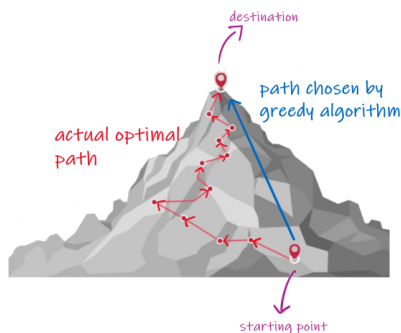


# Greedy and DP

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## Greedy approach -

- to achieve optimization we introduced greedy method.
- It is a problem solving paradigm
- It follows principal of "making the choice that seems best at that point of time".
- It is very likely that a greedy algorithm may not give the best solution
- In greedy method once you make a choice you never backtrack it.
- We can not determine whether our choice is optimal or not
- There will be only one optimal solution
- Analyzing the run time of a greedy algorithm is also easier
- Failing of greedy method,



Greedy method will say that to go on top go straight up on mountain but it is not possible to do.

- Ex. You have 10 hrs and you want to learn max courses in this time  
Physics class - 1hr  
Tennis - 3hrs  
Chemistry - 2hrs  
Cooking - 4hrs  
Cricket - 5hrs

Using greedy method we will choose those courses having min time,  
Physics + Chemistry + cooking + tennis =  $1+3+2+4 = 10$

## Control abstraction for Greedy algo -

- Algo\_greedy(L,n)
- L - List of solutions, n - size of solution
- for i = 1 to n  
  choice = select(L)  
  if(feasible(choice))  
    solution = choice + solution
- end

## Applications of Greedy -

- Knapsack
- Minimum Spanning Tree
- Shortest path
- Job scheduling
- Huffman

### Fractional Knapsack -

#### Algorithm

```

fractional_knapsack()
{
    P = 0
    for i to n
        compute (V/W)
    for i to n
        sort by (V/W) ratio
    for i to n
        if (M > 0 && Wi ≤ M)
            M = M - Wi
            P = P + Vi
        else
            break
    if (M > 0)
        P = P + Vi * (M / Wi)
}

```

Time Complexity:  $O(N * \log N)$

Auxiliary Space:  $O(N)$

#### Example

	1	2	3
V	25	24	15
W	12	15	10
$(\frac{V}{W})$	1.4	1.6	1.5

Sort by  $(\frac{V}{W}) \rightarrow [2, 3, 1]$   
in decreasing order

M = 20  
for 2,  $M = 20 - 15 = 5$   
 $P = 0 + 24 = 24$

for 3,  $M < W_3$   
 $\therefore P = P + V_3 \left( \frac{M}{W_3} \right)$   
 $\therefore P = 24 + 15 \left( \frac{5}{10} \right)$   
 $\therefore \boxed{P = 31.5}$   
Most optimal soln.

$$\text{for } 2, \quad H = 20 - 15 = 5$$

$$P = 0 + 24 = 24$$

### Job Scheduling -

- deadlines to perform the jobs
- profits associated with this jobs
- take a sequence/array with the size of max deadline given
- start with max profit job and so on
- bounds for placing the job is deadline and initial bound that starting point
- keep the job farthest from initial bound and within deadline
- Time complexity -  $O(n^2)$
- Space complexity - Extra space used by sequence array

	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$	$J_6$	$J_7$
D	4	2	2	6	5	3	1
P	60	40	20	70	50	30	10
	②	⑦	⑥	①	⑤	③	

0	1	2	3	4	5	6	
	$J_3$	$J_2$	$J_6$	$J_1$	$J_5$	$J_4$	✓

$$P = 70 + 60 + 50 + 40 + 30 + 20 = \underline{270}$$

	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$	$J_6$
D	5	3	3	2	4	2
P	15	10	12	20	8	5
	②	④	③	①	⑤	

seq/arr  $\rightarrow$ 

0	1	2	3	4	5
$J_2$	$J_4$	$J_3$	$J_5$	$J_1$	

 ✓

$p = 20 + 15 + 12 + 10 + 8 = 65$

Activity selection problem -

a	1	2	3	4	5	6	7	8	9
s	1	2	4	1	5	8	9	11	13
f	3	5	7	8	9	10	11	14	16
	✓	x	✓	x	x	✓	x	✓	x

$A = \{a_1, a_3, a_6, a_8\}$

$s_1 = 1, f_1 = 3$   
 $s_3 = 4, f_3 = 7$   
 $s_6 = 8, f_6 = 10$   
 $s_8 = 11, f_8 = 14$

$i = 2$   
 if  $s[i] \geq f[i-1] \rightarrow$  accept  
 else  $\rightarrow$  reject





1. Include the current item in the knapsack and recur for remaining items with knapsack's decreased capacity. If the capacity becomes negative, do not recur or return -INFINITY.
2. Exclude the current item from the knapsack and recur for the remaining items.
3. Finally, return the maximum value we get by including or excluding the current item.

```
int include = v[n] + knapsack(v, w, n - 1, W - w[n]);
int exclude = knapsack(v, w, n - 1, W);
```

DP -

- DP is a problem solving approach or programming paradigm.
- It solves given problem by dividing it into sub problems using recursion.
- It stores results of subproblems to avoid re-computation of subproblems.

Characteristics Components of DP -

Overlapping Subproblem -

In DP we store results of subproblems to avoid re calculations. But if a problem does not have common subproblem or overlapping subproblem then DP can't be applied to it.

Ex. Binary Search - it does not have any overlapping subproblem

Optimal Substructure -

An optimal solution can be found using optimal solutions of its subproblems.

If node x lies in the shortest path from a source node U to destination node V then the shortest path from U to V is a combination of the shortest path from U to X and the shortest path from X to V.

DP control abstraction -

- Control abstraction for dynamic programming is shown below :

**Algorithm DYNAMIC\_PROGRAMMING (P)**

**if solved(P) then**

**return lookup(P)**

If P is already solved, then  
retrieve stored answer

**else**

    Ans ← SOLVE(P)

**store (P, Ans)**

**end**

**Function SOLVE(P)**

**if sufficiently small(P) then**

    solution(P)      // Find solution for sufficiently small problem

**else**

    Divide P into smaller sub-problems  $P_1, P_2, \dots, P_n$

    Ans<sub>1</sub> ← DYNAMIC\_PROGRAMMIN(P<sub>1</sub>)

    Ans<sub>2</sub> ← DYNAMIC\_PROGRAMMIN(P<sub>2</sub>)

    ...

    Ans<sub>n</sub> ← DYNAMIC\_PROGRAMMIN(P<sub>n</sub>)

**return(combine(Ans<sub>1</sub>, Ans<sub>2</sub>, ..., Ans<sub>n</sub>))**

**end**

Combine solutions of  
smaller sub problems in  
order to achieve the  
solution to larger  
problem