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Simulations of an impulsive model for the growth of fruit trees

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Abstract. Mediterranean agricultural systems have been severely affected because of the decrease in rainfall and more frequent and severe droughts due to the global warming phenomenon. The current and future scenario of water deficit could have a negative effect on the growth and development rates of the fruit trees, reflected in the drop of production. To help to face this problem, this work presents a mathematical simulation model of fruit growth with two-time scales: a continuous scale that governs the dynamics of fruit growth and a discrete scale representing the period of time in which the system is intervened with irrigation supply. The results obtained in the simulations allow us to describe and understand the physical phenomena involved in the growth dynamics of fruit trees. In addition, show the importance of the water resource for the growth and development of fruit trees; therefore, a scenario of water deficit would compromise the production and existence of fruit trees.

1. Introduction

The scarcity of water generated by climate change is modifying the production and conservation of agricultural systems. The increase in global temperature [1, 2], along with diminishing water supply, is altering in a negative way the growth and production cycles of fruit trees growing under a Mediterranean-type climate. In Mediterranean semi-arid production conditions, all fruit trees are produced under irrigation. Irrigation is defined as "the application of water to a soil profile, in a timely and uniform manner, to replace the water consumed by the crop between two consecutive irrigations" [3].

Thus, especially in semi-arid areas, the reduction in the water supply for irrigation and the increase of dry and hot weather conditions, are dramatically increasing the risk of fruit production because of their effects on the growth and development rates [4, 5]. Consequently, understanding the interactions between irrigation and fruit trees production is essential to the fruit growers to take decisions to sustainably manage the fruit trees production, especially right now where environmental conditions are adverse.

Crop growth modeling has been developed widely with engineering-type models, supported by biophysical sub-models based, for example, on the second Newton's law, and Darcy's laws, that combined with a numeric fit, provide essential information for support the water management at field [6–8]. Some models of this type are crop yield prediction models [9–11], models of crop response to environmental factors [12, 13] and simulation models such as world food studies (WOFOST), used in [14] to simulate the growth of jujube under different irrigation treatments.

Engineering models are characterized by using statistical tools and computer programs to find trends and validate field data. Scientific models are also evidenced that contribute to understanding the response of crops to environmental factors. Among them, those exposed in [15] stand out. In [16] a study of the growth dynamics of fruit trees under a water scarcity scenario is presented.

The construction of a mathematical model implies analysis and understanding of the problem to be modeled, identifying the variables and parameters involved, and the relationships between them, in order to reflect reality simply but precisely. However, as far as the authors are aware, there is no evidence of a dynamic model that describes the interaction between the 3 variables: energy, biomass of the fruit tree, and water, incorporating the supply of irrigation. This article presents the simulation of an impulsive model for the growth of fruit trees with different irrigation strategies.

2. Mathematical modeling

The models for the growth of fruit trees involve the interaction between different variables such as genotype, crop management, type of soil, salinity, environmental factors, water, energy, among other variables [15–19]. For the construction of an impulsive mathematical model for the growth of fruit trees, we consider the following assumptions:

(i) The growth dynamics is governed by an interaction between the variables energy, water, vegetative growth.
(ii) The fruit tree responds instantly to the irrigation application.
(iii) We assumed an adult fruit tree (older than 5 years) and a suitable soil surface for the growth of the fruit tree.
(iv) Ideal agronomic management conditions.
(v) Optimal environmental conditions are assumed, that is, the energy of the system is constant.

The parameters that intervene in the dynamics: water, energy and vegetative growth in our model are summarized in Table 1.

Table 1. Parameters used in the fruit tree growth model.

Parameter	Meaning
q	Accumulated energy constant
r	Fruit trees intrinsic growth rate
N	Fruit carrying capacity
I	Irrigation water amount
β	Evapotranspiration rate
γ	Photosynthetic contribution rate
ω	Mortality rate of fruit trees

We denote $E = E(t)$ the solar radiation at time t, $W = W(t)$ the water amount in the soil in time t and $C = C(t)$ the fruit biomass concentration in time t, both state variables. Under assumption (iv), the fruit growth dynamics is reduced to two-state variables W and C. The variation of the amount of water in the system with respect to the time is denoted by $W'(t)$. When considering a scenario without precipitation, this is $p = 0$, the water variation presents output from the system due to environmental effects (evapotranspiration) at a rate β. At the same time, there is another water outlet from the system corresponding to promoting the growth of the fruit tree at a rate r in the sistem.

The variation in biomass growth in the system with respect to the time is denoted by $C'(t)$ corresponds to the input by water-biomass growth interaction at a rate r, in addition, this variation is also positively affected due to the water-energy-biomass growth interaction at a rate γ. in addition, there is an exit to an ω rate corresponding to loss or natural death of the crop. This behavior applies to a continuous time scale (long duration) that governs the growth dynamics of the fruit tree. However, this model also presents a short time scale (pulse) in discrete time that represents the events in which water enters the system through irrigation supply.

The dynamic relationship between the variables energy, amount of water, and concentration of biomass of the fruit tree is represented in Figure 1; where: E = Energy; W = Water amount; C = Fruit biomass concentration; r = Fruit trees intrinsic growth rate; N = Fruit carrying capacity; β = Evapotranspiration rate; γ = Photosynthetic contribution rate; ω = Mortality rate of fruit trees; p = Rainfall rate; I = Irrigation amount, adapted from [16].

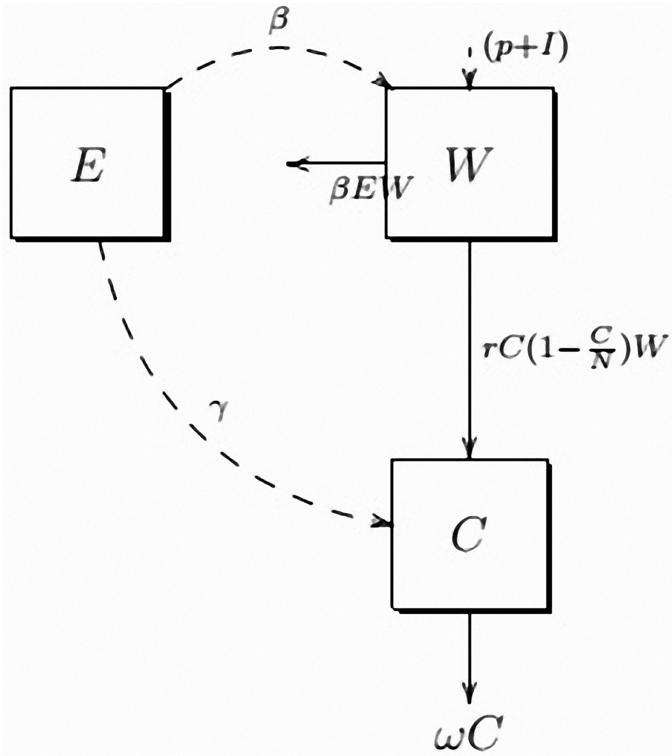


Figure 1. Compartment diagram of dynamics.

Therefore, a model is proposed that represents the growth dynamics of fruit trees, where the supply of irrigation is evidenced in the form of a pulse. This model can be described by the impulsive differential system of equations, Equation (1).

$$\begin{cases} W'(t) &= -\beta q W(t) - rC(t)(1-\frac{C(t)}{N})W(t) & \\ C'(t) &= rC(t)(1-\frac{C(t)}{N})W(t) + \frac{\gamma q C(t)W(t)}{(C+1)(W+1)} - \omega C(t) & t \neq nT. \\ \Delta W(nT) &= I & \\ \Delta C(nT) &= 0 & t = nT. \end{cases} \quad (1)$$

3. Results

In the different simulations, the state variables are represented. In Figure 2, the amount of water in the system (blue line) shows the behavior of this variable with the implementation of an irrigation strategy that considers the application of the same amount of irrigation water in 8 regular periods during the season. Figure 2 shows how the amount of water increases in the moments $t = nt$ in which the irrigation intervention is carried out, subsequently the loss of this is observed due to the effects of evapotranspiration and the demand of the fruit tree. For its part, the growth curve of the biomass of the fruit tree (red segmented line) shows the response that this variable has to the intervention of the irrigation system.

Figure 2 shows a growth of the biomass of the fruit tree as a response to the availability of water, showing greater growth in those stages in which irrigation has been applied, that is, in the moments in which there is the availability of water resources for satisfying the demand of the fruit tree. In Figure 2 the blue line corresponds to the amount of water in the soil ($W(t)$) , and the segmented red line to the biomass concentration of the fruit tree ($C(t)$). The initial conditions are $W(0) = 0.6$, $C(0) = 1$ and parameter values $r = 0.043$, $q = 0.5$, $N = 3000$, $I = 1$, $\beta = 0.06$, $\gamma = 0.001$, $\omega = 0.00001$.

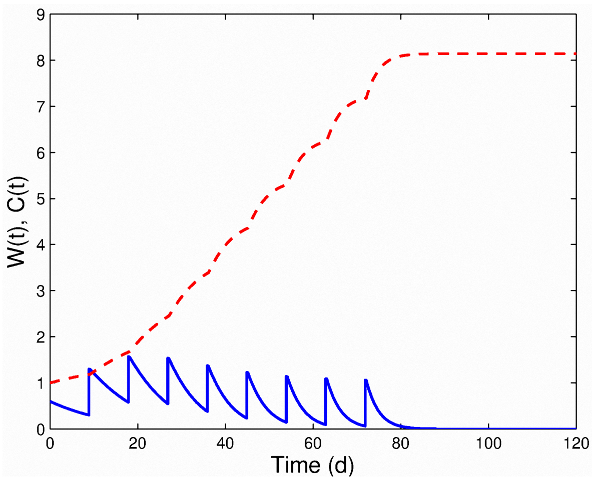


Figure 2. Simulations of the soil water amount (W) and dynamic of growth of the fruit tree biomass (C).

Figure 3 shows the response of biomass growth to different environmental conditions. In this simulation, the irrigation strategy implemented in Figure 2 is preserved. However, the response of the growth of the biomass of the fruit tree is simulated for different values of accumulated energy within the system. The established values are $q = 0.1$, $q = 0.5$, $q = 1$, blue, red and green line respectively. Here, the initial condition is $C(0) = 1$ and parameter values $r = 0.043$, $N = 3000$, $I = 1$, $\beta = 0.06$, $\gamma = 0.001$, $\omega = 0.00001$.

In this regard, the results presented in Figure 3 show how environmental conditions affect the growth and production of fruit trees. The blue line represents the response of the fruit tree to small values of accumulated energy, that is, it presents favorable environmental conditions for the growth and development of the fruit tree.

On the other hand, when considering an average value of accumulated energy (red line), a growth of the fruit tree is evident, however, a maximum yield is not presented. The green line presents an adverse condition in which a loss of crop yield is shown because it is a scenario with an irrigation strategy that does not satisfy the water demands of the fruit tree generated by strong environmental conditions.

The amount of water in the system is presented in Figure 4(a), which shows two different irrigation strategy scenarios. The first scenario (blue line) considers the application of the same amount of water for 8 regular periods of time. The (red line) represents the second scenario in which a fixed amount I of irrigation water is applied in regular longer periods. Both scenarios show the decrease of this variable over time, with the second scenario presenting a more pronounced loss. Here, the initial conditions are $W(0) = 0.6$, $C(0) = 1$ and parameter values $r = 0.043$, $q = 0.5$, $N = 3000$, $I = 1$, $\beta = 0.06$, $\gamma = 0.001$, $\omega = 0.00001$. In addition, the biomass of the fruit tree is represented in Figure 4(b), this shows the response of the fruit tree to the different irrigation strategies. With $W(0) = 0.6, C(0) = 1$ and parameter values $r = 0.043$, $q = 0.5$, $N = 3000$, $I = 1$, $\beta = 0.06$, $\gamma = 0.001$, $\omega = 0.00001$.

In Figure 4(b) the response of fruit growth (blue line) to the first scenario shows that the application of irrigation in regular periods of time enhances the crop, however, there is evidence of alterations in fruit growth, more appreciable in the latter irrigation applications, this occurs because the water demand is not satisfied. The response of fruit growth to the second irrigation strategy (red line) shows that the application of irrigation between long periods of time extends the water requirement of the fruit tree and affects growth by decreasing its biomass.

These results agree with those presented in [20] who used the farming systems model agricultural production systems simulator (APSIM), for simulating and validating crop yield and field water balance of the wheat-corn rotation in the North China plain. The result indicates that an Increased amount of irrigation led to increased crop yield. The simulations presented contribute to the knowledge and description of the fruit growth phenomenon. This methodology broadens the approach to the study of crop modeling, especially fruit trees whose complex dynamics have been studied from physical aspects.

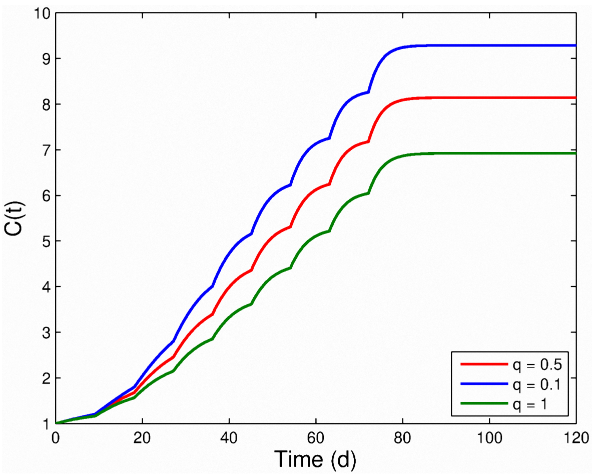
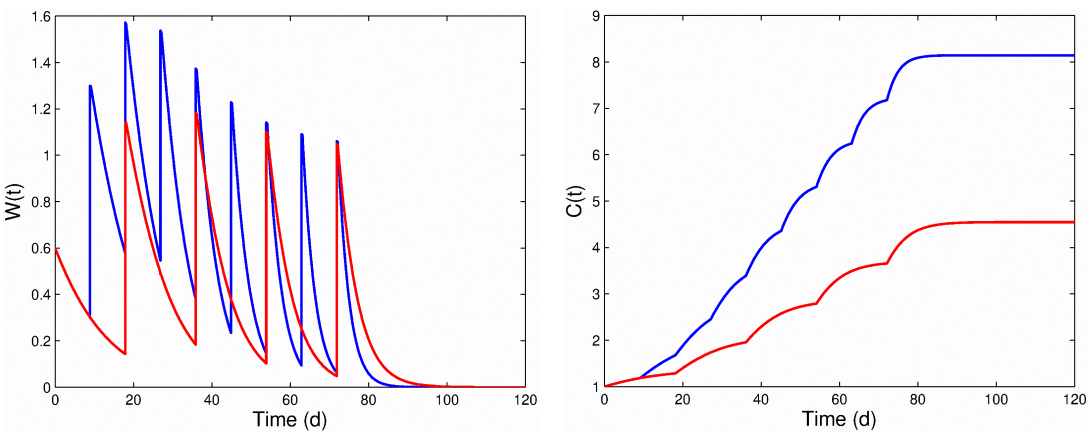


Figure 3. Simulations of dynamic of growth of the fruit tree biomass C.



(a) (b)

Figure 4. Simulations of: (a) the soil water amount $W(t)$, and (b) the dynamic of growth of the fruit tree biomass $C(t)$.

4. Conclusion

In this work, simulations of an impulsive system of nonlinear differential equations were presented to describe the response of fruit tree growth to the application of water by irrigation. The increase in temperature, the irrigation times and the amount of irrigation to be supplied, are some of the factors that were considered in the different simulations, with the aim of showing how the variation of any of them can compromise the growth of the fruit trees. Therefore, the different simulated scenarios respond to concerns that must be addressed when implementing an irrigation strategy that contributes to improving the production of fruit trees.

The results of this work showed the positive effects of the implementation of adequate irrigation scheduling, which satisfies the water demands of the fruit trees considering the adverse climatic conditions that arise. The novelty of this work is that it offers an easy-to-use mathematical qualitative model to numerical simulate the effects of different water supply scenarios on fruit production as an alternative to other complex engineering models.

Therefore, we can conclude that in the current scenarios of decreased rainfall and frequent drought, the application of irrigation water becomes a prevailing need for the production, conservation, and persistence of fruit trees. However, it is necessary to continue with the study of the growth dynamics of fruit trees using impulsive models in which the supply of irrigation variables overtime is considered and to analyze how this affects the development and productivity of fruit trees.

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