Convergence of $\frac{F_{n-2}}{F_n}$

$$
\begin{aligned}
\frac{F_{n-2}}{F_n} &= \frac{F_{n-2}}{F_{n-2} + F_{n-1}} \\
&= \left(\frac{F_{n-2}}{F_{n-2}}\right) \frac{1}{1 + \frac{F_{n-1}}{F_{n-2}}} \\
&= \frac{1}{1 + \frac{F_{n-2}+F_{n-3}}{F_{n-2}}} \\
&= \frac{1}{2 + \frac{F_{n-3}}{F_{n-2}}} \\
&= \frac{1}{2 + \frac{1}{1+\frac{F_{n-4}}{F_{n-3}}}} \\
&= \dots
\end{aligned}
$$

Clearly there is a continued fraction of the form

$$
\frac{F_{n-(k+1)}}{F_{n-k}} = \frac{1}{1 + \frac{F_{n-(k+2)}}{F_{n-(k+1)}}}
$$

which expands to

$$
\frac{1}{1 + \frac{1}{1+\dots}}
$$

(TODO) which converges to $1 - \phi$.
For a large enough n

$$
\begin{aligned}
\frac{1}{2 + \frac{F_{n-3}}{F_{n-2}}} &= \frac{1}{2 + (\phi - 1)} \\
&= \frac{1}{1 + \phi}
\end{aligned}
$$

Given $n = F_i$, estimate i

Given the above convergence, F_n can be computed from F_{n-2}:

$$\frac{F_{n-2}}{F_n} = \frac{1}{1+\phi}$$
$$F_n = (1+\phi)\,F_{n-2}$$

Therefore, starting from F_1 any F_{2k+1} can be reached[1] by performing $(1+\phi)^k F_1$, $k \geq 0$.

So to find the number of steps from some $n = F_{2k+1}$ to F_1, solve for k:

$$n\left(\frac{1}{1+\phi}\right)^k = 1$$
$$k = \frac{\log\left(\frac{1}{n}\right)}{\log\left(\frac{1}{1+\phi}\right)}$$

To get the actual index perform $2 * f(n)$ as $f(n)$ skips half the indices.

TODO: Verify indices are correct, assuming $F_1 = 1, F_2 = 1$.

[1]Requires $k > 40$, give or take.