Aerospace Engineering and Engineering Mechanics Department

ASE 389P.4 Methods of Orbit Determination

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Outline

The Four Fields

Gravitational Effects

Non-Gravitational Effects

Celestial Mechanics

Non-Gravitational Astrodynamics

The Four Fields

There are Four (External) Fields that affect Space Object motion

- (1) Gravitational
  - Central-Body
  - Non-Sphericity
  - 3rd Body
  - Relativity
- Non-Gravitational
  - (2) Radiative (e.g. Solar and Earth Radiation)
  - (3) Particulates (e.g Micrometeoroid, Thermospheric Density)
  - (4) Electro-Magnetic (e.g. Lorentz Forces)

Modeling Gravitational Effects

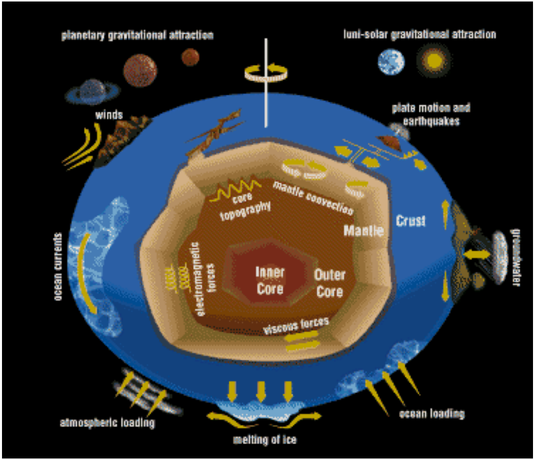


Figure: Modeling the Earth

Gravitational Potential for Central Body

Let

$$V = \frac{\mu}{|\underline{r}|}$$

We say that for V, if $\nabla V = \underline{a}$ then $\underline{a}$ is a conservative vector field

$$\nabla V = \frac{\partial V}{\partial x}\underline{\hat{i}} + \frac{\partial V}{\partial y}\underline{\hat{j}} + \frac{\partial V}{\partial z}\underline{\hat{k}}$$

Let

$$\underline{r} = x\underline{\hat{i}} + y\underline{\hat{j}} + z\underline{\hat{k}}$$

Then

$$|\underline{r}| = \sqrt{\underline{r} \bullet \underline{r}} = \sqrt{x^2 + y^2 + z^2}$$

And

$$\nabla V = \underline{a} = -\frac{\mu}{|\underline{r}|^3}\,\underline{r}$$

Is $\underline{a}$ a conservative vector field? If Curl $\underline{a} = \underline{0}$ it is!

Gravitational Potential for Central Body

Curl $\underline{a} = \nabla \times \underline{a}$

$$\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]$$

and

$$\underline{a} = -\frac{\mu}{|\underline{r}|^3}\,[x, y, z]$$

So, Curl $\underline{a}$ is

$$\begin{bmatrix} \nabla_2\, a_3 - \nabla_3\, a_2 \\ \nabla_3\, a_1 - \nabla_1\, a_3 \\ \nabla_1\, a_2 - \nabla_2\, a_1 \end{bmatrix}^T$$

Looking at the first component

$$\nabla_2\, a_3 - \nabla_3\, a_2 = 3\frac{\mu\, z\, y}{|\underline{r}|^5} - 3\frac{\mu\, y\, z}{|\underline{r}|^5}$$

...and the other components follow in similar fashion, and indeed Curl $\underline{a} = \underline{0}$

Gravitational Potential and Anomalies

If we bring in more reality, then $V = \frac{\mu}{|\underline{r}|} + V'$

$$V' = \frac{\mu}{r} \sum_{l=0}^{\infty} \left(\frac{R_E}{r}\right)^l \sum_{m=0}^{l} \overline{P}_{lm}(sin\phi) \left[\overline{C}_{lm} \cos m\lambda + \overline{S}_{lm} \sin m\lambda\right]$$

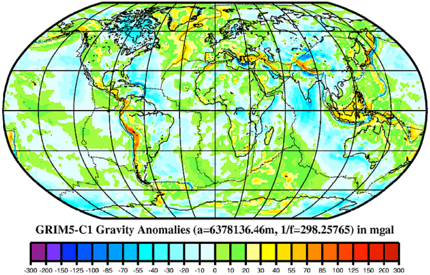
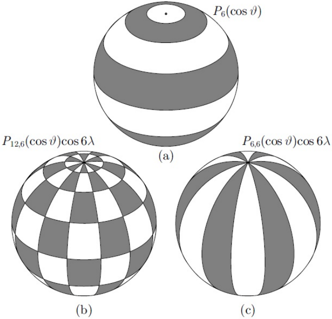


Figure: GRIM5-1C Combined Gravity Solution

Modeling Gravitational Effects

$$V' = \frac{\mu}{r} \sum_{l=0}^{\infty} \left(\frac{R_E}{r} \right)^l \sum_{m=0}^{l} \overline{P}_{lm}(sin\phi) \left[\overline{C}_{lm} \cos m\lambda + \overline{S}_{lm} \sin m\lambda \right]$$



The kinds of spherical harmonics: (a) zonal, (b) tesseral, (c) sectorial

Modeling Gravitational Effects

$$V' = \frac{\mu}{r} \sum_{l=0}^{\infty} \left(\frac{R_E}{r} \right)^l \sum_{m=0}^{l} \overline{P}_{lm}(sin\phi) \left[\overline{C}_{lm} \cos m\lambda + \overline{S}_{lm} \sin m\lambda \right]$$

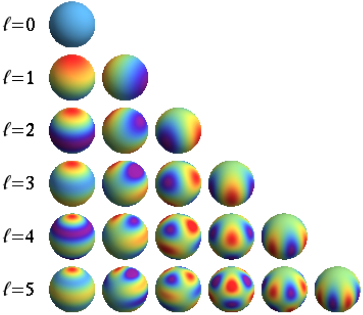


Figure: Types of Spherical Harmonics

Modeling Gravitational Effects

$$V' = \frac{\mu}{r}\sum_{l=0}^{\infty}\left(\frac{R_E}{r}\right)^l \sum_{m=0}^{l} \overline{P}_{lm}(sin\phi)\left[\overline{C}_{lm}\cos m\lambda + \overline{S}_{lm}\sin m\lambda\right]$$

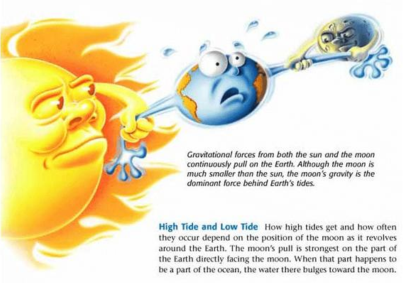


Figure: Tides

Tidal Effects on Length of Day

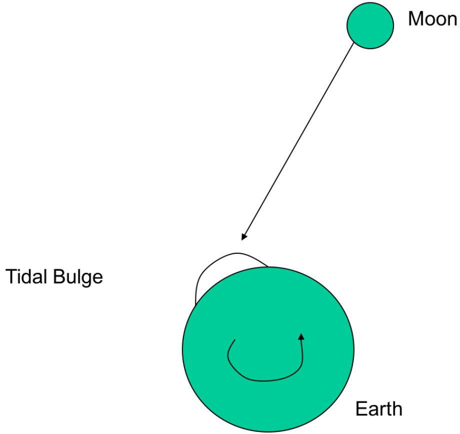


Figure: Tides

- There is no "absolute" frame of reference. Every time you measure an object's velocity, or its momentum, or how it experiences time, it's always in relation to something else
- The speed of light is the same no matter who measures it or how fast the person measuring it is going
- Nothing can go faster than light

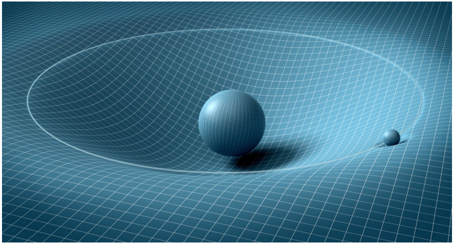


Figure: General Relativity

Radiative Fields

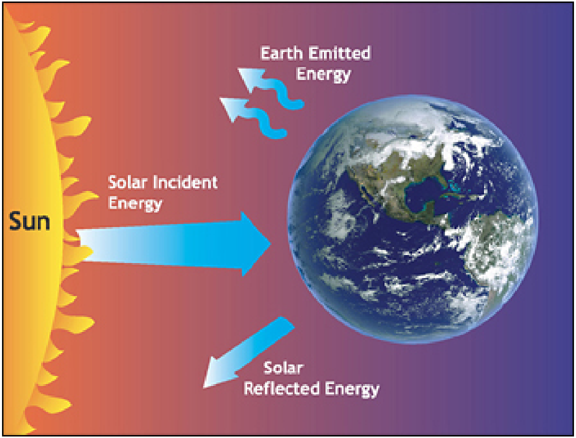


Figure: Radiative Fields

Particulates Field

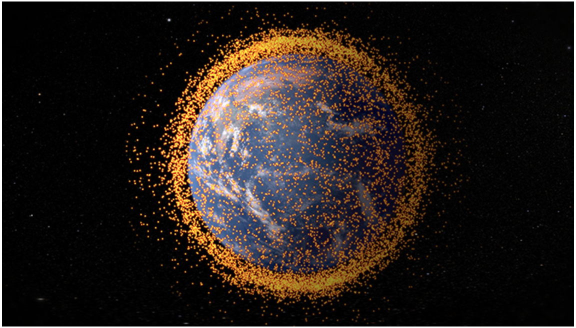


Figure: Micrometeoroid Field

Particulates Field

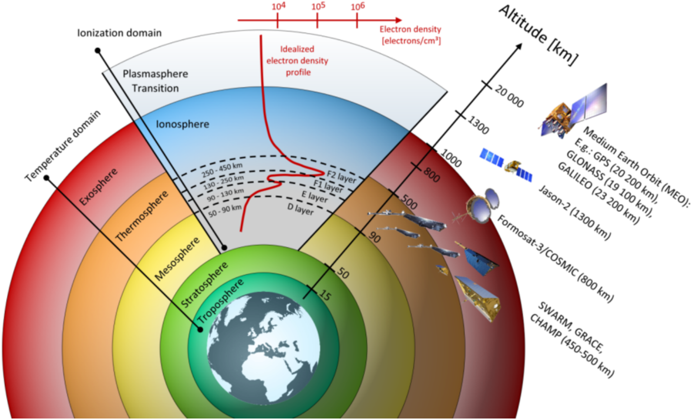


Figure: Atmospheric Field

Electromagnetic Field

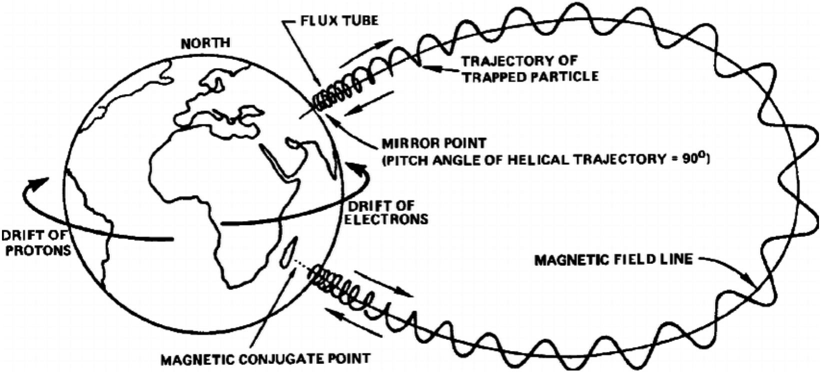


Figure: Electromagnetic Field

Historical Background

- Nicholas Copernicus (1473-1543)
  - De Revolutionibus Orbium Coelestium (1543)
  - Heliocentric (circular)
- Galileo Galilei (1564-1642)
  - Agreed with Copernicus
  - Performed gravity experiments at the tower of Pisa
- Tycho Brahe (1546-1601)
  - Detailed observations used by Kepler

Historical Background

- Johannes Kepler (1571-1630)
  - Harmonices Mundi Libri V (1619)
  - Kepler's 3 observations (kinematic)
    - Planet orbits are ellipses with Sun at one focus
    - Planet-Sun line sweeps equal areas in equal times
    - Square of the period is proportional to cube of its mean distance to the sun (semi-major axis)
- Isaac Newton (1642-1727)
  - Philosophiae Naturalis Principia Matematica (1687)
  - Newton's 3 laws (kinetic)
    - Every body continues in its state of rest, or of uniform motion in a right (straight) line, unless it is compelled to change that state by forces impressed upon it
    - The change of motion is proportional to the motive force impressed and is made in the direction of the right line in which that force is impressed
    - To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal and directed to contrary parts

Historical Background

Figure: Field of Miracles, Pisa

Historical Background

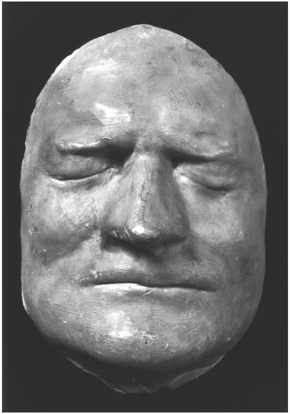


Figure: Death Mask of Sir Isaac Newton

Historical Background

Figure: Church of Our Lady Before Tyn, Prague

Historical Background

Figure: Johannes Kepler's House, Prague

Kepler's 1st and 3rd Observations

$$P^2 = \frac{4\pi^2}{\mu} a^3$$

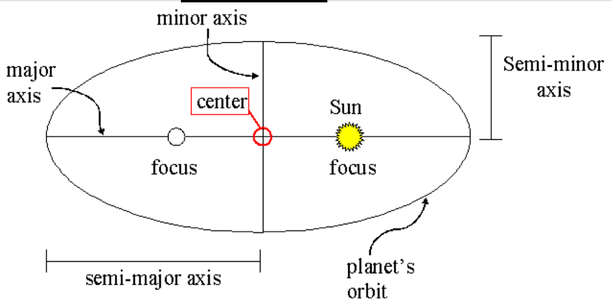


Figure: Kepler's 1st and 3rd Observatons

Kepler's 2^{nd} Observation

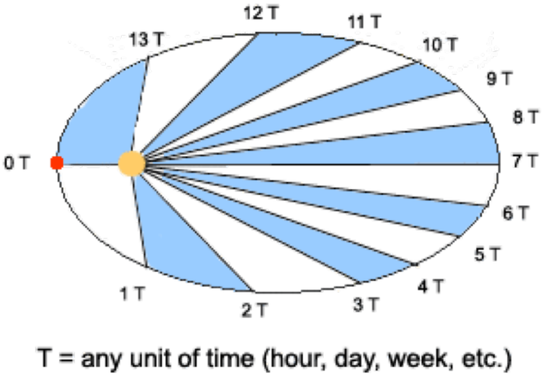


Figure: Kepler's 2^{nd} Observaton

Galileo's Gravity Experiments in Pisa

Two objects of different masses released at the same time, hit the Earth's surface at the same time, regardless of height, neglecting friction. The acceleration on both objects is constant!

$$\underline{F}_g = -m\, a_{const}\, \frac{\underline{r}}{\|\underline{r}\|}$$

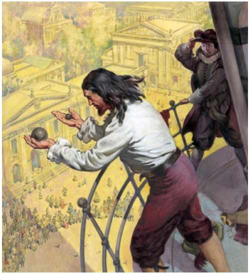


Figure: Gravity Experiments in Pisa

From Kepler we have

$$\mathbb{P}^2 \propto a^3$$

And Newton knew, from Galileo, that at the Earth's surface

$$g \approxeq 9.8\frac{m}{s^2}$$

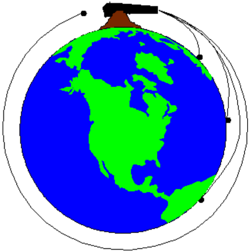


Figure: What's An Orbit?

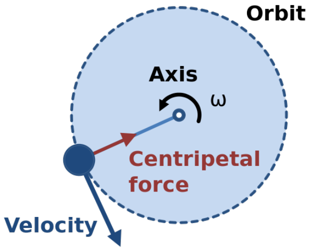


Figure: Centripetal Force

Inverse Squared Law of Gravity

Newton posited that the same force acting on the apple was acting on the moon, but he'd have to reconcile slower orbital rate of moon. How? He noted that the moon was about 60 times further than the apple from the Earth and in order for the relationships to hold:

$$F_g \propto \frac{1}{60^2}$$

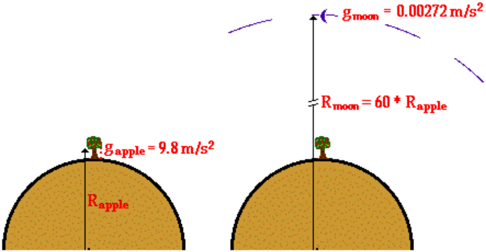


Figure: Inverse Squared Law

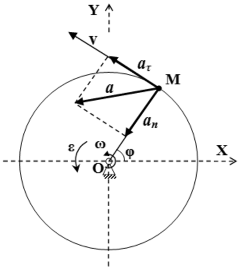
Orbit Plane Kinematics

Assume we have an Inertial Reference Frame A, then

$$^{A}\underline{a} = a_x\underline{X} + a_y\underline{Y} + a_z\underline{Z}$$

And a Rotating Reference Frame B attached to point M, so

$$^{B}\underline{a} = a_n\underline{n} + a_t\underline{t} + a_l\underline{l}$$

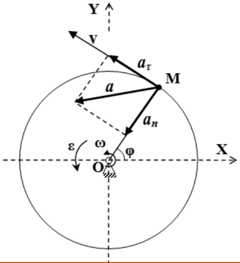


Let

$$\underline{n} = -\frac{\underline{a}_n}{\|\underline{a}_n\|} \quad \underline{t} = \frac{\underline{a}_t}{\|\underline{a}_t\|} \text{ and } \underline{Z} \parallel \underline{l} \text{ are unit vectors}$$

Frame B rotates *w.r.t* frame A at a constant speed $\dot{\psi} = \omega$ about point O such that

$$^{A}\underline{\omega}^{B} = \dot{\psi}\,\underline{l}$$



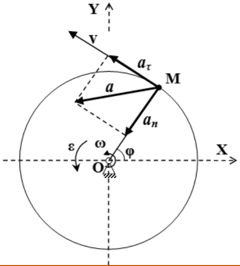
Orbit Plane Kinematics

Let the vector connecting point O to M be $\underline{r}^{OM}$ such that $r = \|\underline{r}\|$ and

$$\underline{r}^{OM} = r\,\underline{n} = r\cos\psi\,\underline{X} + r\sin\psi\,\underline{Y}$$

The Basic Kinematic Equation or Transport Theorem states that

$$\frac{{}^{A}d(\underline{a})}{dt} = \frac{{}^{B}d(\underline{a})}{dt} + {}^{A}\underline{\omega}^{B} \times (\underline{a})$$



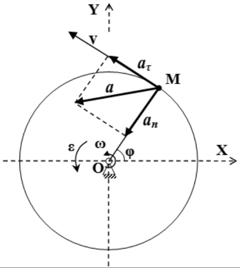
Orbit Plane Kinematics

Applying the Basic Kinematic Equation to our problem

$$\frac{{}^{A}d(\underline{r}^{OM})}{dt} = \frac{{}^{B}d(\underline{r}^{OM})}{dt} + {}^{A}\underline{\omega}^{B} \times (\underline{r}^{OM})$$

$$\frac{{}^{A}d(\underline{r}^{OM})}{dt} = [\dot{r}\,\underline{n} + r\,\dot{\underline{n}}] + \left(\dot{\psi}\,\underline{l}\right) \times\ r\underline{n}$$

$$\boxed{{}^{A}\underline{v}^{OM} = \dot{r}\,\underline{n} + r\,\dot{\psi}\underline{t}}$$



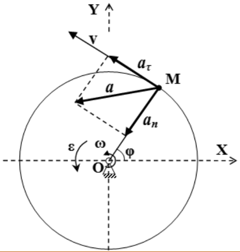
Orbit Plane Kinematics

Applying the Basic Kinematic Equation to ${}^A\underline{v}^{OM}$

$$\frac{{}^A d({}^A\underline{v}^{OM})}{dt} = \frac{{}^B d({}^A\underline{v}^{OM})}{dt} + {}^A\underline{\omega}^B \times ({}^A\underline{v}^{OM})$$

$$\frac{{}^A d({}^A\underline{v}^{OM})}{dt} = \ddot{r}\underline{n} + \dot{r}\dot{\underline{n}} + \left(\dot{r}\dot{\psi} + r\ddot{\psi}\right)\underline{t} + r\dot{\psi}\dot{\underline{t}} + \dot{\psi}\underline{l} \times \left[\dot{r}\underline{n} + r\dot{\psi}\underline{l}\right]$$

$$\boxed{{}^A\underline{a}^{OM} = \left(\ddot{r} - r\dot{\psi}^2\right)\underline{n} + \left(r\ddot{\psi} + 2\dot{r}\dot{\psi}\right)\underline{t}}$$



Newton's Universal Law of Gravitation

For 2-body dynamics we assume that the body at point M is only under the influence of the gravitational acceleration due to the body at point O

$$\boxed{{}^{A}\underline{a}^{OM} = \left(\ddot{r} - r\dot{\psi}^2\right)\underline{n} + \left(r\ddot{\psi} + 2\dot{r}\dot{\psi}\right)\underline{t}}$$

$$\underline{a}_{cent} = -r\dot{\psi}^2\underline{n} = -r\omega^2\underline{n} = -r\left(\frac{2\pi}{\mathbb{P}}\right)^2\underline{n}$$

$$\frac{\mathbb{P}^2}{r}\left(\underline{F}_{cent}\right) = -4\pi^2 M_2$$

Recall from Kepler that $\dfrac{\mathbb{P}^2}{r^3} = K$ so, to reconcile we say that

$$\frac{\mathbb{P}^2}{r}\left(\frac{GM_1M_2}{r^2}\right) = -4\pi^2 M_2$$

$$\boxed{\underline{F}_{grav} = \frac{GM_1M_2}{r^2}}$$

Keplerian (2-Body) Motion

Let us now derive the equations for Keplerian motion from Newton's Law of Gravitation

$$\underline{F}_{grav} = \frac{GM_1M_2}{r^2}$$

First, assume $M_1 \gg M_2$ and the only force is graviational acting along the radial direction, so that

$$\ddot{\underline{r}} = -\frac{GM_1\underline{r}}{r^3}$$

and is the acceleration of body M_2 due to the gravitational influence of body M_1

Keplerian (2-Body) Motion

Let $\underline{h} = \underline{r} \times \underline{v}$ which we will call the specific angular momentum vector of the orbit.

$$\frac{d(\underline{h})}{dt} = \frac{d(\underline{r} \times \dot{\underline{r}})}{dt} = \dot{\underline{r}} \times \dot{\underline{r}} + \underline{r} \times \ddot{\underline{r}} = 0 = constant$$

since

$$\underline{r} \times \ddot{\underline{r}} = \underline{r} \times -\frac{GM_1 \underline{r}}{r^3} = 0$$

This implies that the orbital plane is fixed under the assumption of Keplerian (2-Body) motion, and the orbital energy is conserved (i.e. a changing orbital energy would yield a changing specific angular momentum vector)!

Keplerian (2-Body) Motion

$$\underline{h} \times \ddot{\underline{r}} = (\underline{r} \times \dot{\underline{r}}) \times \ddot{\underline{r}} = (\underline{r} \times \dot{\underline{r}}) \times -\frac{GM_1 \underline{r}}{r^3} = \frac{GM_1 \underline{r}}{r^3} \times (\underline{r} \times \dot{\underline{r}})$$

...which comes from vector triple product identities and thus

$$\underline{h} \times \ddot{\underline{r}} = \left(\frac{GM_1 \underline{r}}{r^3} \bullet \dot{\underline{r}}\right) \underline{r} + \left(-\frac{GM_1 \underline{r}}{r^3} \bullet \underline{r}\right) \dot{\underline{r}} = -\frac{GM_1}{r^3} \left[(\underline{r} \bullet \underline{r}) \dot{\underline{r}} - (\underline{r} \bullet \dot{\underline{r}}) \underline{r}\right]$$

Now,

$$\frac{d}{dt}\left(\frac{\underline{r}}{\|\underline{r}\|}\right) = \frac{1}{r} \dot{\underline{r}} - \frac{\dot{r}}{r^2} \underline{r} = \frac{1}{r^3} \left[(\underline{r} \bullet \underline{r}) \dot{\underline{r}} - (\underline{r} \bullet \dot{\underline{r}}) \underline{r}\right]$$

...noting that the term

$$-\frac{(\underline{r} \bullet \dot{\underline{r}}) \underline{r}}{r^3} = -\frac{(\hat{\underline{r}} \bullet \dot{\underline{r}}) \underline{r}}{r^2} = -\frac{\dot{r}}{r^2} \underline{r}$$

where $\hat{\underline{r}} \bullet \dot{\underline{r}}$ is the orbiting body's velocity projected onto the body's orbital position vector (i.e. the range-rate $\dot{r}$)

Keplerian (2-Body) Motion

Thus,

$$\boxed{\underline{h} \times \ddot{\underline{r}} = -GM_1 \frac{d}{dt}\left(\frac{\underline{r}}{||\underline{r}||}\right)}$$

If we integrate this equation on both sides, once, $w.r.t.$ time

$$\underline{h} \times \dot{\underline{r}} = -GM_1 \frac{\underline{r}}{||\underline{r}||} - \underline{A}$$

Where $\underline{A}$ is a constant of integration, in our case, called the **Laplace Runge Lenz** vector. Dotting both sides by $\underline{r}$ and applying the scalar triple product identity we have

$$\underline{r} \bullet (\underline{h} \times \dot{\underline{r}}) = -GM_1 \frac{\underline{r}}{||\underline{r}||} \bullet \underline{r} - \underline{A} \bullet \underline{r}$$

which we can re-write as (via Circular Shift properties)

$$-||\underline{h}|| \bullet (\underline{r} \times \dot{\underline{r}}) = -GM_1 \frac{\underline{r}}{||\underline{r}||} \bullet \underline{r} - \underline{A} \bullet \underline{r}$$

Keplerian (2-Body) Motion

Leaving us with,

$$-\|\underline{h}\|^2 = -GM_1\|\underline{r}\| - \|\underline{A}\|\|\underline{r}\|\cos\nu$$

If we let $P = \dfrac{\|\underline{h}\|^2}{GM_1}$ and $\|\underline{e}\| = \dfrac{\|\underline{A}\|}{GM_1}$, and $GM_1 = \mu$

$$P = r + e\,r\cos\nu$$

$$\boxed{r = \frac{P}{1 + e\ \cos\nu}}$$

Which is the Polar form of a Conic Section. Before we go into the conics, let's derive a couple more importnt relationships

Conic Sections

Given:

$$r = \frac{P}{1 + e\ \cos\nu}$$

The closest and farthest points of the orbit are $\underline{r}_{min} = \dfrac{P}{1+e}$ and $\underline{r}_{max} = \dfrac{P}{1-e}$

then the mean distance, called the semi-major axis, is

$$a = \frac{1}{2}\left(\underline{r}_{min} + \underline{r}_{max}\right) = \frac{1}{2}\left(\frac{P}{1+e} + \frac{P}{1-e}\right) = \frac{1}{2}\left(\frac{P(1-e) + P(1+e)}{(1+e)(1-e)}\right)$$

$$a = \frac{P}{1-e^2}$$

Since $P = \dfrac{h^2}{\mu}$

$$\boxed{a = \frac{h^2}{\mu(1-e^2)}}$$

Conic Sections

Recall:

$$\underline{h} \times \underline{\dot{r}} = -GM_1 \frac{\underline{r}}{||\underline{r}||} - \underline{A}$$

If we square the vector magnitudes on both sides

$$||\underline{h} \times \underline{\dot{r}}||^2 = ||-GM_1 \frac{\underline{r}}{||\underline{r}||} - \underline{A}||^2$$

by using the vector dot product, we have

$$(\underline{h} \times \underline{\dot{r}}) \bullet (\underline{h} \times \underline{\dot{r}}) = \left(-GM_1 \frac{\underline{r}}{||\underline{r}||} - \underline{A}\right) \bullet \left(-GM_1 \frac{\underline{r}}{||\underline{r}||} - \underline{A}\right)$$

Then

$$||\underline{h}||^2 ||\underline{v}||^2 = \mu^2 + 2\mu \frac{\underline{r}}{||\underline{r}||} \underline{A} - ||\underline{A}||^2 = \mu^2 \left(1 + \frac{2A}{\mu} \underline{\hat{A}}\, \underline{\hat{r}} + \left(\frac{A}{\mu}\right)^2\right)$$

Noting that $\underline{A} = \underline{\hat{A}}\ ||\underline{A}||$, that $\underline{\hat{A}} \bullet \underline{\hat{r}} = \cos \nu$ and $e = \dfrac{A}{\mu}$

Vis Viva Equation

Then:

$$h^2 v^2 = \mu^2 \left(1 + 2\, e\ \cos\nu + e^2\right) = \mu^2 \left(2\left(1 + e\ \cos\nu\right) - \left(1 - e^2\right)\right)$$

recalling that $\dfrac{1}{a} = \dfrac{\mu(1-e^2)}{h^2}$ and $P = \dfrac{h^2}{\mu}$ and $r = \dfrac{P}{1 + e\ \cos\nu}$ Then

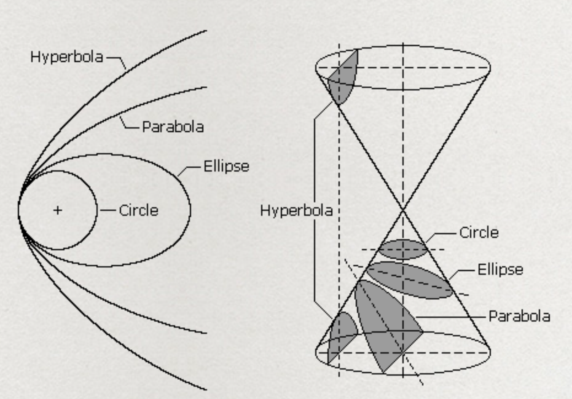
$$v^2 = \mu \left(\frac{2\mu\left(1 + e\ \cos\nu\right)}{h^2} - \frac{\mu\left(1 - e^2\right)}{h^2}\right) = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

Dividing by 2 and re-arranging leaves us with the Vis-Viva (Latin for "Life Force") equation

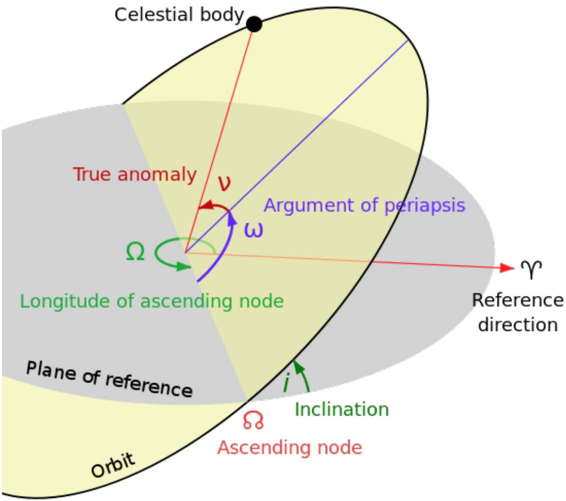
$$\boxed{\varepsilon = \left(\frac{v^2}{2} - \frac{\mu}{r}\right) = -\frac{\mu}{2a}}$$

$$\varepsilon = \begin{cases} < 0 & \text{for} \quad \text{circular and elliptical orbit} \\ = 0 & \text{for} \quad \text{parabolic orbit} \\ > 0 & \text{for} \quad \text{hyperbolic orbit} \end{cases}$$

Conic Sections



Orbits



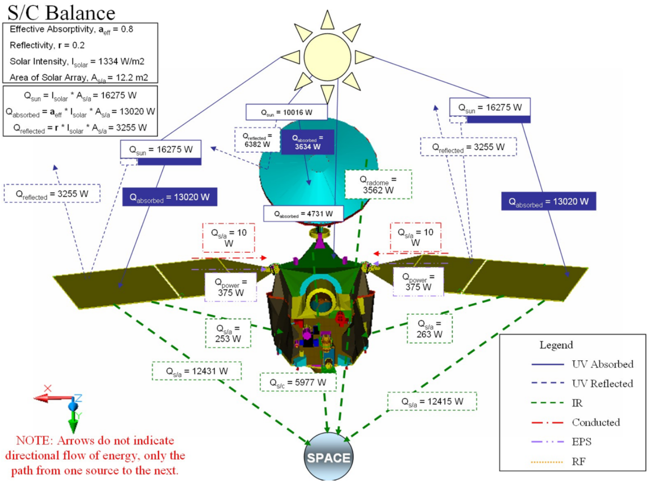
Celestial body
True anomaly
ν
Argument of periapsis
ω
Ω
Longitude of ascending node
♈
Reference
direction
Plane of reference
i
Inclination
☊
Ascending node
Orbit

There are Four (External) Fields that affect Space Object motion

- (1) Gravitational
  - Central-Body
  - Non-Sphericity
  - 3rd Body
  - Relativity
- Non-Gravitational
  - (2) Radiative (e.g. Solar and Earth Radiation)
  - (3) Particulates (e.g Micrometeoroid, Thermospheric Density)
  - (4) Electro-Magnetic (e.g. Lorentz Forces)

There are Three Body Effects (non-gravitational) that affect Space Object motion

- (1) Radiative (e.g. Thermal Emissivity)
- (2) Particulates (e.g Thruster Firings, Outgasing)
- (3) Electro-Magnetic (e.g. Coulomb Forces)



S/C Balance
Effective Absorptivity, a_{eff} = 0.8
Reflectivity, r = 0.2
Solar Intensity, I_{solar} = 1334 W/m2
Area of Solar Array, $A_{s/a}$ = 12.2 m2
$Q_{sun} = I_{solar} * A_{s/a}$ = 16275 W
$Q_{absorbed} = a_{eff} * I_{solar} * A_{s/a}$ = 13020 W
$Q_{reflected} = r * I_{solar} * A_{s/a}$ = 3255 W
Q_{sun} = 16275 W
$Q_{reflected}$ = 3255 W
$Q_{absorbed}$ = 13020 W
Q_{sun} = 10016 W
$Q_{reflected}$ = 6382 W
$Q_{absorbed}$ = 3634 W
Q_{radome} = 3562 W
$Q_{absorbed}$ = 4731 W
Q_{sun} = 16275 W
$Q_{reflected}$ = 3255 W
$Q_{absorbed}$ = 13020 W
$Q_{s/a}$ = 10 W
$Q_{s/a}$ = 10 W
Q_{power} = 375 W
Q_{power} = 375 W
$Q_{s/a}$ = 253 W
$Q_{s/a}$ = 263 W
$Q_{s/a}$ = 12431 W
$Q_{s/c}$ = 5977 W
$Q_{s/a}$ = 12415 W
SPACE
X
Z
Y
NOTE: Arrows do not indicate directional flow of energy, only the path from one source to the next.
Legend
UV Absorbed
UV Reflected
IR
Conducted
EPS
RF

Outgassing

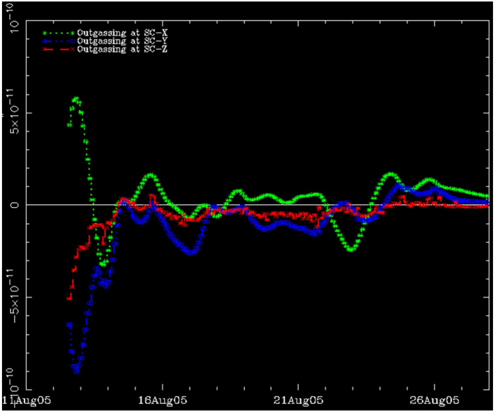
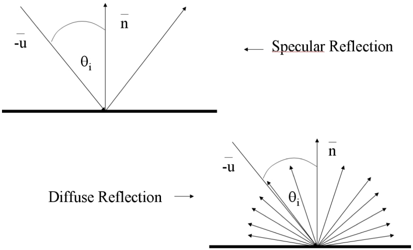


Figure: Acceleration in $\frac{km}{s^2}$ vs Time

Spacecraft Albedo



Consider that the surface element:

- is exposed to direct radiation
- reflects a fraction γ of the incoming photons, out of which another fraction β are specularly reflected (i.e. symmetrically with respect to $\underline{\hat{n}}$) and $(1-\beta)$ are diffusely reflected (reradiates uniformly in all directions)
- absorbs and reradiates a fraction $\kappa(1-\gamma)$ of the incident flux of energy (i.e. the emissivity is κ)

Radiation Pressure Force

The force due to solar radiation pressure is then:

$$\underline{F}_{SRP} = -\frac{C_1}{d^2}\left\{B(\theta_i)\,\underline{\hat{n}} + (1-\mu)\cos^2\theta_i\,\underline{\hat{u}}\right\}\; dA$$

where

$$B(\theta_i) = 2\nu\cos\theta_i + 4\mu\cos^2\theta_i$$

The diffuse term is

$$\nu = \frac{1}{3}\left[(1-\beta)\gamma + \kappa(1-\gamma)\right]$$

The specular term is

$$\mu = \frac{1}{2}\beta\gamma$$

and C_1 is the force per unit area, due to the incident soalr radiation, on an element normal to the surface. d is the heliocentric distance of the surface element in question, in Astronomical Units.

Body-Generated Radiation Acceleration

The acceleration due to body-generated radiation is:

$$a_{rad} = \frac{P}{m\,c}$$

where P is power in Watts, m is mass of the object in kilograms, and c is the speed of light in a vacuum in meters per second.

This could be used to compute the acceleration due to thermal emissions or even radio communications transmissions out of the spacecraft antenna.

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