HW 1

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3)

$$\nabla U = \frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial}{\partial x}\hat{k} \qquad U = \frac{\mu}{R}$$

$$R = \sqrt{\underline{R}\cdot\underline{R}} \qquad \underline{R} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$\frac{\partial U}{\partial x} = \frac{\partial \frac{\mu}{\sqrt{x^2+y^2+z^2}}}{\partial x} = \mu \frac{\partial \frac{1}{\sqrt{x^2+y^2+z^2}}}{\partial x} \qquad n =$$

$$= \mu \frac{\partial \frac{1}{n}}{\partial n} \cdot \frac{\partial \sqrt{x^2+y^2+z^2}}{\partial x}$$

$$= \mu \left(-\frac{1}{n^2}\right) \cdot \frac{\partial \sqrt{x^2+y^2+z^2}}{\partial x}$$

$$= \frac{-\mu x}{(x^2+y^2+z^2)^{3/2}}\hat{i}$$

$$\frac{\partial U}{\partial y} = \frac{-\mu y}{(x^2+y^2+z^2)^{3/2}}\hat{j} \qquad \frac{\partial U}{\partial z} = \frac{-\mu}{(x^2+y^2+z^2)^{3/2}}\hat{k}$$

$$\nabla U = \frac{-\mu x}{(x^2+y^2+z^2)^{3/2}}\hat{i} - \frac{\mu y}{(x^2+y^2+z^2)^{3/2}}\hat{j} - \frac{\mu z}{(x^2+y^2+z^2)^{3/2}}\hat{k}$$

$$\boxed{\nabla U = -\mu \frac{\underline{R}}{R^3}}$$