HW 2 1) a.

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$$r = \sqrt{x^2+y^2+z^2}$$

$$U = \frac{\mu}{r}\left[1 - J_2\left(\frac{R}{r}\right)^2\left(\frac{3}{2}\left(\frac{z}{r}\right)^2 - \frac{1}{2}\right)\right]$$

$$\frac{dU}{dx} = \frac{dU}{dr}\cdot\frac{dr}{dx}$$

$$U = \frac{\mu}{r} - J_2\frac{\mu}{r}\left(\frac{R}{r}\right)^2\left(\frac{3}{2}\left(\frac{z}{r}\right)^2 - \frac{1}{2}\right)$$

$$= \frac{\mu}{r} - J_2\frac{R^2\mu}{r^3}\left(\frac{3}{2}\left(\frac{z}{r}\right)^2 - \frac{1}{2}\right)$$

$$= \mu r^{-1} - \frac{3J_2R^2\mu z^2}{2}\frac{1}{r^5} + \frac{J_2R^2\mu}{2}\frac{1}{r^3}$$

$$\frac{d}{dr}\frac{1}{r} = -\frac{1}{r^2} \qquad \frac{d}{dr}\frac{1}{r^3} = -\frac{3}{r^4}$$

$$\frac{d}{dr}\frac{1}{r^5} = -\frac{5}{r^6}$$

$$\frac{dU}{dr} = -\frac{\mu}{r^2} + \frac{15J_2R^2\mu z^2}{2r^6} - \frac{3J_2R^2\mu}{2r^4}$$

$$\frac{dr}{dx} = \frac{x}{r}$$

$$\frac{dU}{dx} = \frac{x}{r}\left(-\frac{\mu}{r^2} + \frac{15J_2R^2\mu z^2}{2r^6} - \frac{3J_2R^2\mu}{2r^4}\right)$$

$$\boxed{\frac{dU}{dx} = \frac{-\mu x}{r^3} + \frac{15J_2R^2\mu z^2 x}{2r^7} - \frac{3J_2R^2\mu x}{2r^5}}$$