Flight Model

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1 Lift

The buoyant force on a balloon of diameter d with density of content ρ_{in} and environmental density ρ_{out}:

$$F_b = (\rho_{out} - \rho_{in})g\frac{\pi d^3}{6}, \tag{1}$$

where content is Helium with density (at STP):

$$\rho_{in} = 0.1786 \quad [\mathrm{kg\,m^{-3}}]. \tag{2}$$

The environment is standard atmosphere taken from [1] at zero altitude:

$$\rho_{out} = 1.225 \quad [\mathrm{kg\,m^{-3}}]. \tag{3}$$

under following conditions:

$$\begin{aligned} p &= 1.013\,25 \times 10^5 & [\mathrm{kg\,m^{-3}}]. \\ T &= 288.15 & [\mathrm{K}]. \end{aligned} \tag{4}$$

Under the conditions Helium density is:

$$\rho_{in} = 0.1786 \cdot \frac{1.013\,25 \times 10^5}{1 \times 10^5} \cdot \frac{273.15}{288.15} = 0.1715 \quad [\mathrm{kg\,m^{-3}}]. \tag{5}$$

Let's simplify equation (1):

$$F_b = k_0 d^3, \tag{6}$$

where k_0 is:

$$k_0 = \frac{\pi}{6} \cdot 9.81 \cdot (1.225 - 0.1715) = 5.411. \quad [\mathrm{N\,m^{-3}}] \tag{7}$$

For example balloons with diameters 0.36, 0.70 and 1.15 m get following lifts:

$$\begin{aligned} F_s &= 5.411 \cdot 0.36^3 = 0.2525[\mathrm{N}] = 25.74[\mathrm{gf}]; \\ F_m &= 5.411 \cdot 0.70^3 = 1.856[\mathrm{N}] = 189.3[\mathrm{gf}]; \\ F_b &= 5.411 \cdot 1.15^3 = 8.230[\mathrm{N}] = 839.2[\mathrm{gf}]. \end{aligned} \tag{8}$$

These balloons have following volumes:

$$\begin{aligned} V_s &= 24.43 \quad [\mathrm{L}]; \\ V_m &= 179.6 \quad [\mathrm{L}]; \\ V_b &= 796.3 \quad [\mathrm{L}], \end{aligned} \tag{9}$$

which is approximately $\frac{1}{57}$, $\frac{1}{7}$ and $\frac{2}{3}$ of 10 L tank.

References

[1] NOAA, NASA, USAF, US. Standard Atmosphere, 1976