{i-1}}, \dots, s_{i-L})$$

same

$$= \sum_{s_{i-1},\tilde{s}_{i-1}} \underbrace{P(r_i \geq \tau|_{\tilde{s}_{i-1}}} \,|\, s_i=0, s_{i-1}, \tilde{s}_{i-1}) \quad (11)$$

Because when $r_i \geq \tau|_{\tilde{s}_{i-1}}$, then $\hat{s}_i = 1$ detection Rule

$$= \sum_{s_{i-1},\tilde{s}_{i-1}} P(11)\, \underbrace{P(\tilde{s}_{i-1}, s_{i-1}, \dots, s_{i-L})}$$

$$= \sum_{s_{i-1},\tilde{s}_{i-1}} P(11)\, P(\tilde{s}_{i-1}|s_{i-1})\, \underbrace{P(s_{i-1})\, P(s_{i-2}, \dots, s_{i-L})}$$

$\rightarrow$ as all are independent

$$= \sum P(11)\, P(\tilde{s}_{i-1}|s_{i-1})\, P(s_{i-1})\, P(s_{i-2}) \cdots P(s_{i-L})$$

$$= \sum P(11)\, P(\tilde{s}_{i-1}|s_{i-1})\, \frac{1}{2} \times \frac{1}{2} \times \cdots \times \frac{1}{2}$$

$$= \frac{1}{2^L} \sum_{s_{i-1},\tilde{s}_{i-1}} \underbrace{P(r_i \geq \tau|_{\tilde{s}_{i-1}} \,|\, s_i=0, s_{i-1}, \tilde{s}_{i-1})}\, P(\tilde{s}_{i-1}|s_{i-1})$$

$$= \frac{1}{2^L} \sum_{s_{i-1},\tilde{s}_{i-1}} Q\left(\lambda|_{s_{i-1}, s_i=0}, [\tau|_{\tilde{s}_{i-1}}]\right)\, \underbrace{P(\tilde{s}_{i-1}|s_{i-1})}$$

$m = P(\tilde{s}=1|s_i=0)$

turned into ψ.

as when

	s_{i-1}	$\tilde{s}_{i-1}$
m	0	ϕ
$1-m$	0	0
n	1	0
$1-n$	1	1

$$P(\tilde{s_i}=0|s_i=1) = \sum_{s_{i-1},\tilde{s_{i-1}}} P(\tilde{s_i}=0 \mid s_i=1, s_{i-1}, \tilde{s_{i-1}})\, P(\tilde{s_{i-1}}, s_{i-1}, s_{i-2}, \ldots s_{i-L})$$

$$= \sum_{s_{i-1},\tilde{s_{i-1}}} P(\underbrace{r < \tau|_{\tilde{s_{i-1}}}} \mid s_i=1, s_{i-1}, \tilde{s_{i-1}})\, P(\tilde{s_{i-1}}, s_{i-1}, s_{i-2}, \ldots, s_{i-L})$$

as when $r_i < \tau|_{\tilde{s_{i-1}}}$, then $\tilde{s_i}=0$; detection Rule

$$= \sum_{s_{i-1},\tilde{s_{i-1}}} P(11)\, P(\tilde{s_{i-1}} \mid s_{i-1})\, \underbrace{P(s_{i-1})\, P(s_{i-2}) \ldots P(s_{i-L})}_{\frac{1}{2^L}}$$

$$= \frac{1}{2^L} \sum_{s_{i-1},\tilde{s_{i-1}}} P(r < \tau|_{\tilde{s_{i-1}}} \mid s_i=1, s_{i-1}, \tilde{s_{i-1}})\, P(\tilde{s_{i-1}} \mid s_{i-1})$$

$$= \frac{1}{2^L} \sum_{s_{i-1},\tilde{s_{i-1}}} \left(1 - Q\left(\lambda|_{s_{i-1}, s_i=1}, [\tau|_{\tilde{s_{i-1}}}]\right)\right) \underbrace{P(\tilde{s_{i-1}} \mid s_{i-1})}_{\text{turned into } \psi.}$$

$\uparrow$

$P(\tilde{s_i}=0 \mid s_i=1) = n$

	s_{i-1}	$\tilde{s_{i-1}}$
m	0	1
$1-m$	0	0
n	1	0
$1-n$	1	1

Hence proved

$$\boxed{P_e = \frac{m+n}{2}}$$

Scanned by CamScanner

Figure 7 → derivation of eqn :→

probability of error in a poisson process =

$$P_e = \frac{1}{2} - \frac{1}{2}\sum_{K=0}^{x_0} \frac{R_0^K e^{-R_0} - R_1^K e^{-R_1}}{K!} \quad \text{—(A)}$$

In our case, & their case,

$$R_0 = \lambda|_{s_i=0} = C_0 s_i + \frac{\sum_{j=1}^{L} C_j}{2} + \lambda_0 T = \frac{\sum C_j}{2} + \lambda_0 T$$

$$R_1 = \lambda|_{s_i=1} = C_0 s_i + \frac{\sum C_j}{2} + \lambda_0 T = C_0 + \frac{\sum C_j}{2} + \lambda_0 T$$

$$x_0 = \tau \begin{cases} \dfrac{C_0}{\ln\left(1 + \dfrac{C_0}{\frac{\sum C_j}{2} + \lambda_0 T}\right)}, & \text{sub-optimal} \\ \tau^* \text{ in } (\tau^*, P_e^*) = \min_{\tau} P_e(\tau), & \text{Optimal} \end{cases}$$

$$SNR = 10 \log_{10}\left(\frac{C_0}{2\lambda_0 T}\right) \rightarrow \text{in dB}$$

$$\boxed{C_0 = 2\lambda_0 T \, 10^{SNR/10}} \rightarrow \text{SNR changes, } C_0 \text{ changes}$$

deriving C_0 from each SNR & putting it into the eqn — (A)

$$P_e = \frac{1}{2} - \frac{1}{2}\sum_{K=0}^{\tau} \frac{\left(\frac{\sum C_j}{2} + \lambda_0 T\right)^K e^{-\left(\frac{\sum C_j}{2} + \lambda_0 T\right)} - \left(C_0 + \lambda_0 T + \frac{\sum C_j}{2}\right)^K e^{-\left(C_0 + \frac{\sum C_j}{2} + \lambda_0 T\right)}}{K!}$$

substituting value of C_0 for every SNR & calculating each corresponding BER and plotting the values