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Bayesian Statistics
Assignment 1

Question 1: The Galenshore distribution

Point a.

$Y|\theta \sim \text{Galenshore}\,(a, \theta)$ is such that $p(y|\theta)$ is a density in the exponential family indeed

$$p(y|\theta) = \frac{2}{\Gamma(a)}\theta^{2a}y^{2a-1}e^{-\theta^2 y^2}\mathbb{1}_{\{y>0\}}, \theta > 0, a > 0.$$

Then definining $\phi \overset{\text{def}}{=} \theta^2$ one has

$$p(y|\phi) = h(y)c(\phi)e^{\phi t(y)} \text{ with } h(y) = \frac{2y^{2a-1}}{\Gamma(a)}\mathbb{1}_{\{y>0\}}, c(\phi) = \phi^a, t(y) = -y^2,$$

hence, by the easy shape of a distribution in the exponential family, we can state that a class of conjugate priors for $p(y|\phi)$ is such that

$$p(\phi) \propto c(\phi)^{n_0}e^{\phi n_0 t_0} = \phi^{an_0}e^{\phi n_0 t_0}.$$

If ϕ has density $p(\phi)$ and we want to obtain the density of $\theta = \sqrt{\phi}$ it is sufficient to define the map $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that $f(x) = \sqrt{x}$ and recall that

$$p_\theta(\theta) = p_\phi(f(\phi)) = p_\phi(f^{-1}(\theta)\left|\frac{df^{-1}(\theta)}{d\theta}\right|.$$

Observing that $\left|\frac{df^{-1}(\theta)}{d\theta}\right| = \left|\frac{d\theta^2}{d\theta}\right| = 2\theta$

$$p(\theta) \overset{\text{def}}{=} p_\theta(\theta) = p_\phi(\theta^2)2\theta \propto \theta^{2an_0}e^{\theta^2 n_0 t_0}2\theta.$$

Remark

Observing the three parameters a, n_0 and t_0 we can say

- $a > 0$ by hypotesis;
- $n_0 > 0$ because it represents the *prior sample size* ($p(\theta)$ has the same kernel of $p(y|\theta)$ after n_0 observations);
- $t_0 < 0$ because it is the *prior guess* that we make for t, with $t(y) = -\frac{y^2}{2}, \forall y \in \mathbb{R}^+ : t_0 = \frac{\sum_{i=1}^n t(y_i)}{n} = -\frac{\sum_{i=1}^n y_i^2}{n} < 0$.

Hence we have

$$an_0 + 1 > 0 \text{ and } -n_0 t_0 > 0.$$

So we can rewrite

$$p(\theta) \propto 2\theta^{2an_0+1}e^{-\left(\sqrt{-n_0 t_0}\right)^2\theta^2}$$

and recognizing the kernel of a Galenshore distribution we can write explicitly

$$p(\theta) = 2\theta^{2(an_0+1)-1}e^{-\left(\sqrt{-n_0t_0}\right)^2\theta^2} \cdot \underbrace{\frac{\left(\sqrt{-n_0t_0}\right)^{2(an_0+1)}}{\Gamma(an_0+1)}}_{\text{it does not depends on } \theta} \underbrace{\mathbb{1}_{\theta>0}}_{\text{by hypotesis}} =$$

$$= \frac{2}{\Gamma(an_0+1)}\left(\sqrt{-n_0t_0}\right)^{2(an_0+1)}\theta^{2(an_0+1)-1}e^{-\left(\sqrt{-n_0t_0}\right)^2\theta^2}\mathbb{1}_{\theta>0}$$

$$\Downarrow$$

$$\theta \sim \text{Galenshore}\left(an_0+1, \sqrt{-n_0t_0}\right).$$

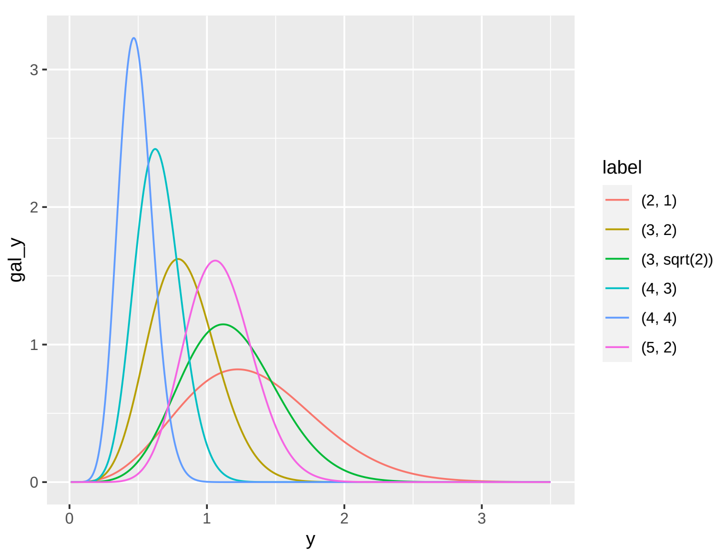
Finally we plot a few of these densities Galenshore $(an_0+1, \sqrt{-n_0t_0})$ sampled with the following code:

- $n_0 = 1, t_0 = -1, a = 1 \implies$ Galenshore $(2, 1)$;
- $n_0 = 2, t_0 = -1, a = 1 \implies$ Galenshore $\left(3, \sqrt{2}\right)$;
- $n_0 = 2, t_0 = -2, a = 1 \implies$ Galenshore $(3, 2)$;
- $n_0 = 2, t_0 = -2, a = 2 \implies$ Galenshore $(5, 2)$;
- $n_0 = 3, t_0 = -3, a = 1 \implies$ Galenshore $(4, 3)$;
- $n_0 = 3, t_0 = -4, a = 1 \implies$ Galenshore $(4, 4)$.

```
dgalenshore = function(y, a, theta) {
    (2 / gamma(a)) * theta^(2 * a) * y^(2 * a - 1) * exp(-(theta^2) * y^2)
}

y = seq(0.01, 3.5, length = 1000)
df = rbind(
    data.frame(y = y, gal_y = dgalenshore(y, 2, 1), label = "(2, 1)"),
    data.frame(y = y, gal_y = dgalenshore(y, 3, sqrt(2)), label = "(3, sqrt(2))"),
    data.frame(y = y, gal_y = dgalenshore(y, 3, 2), label = "(3, 2)"),
    data.frame(y = y, gal_y = dgalenshore(y, 5, 2), label = "(5, 2)"),
    data.frame(y = y, gal_y = dgalenshore(y, 4, 3), label = "(4, 3)"),
    data.frame(y = y, gal_y = dgalenshore(y, 4, 4), label = "(4, 4)")
)
```

Then we plot all of them at the same time:

```
ggplot(df, aes(y, gal_y, group = label, color = label)) +
geom_line() + coord_fixed(ratio = 1)
```



Point b.

Let's define $b \stackrel{\text{def}}{=} an_0 + 1$ and $c = \sqrt{-n_0 t_0}$ for coinciseness.
Recalling $\theta \sim \text{Galenshore}(b, c)$ and $Y_i|\theta \sim \text{Galenshore}(a, \theta), \forall i \in 1:n$ and defining $\text{SS}(y_{1:n}) \stackrel{\text{def}}{=} \sum_{i=1}^n y_i^2$ we have

$$\begin{aligned} p(\theta|y_{1:n}) = p(\theta)p(y_{1:n}|\theta) \propto \\ &\propto (\theta^{2b-1} e^{-c^2\theta^2})(\theta^{2na} e^{-\theta^2 \sum_{i=1}^n y_i^2}) \propto \\ &\propto \theta^{2(an+b)-1} e^{-(c^2 + \text{SS}(y_{1:n}))\theta^2}. \end{aligned}$$

Hence we recognize the kernel of a $\text{Galenshore}\left(an + b, \sqrt{c^2 + \text{SS}(y_{1:n})}\right)$

$$\implies \theta|Y_{1:n} \sim \text{Galenshore}\left(a(n + n_0) + 1, \sqrt{\text{SS}(y_{1:n}) - n_0 t_0}\right).$$

Point c.

$$\begin{aligned} \frac{p(\theta_a|y_{1:n})}{p(\theta_b|y_{1:n})} &= \frac{2\Gamma(a(n+n_0)+1)}{\Gamma(a(n+n_0)+1)2} (\text{SS}(y_{1:n}) - n_0 t_0)^{(a(n+n_0)+1)(1-1)} \left(\frac{\theta_a}{\theta_b}\right)^{2a(n+n_0)+1} e^{-(\text{SS}(y_{1:n}) - n_0 t_0)\left(\theta_a^2 - \theta_b^2\right)} = \\ &= \left(\frac{\theta_a}{\theta_b}\right)^{2a(n+n_0)+1} e^{-\left(\sum_{i=1}^n y_i^2 - n_0 t_0\right)\left(\theta_a^2 - \theta_b^2\right)}. \end{aligned}$$

Hence

$$\mathbb{P}(\theta \in A | Y_{1:n} = y_{1:n}) = \mathbb{P}\left(\theta \in A | \sum_{i=1}^n Y_i^2 = \sum_{i=1}^n y_i^2\right), \forall A$$

and then, by definition $\text{SS}(Y_{1:n}) \stackrel{\text{def}}{=} \sum_{i=1}^n Y_i^2$ is a sufficient statistic.

Point d.

Recalling that $\theta|Y_{1:n} \sim \text{Galenshore}\left(a(n + n_0) + 1, \sqrt{\text{SS}(y_{1:n}) - n_0 t_0}\right)$ and that if $X \sim \text{Galenshore}(a, \theta) \implies \mathbb{E}[X] = \frac{\Gamma\left(a + \frac{1}{2}\right)}{\theta \Gamma(a)}$ we have

$$\mathbb{E}[\theta|y_{1:n}] = \frac{\Gamma\left(a(n+n_0) + frac32\right)}{\left(\sqrt{\text{SS}(y_{1:n}) - n_0 t_0}\right)\Gamma(a(n+n_0)+1)}$$

Point e.

With the usual notation $b \stackrel{\text{def}}{=} an_0 + 1$ and $c \stackrel{\text{def}}{=} \sqrt{-n_0 t_0}$:

$$\begin{aligned} p(y_{n+1}|y_{1:n}) &= \int_0^\infty p(y_{n+1}|\theta)p(\theta|y_{1:n}) d\theta = \\ &= \int_0^\infty \frac{2}{\Gamma(a)} \theta^{2a} y_{n+1}^{2a-1} e^{-\theta^2 y_{n+1}^2} \cdot \frac{2}{\Gamma(an+b)} \left(c^2 + \text{SS}(y_{1:n})\right)^{an+b} \theta^{2(an+b)-1} e^{-(c^2 + \text{SS}(y_{1:n}))\theta^2} d\theta = \\ &= \frac{4}{\Gamma(a)\Gamma(an+b)} y_{n+1}^{2a-1} \left(c^2 + \text{SS}(y_{1:n})\right)^{an+b} \int_0^\infty \underbrace{\theta^{2(a+an+b)-1} e^{-(c^2 + \text{SS}(y_{1:n}) + y_{n+1}^2)\theta^2}}_{\text{kernel of a Galenshore}\left(a+an+b, \sqrt{c^2 + \text{SS}(y_{1:n}) + y_{n+1}^2}\right)} d\theta \end{aligned}$$

$$\Downarrow$$

$$\int_0^\infty \theta^{2(a+an+b)-1} e^{-(c^2+\mathrm{SS}(y_{1:n})+y_{n+1}^2)\theta^2} = \frac{\Gamma(a+an+b)}{2}\left(\frac{1}{c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2}\right)^{a+an+b}$$

$$\Downarrow$$

$$\begin{aligned}
p(y_{n+1}|y_{1:n}) &= \frac{4}{\Gamma(a)\Gamma(an+b)} y_{n+1}^{2a-1}\left(c^2+\mathrm{SS}\left(y_{1:n}\right)\right)^{an+b}\frac{\Gamma(a+an+b)}{2}\left(\frac{1}{c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2}\right)^{a+an+b} = \\
&= \frac{2}{y_{n+1}}\frac{\Gamma(a+an+b)}{\Gamma(a)\Gamma(an+b)}\left(\frac{y_{n+1}^2}{c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2}\right)^{a}\left(\frac{c^2+\mathrm{SS}\left(y_{1:n}\right)}{c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2}\right)^{an+b} = \\
&= \frac{2y_{n+1}\left(c^2+\mathrm{SS}\left(y_{1:n}\right)\right)}{\left(c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2\right)^2}\cdot \\
&\quad \cdot \underbrace{\frac{1}{B(a,an+b)}\left(\frac{y_{n+1}^2}{c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2}\right)^{a-1}\left(1-\frac{y_{n+1}^2}{c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2}\right)^{(an+b)-1}}_{\text{density of } X\sim B(a,an+b) \text{ evaluated on } \frac{y_{n+1}^2}{c^2+\mathrm{SS}(y_{1:n})+y_{n+1}^2}} = \\
&= \frac{2y_{n+1}\left(c^2+\mathrm{SS}\left(y_{1:n}\right)\right)}{\left(c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2\right)^2} p_X\left(\frac{y_{n+1}^2}{c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2}\right).
\end{aligned}$$

Now one should note that this is a differentiable transformation of $\mathbb{R}^+$ of an unknown random variable. Indeed if one try to derive, with respect to y_{n+1}, the variable of the density of X in our last expression obtains

$$\begin{aligned}
\frac{d}{dy_{n+1}}\frac{y_{n+1}^2}{c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2} &= \frac{2y_{n+1}(c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2)-y_{n+1}^2 2y_{n+1}}{\left(c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2\right)^2} = \\
&= \underbrace{\frac{2y_{n+1}(c^2+\mathrm{SS}\left(y_{1:n}\right))}{\left(c^2+\mathrm{SS}\left(y_{1:n}\right)+y_{n+1}^2\right)^2}}_{>0 \text{ indeed } y_{n+1}\in\mathbb{R}^+ \text{ and the other terms are squared}}.
\end{aligned}$$

Hence if we define $f^{-1}:\mathbb{R}^+\to\mathbb{R}^+$ such that $f^{-1}(x)=\dfrac{x^2}{c^2+\mathrm{SS}\left(y_{1:n}\right)+x^2}$ we can state

$$\begin{aligned}
p(y_{n+1}|y_{1:n}) &= \left|\frac{d}{dy_{n+1}}f^{-1}(y_{n+1})\right| p_X(f^{-1}(y_{n+1}) = \\
&= p_{f(X)}(y_{n+1}).
\end{aligned}$$

Let's compute f explicitly

$$f^{-1}(x)=\frac{x^2}{c^2+\mathrm{SS}\left(y_{1:n}\right)+x^2} = 1-\frac{c^2+\mathrm{SS}\left(y_{1:n}\right)}{c^2+\mathrm{SS}\left(y_{1:n}\right)+x^2}$$

hence

$$\begin{aligned}
x = 1-\frac{c^2+\mathrm{SS}\left(y_{1:n}\right)}{c^2+\mathrm{SS}\left(y_{1:n}\right)+f(x)^2} &\iff 1-x=\frac{c^2+\mathrm{SS}\left(y_{1:n}\right)}{c^2+\mathrm{SS}\left(y_{1:n}\right)+f(x)^2} \iff \\
&\iff \frac{f(x)^2}{c^2+\mathrm{SS}\left(y_{1:n}\right)}+1=\frac{1}{1-x} \iff \\
&\iff f(x)=\sqrt{\frac{x}{1-x}}\sqrt{c^2+\mathrm{SS}\left(y_{1:n}\right)}.
\end{aligned}$$

This leads us to conclude (substituting again $b = an_0 + 1$ and $c = \sqrt{-n_0 t_0}$) that

$$Y_{n+1}|Y_{1:n} \sim f\left(B(a, a(n+n_0)+1)\right), \text{ with } f : \mathbb{R}^+ \to \mathbb{R}^+, f(x) = \sqrt{\frac{x}{1-x}}\sqrt{\mathrm{SS}\left(y_{1:n}\right) - n_0 t_0}.$$

Question 2: Tumor Counts

Part 1: Tumor Counts

theta_A = Gamma(120, 10) theta_B = Gamma(12, 1)

Point a.

```
load(file = 'dataAssignment1.RData')
library(tidyverse)
library(gridExtra)
library(grid)
library(ggplot2)
library(lattice)

#posterior parameters

a_n = 120.0 + sum(y.a)
b_n = 10.0 + length(y.a)

c_n = 12 + sum(y.b)
d_n = 1 + length(y.b)

sprintf("posterior of A : Gamma(%i, %i)", a_n, b_n)
```

```
## [1] "posterior of A : Gamma(237, 20)"
```

```
sprintf("posterior of B : Gamma(%i, %i)", c_n, d_n)
```

```
## [1] "posterior of B : Gamma(125, 14)"
```

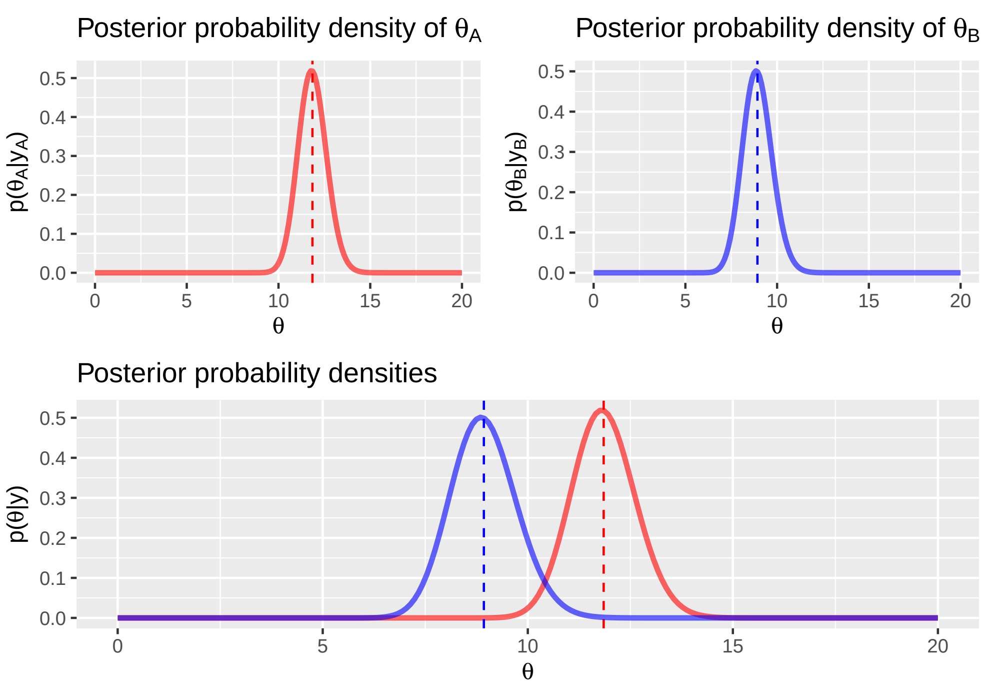
```
#posterior means

mean_A = a_n / b_n
mean_B = c_n / d_n

#posterior density and quantile interval for A

y.a.sum = sum(y.a)
n.a = length(y.a)
alpha = 0.05
gamma.values = seq(0, 20, length= 200)

quant.interval.a = qgamma(c(alpha/2,1-alpha/2), shape = a_n, rate = b_n)
quant.interval.b = qgamma(c(alpha/2,1-alpha/2), shape = c_n, rate = d_n)

post.values.a = dgamma(gamma.values, a_n, b_n)
post.values.b = dgamma(gamma.values, c_n, d_n)

post.data = data.frame(gamma.values, post.values.a, post.values.b)
```

```
#posterior density plots

post.data %>% ggplot()+
  geom_line(aes(x = gamma.values, y = post.values.a), col = "red", alpha = 0.6, size = 1.2)+
  geom_vline(xintercept = mean_A, col = "red", linetype = 2)+
  scale_color_discrete(guide = "none") -> p1

post.data %>% ggplot()+
  geom_line(aes(x = gamma.values, y = post.values.b), col = "blue", alpha = 0.6, size = 1.2)+
  geom_vline(xintercept = mean_B, col = "blue", linetype = 2)+
  scale_color_discrete(guide = "none") -> p2
```

```
p3 <- p1 +
  geom_line(aes(x = gamma.values, y = post.values.b), col = "blue", alpha = 0.6, size = 1.2)+
  geom_vline(xintercept = mean_B, col = "blue", linetype = 2)+
  scale_color_discrete(guide = "none")+
  xlab(expression(theta)) +
  ylab(expression(paste("p(",theta,"|",y,")")))+
  ggtitle("Posterior probability densities")
```

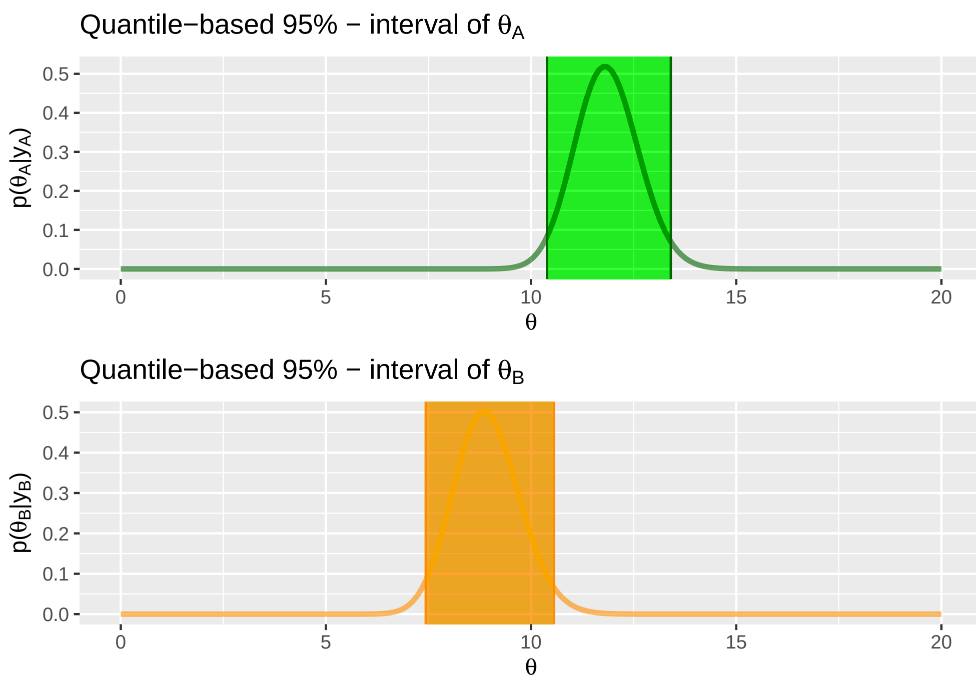
```
## Scale for 'colour' is already present. Adding another scale for 'colour',
## which will replace the existing scale.
```

```
p1 <- p1 +
  xlab(expression(theta)) +
  ylab(expression(paste("p(",theta[A],"|",y[A],")")))+
  ggtitle(expression(paste("Posterior probability density of ", theta[A])))

p2 <- p2 +
  xlab(expression(theta)) +
  ylab(expression(paste("p(",theta[B],"|",y[B],")")))+
  ggtitle(expression(paste("Posterior probability density of ", theta[B])))

grid.arrange(arrangeGrob(p1, p2, ncol = 2), p3, nrow = 2)
```




```
#quantile-based intervals plots

post.data %>% ggplot()+
  geom_line(aes(x = gamma.values, y = post.values.a), col="darkgreen", alpha = 0.6, size=1.2)+
  geom_vline(xintercept = quant.interval.a[1], col = "darkgreen")+
  geom_vline(xintercept = quant.interval.a[2], col = "darkgreen")+
  geom_rect(aes(xmin = quant.interval.a[1], xmax = quant.interval.a[2], ymin = -Inf, ymax = Inf),
    fill = "green", alpha = 0.002)+
  xlab(expression(theta)) +
  ylab(expression(paste("p(",theta[A],"|",y[A],")")))+
  ggtitle(expression(paste("Quantile-based 95% - interval of ", theta[A])))+
  scale_color_discrete(guide = "none") ->q1

post.data %>% ggplot()+
  geom_line(aes(x = gamma.values, y = post.values.b), col = "darkorange", alpha = 0.6, size = 1.2)+
  geom_vline(xintercept = quant.interval.b[1], col = "darkorange")+
  geom_vline(xintercept = quant.interval.b[2], col = "darkorange")+
  geom_rect(aes(xmin = quant.interval.b[1], xmax = quant.interval.b[2], ymin = -Inf, ymax = Inf),
    fill = "orange", alpha = 0.002)+
  xlab(expression(theta)) +
```

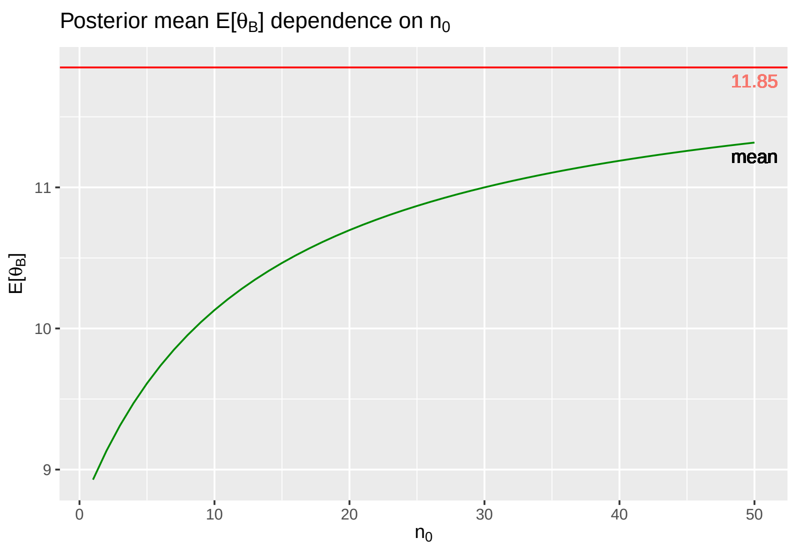
```
  ylab(expression(paste("p(",theta[B],"|",y[B],")")))+
  ggtitle(expression(paste("Quantile-based 95% - interval of ", theta[B])))+
  scale_color_discrete(guide = "none") -> q2


grid.arrange(q1, q2, nrow = 2)
```



Point b.

```
mean.b = rep(0, 50)

n_0 = 1:50

for (i in n_0){
  a = 12*i + sum(y.b)
  b = i + length(y.b)
  mean.b[i] = a/b
}

mean_varying <- data.frame(n_0, mean.b)
```

```
mean_varying %>% ggplot(aes(x = n_0, y = mean.b))+
  geom_line(col = "green4")+
  xlab(expression(n[0]))+
  ylab (expression(paste("E[",theta[B], "]")))+
  ggtitle(expression(paste("Posterior mean ", "E[",theta[B], "]", " dependence on ", n[0])))+
  geom_hline(yintercept = mean_A, col = "red")+
  scale_color_discrete(guide = "none")+
  geom_text(aes(50, mean_A, label = mean_A, vjust = +1.5, col = "red"))+
  geom_text(aes(50, mean.b[50], label = "mean", vjust = +1.5))
```



Point c.

Part 2: Tumor Counts Comparison

Point d.

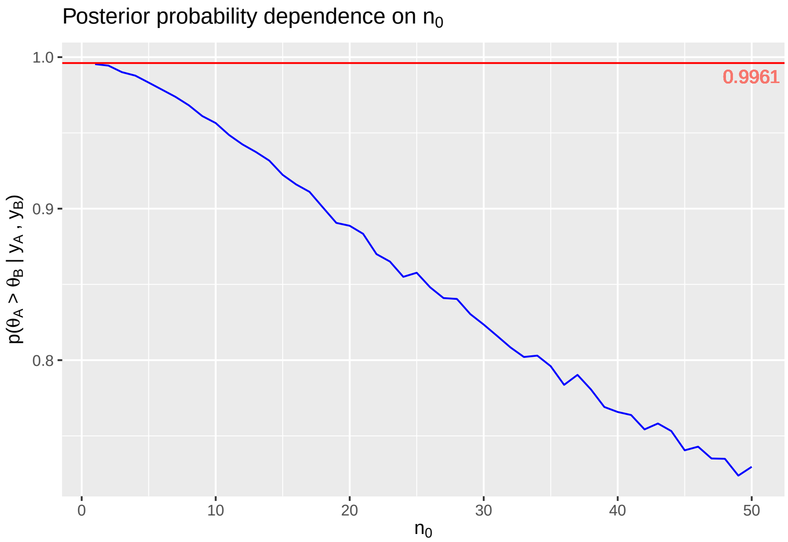
```r
S = 10000

#Monte Carlo estimate with original p(theta[B])
sample.a = rgamma(S, a_n, b_n)
sample.b = rgamma(S, c_n, d_n)

mc1 = sum(sample.a > sample.b) / S

sprintf("The Monte Carlo estimate given the original prior of theta[B] is: %f", mc1)
```

```
## [1] "The Monte Carlo estimate given the original prior of theta[B] is: 0.996100"
```

Point e.

```r
#Monte Carlo estimate with varying p(theta[B])

mc2 = rep(0, 50)

for (i in n_0){
  a = 12*i + sum(y.b)
  b = i + length(y.b)
  sample2.b = rgamma(S, a, b)
  mc2[i] = sum(sample.a > sample2.b) / S
}

mc1_varying <- data.frame(n_0, mc2)
```

```r
mc1_varying %>% ggplot(aes(x = n_0, y = mc2))+
  geom_line(col = "blue")+
  xlab(expression(n[0]))+
  ylab (expression(paste("p(",theta[A], " > ",theta[B], " | ",y[A], " , ", y[B], ")")))+
  ggtitle(expression(paste("Posterior probability dependence on ", n[0])))+
  geom_hline(yintercept = mc1, col = "red")+
  scale_color_discrete(guide = "none")+
  geom_text(aes(50, mc1, label = mc1, vjust = +1.5, col = "red"))
```



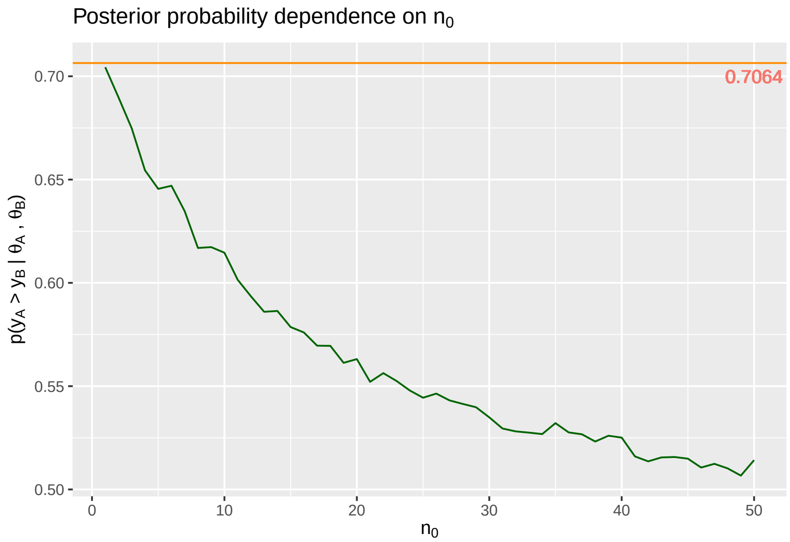
Point f.

```r
#Monte Carlo estimate of the posterior predictive distribution (original theta[B])

y.post.a = rep(0, S)
y.post.b = rep(0, S)

for (i in 1:S){
  y.post.a[i] = rpois(1, sample.a[i])
  y.post.b[i] = rpois(1, sample.b[i])
}

mc3 = sum(y.post.a > y.post.b)/S
sprintf("The Monte Carlo estimate given the original prior of theta[B] is: %f", mc3)
```

```
## [1] "The Monte Carlo estimate given the original prior of theta[B] is: 0.706400"
```

```r
#Monte Carlo estimate of the posterior predictive distribution (varying theta[B])

mc4= rep(0, 50)

for (i in 1:50){
  y.post2.b = rep(0, S)
  a = 12*i + sum(y.b)
  b = i + length(y.b)
  sample2.b = rgamma(S, a, b)
  for (j in 1:S){
  y.post2.b[j] = rpois(1, sample2.b[j])
  }
  mc4[i] = sum(y.post.a > y.post2.b)/S
}

mc3_varying <- data.frame(n_0, mc4)
```

```r
mc3_varying %>% ggplot(aes(x = n_0, y = mc4))+
  geom_line(col = "darkgreen")+
  xlab(expression(n[0]))+
  ylab (expression(paste("p(",y[A], " > ",y[B], " | ", theta[A], " , ", theta[B], ")")))+
  ggtitle(expression(paste("Posterior probability dependence on ", n[0])))+
  geom_hline(yintercept = mc3, col = "darkorange")+
  scale_color_discrete(guide = "none")+
  geom_text(aes(50, mc3, label = mc3, vjust = +1.5, col = "orange"))
```

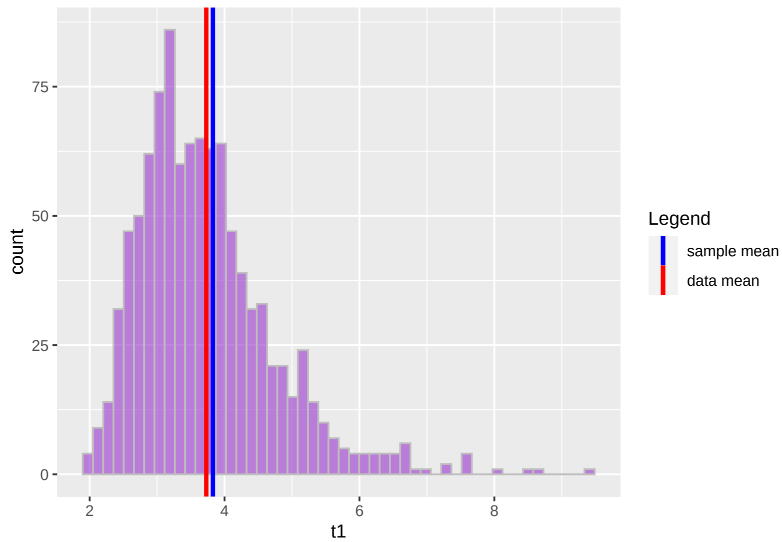


Part 3: Posterior predictive checks

Point g.

```
#generating the samples case theta[A]

K = 1000
sample.theta.a = rgamma(K, a_n, b_n)
t1 = rep(0, K)
for (i in 1:K){
  post.pred.a = rpois(10, sample.theta.a[i])
  t1[i] = sum(post.pred.a) / (10*sd(post.pred.a))
}

t2 =  sum(y.a) / (10*sd(y.a))

t.a <- data.frame(1:K, t1)

colors <- c("data mean" = "blue", "sample mean" = "red")
```

```
t.a %>% ggplot()+
  geom_histogram(aes(t1), bins = 50, fill = "darkorchid", alpha = 0.6, col = "grey")+
  geom_vline(aes(xintercept = t2, col = "sample mean"), size = 1.2) +
  geom_vline(aes( xintercept = mean(t1), col = "data mean"), size = 1.2)+
  scale_color_manual(name = "Legend", values = c( "sample mean" = "blue", "data mean" = "red"),
    labels = c("sample mean", "data mean"))
```



Point h.

```
#generating the samples case theta[B]

K = 1000
sample.theta.b = rgamma(K, c_n, d_n)
t3 = rep(0, K)
for (i in 1:K){
  post.pred.b = rpois(10, sample.theta.b[i])
  t3[i] = sum(post.pred.a) / (10*sd(post.pred.b))
}

t4 =  sum(y.a) / (10*sd(y.a))

t.b <- data.frame(1:K, t3)
```

```
t.b %>% ggplot()+
  geom_histogram(aes(t3), bins = 50, fill = "darkorchid", alpha = 0.6, col = "grey")+
  geom_vline(aes(xintercept = t4, col = "sample mean"), size = 1.2) +
  geom_vline(aes( xintercept = mean(t3), col = "data mean"), size = 1.2)+
  scale_color_manual(name = "Legend", values = c( "sample mean" = "blue", "data mean" = "red"),
    labels = c("sample mean", "data mean"))
```