Hw1

February 2, 2023

0.1 Tory Smith

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
```

1) Given the Earth orbiting spacecraft position and velocity vectors in Cartesian coordinates

[ ]:
```
R=np.array([-2436.45, -2436.45, 6891.037]) #[km]
R_dot = np.array([5.088611, -5.088611, 0.0]) #[km/s]
mu = 398600.5 #[km^3/s^2]
```

[ ]:
```
h = np.cross(R, R_dot)
A = -1*(np.cross(h, R_dot) + mu*R/np.linalg.norm(R))
P = np.linalg.norm(h)**2/mu
n = np.cross(np.array([0, 0, 1]), h)
e = np.cross(R_dot/mu, h-R/np.linalg.norm(R))
```

[ ]:
```
#eccentricity
ecc = np.linalg.norm(A)/mu
```

[ ]:
```
#semi-major-axis
a = P/(1-ecc**2)
```

[ ]:
```
#inclination
inc = np.arccos(np.dot(h/np.linalg.norm(h), np.array([0, 0, 1])))
```

[ ]:
```
#right ascension of the ascending node
raan = np.arccos(np.dot(np.array([1, 0, 0]), n/np.linalg.norm(n)))
```

[ ]:
```
#argument of perigee
w = np.arccos(np.dot(n, e)/(np.linalg.norm(n)*np.linalg.norm(e)))
```

[ ]:
```
#true anamoly
nu = np.arccos(np.dot(R, e)/(np.linalg.norm(R)*np.linalg.norm(e)))
```

[ ]:
```python
print("Semi-Major Axis:", a, "[km]", "Eccentricity:", ecc, "Inclination:", inc,␣
↪"[rad]", "\nRAAN:", raan, "[rad]", "Argument of Perigee:", w,"[rad]", "True␣
↪Anamoly:", nu, "[rad]")
```

```
Semi-Major Axis: 7712.184983762813 [km] Eccentricity: 0.0009994359212409886
Inclination: 1.1071322171865605 [rad]
RAAN: 2.356194490192345 [rad] Argument of Perigee: 1.5707963267948966 [rad] True
Anamoly: 1.80360998372462e-05 [rad]
```

2)Convert the Keplerian elements from Problem 1 back to position and velocity

[ ]:
```python
p = a*(1-ecc**2)
r = p / (1 + ecc * np.cos(nu))
```

[ ]:
```python
#rotation matrices
def c_zxz(gamma, theta, phi):
    z_phi = np.array([[np.cos(phi), np.sin(phi), 0], [-np.sin(phi), np.
↪cos(phi), 0], [0, 0, 1]])
    x_theta = np.array([[1, 0, 0], [0, np.cos(theta), np.sin(theta)], [0, -np.
↪sin(theta), np.cos(theta)]])
    z_gamma = np.array([[np.cos(gamma), np.sin(gamma), 0], [-np.sin(gamma), np.
↪cos(gamma), 0], [0, 0, 1]])
    return np.dot(np.dot(z_phi, x_theta), z_gamma)
```

[ ]:
```python
phi = raan
gamma = inc
theta = w+nu
C = c_zxz(gamma, theta, phi)
print("Position Vector: ", r*C[:, 0])
print("Velocity Vector: ", np.sqrt(mu/p)*C[:, 2])
```

```
Position Vector:  [-2436.36211551 -2436.53788449  6891.03699888]
Velocity Vector:  [ 5.08353034e+00 -5.08353034e+00 -1.29665085e-04]
```

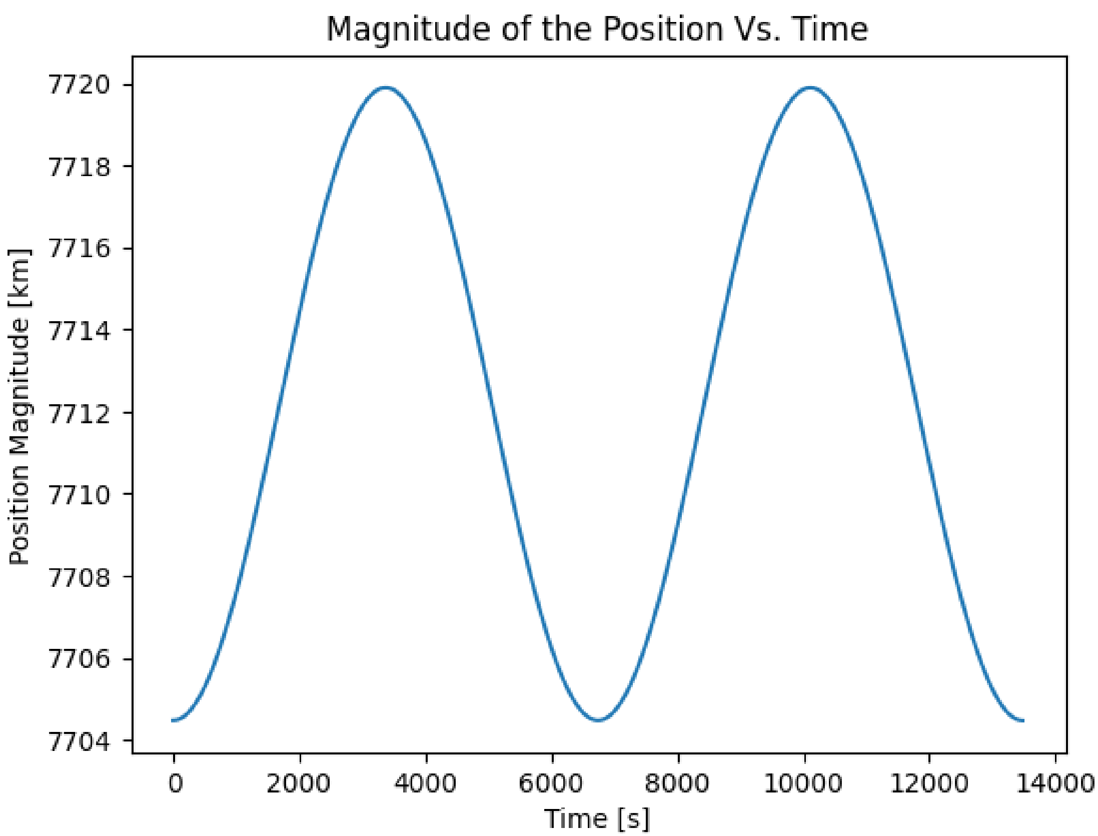
3) Given the gravity potential function U = /R, solve for the two-body acceleration due to gravity - see attached derivation

4) Develop the necessary code to numerically integrate the equations of motion using the position and velocity from Problem 1 as the initial conditions. Compute the future position and velocity at 20- second intervals for two full orbits. Plot the magnitude of the position, velocity, and acceleration as a function of time for two full orbits and provide the figure.

[ ]:
```python
#calculate the period
period = 2*np.pi*np.sqrt(a**3/mu)
t = np.arange(0, period*2, 20)

R=np.array([-2436.45, -2436.45, 6891.037]) #[km]
R_dot = np.array([5.088611, -5.088611, 0.0]) #[km/s]
```

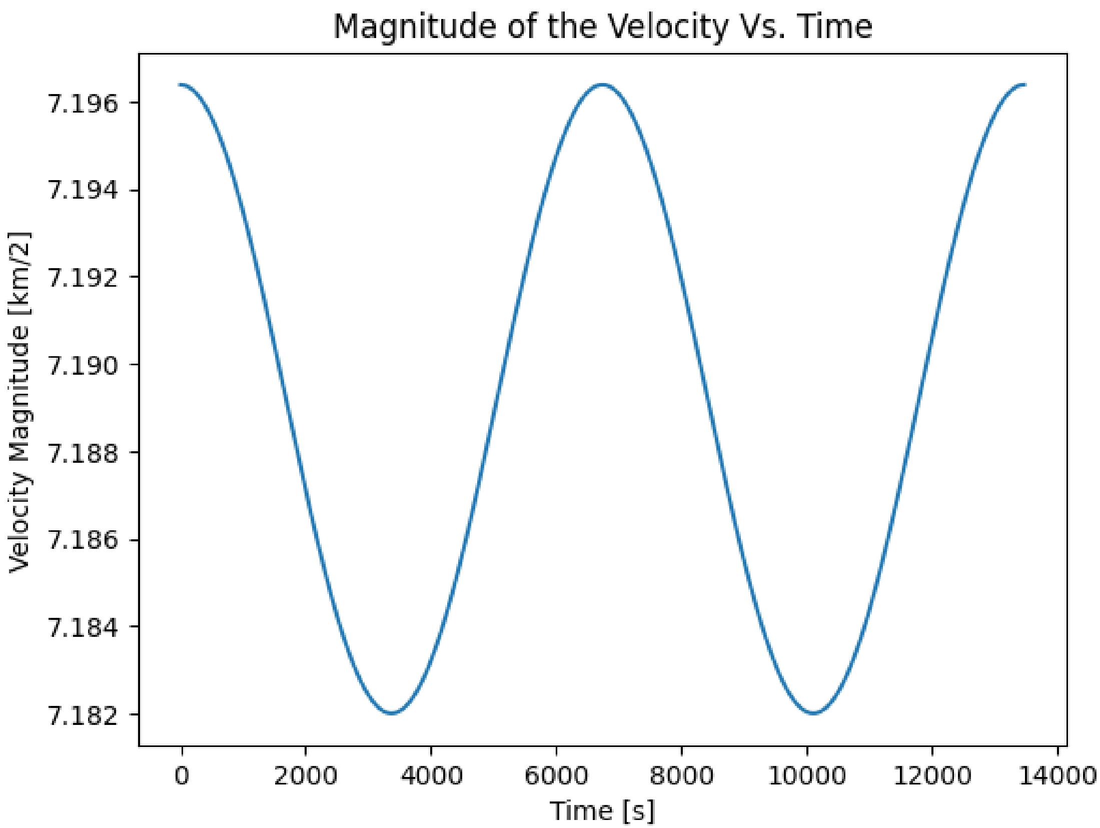
```
#equations of motion
def satellite_motion(y, t):
    mu = 398600.5 #[km^2/s^3]
    dydt = np.concatenate([y[3:6], -mu*y[0:3]/np.linalg.norm(y[0:3])**3])
    return dydt
#initial Conditions
y0 = np.concatenate([R, R_dot])

#numeric integration
sol = odeint(satellite_motion, y0, t)


plt.plot(t, [np.sqrt(sol[i, 0]**2 + sol[i, 1]**2 + sol[i, 2]**2) for i in␣
 ↪range(len(t))])
plt.title('Magnitude of the Position Vs. Time')
plt.xlabel('Time [s]')
plt.ylabel('Position Magnitude [km]')
```

```
[ ]: Text(0, 0.5, 'Position Magnitude [km]')
```



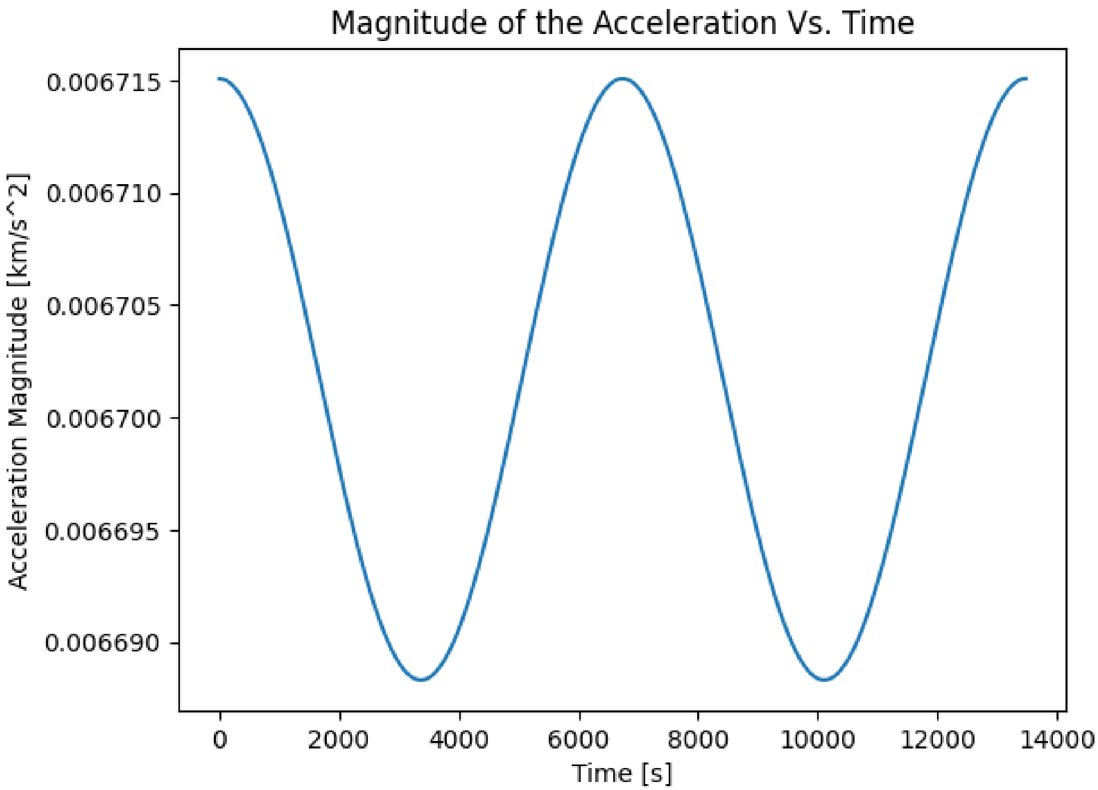
[ ]:
```
plt.plot(t, [np.linalg.norm(sol[i, 3:6]) for i in range(len(t))])
plt.title('Magnitude of the Velocity Vs. Time')
plt.xlabel('Time [s]')
plt.ylabel('Velocity Magnitude [km/2]')
```

[ ]:
```
Text(0, 0.5, 'Velocity Magnitude [km/2]')
```



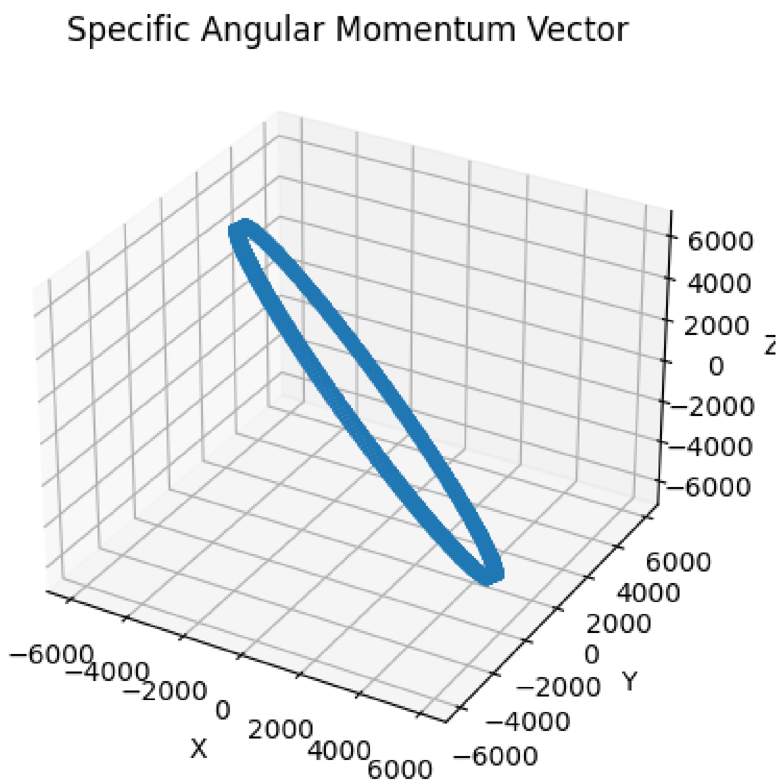
[ ]:
```
-mu*y[0:3]/np.linalg.norm(y[0:3])**3
plt.plot(t, [np.linalg.norm(-mu*sol[i, 0:3]/np.linalg.norm(sol[i, 0:3])**3) for
 ↪i in range(len(t))])
plt.title('Magnitude of the Acceleration Vs. Time')
plt.xlabel('Time [s]')
plt.ylabel('Acceleration Magnitude [km/s^2]')
```

[ ]:
```
Text(0, 0.5, 'Acceleration Magnitude [km/s^2]')
```



Compute the specific orbital angular momentum vector for these two full orbits and plot that as well, as a function of time, as a 3D scatter plot (h = R X V). Assume that the motion is only due to the accelerations derived from Eq(3)

[ ]:
```
ax = plt.figure().add_subplot(projection='3d')
h = np.array([np.cross(sol[i, 0:3], sol[i, 3:6]) for i in range(len(t))])
ax.quiver(sol[0:len(t), 0], sol[0:len(t), 1], sol[0:len(t), 2], h[:, 0], h[:,␣
 ↪1], h[:, 2], normalize=True, length=500)
plt.title('Specific Angular Momentum Vector')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

plt.show()
```

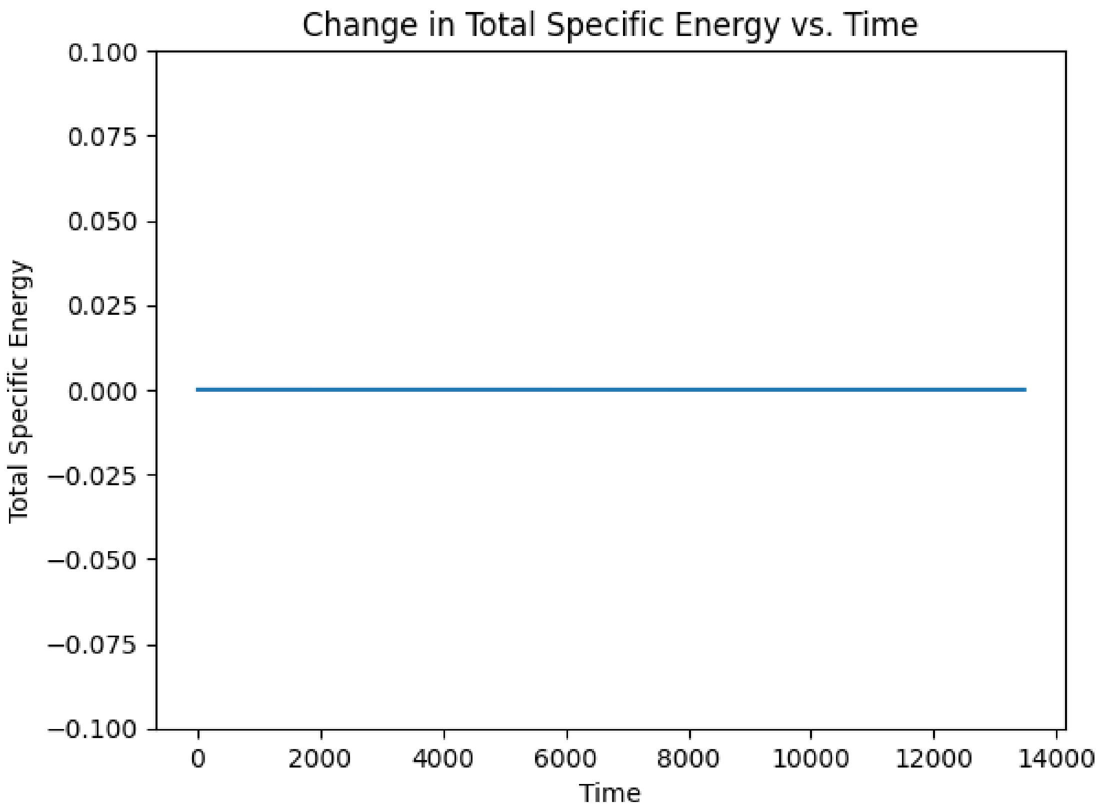


5) Compute the specific kinetic energy and specific potential energy (hint: Vis-Viva equation) as a function of time and plot the change in total specific energy to show that it remains constant over the two orbits. (i.e. plot dE = E(t) −E(t0)).

[ ]:
```
#calculate the total specific energy
def E(r, r_dot, mu):
    KE = np.dot(r_dot, r_dot)/2
    PE = mu/np.linalg.norm(r)
    return KE - PE

#r from the numeric integration solver
r = np.array([sol[0:len(t), 0], sol[0:len(t), 1], sol[0:len(t), 2]])
#r_dot from the numeric integration solver
r_dot = np.array([sol[0:len(t), 3], sol[0:len(t), 4], sol[0:len(t), 5]])

#change in total specific energy
dE = [E(r.T[i], r_dot.T[i], mu) for i in range(len(r.T))] - E(r.T[0], r_dot.
 ↪T[0], mu)
plt.plot(t, dE)
plt.title('Change in Total Specific Energy vs. Time')
plt.xlabel('Time')
plt.ylabel('Total Specific Energy')
```

```python
plt.ylim(-0.1, 0.1)
```

[ ]: (-0.1, 0.1)



Why is the change in total specific energy constant?
The total specific energy remains constant due to the law of conservation of energy. Slight errors in the values may be due to numeric integration.

HW 1

Wednesday, February 1, 2023 3:55 PM

3)

$$\nabla U = \frac{\partial U}{\partial x} i + \frac{\partial U}{\partial y} j + \frac{\partial}{\partial x} k \qquad U = \frac{\mu}{R}$$

$$R = \sqrt{\underline{R} \cdot \underline{R}} \qquad \underline{R} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$\frac{\partial U}{\partial x} = \frac{d \frac{\mu}{\sqrt{x^2 + y^2 + z^2}}}{\partial x} = \mu \frac{d \frac{1}{\sqrt{x^2+y^2+z^2}}}{\partial x} \qquad n =$$

$$= \mu \frac{d \frac{1}{n}}{\partial n} \cdot \frac{\partial \sqrt{x^2+y^2+z^2}}{\partial x}$$

$$= \mu - \frac{1}{n^2} \cdot \frac{\partial \sqrt{x^2+y^2+z^2}}{\partial x}$$

$$= \frac{-\mu x}{(x^2 + y^2 + z^2)^{3/2}} \hat{i}$$

$$\frac{\partial U}{\partial y} = \frac{-\mu y}{(x^2+y^2+z^2)^{3/2}} \hat{j} \qquad \frac{\partial U}{\partial z} = \frac{-\mu}{(x^2+y^2+z^2)^{3/2}} \hat{k}$$

$$\nabla U = \frac{-\mu x}{(x^2+y^2+z^2)^{3/2}} \hat{i} - \frac{\mu y}{(x^2+y^2+z^2)^{3/2}} \hat{j} - \frac{\mu z}{(x^2+y^2+z^2)^{3/2}} \hat{k}$$

$$\boxed{\nabla U = -\mu \frac{\underline{R}}{R^3}}$$