Simulations of an impulsive model for the growth of fruit trees

Theme 08 - Introduction to Systems Biology
Reproduce research

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Abstract

Droughts have affected the agriculture in the mediterranean in an increasingly severe matter. The decrease in rainfall and the effects of global warming have taken their toll on many orchards, especially fruit producing ones. Since the water supply of these orchards is always artificial because of the aforementioned factors, dwindling water capacities in reservoirs is a serious issue. This study aims to provide an insight into the effects of different irrigation patterns on the growth of these fruit trees. Without a sustainable plan for irrigation, whole populations of fruit trees might perish under a critical water deficit.

Summary

List of Abbreviations

ODE Ordinary Differential Equation

Contents

1 Introduction

1.1 Purpose

The effects of climate change are an existential threat to planet earth. This paper's purpose is to reproduce and expand on the research done by E Duque-Marin to get a further understanding of the dynamics and different scenarios of fruit tree growth, and by extension shed light on a part of this humongous problem that is already cropping up around the world.

1.2 Theory

The growth of the fruit tree is highly impacted by the different variables in- and outside the tree. To construct an impulsive model, an order of assumptions must be made:

1. The growth dynamics is governed by an interaction between the variables energy, water, vegetative growth.
2. The fruit tree responds instantly to the irrigation application.
3. The model concerned an adult fruit tree (older than 5 years) with a suitable soil surface for the growth of the fruit tree.
4. Ideal agronomic management conditions.
5. Optimal environmental conditions: The energy of the system is constant.

The parameters concerning the model are summarized in Table 1.

Table 1: Parameters used for the model

Parameter	**Meaning**
q	Accumulated energy constant
r	Fruit trees intrinsic growth rate
N	Fruit carrying capacity
I	Irrigation water amount
β	Evapotranspiration rate
γ	Photosynthetic contribution rate
ω	Mortality rate of fruit trees

With that, the state variables can be denoted as following:

1. $E = E(t)$ the solar radiation at time t;
2. $W = W(t)$ the water amount in the soil at time t;
3. $C = C(t)$ the fruit biomass concentration at time t.

Under assumption 5, the state variables are reduced to only W and C. The variation of the amount of water in the system is denoted by $W'(t)$ and the variation in biomass is denoted by $C'(t)$. Considering there is no rainfall (p = 0), the water variation output is due to evapotranspiration at rate β. Besides that, water is crucial for promoting the growth of the fruit tree at rate r.

C'(t) corresponds to the water input at a rate r; in addition, it is also positively affected due to the water-energy-biomass growth interaction at a rate γ. There is an exit at an ω rate for the loss of natural death of the crop. This behavior applies to a continuous timescale (long duration), that governs the growth dynamics of the fruit tree. However, this model also presents a short timescale (pulse) in discrete time that represents the events in which water enters the system through irrigation supply.

The dynamics between the variables energy, amount of water, and concentration of biomass of the fruit tree are shown in Figure 1, adapted from **??**; where: E = Energy; W = Water amount; C = Fruit biomass concentration; r = Fruit trees intrinsic growth rate; N = Fruit carrying capacity; β = Evapotranspiration rate; γ = Photosynthetic contribution rate; ω = Mortality rate of fruit trees; p = Rainfall rate; I = Irrigation amount.

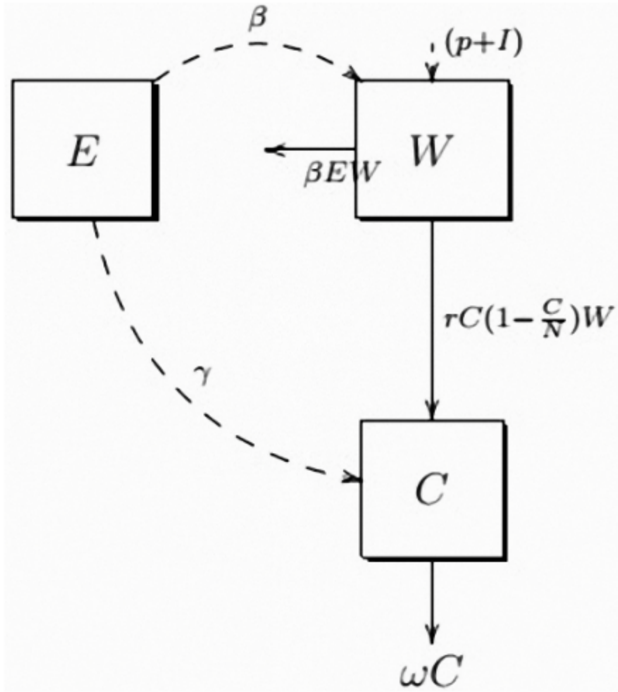


Figure 1: Diagram of dynamics

A model can be proposed that represents the growth dynamics of fruit trees, where the supply of irrigation is evidenced in the form of a pulse, and thus be described by impulsive differential equations 1.2.

Equations:

$$\begin{array}{ll} W'(t) = -\beta q W(t) - rC(t)(1 - \frac{C(t)}{N} W(t) & \\ C'(t) = rC(t)(1 - \frac{C(t)}{N})W(t) + \frac{\gamma q C(t) W(t)}{(C+1)(W+1)} - \omega C(1) & t \neq nT \\ \Delta W(nT) = I & \\ \Delta C(nT) = 0 & t = nT \end{array} \tag{1.2}$$

2 Materials and Methods

Since no datasets were used in the production of the results, the simulation data had to be generated. This data was plotted using R graphics.

2.1 Materials

The simulation data was obtained using the ODE (ordinary differential equation) function of the deSolve package. This function takes differential equations and parameters and calculates the output of these functions. Aforementioned equations can ben found in the theory section. Other packages like ggplot2, ggpubr, formatR and scales were used in data visualisation. (2)

Table 2: Software and packages

Software	**Package**	**Version**
R		4.0.4
	deSolve	1.32
	formatR	1.11.1
	ggplot2	3.3.5
	ggpubr	0.4.0
	scales	1.1.1

The simulation data consists of an index, the *time*, water level in the soil (W) and biomass (C). (3)

Table 3: The simulation data

	time	W	C
1	0.00000	0.600000	1.00000
2	1.00000	0.556209	1.02602
3	2.00000	0.514991	1.05076
4	3.00000	0.476281	1.07421
5	4.00000	0.440001	1.09637

2.2 Methods

The differential equations were translated into a mathematical model that deSolve could understand. The last two equations also needed to be incorporated; this was done through if/else statements. After which several tests were run to ensure parity with the described model in the paper. See Appendix B. File: results1.

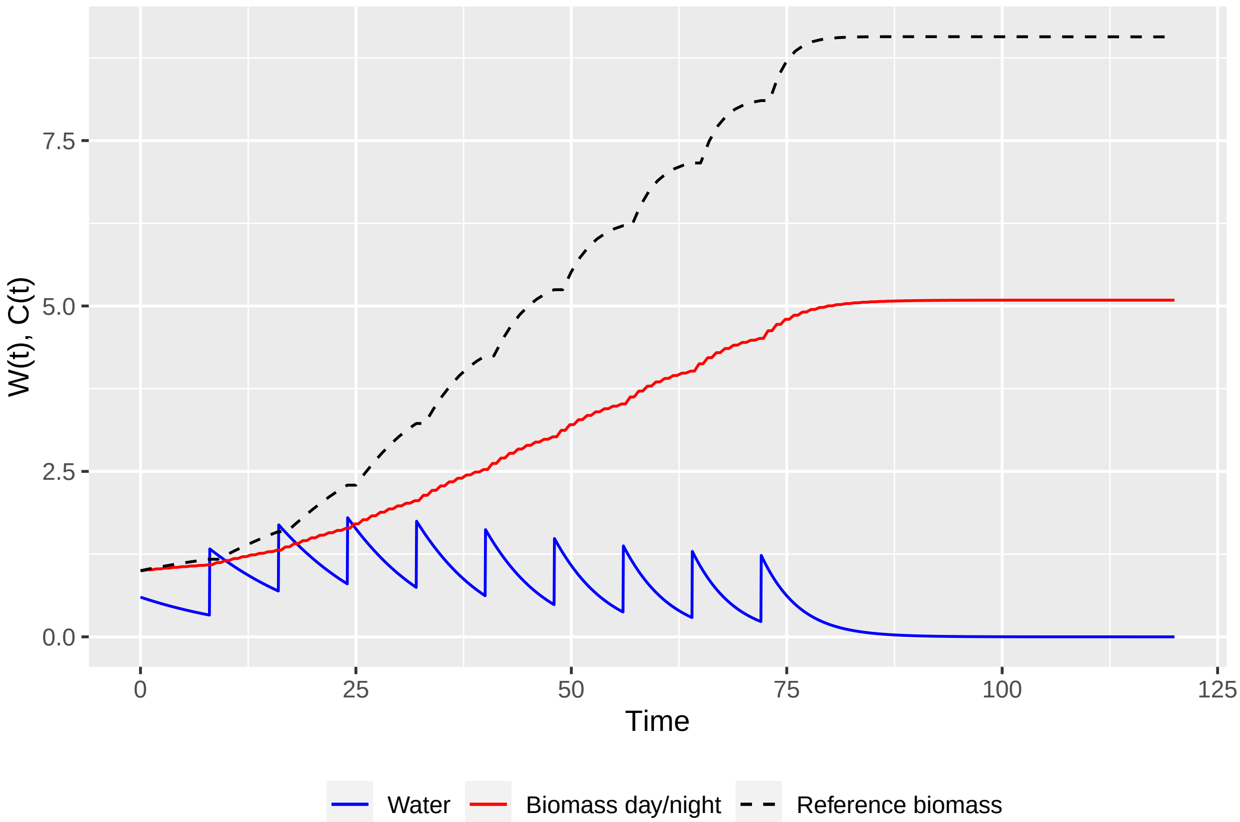
After this, the model was updated to use 1h steps instead of 1 day. This lead to the model being adapted to incorporate different growth rates for day and night, for even more precise data. See Appendix C. File: results2.

Another few models were made to compare different irrigation intervals, as well as a model for watering the soil as soon as almost all water was drained from it. See Appendix D and Appendix E. Files: results3 & results4.

3 Results

3.1 Replication

The growth of the fruit trees can be shown in multiple simulations, tweaking a few variables to see the differences between these. In Figure 2a, the courses of the two state variables are shown: The water amount in the soil (blue) and the growth of the fruit tree's biomass (red). Figure 2b shows what happens with the growth of the fruit tree if the parameter q changes, for $q = 0.1$ in red, $q = 0.5$ in black (the default value), and $q = 1$ in blue.



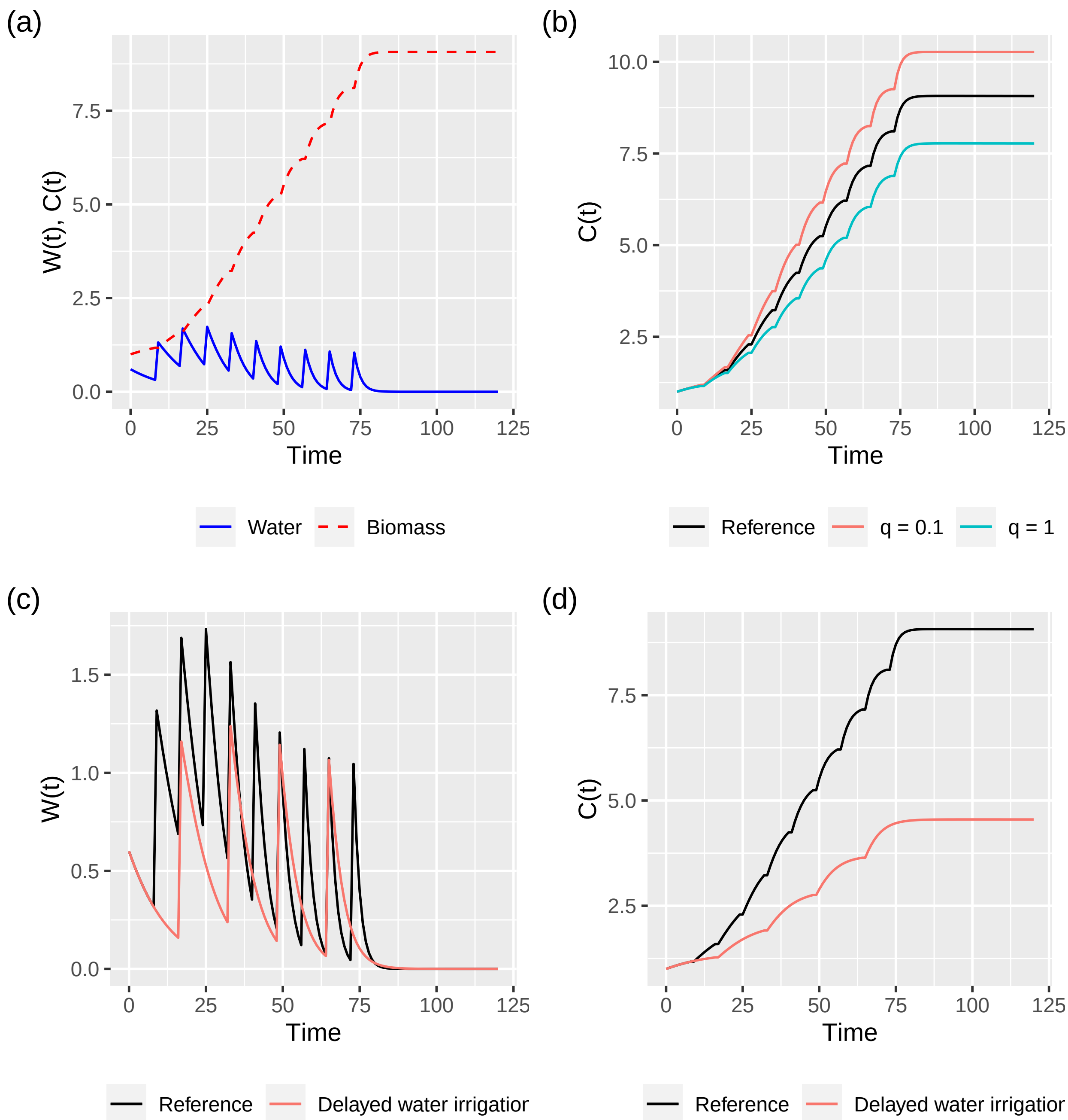
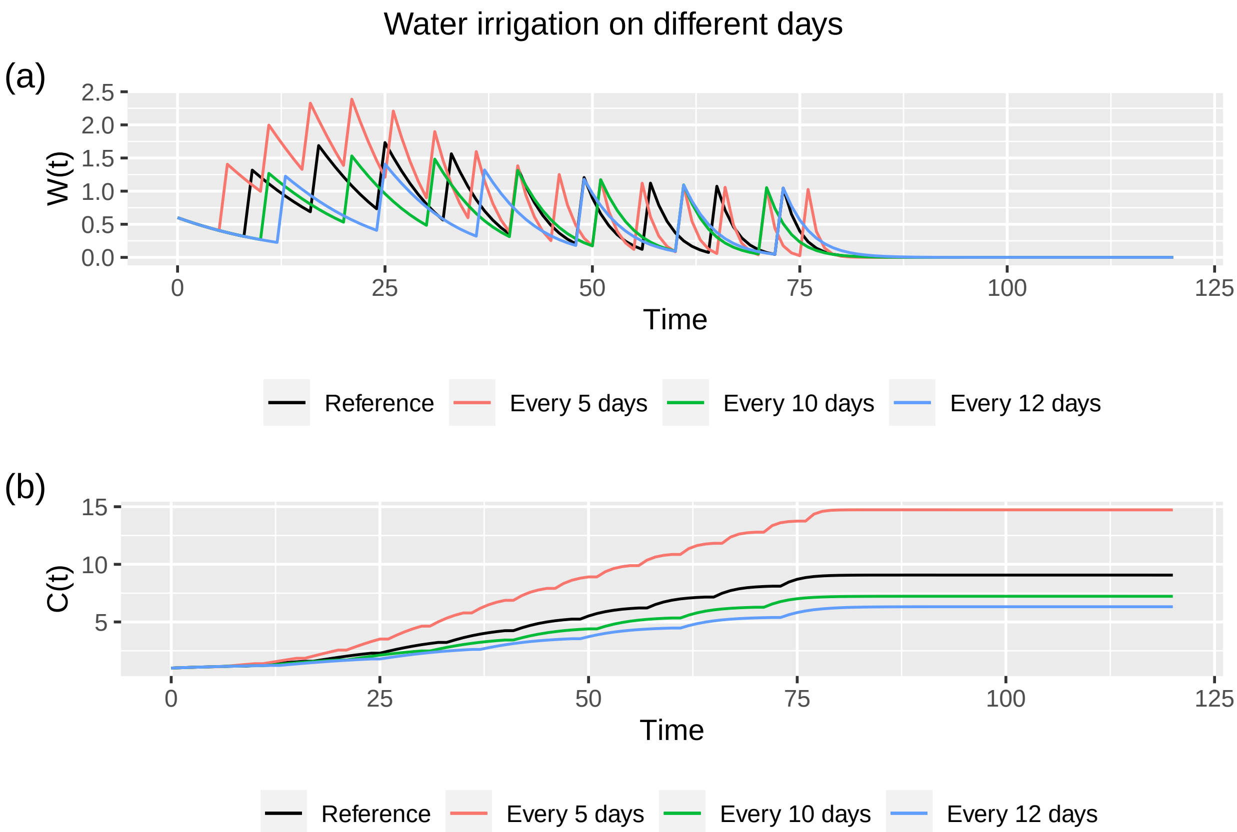
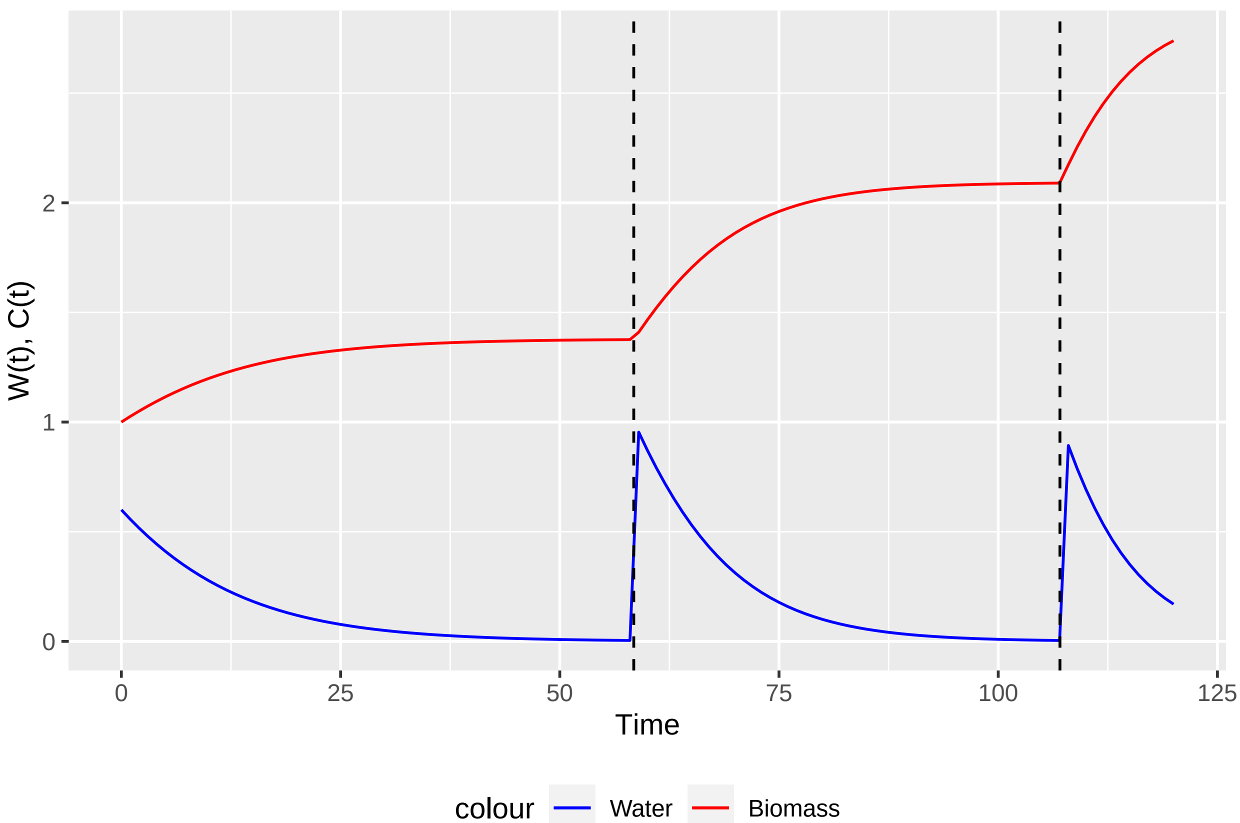


Figure 2: The different simulations



Water irrigation on different days
(a)
W(t)
2.5
2.0
1.5
1.0
0.5
0.0
0
25
50
75
100
125
Time
Reference
Every 5 days
Every 10 days
Every 12 days
(b)
C(t)
15
10
5
0
25
50
75
100
125
Time
Reference
Every 5 days
Every 10 days
Every 12 days



W(t), C(t)
2
1
0
0
25
50
75
100
125
Time
colour
Water
Biomass

4 Conclusion

5 Discussion

6 References

7 Appendix