ASE 389P.4: Methods of Orbit Determination
Homework 5: Setting Up the Term Project

Assigned:	Thursday, March 23, 2023
Due:	Thursday, April 6, 2032 @ 12:30pm

With this assignment, you will start preparing your software for the final project. This project will consist of deriving the theory and algorithms and developing a computer program to establish the trajectory of an Earth-orbiting satellite. The assumptions for the study are:

- Three tracking stations taking apparent range and range-rate data are available for tracking the satellite. Apparent quantities imply that the one-way light time between signal transmission and reception were modeled into the measurement (i.e. it is an effect that you will have to deal with).
- The force model used to generate the truth is the EGM96 gravity field of degree and order 20, attitude-dependent solar radiation pressure, and atmospheric drag.
- The satellite is a box-wing shaped with one Sun-pointed solar panel with known component sizes, material properties, and orientation. The spacecraft -Z axis (in the spacecraft body refernce frame) is always Nadir-pointed and has the antenna.

To make sure your software is properly setup for the project and future assignments, you will compare the numeric values generated to solutions posted online.

Problems

For the state vector containing

$$\boldsymbol{X} = \begin{bmatrix} \underline{r} \\ \underline{\dot{r}} \\ C_D \\ C_{solar} \\ \cdots \\ \underline{b}_{Range_i} \end{bmatrix} \tag{1}$$

where $\underline{b}_{Range_i}$ are the Range biases for each tracking station, $i = 1, ..., n$.

1. Derive the $\underline{\underline{A}}_{n\times n}$ and $\underline{\underline{\tilde{H}}}_{m\times n}$ matrices for the linearized system and implement the partials in your computer language of choice. We recommend using a symbolic solver, e.g., MATLAB's symbolic toolbox, due to the large number of partial derivatives required. Compare to the numeric solutions online (found on canvas) at t_0. Compute the relative difference for each non-zero element, for example (in MATLAB)

   ```
   >> relDiff = abs((yourHtilde-solutionHtilde)./solutionHtilde)
   ```

   For the $\underline{\underline{\tilde{H}}}$ solution, provide the numeric values for the relative difference in your write-up. For the $\underline{\underline{A}}$ matrix, include a histogram of the exponents, e.g., (in MATLAB)

   ```
   >> hist(reshape(log10(abs(relDiff)),n*m,1))
   ```

2. Integrate position, velocity, and $\underline{\underline{\Phi}}_{(t_i,t_0)}$ from $t = 0, \ldots, 21600$ seconds. Store the results in 60 second intervals. Compare your results with those on the web (found on ther Canvas site) and compute the relative difference of the top left relevant portion of $\underline{\underline{\Phi}}_{\Phi(21600,0)}$. Like the previous question, provide a histogram of the exponents for the STM comparison.

3. Calculate the predicted range and range-rate for the appropriate tracking station at each observation time (use the data provided on the Canvas site). Compare and plot the range and range-rate residuals (post-fit). Calculate the range residual RMS and range-rate residual RMS using:

$$RMS = \sqrt{\frac{\sum_{i=0}^{n} \left(Y_{\text{Obs}} - Y_{\text{Comp}}\right)^2}{n}}$$

   Provide the values in your write-up.

Initial Satellite State

Approximate initial conditions for the satellite are (note that the state is in meters!)

	Position (m)	**Velocity (m/s)**
i	6990077.798814194	-1675.13972506056
j	1617465.311978378	7273.72441330686
k	22679.810569245355	252.688512916741

Station Position in Body-Fixed Coordinate System

The station coordinates are

Number	Descirption	X_s (m)	Y_s **(m)**	Z_s **(m)**
1	Atoll	-6143584	1364250	1033743
2	Diego Garcia	1907295	6030810	-817119
3	Arecibo	2390310	-5564341	1994578

where the coordinates for each tracking station approximately known to say 1 meter.

Earth Orientation

The epoch for your initial conditions is 1 Feb 2018, 05:00:00 UTC.

You will have to deal with Earth precession, nutation, polar motion, etc. for this project. Homework 4 prepared you for that. In essence you have to deal with IAU-76/FK5 mapping of position from ECEF (ITRF) to ECI (GCRF).

NOTE: This method uses the IAU-1976 Precession Model & IAU-1980 Theory of Nutation.

Earth Gravitational Model (EGM-96)

You do not need to estimate any of the gravitational force model terms. However, in order to get the best results in your Spacecraft Navigation and Orbit Determination, you are encouraged to implement a process noise model covered in lecture (summarized below) via the State Noise Compensation model. You will not be able to perfectly model the dynamics and thus your uncertainty must accommodate this dynamic mismodeling.

$$\underline{\underline{\bar{P}}}_i = \underline{\underline{\Phi}}_{(t_i,t_{i-1})}\ \underline{\underline{\hat{P}}}_{i-1}\ \underline{\underline{\Phi}}^T_{(t_i,t_{i-1})} + \underline{\underline{\Gamma}}_{(t_i,t_{i-1})}\ \underline{\underline{Q}}_{i-1}\ \underline{\underline{\Gamma}}^T_{(t_i,t_{i-1})} \tag{2}$$

where

$$\underline{\underline{\Gamma}}_{(t_i,t_{i-1})}\ \underline{\underline{Q}}_{i-1}\ \underline{\underline{\Gamma}}^T_{(t_i,t_{i-1})} = \Delta t^2 \begin{bmatrix} \frac{\Delta t^2}{4}\,\sigma^2_{\ddot{X}} & 0 & 0 & \frac{\Delta t}{2}\,\sigma^2_{\ddot{X}} & 0 & 0 \\ 0 & \frac{\Delta t^2}{4}\,\sigma^2_{\ddot{Y}} & 0 & 0 & \frac{\Delta t}{2}\,\sigma^2_{\ddot{Y}} & 0 \\ 0 & 0 & \frac{\Delta t^2}{4}\,\sigma^2_{\ddot{Z}} & 0 & 0 & \frac{\Delta t}{2}\,\sigma^2_{\ddot{Z}} \\ \frac{\Delta t}{2}\,\sigma^2_{\ddot{X}} & 0 & 0 & \sigma^2_{\ddot{X}} & 0 & 0 \\ 0 & \frac{\Delta t}{2}\,\sigma^2_{\ddot{Y}} & 0 & 0 & \sigma^2_{\ddot{Y}} & 0 \\ 0 & 0 & \frac{\Delta t}{2}\,\sigma^2_{\ddot{Z}} & 0 & 0 & \sigma^2_{\ddot{Z}} \end{bmatrix} \tag{3}$$

Atmospheric Drag Model

The satellite cross-sectional area must be computed from you box-wing model including the solar panel orientation. An approximate value for C_D is 1.88. The exponential density model is given by

$$\begin{aligned} \rho &= \rho_0 e^{-(r-r_0)/H} \\ \rho_0 &= 3.614 \times 10^{-13}\ \text{kg/m}^3 \\ r_0 &= (700000.0 + R_{\text{Earth}})\ \text{m} \\ H &= 88667.0\ \text{m} \\ r &= \text{magnitude of the satellite radius vector} \end{aligned}$$

Spacecraft Model Properties

The spacecraft has a mass of 2000 kg. The solar-panel is double gimbaled and always Sun-Pointed, and the -Z axis of the spacecraft bus is always NADIR pointed.

Component	**Area**	**Coating**
+X/-X Face	$6m^2$	MLI Kapton
+Y/-Y Face	$8m^2$	MLI Kapton
+Z/-Z Face	$12m^2$	White Paint/Germanium Kapton
Solar Panel	$15m^2$	Solar Cells

Material	C_d	C_s
Bulk S/C MLI Kapton	0.04	0.59
White Paint	0.80	0.04
Germanium Kapton	0.28	0.18
Solar Cells	0.04	0.04

Observations

The observation data file has been uploaded to canvas under Files, Final Project. The columns of the data file have been populated as follows:
1. Observation Station ID
2. Observation time in seconds past epoch
3. Apparent range in kilometers
4. Apparent range rate in kilometers/second

The apparent range and range rate data was simulated with noise. The noise has zero mean and standard deviation of 5 m in range and 1 mm/s in range rate.

Constants

Earth Gravitational Parameter, $\mu = 398600.4415\ km^3/s^2$
Earth Radius, $R_{Earth} = 6378.1363\ km$
Sun's Gravitational Parameter, $\mu_{Sun} = 132712440018\ km^3/s^2$
1 Astronomical Unit $= 149597870.7\ km$
Moon's Gravitational Parameter, $\mu_{Moon} = 4902.800066\ km^3/s^2$
Earth's eccentricity, $e_{Earth} = 0.081819221456$
Earth's rotational velocity, $\omega_{Earth} = 7.292115146706979e - 5\ rad/s$;