The one-way range instrumentation measures $\tilde{\rho}$ by determining t_T and t_R. Equation (3.3.7) shows that the measured quantity is a biased range. A useful alternate form of Eq. (3.3.7) is

$$\tilde{\rho} = \rho + c\delta t_R - c\delta t_T + \delta\rho_{\text{atm}} + \epsilon \tag{3.3.8}$$

where δt_R is the receiver clock correction and δt_T is the transmitter clock correction. For example, from Eq. (3.3.4), $\delta t_R = a_R + b_R(T - T_0)$ plus higher order and random components.

From Eq. (3.3.7) the measured quantity, $\tilde{\rho}$, is related to the satellite position at time T_T and the receiver position at time T_R, or

$$\tilde{\rho} = [(X - X_I)^2 + (Y - Y_I)^2 + (Z - Z_I)^2]^{1/2} + \rho_b + \delta\rho_{\text{atm}} + \epsilon \tag{3.3.9}$$

where (X, Y, Z) represents the true position of the satellite at time T_T and the true instrument components $(X, Y, Z)_I$ are determined at T_R.

A computed pseudorange, $\tilde{\rho}_c$, would be formed with an assumed, or nominal, ephemeris for the satellite and coordinates of the instrument and other parameters in Eq. (3.3.9). A *residual* would be obtained from the difference, $\tilde{\rho} - \tilde{\rho}_c$. Such a residual is required in orbit determination.

It is significant to note that the true range, ρ, is formed as the magnitude of the difference between two position vectors, each of which has a different time attached to it. As a consequence, these two vectors must be expressed in the same reference frame. If the reference frame is nonrotating, the resulting path is simply the straight line distance between the two vectors. If an Earth-fixed frame is used, for example, the path appears to be curved and it will be necessary to account for this curvature. Unless otherwise stated, it will be assumed that $\rho(T_T, T_R)$ will be formed using a nonrotating reference frame.

Measurement model should account for light time delay

Light time and aberration

Geometric Range: Effect of Light Time and Stellar Aberration

Geometric position of the object now (time = T)

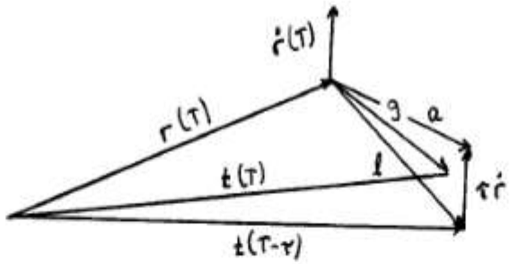
$$\underline{g}(T) = \underline{t}(T) - \underline{r}(T)$$

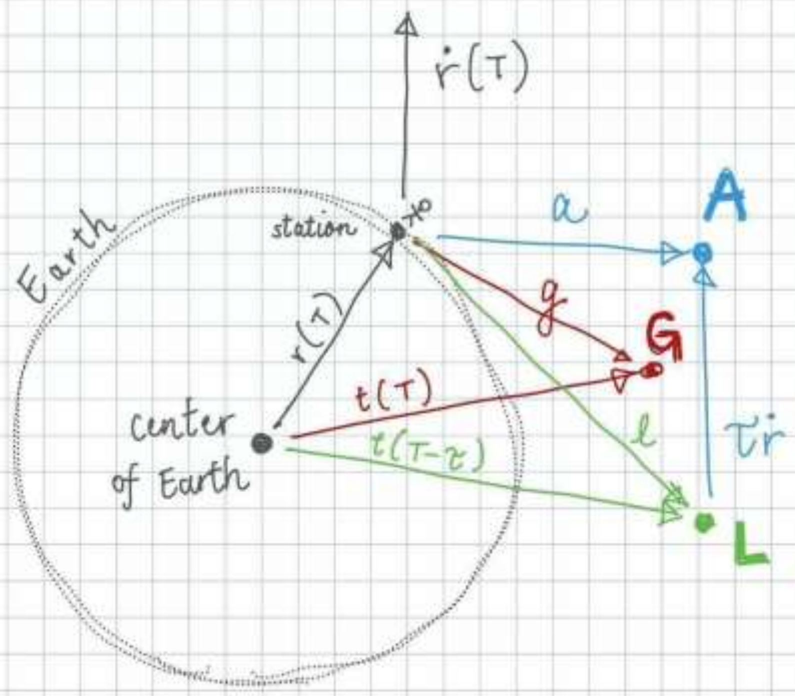
True position of the object now (time = T)

$$\underline{l}(T) = \underline{t}(T - \tau) - \underline{r}(T)$$

Apparent position of the object now (time = T)

$$\underline{a}(T) = \underline{l}(T) + \tau\dot{\underline{r}}(T)$$





A = "apparent" light-time corrected + aberration state (of satellite) $\quad a(T) = l(T) + \tau\dot{r}(T)$

G = "geometric" propagated state $\quad g(T) = t(T) - r(T)$

L = "true" light time-corrected state (of propagated state) $\quad l(T) = t(T-\tau) - r(T)$

4. Include Light Time Correction

The measurements given to you are light time corrected aka apparent range and range rate measurements, as opposed to instantaneous measurements. This means that the time taken by the signal to travel from the satellite to the ground station has been taken into account. If you don't correct for this in the final project, you will be off on the order of 50 meters. You can neglect atmospheric extinction and delays.

1. Calculate initial guess for light time using the range at the current time.

$$lt = \rho/c$$

ρ = computed range

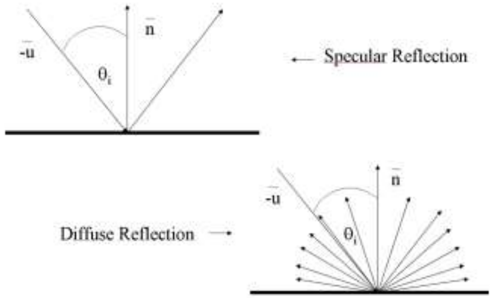
c is the speed of light.

While Δ > *tolerance*

2. $t_{-lt} = t - lt$ (Compute light time corrected time, where t is the current time)
3. $JD_{-lt} = JD - lt/86400$ (Compute light time corrected Julian Date)
4. $X\_lt = f(t, t_{-lt})$ (Propagate the Satellite state backwards from the current time to the light time corrected time).
5. Get Transformation Matrix at JD_{-lt} and convert the satellite state X_lt into Earth Fixed System (or convert the station coordinates to ECI corresponding to JD_{-lt})
6. Calculate new guess for light time by computing the range.
7. Compute the difference in the satellite position (in Earth fixed system) between the current iteration of the while loop and the previous iteration). This is **Δ**
8. If the difference is not within the tolerance, repeat. Use new guess for light time in Step 2. Set the tolerance to a really small value – like 1mm.

Use the last value of X_lt for your range and range rate calculations.

Spacecraft Albedo



Consider that the surface element:

- is exposed to direct radiation
- reflects a fraction γ of the incoming photons, out of which another fraction β are specularly reflected (i.e. symmetrically with respect to $\underline{\hat{n}}$) and $(1-\beta)$ are diffusely reflected (reradiates uniformly in all directions)
- absorbs and reradiates a fraction $\kappa(1-\gamma)$ of the incident flux of energy (i.e. the emissivity is κ)

The total force associated with the solar radiation pressure is (cf. Borderies & Longaretti, 1990):

$$\mathbf{F}_{srp} = -\frac{C_1}{d^2}\left\{B(\theta_i)\,\hat{n} + (1-\mu)\cos^2\theta_i\,\hat{u}\right\}dA \tag{2}$$

where:

$C_1 \equiv$ the force per unit area, due to the incident solar radiation, on an element normal to the surface ($C_1 = \frac{\Phi_\oplus}{c}$).

$\Phi_\oplus \equiv$ the solar constant or energy which falls per unit time, per unit area perpendicular to the solar rays, outside of the atmosphere of the Earth at 1 AU (~ 1.50E13 m). ($\Phi_\oplus$ ~ 1367 W/m^2).

$c \equiv$ speed of light in a vacuum.

$d \equiv$ heliocentric distance (in AUs) of the surface element in question.

$$B(\theta_i) = 2\upsilon\cos\theta_i + 4\mu\cos^2\theta_i \tag{3}$$

$$\upsilon = \frac{1}{3}\left[(1-\beta)\gamma + \kappa(1-\gamma)\right] \text{ (diffuse term)} \tag{4}$$

$$\mu = \frac{1}{2}\beta\gamma \text{ (specular term)} \tag{5}$$

The values supplied by LMA are:

$$C_{diffuse\ LMA} = (1-\beta)\gamma \qquad \kappa = \varepsilon \text{ (i.e. emissivity)}$$

$$C_{specular\ LMA} = \beta\gamma$$