

# BEF Unified Architecture: State Over Time + HSSA (Hash Shielded Streaming Accumulator)

A detailed architectural description of the BEF pipeline (BICEP  $\rightarrow$  ENNC++  $\rightarrow$  FusionAlpha) and the accompanying cryptographic accumulator for temporal traces.

## Executive Summary

This document specifies the BEF pipeline from data generation to planning, then formalizes and implements a cryptographic accumulator (HSSA) that binds epoch traces and enables  $O(1)$  global checks with quantified error. We include math definitions, algorithms, encoding rules, GPU kernel design, JSON schemas, benchmarks, and verification strategies.

## 1. BICEP: State Over Time Engine

**Purpose:** *generate temporal traces via SDE simulation with reproducible stochasticity and controlled numerical schemes.*

- 1 Integrators: Euler–Maruyama (Itô), Heun midpoint (Stratonovich), Milstein (with derivative terms planned).
- 2 Noise: counter-based PRNG; Brownian increments scaled by  $\sqrt{dt}$ ; variance reduction via antithetic pairs.
- 3 Models: GBM, OU, double well; extendable to domain-specific drifts/diffusions.
- 4 State layout: dense vectors (n algebra DVector), time stamps, path/ensemble containers.
- 5 I/O: structured exports to CSV/Parquet; for crypto, exports as flattened integer sequences (mod  $p$ ) with chunk metadata.
- 6 Determinism knobs: seeds,  $dt$ , stride, chunk boundaries.

Dataflow: Each simulation produces a sequence of states  $x_t$ . We derive integers  $v[i]$  from  $x_t$  (e.g., quantized features, recorded transitions) and partition into fixed-size chunks for downstream modules.

## 2. ENNC++: Entangled Neural Network Encoder

**Purpose:** *encode sequences/traces into latent summaries; learn committor functions, sequence tasks, or domain-specific features.*

- 1 Cell dynamics:  $\psi_{t+1} = \tanh(W_x x_t + W_h h_t + (E - \lambda I) \psi_t + b)$  with  $E = L L^T$  (PSD).
- 2 Collapse/attention:  $\alpha = \text{softmax}(W_m \psi)$ ,  $y = \alpha^T \psi$ ; temperature control; entropy as uncertainty.

- 3 Training: BPTT, Adam/AdamW, schedulers (cosine/linear), gradient clipping, regularization.
- 4 Validation: finite difference grad checks ( $1e-10$ ), PSD eigenvalue checks, softmax stability tests.
- 5 Performance: OpenMP + Eigen vectorization; zero copy; small allocations; configurable batch/seq lengths.
- 6 I/O contracts: CSV/NPY readers; toy generators (parity, copy, double well); can ingest per chunk features (e.g., HSSA sketch\_vec).

Integration: ENN can be used to learn summaries over chunks (e.g., map sketch\_vec → committor or risk scores). It is optional for auditing: the cryptographic layer stands alone.

### 3. FusionAlpha: Graph Planning & Contradiction

***Purpose: build state-time graphs and propagate priors with severity/risk controls; focus attention on contradiction-heavy regions.***

- 1 Graph builders: HumanoidMaze (positions), AntSoccer (ball/agent), Puzzle (toggle dynamics).
- 2 Priors: committor/confidence from ENN; boundary conditions (goal nodes); density priors.
- 3 Propagation: severity-scaled diffusion/propagation steps; risk-sensitive variants; pick\_next\_node by argmax  $q$ .
- 4 Contradiction: per node scores where local checks disagree with global trends; use to bias sampling/audits.
- 5 Bindings: Python + Rust (bindings crate); demos and integration tests.

Integration: HSSA per chunk fingerprints can be node features; FusionAlpha can propose where to audit Merkle paths when contradiction is high.

## 4. Cryptographic Accumulator (HSSA)

Goal: bind a temporal epoch trace with a Merkle transcript and a  $k$ -dimensional polynomial fingerprint; enable  $O(1)$  global checks with explicit SZ error.

### Definitions

Field  $p$  (default  $2^{61}-1$ ). Trace  $v \in \mathbb{F}^L$ . Chunks  $C_j$  of size  $B$ .  $\text{Root}_j = \text{Merkle}(\text{SHA256}(\text{encode\_u256}(v[i] \bmod p)) \text{ over } C_j)$ . Transcript  $T = \text{Root}_0 \parallel \dots \parallel \text{Root}_{\{m-1\}} \parallel \text{encode\_u256}(L)$ . Challenges  $r_j = \text{SHA256}(T \parallel \text{encode\_u32}(j)) \bmod p$ ,  $j \in [0..k-1]$ ,  $r_j \neq 0$ . Polynomial  $f(X) = \sum (v[i] \bmod p) \cdot X^i$ . Fingerprints  $S_j = f(r_j) \bmod p$ .

### Commit/Open/Verify

- 1 Commit( $v$ ): output  $(p, L, \text{Roots}, r, S)$  and optionally per-chunk sketch\_vec.
- 2 Open( $i$ ): return  $(v[i], j, \text{Merkle path to Root}_j)$ .
- 3 Verify: check Merkle path; optional audit: recompute  $\sum v[i] \cdot r_j^i$  for a sample or entire vector; compare to  $S_j$ .

### Security: Binding

If  $v \neq v'$  produce same  $(\text{Roots}, L, S)$ , then either Merkle collision or  $\delta(X) = f - f'$  vanishes at all  $r_j$ . Schwartz–Zippel:  $\Pr[\delta(r_j) = 0 \ \forall j] \leq (\deg \delta / p)^k \leq (\min\{t, L\}/p)^k$ . Overall:  $\text{Adv}_{\text{forge}} \leq \text{Adv}_{\text{crh}} + (\min\{t, L\}/p)^k$ .

## 5. Encoding and Chunking

Deterministic transcript: fixed  $B$ ; stable hashing; big-endian encodings; contiguous root list; length as  $32$ -byte big-endian. Last chunk shorter allowed; order bound by root ordering.

## 6. Verification Strategies

- 1 Equality: compare  $S$  across parties for identical transcript ( $O(1)$ ).
- 2 Full audit: recompute  $S_j$  in  $O(L)$ .
- 3 Partial audit: sample indices; verify Merkle paths; add contributions; combine with SZ error.

## 7. CUDA Engineering

- 1 Mod prime  $2^{61}-1$ :  $128$ -bit products, fold reduction.
- 2  $r^i$  builder: anchors $\times$ base tiling; device-resident  $r^i$  vector;  $O(L)$ .

- 3 Per■chunk dot: coalesced loads, shared memory reductions per block.
- 4 Multi■challenge: loop over k; reuse builder; option to fuse later.
- 5 Memory:  $r^i$  length L; values streamed by chunk; per■chunk outputs sketch\_vec.

Benchmarks: GTX1650 L=12.5M  $\rightarrow r^i \approx 44.9\text{ms}$ ; accumulate  $\approx 33.6\text{ms}$ ; CPU  $\approx 69.1\text{s}$ . A100 L=12.5–20.5M  $\rightarrow r^i \approx 4\text{--}7\text{ms}$ ; accumulate  $\approx 2\text{--}4\text{ms}$ ; CPU  $\approx 53\text{--}92\text{s}$ .

## 8. JSON Schemas and Tools

```
bef_trace_v1: {schema, trace_id, field_modulus, vector_length, chunk_length, chunks:[{chunk_index, offset, values}]}
```

```
bef_sketch_v1: {schema, trace_id, field_modulus, seed, length, challenges:[...],  
global_sketch_vec:[...], chunks:[{chunk_index, offset, length, root_hex, sketch_vec:[...]}],  
timing_ms:{cuda_rpow, cuda_chunks}}
```

Tool: sketch\_trace.py takes bef\_trace\_v1 and writes bef\_sketch\_v1 with chosen (p,k).

## 9. Threat and Update Model

Adversary may attempt (a) change v but keep T (Merkle collision), or (b) change both v and T and hope sketches collide (SZ). Epoch discipline: freeze T per batch; derive fresh  $r_i$ ; do not reuse r across epochs; cache  $r^i$  within epoch.

## 10. Comparison Axes (Plain Terms)

- 1 Merkle tries:  $O(\log N)$  index proofs; no  $O(1)$  global check; HSSA adds  $S_{\text{Merkle}}$ .
- 2 Vector commitments: succinct per-index; HSSA offers global SZ checks under CRH/RO (no setup).
- 3 Polynomial commitments: evaluation proofs; HSSA publishes evaluations for equality/audits (no setup).
- 4 Streaming fingerprints: unauthenticated; HSSA authenticates via transcript.

## 11. Colab/Runtime Notes

Use GPU runtime; install ninja; clear Torch extension cache if needed. demo\_cuda(num\_chunks, chunk\_len, seed, num\_challenges). sketch\_trace.py --modulus p --num-challenges k. Recommended  $p=2^{61}-1$ ,  $k=4$ .

## 12. Roadmap

- 1 Formalize partial audit detection probabilities (sampling + SZ).
- 2 Adaptive/multi-epoch analysis in QROM.
- 3 Fused multi-r kernel for higher k.
- 4 Alternative fast mod primes (e.g.,  $2^{64}-2^{32}+1$ ), quantify constants.

This document unifies system and cryptography views with explicit interfaces, parameters, and performance, enabling reproducible evaluation and fair comparison.

