Aerospace Engineering and Engineering Mechanics Department

ASE 389P.4 Methods of Orbit Determination

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Outline

Notation and Definitions

$\underline{r}$ is a generalized vector (i.e. uncoordinatized)

${}^{B}\underline{r}$ is a vector r coordinatized in reference frame B

$\underline{\underline{C}}$ is a generalized matrix (i.e. uncoordinatized)

$\underline{\underline{R}}_{BA}$ is a rotation matrix from frame A to frame B

Vectors: directed line segment (i.e. an abstract quantity)

Reference Frame: a construct composed of 3 orthonormal vectors defined in a dextral sense

Fundamentals of Classical Mechanics

We will make the following Basic Assumptions

- Space and Time: Space is 3-dimensional (i.e. $\mathbb{R}^3$) and Time is 1-dimensional (i.e. $\mathbb{R}^1$)
- Galileo's Principle of Relativity
  - An inertial reference frame exists (i.e. natural laws of motion can be applied)
  - Any non-accelerating frame w.r.t. this inertial frame is also inertial
- Newtons Principle of Determinacy: given <u>all</u> initial position and velocity parameters at a specific time, along with the forces acting on the system, its motion at all future times is known (i.e. causal)

Reference Frames

If we want to known "it" we must measure "it"; if we want to understand "it" we must predict "it." We are interested in making measurements of quantities and events, and this by definition implies making comparisons. In order for these to be consistent and meaningful, we create a construct called Reference Frame in which to base our measurements.

Information is always relative!

Reference Frames

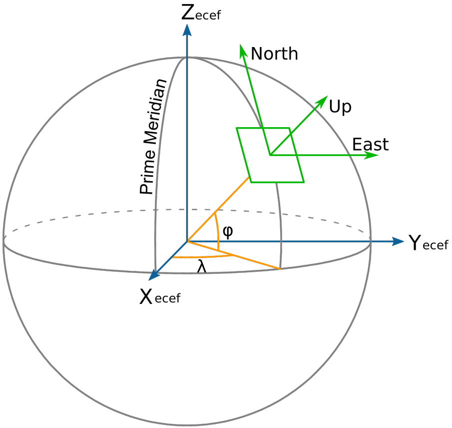


Figure: Earth Centered Earth Fixed and East-North-Up Reference Frames from Wikipedia

Vector Representation

Assume we have a Reference Frame F, then

$$^{F}\underline{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

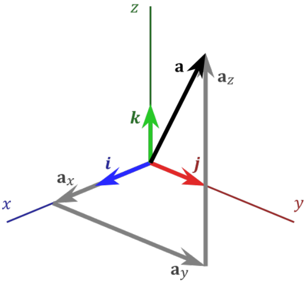


Figure: Vector Representation from Wikipedia

Vector Spaces

A space consisting of vectors $\{\underline{v}_1, \ldots, \underline{v}_n\}$ which satisfy 2 basic rules: a set that is closed under finite

- Vector Addition
- Scalar Multiplication

We shall specialize this space for $\mathbb{R}^n$ to include multiplication by vectors (i.e. dot product and cross product)

Addition

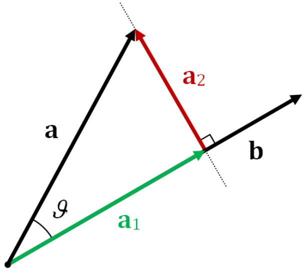
- $\underline{a} + \underline{b} = \underline{b} + \underline{a}$ commutative
- $(\underline{a} + \underline{b}) + \underline{c} = \underline{a} + (\underline{b} + \underline{c})$ associative
- $\exists$ a zero vector such that
  - $\underline{a} + \underline{0} = \underline{a}$
  - $\underline{a} + (-\underline{a}) = \underline{0}$

Multiplication

- $a(\underline{a}+\underline{b}) = a\underline{a} + a\underline{b}$ distributive on vector addition
- $(a+b)\underline{a} = a\underline{a} + b\underline{a}$ distributive on scalar addition

Specialized for $\mathbb{R}^n$:

- Dot Product: $\underline{a} \bullet \underline{b} = \|\underline{a}\|\|\underline{b}\| \cos\theta = \|\underline{a}_1\|\|\underline{b}\|$
  - Scalar orthogonal projection of $\underline{a}$ onto $\underline{b}$



Multiplication Continued

- Cross Product: $\underline{a} \times \underline{b} = \|\underline{a}\|\|\underline{b}\| \sin\theta \ \hat{\underline{n}} \perp$

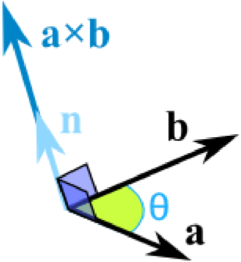


Figure: Vector Cross Product from Wikipedia

We have two reference frames: A and B

A has orthonormal unit vectors $\underline{\hat{a}}_1, \underline{\hat{a}}_2, \underline{\hat{a}}_3$

B has orthonormal unit vectors $\underline{\hat{b}}_1, \underline{\hat{b}}_2, \underline{\hat{b}}_3$

We wish to transform a vector ${}^B\underline{v} = v_{1b}\underline{\hat{b}}_1 + v_{2b}\underline{\hat{b}}_2 + v_{3b}\underline{\hat{b}}_3$ represented in frame B to a vector ${}^A\underline{v} = v_{1a}\underline{\hat{a}}_1 + v_{2a}\underline{\hat{a}}_2 + v_{3a}\underline{\hat{a}}_3$ represented in frame A via a Direction Cosine Matrix (DCM) or simply a Rotation Matrix. There are several ways to do this, and we will only cover two:

- Projection
- Euler Rotation Sequence

$$ {}^{A}\underline{v} = \underline{\underline{C}}_{AB} \, {}^{B}\underline{v} $$

$\underline{\underline{C}}_{AB}$ is the Direction Cosine Matrix (DCM) from frame B to frame A. From a Projection approach we can write:

$$ \hat{\underline{b}}_1 = (\hat{\underline{b}}_1.\hat{\underline{a}}_1)\,\hat{\underline{a}}_1 + (\hat{\underline{b}}_1.\hat{\underline{a}}_2)\,\hat{\underline{a}}_2 + (\hat{\underline{b}}_1.\hat{\underline{a}}_3)\,\hat{\underline{a}}_3 $$

$$ \hat{\underline{b}}_2 = (\hat{\underline{b}}_2.\hat{\underline{a}}_1)\,\hat{\underline{a}}_1 + (\hat{\underline{b}}_2.\hat{\underline{a}}_2)\,\hat{\underline{a}}_2 + (\hat{\underline{b}}_2.\hat{\underline{a}}_3)\,\hat{\underline{a}}_3 $$

$$ \hat{\underline{b}}_3 = (\hat{\underline{b}}_3.\hat{\underline{a}}_1)\,\hat{\underline{a}}_1 + (\hat{\underline{b}}_3.\hat{\underline{a}}_2)\,\hat{\underline{a}}_2 + (\hat{\underline{b}}_3.\hat{\underline{a}}_3)\,\hat{\underline{a}}_3 $$

Another way to write this is:

$$\underline{\hat{b}}_1 = C_{11}\,\underline{\hat{a}}_1 + C_{12}\,\underline{\hat{a}}_2 + C_{13}\,\underline{\hat{a}}_3$$

$$\underline{\hat{b}}_2 = C_{21}\,\underline{\hat{a}}_1 + C_{22}\,\underline{\hat{a}}_2 + C_{23}\,\underline{\hat{a}}_3$$

$$\underline{\hat{b}}_3 = C_{31}\,\underline{\hat{a}}_1 + C_{32}\,\underline{\hat{a}}_2 + C_{33}\,\underline{\hat{a}}_3$$

thus

$$\underline{\underline{C}}_{AB} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Reference Frame Transformations

Some properties of:

$$\underline{\underline{C}}_{AB} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$0 \leq C_{ij} \leq 1$

$\underline{\underline{C}}_{AB}{}^{-1} = \underline{\underline{C}}_{AB}{}^{T}$ (Orthonormal)

$det(\underline{\underline{C}}_{AB}) \; = \; det(\underline{\underline{I}}) \; = \; 1$

where $\underline{\underline{I}}$ is the Identity Matrix

Euler Rotations

We have two reference frames: *A* and *B* with orthonormal unit vectors $\underline{\hat{x}}, \underline{\hat{y}}, \underline{\hat{z}}$ and $\underline{\hat{i}}, \underline{\hat{j}}, \underline{\hat{k}}$ respectively where these frames only differ by one rotation about $\underline{\hat{z}}$ by some angle γ so that $\underline{\hat{z}}$ and $\underline{\hat{k}}$ are colinear.

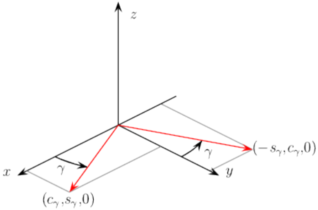


Figure: Rotation About $\underline{\hat{z}}$ from Wikipedia

We can write this relationship as:

$$\hat{\underline{i}} = \cos\gamma\,\hat{\underline{x}} + \sin\gamma\,\hat{\underline{y}} + 0\,\hat{\underline{z}}$$

$$\hat{\underline{j}} = -\sin\gamma\,\hat{\underline{x}} + \cos\gamma\,\hat{\underline{y}} + 0\,\hat{\underline{z}}$$

$$\hat{\underline{k}} = 0\,\hat{\underline{x}} + 0\,\hat{\underline{y}} + 1\,\hat{\underline{z}}$$

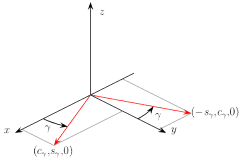


Figure: Rotation About $\hat{\underline{z}}$ from Wikipedia

Reference Frame Transformations

In Matrix form:

$$\begin{bmatrix} \hat{\underline{i}} \\ \hat{\underline{j}} \\ \hat{\underline{k}} \end{bmatrix} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\underline{x}} \\ \hat{\underline{y}} \\ \hat{\underline{z}} \end{bmatrix}$$

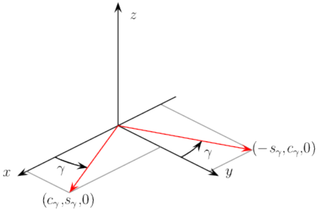


Figure: Rotation About $\hat{\underline{z}}$ from Wikipedia

And thus:

$$\underline{\underline{C}}_3(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

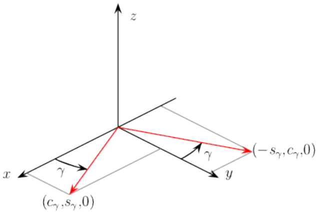


Figure: Rotation About $\underline{\hat{z}}$ from Wikipedia

The other Rotation Matrices are:

$$\underline{\underline{C}}_1(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix} \quad \underline{\underline{C}}_2(\gamma) = \begin{bmatrix} \cos\gamma & 0 & -\sin\gamma \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\gamma \end{bmatrix}$$

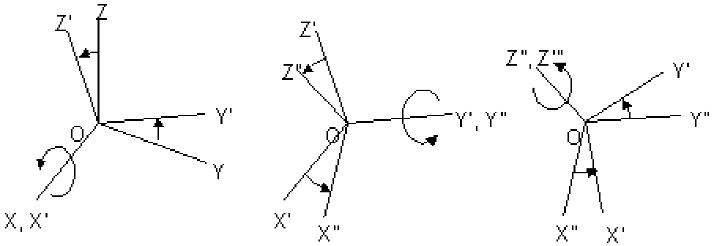


Figure: Euler Rotations from Wikipedia

Reference Frame Transformations

The Classical Euler Rotation Sequence is 3-1-3:

$$\underline{\underline{C}}_{313}(\gamma,\theta,\phi) =$$
$$\begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{C}}_{313}(\gamma,\theta,\phi) =$$
$$\begin{bmatrix} \cos\phi\cos\gamma - \sin\phi\cos\theta\sin\gamma & \cos\phi\sin\gamma + \sin\phi\cos\theta\cos\gamma & \sin\phi\sin\theta \\ sin\phi\cos\gamma - \cos\phi\cos\theta\sin\gamma & -\sin\phi\sin\gamma + \cos\phi\cos\theta\cos\gamma & \cos\phi\sin\theta \\ \sin\theta\sin\gamma & -\sin\theta\cos\gamma & \cos\theta \end{bmatrix}$$

A few points to make regarding Euler Rotation Sequences:

- There are a total of 12 different sequences
  - 6 are symmetrical
- All of them have a singularity when deriving them from a DCM
  - when all of the angles cannot be uniquely determined
  - symmetrical sequences (e.g. 3-1-3) experience this when the 2nd rotation angle is π radians
  - asymmetrical sequences (e.g.3-2-1) experience this when the 2nd rotation angle is $\frac{\pi}{2}$ radians
  - the singularity occurs when the second of the three Euler angles aligns the first and third rotation axes
  - happens when the first and last rotations are by the same angle

Euler Sequence Singularities

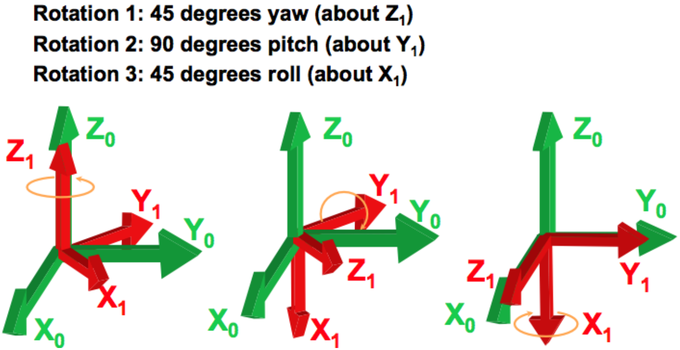


Figure: 3-2-1 Sequence

Euler Sequence Singularities

Rotation 1: 90 degrees yaw (about Z_1)
Rotation 2: 90 degrees pitch (about Y_1)
Rotation 3: 90 degrees roll (about X_1)

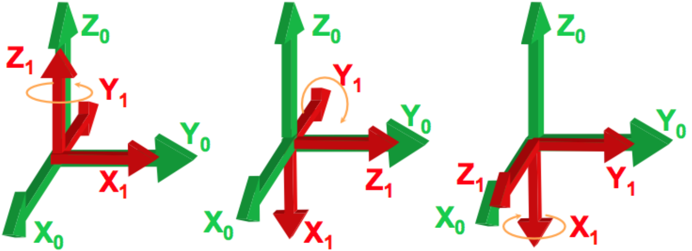


Figure: 3-2-1 Sequence

Earth Centered Inertial (ECI) Reference Frame

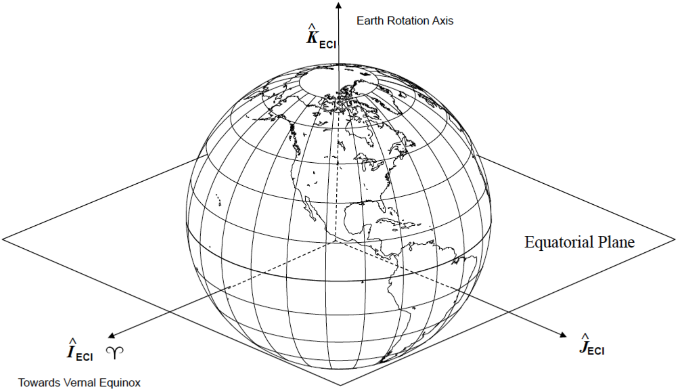


Figure: Earth Centered Inertial Frame

Image from David Vallado, "Fundamentals of Astrodynamics and Applications"

Earth Centered Inertial (ECI) Reference Frame

A few points to make with the ECI frame:

- ECI is a Dextral reference frame
- ${}^{ECI}\hat{\underline{i}}$ points toward Vernal Equinox in the True Equatorial Plane
- ${}^{ECI}\hat{\underline{k}}$ is co-linear with Earth's Axis of Rotation
- ${}^{ECI}\hat{\underline{j}}$ satisfies a dextral system and is in Earth's True Equatorial Plane
- ECI is not a true inertial frame as its origin is translating about the Solar System Barycenter and its axis of rotation is precessing and nutating
- ECI is a generic term as the International Earth Rotation Service (IERS) has a formal definition and it's called the Geocentric Celestial Reference Frame (GCRF)

Precession and Nutation

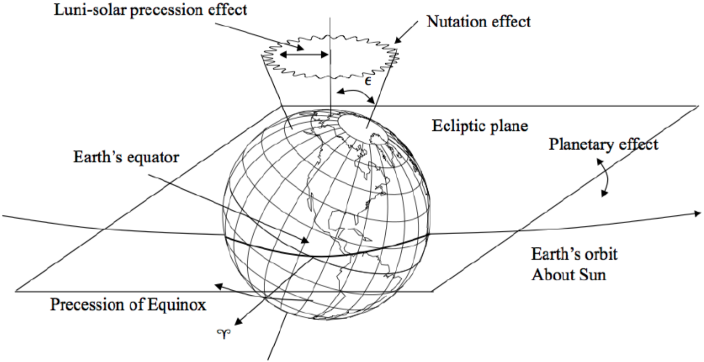


Figure: Precession and Nutation

Image from David Vallado, "Fundamentals of Astrodynamics and Applications"

Precession and Nutation

- Luni-Solar Precession - 50 arcseconds per year, period of 26,000 years
  - due to the torques of the Moon and the Sun on the Earth's equatorial bulge
- Planetary Precession - precession of 12 arcseconds/century and decrease of the obliquity of the ecliptic of 47 arcseconds/century
  - due the planetary perturbations on the Earth's orbit, causing changes in the ecliptic
- Nutation - amplitude 9 arcseconds, occurs at orbital periods of the Sun and the Moon (13.7 days, 27.6 days, 6 months, 1 year, 18.6 years, etc.) 18.6 year motion is largest - 20 arcseconds amplitude (0.5 km)
- Precession and Nutation would not occur if the Earth were of uniform/homogeneous mass and density

Earth Centered Earth Fixed (ECEF) Reference Frame

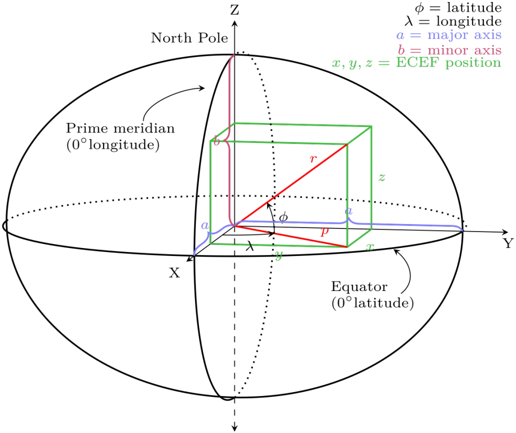


Figure: Earth Centered Earth Fixed Frame

Earth Centered Earth Fixed (ECEF) Reference Frame

- Origin at earth center of mass
- International Terrestrial Reference Frame (ITRF) is the formal terminology
  - ${}^{ECEF}\underline{\hat{z}}$ axis through IERS Reference Pole (IRP): average position of rotation axis between 1900 and 1905 (north pole)
  - ${}^{ECEF}\underline{\hat{x}}$ equatorial through a reference meridian (Greenwich meridian, 0 deg longitude)
  - ${}^{ECEF}\underline{\hat{y}}$ - makes it dextral (90 degrees East of ${}^{ECEF}\underline{\hat{x}}$)
- Realization is based on a selection of a globally distributed set of points.
- WGS 84 - World Geodetic System 1984 is native GPS Reference Frame
- Offset between ECI and ECEF (simplified)
  - θ angle between vernal equinox and Greenwich meridian is Greenwich Apparent Sidereal Time, GAST (a simplification is to use Greenwich Mean Sidereal Time, GMST)
  - Polar motion
    - the instantaneous axis of rotation moves around wrt the Earth's crust (polar motion)
    - xp, yp, represent the offset of the instantaneous axis of rotation (the Celestial Intermediate Pole) from the International Reference Pole (a fixed point on the Earth surface that serves as ${}^{ECEF}\underline{\hat{z}}$, the north pole)

ECI and ECEF Relationship

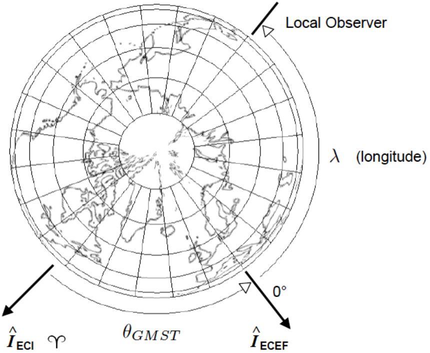


Figure: ECI and ECEF Relative Geometry

Image from David Vallado, "Fundamentals of Astrodynamics and Applications"

Polar Motion

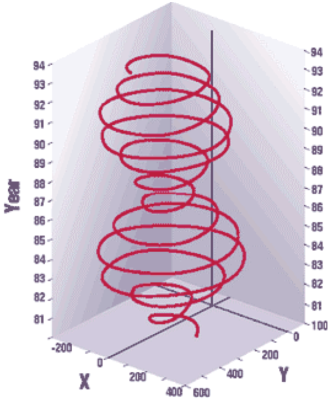


Figure: Polar Motion in Milliarcseconds

Photo courtesy of Dr. Steve Nerem: Satellite Geodesy at CU

Polar Motion

- Linear drift of the rotation pole of 3-4 milliarcseconds/year in a direction between Greenland and Hudson Bay (due to post glacial rebound)
- Long period wobble (approx. 30 years) of amplitude 30 milliarcseconds (cause unknown)
- Annual Wobble (amplitude of 0.1 arcseconds - 3 meters on the Earth's surface), 75% caused by annual variation in the inertia tensor of the atmosphere, rest by mass variations in snow, ice, ground water, etc
- Chandler Wobble (430 day period), 6 meters amplitude. Normal mode of the Earth. Caused by atmospheric and oceanic effects

The Full Precise Sequence:

$$^{GCRF}\underline{r} = \underline{\underline{P}}\,\underline{\underline{N}}\,\underline{\underline{S}}\,\underline{\underline{M}}\,^{ITRF}\underline{r}$$

- $\underline{\underline{M}}$ - Polar Motion: rotates the terrestrial (ITRF) frame from the conventional pole to the celestial ephemeris pole
- $\underline{\underline{S}}$ - Sidereal Time: rotates the terrestrial frame from the Greenwich Meridian to the true equinox of date
- $\underline{\underline{N}}$ - Nutation: rotates the celestial frame from the true equinox of date to the mean equinox of date
- $\underline{\underline{P}}$ - Precession: rotates the celestial frame from the mean equinox of date to the GCRF

ECEF to ECI Reference Frame Transformations

The Simplified Approximation Sequence:

$$^{ECI}\underline{r} = \begin{bmatrix} \cos -\theta_{GMST} & \sin -\theta_{GMST} & 0 \\ -\sin -\theta_{GMST} & \cos -\theta_{GMST} & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{ECEF}\underline{r}$$

- Use rotation about $^{ECEF}\underline{\hat{z}}$ through Greenwich Mean Sidereal Time (GMST) only
- Accurate to about 100 meters in LEO and 1 km at GEO

Latitude, Longitude, and Height

- ϕ' is what is called the Geocentric Latitude (measured in the meridian plane and from the Earth's center through point P)
- ϕ is what is called the Geodetic Latitude (measured in the meridian plane and normal to the ellipsoid through point P)
- h is called the Geodetic Height (normal to the ellipsoid through point P)

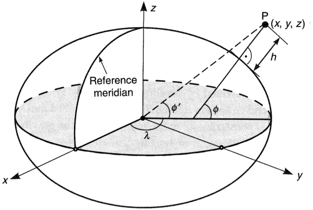


Figure: Latitude, Longitude, and Height

Figure 4.4 from Misra and Enge

Latitude, Longitude, and Height

- The Geodetic frame models the Earth as a reference ellipsoid
  - Buldgier at the equator and flatter at the poles
  - Parallels (Latitudes) are circles and Meridians (Longitudes) are ellipses
  - a mathematical construct that is user-community dependent

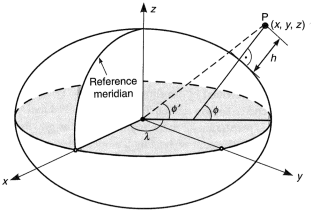


Figure: Latitude, Longitude, and Height

Figure 4.4 from Misra and Enge

Latitude, Longitude, and Height

- Orthometric Height measured from the Reference Geoid to the point of interest on the Earth's surface, along the Plum Line
- Geoid is gravitational equipotential surface that is uneven due to gravitational anomalies (mass cons, loading, etc.)
- Ellipsoidal Height measured from the Reference Ellipsoid (along it's normal) to the point of interest on the Earth's surface

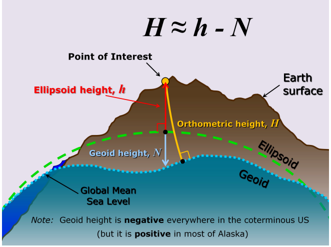


Figure: Geodetic Heights

Latitude, Longitude, and Height

Table: WGS-84 Reference Model Parameters

Parameter	Notation	Value
Semi-major Axis	a	6378137.0 meters
Flattening Factor of the Earth	$\frac{1}{f}$	298.257223563
Nominal Mean Angular Velocity of the Earth	ω	$7292115e^{-11}$ radians/second
Geocentric Gravitational Constant (Mass of Earth's Atmosphere Included)	μ	$3.986004418e^{-14}$ meters^3/second^2

Geodetic Latitude, Longitude, and Height to ECEF

$$e_{\oplus}^2 = 2f - f^2$$

$$R_N = \frac{a}{\sqrt{1 - e_{\oplus}^2 \sin^2 \phi}}$$

Then

$$^{ECEF}\underline{r} = {}^{ECEF}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (R_N + h)\cos\phi\cos\lambda \\ (R_N + h)\cos\phi\cos\lambda \\ (R_N(1 - e_{\oplus}^2) + h)\sin\phi \end{bmatrix}$$

ECEF to Geodetic Latitude, Longitude, and Height

$$\tan\lambda = \frac{y}{x} \quad \rho = \sqrt{x^2 + y^2} \quad r = \sqrt{x^2 + y^2 + z^2}$$

Initial Guess:

$$\phi = \sin^{-1}\frac{z}{r}$$

Loop:

$$R_N = \frac{a}{\sqrt{1 - e_{\oplus}^2 \sin^2\phi}} \quad \tan\phi = \frac{z + R_N\, e_{\oplus}^2 \sin\phi}{\rho}$$

Until: $\phi_{new} - \phi_{old} \leq tolerance\ (e.g.\ 1e^{-8})$

$$h = \frac{\rho}{\phi} - R_N$$

Algorithm from Vallado, D.A., "Fundamentals of Astrodynamics and Applications,"

ECEF to Geodetic Latitude, Longitude, and Height

Try out this Geoid Height Calculator

- Latitude = 30° 17′ 08" N Longitude = 97° 41′ 02" W
- GPS ellipsoidal height = 0 (meters)
- Geoid height = -27.012 (meters)
- Orthometric height (height above mean sea level) = 27.012 (meters)
  - note: orthometric height = GPS ellipsoidal height - geoid height

East-North-Up (ENU) Reference Frame

- ENU plane is perpendicular to the Geodetic Latitude (forms a Tangent Plane at that point)
- Called a Topocentric frame

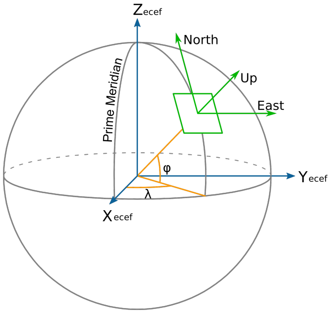


Figure: East-North-Up Reference Frames from Wikipedia

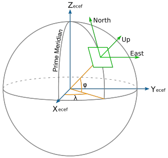
ECEF to ENU Reference Frame

$$\underline{\underline{C}}_{ENU\ ECEF} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{2}-\phi & \sin\frac{\pi}{2}-\phi \\ 0 & -\sin\frac{\pi}{2}-\phi & \cos\frac{\pi}{2}-\phi \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{2}+\lambda & \sin\frac{\pi}{2}+\lambda & 0 \\ -\sin\frac{\pi}{2}+\lambda & \cos\frac{\pi}{2}+\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} cos\frac{\pi}{2}+\lambda & sin\frac{\pi}{2}+\lambda & 0 \\ -\cos\frac{\pi}{2}-\phi\ sin\frac{\pi}{2}+\lambda & \cos\frac{\pi}{2}-\phi\ cos\frac{\pi}{2}+\lambda & \sin\frac{\pi}{2}-\phi \\ \sin\frac{\pi}{2}+\lambda\ \sin\frac{\pi}{2}-\phi & -\cos\frac{\pi}{2}+\lambda\ \sin\frac{\pi}{2}-\phi & \cos\frac{\pi}{2}-\phi \end{bmatrix} =$$

$$\begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\ \cos\lambda & -\sin\phi\ \sin\lambda & \cos\phi \\ \cos\lambda\ \cos\phi & \sin\lambda\ \cos\phi & \sin\phi \end{bmatrix}$$



Azimuth and Elevation

- Azimuth is measured from due North, clockwise, to projection of Line-of-Sight vector onto Horizon plane
- Elevation is measured from Horizon plane to Line-of-Sight vector

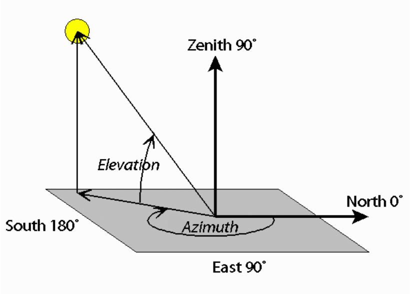


Figure: Azimuth and Elevation

Satellite Visibility

$$^{ECEF}\underline{\hat{e}}_{UP} = \begin{bmatrix} \cos\phi_{gd}\cos\lambda & \cos\phi_{gd}\sin\lambda & \sin\phi_{gd} \end{bmatrix}^T$$

Let $^{ECEF}\underline{r}_{obs}$ = ECEF Position of Observer and

$^{ECEF}\underline{r}_{sat}$ = ECEF Position of Satellite

Then $^{ECEF}\underline{\hat{e}}_{LOS} = \dfrac{^{ECEF}\underline{r}_{sat} - {}^{ECEF}\underline{r}_{obs}}{\|^{ECEF}\underline{r}_{sat} - {}^{ECEF}\underline{r}_{obs}\|}$

$$az = \arctan\left(\frac{^{ECEF}\underline{\hat{e}}_{LOS}(E)}{^{ECEF}\underline{\hat{e}}_{LOS}(N)}\right)$$

$$el = \arcsin(^{ECEF}\underline{\hat{e}}_{UP}.{}^{ECEF}\underline{\hat{e}}_{LOS})$$

$$az = \arctan\left(\frac{{}^{ECEF}\hat{\underline{e}}_{LOS}(E)}{{}^{ECEF}\hat{\underline{e}}_{LOS}(N)}\right)$$

$$el = \arcsin({}^{ECEF}\hat{\underline{e}}_{UP}.{}^{ECEF}\hat{\underline{e}}_{LOS})$$

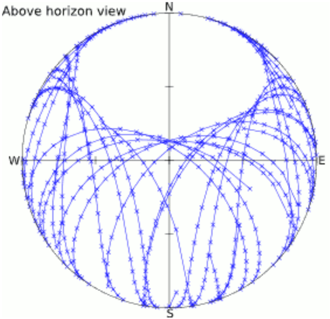


Figure: Satellite Visibility Plot

Fundamentals of Time

Universal Time (UT) some of this came from here

- Greenwich hour angle of a fictitious Sun uniformly orbiting in the equatorial plane, augmented by 12 hours (eliminates ecliptic motion of the Sun).
- UT0 = "raw", uncorrected UT as determined from motions of the stars, thus a function of sidereal time. More modern methods involve GPS satellites.
- UT1 = UT0 corrected for polar motion; Closely approximates mean diurnal motion of the Sun (Solar Time).
- UT2 = UT1 corrected for seasonal variations in the Earth's rotational speed, by adding

  $$\begin{aligned} &+0.022\sin(2\pi t) - 0.017\cos(2\pi t) \\ &-0.007\sin(4\pi t) + 0.006\cos(4\pi t) \end{aligned}$$

  seconds to UT1, where t is the fraction of the year (zero at 1 Jan).UT2 is nowadays considered obsolete.

Dynamic Time (Ephemeris Time)

- derived from planetary motions in the solar system (deduce time from position of planets and equations of motion). Independent variable in the equations of motion.
- Barycentric Dynamic Time (TDB) is based on planetary motions WRT the solar system barycenter. Differs from TDT by at most a few milliseconds. $TDB = TT + 0.001658sec\ \sin(g) + 0.000014sec\ \sin(2g)$ $g = 357.53_d + 0.98560028_d * (JD - 2451545.0)$ (higher order terms neglected; g = Earth's mean anomaly)
- Terrestrial Dynamic Time (TDT) derived from satellite motions around the Earth. Was used 1984-2000 as a time-scale of ephemerides from the Earth's surface. TDT = TAI + 32.184. Replaced ET (Ephemeris Time) in 1984, was replaced by TT (Terrestial Time) in 2001.
- Terrestial Time (TT). Originally used instead of TDT or TDB when the difference between them didn't matter. Was defined in 1991 to be consistent with the SI second and the General Theory of Relativity. Replaced TDT in the ephemerides from 2001 and on.

Fundamentals of Time

- Atomic Time
  - International Atomic Time (Temps Atomique International = TAI) is defined as the weighted average of the time kept by about 200 atomic clocks in over 50 national laboratories worldwide. TAI-UT1 was approximately 0 on 1958 Jan 1. It's based on vibrations of the Cesium atom.
    - TDT = TAI + 32.184sec
- Coordinated Universal Time (UTC)
  - Coordinated Universal Time. Differs from TAI by an integral number of seconds. When needed, leap seconds are introduced in UTC to keep the difference between UTC and UT less than 0.9 s. UTC was introduced in 1972.
    - TAI = UTC + 1s.on (n=integer=33 in early 2006)

Fundamentals of Time

- **GPS time** = TAI - 19 seconds. GPS time matched UTC from 1980-01-01 to 1981-07-01. No leap seconds are inserted into GPS time, thus GPS time is 13 seconds ahead of UTC on 2000-01-01. The GPS epoch is 00:00 (midnight) UTC on 1980-01-06. The differences between GPS Time and International Atomic Time (TAI) and Terrestrial Time (TT), also know as Terrestrial Dynamical Time (TDT), are constant at the level of some tens of nanoseconds while the difference between GPS Time and UTC changes in increments of seconds each time a leap second is added to UTC time scale.
- **GPS week** = a numbering of weeks starting at the GPS epoch 1980-01-06 00:00 GPS time (which back then was equal to UTC). Weeks are numbered from 0 and up until 1023, then it "rolls back" to 0 and are again numbered from 0 and up, etc. One GPS week rollover cycle is therefore 1024 weeks = 7168 days = ca 19.62 years. So far there's been one such GPS week number roll-over, on 1999-08-22 00:00 GPS time - a few older GPS receivers then ceased to show the correct date.

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