#SERVETHELOOP DUMP (FOR THE RLOOP)

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Contents

Abstract. servetheloop notes "dump" - I dump all my notes, including things that I tried but are wrong, here.

Part 1. Eddy Currents, Eddy Current Braking

1. Eddy Currents

Keywords: Eddy currents;

cf. Smythe (1968), Ch. X (his Ch. 10) [2]

Assume Maxwell's "displacement current" is negligible; this is ok if frequencies are such that wavelength λ large compared to dimensions of apparatus L. $\lambda \gg L$ or $\frac{c}{\nu} \gg L$.

I will write down the "vector calculus" formulation of electrodynamics, along side Maxwell's equations, or electrodynamics, over spacetime manifold M. The latter formulation should specialize to the "vector calculus" formulation.

From

$$\text{curl}\mathbf{E} = -\frac{d\mathbf{B}}{dt}\,(SI) \qquad \text{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}\,(cgs) \text{ or } \frac{\partial B}{\partial t} + \mathbf{d}E = 0$$

Suppose $B = \text{curl}A$ or $B = \mathbf{d}A$ (EY 20170528: is this where the assumption above about $\lambda \gg L$ comes in?), then

$$-\frac{\partial B}{\partial t} = \mathbf{d}E \xrightarrow{\int_S} \int_S \mathbf{d}E = \int_{\partial S} E = -\int_S \frac{\partial B}{\partial t} \xrightarrow{B=\mathbf{d}A} -\int_S \frac{\partial}{\partial t}\mathbf{d}A \xrightarrow{\text{flat space}} \int_S \mathbf{d}E = -\int_S \mathbf{d}\frac{\partial A}{\partial t}$$

and so

$$\mathbf{E} = \frac{-\partial \mathbf{A}}{\partial t} \tag{1}$$

up to gauge transformation, if $B = \mathbf{d}\mathbf{A} = \text{curl}\mathbf{A}$

Since this $\mathbf{E}$ field is formed in a conductor, Ohm's law applies. Let's review Ohm's law. Smythe (1968) refers to its 6.02 Ohm's Law - Resistivity section [2]. Indeed, in a lab, the definition of resistance can be defined as this ratio:

$$R_{AB} := \frac{-\int_A^B \mathbf{E}}{I_{AB}} = \frac{V_A - V_B}{I_{AB}} = \frac{\varepsilon_{AB}}{I_{AB}} \tag{2}$$

Moving along, the right way to think about resistivity ρ is to consider conductivity.

Assume current density is linear to $\mathbf{E}$ field (as $\mathbf{E}$ field pushes charges along). This linear response is reasonable.

Also, assume current density $\mathbf{J}$ is uniform over infinitesimal surface area dA (i.e. surface S).

Define

$$\sigma \equiv \quad \textbf{conductivity} \tag{3}$$

Then the empirical relation/equation that underpins *Ohm's law* is

$$\mathbf{J} = \sigma \mathbf{E} \tag{4}$$

and define *resistivity* from there:

$$\sigma := \frac{1}{\rho} \tag{5}$$

where ρ is the *resistivity*.

Thus

$$\sigma \int_A^B -\mathbf{E} = \sigma V_{AB} = \int_A^B \mathbf{J} = I_{AB}\frac{l}{A}$$

$$\frac{1}{\rho}R = \frac{l}{A} \text{ or } \boxed{R = \rho\frac{l}{A}}$$

From Maxwell's Equations,

$$\delta(B - 4\pi\mathbf{M}) = \frac{4\pi}{c}J_{\text{free}} \tag{6}$$

If $B = H + 4\pi M = (1 + 4\pi\chi_m)H = \mu H$, then

$$\delta H = \delta\frac{B}{\mu} = \frac{4\pi}{c}J_{\text{free}} \text{ or } \delta B = \mu\frac{4\pi}{c}J_{\text{free}} \Longleftrightarrow \text{curl}B = \mu J_{\text{free}} \qquad (SI) \tag{7}$$

and if $B = \mathbf{d}A$ and $\mathbf{d}\delta A = 0$.

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I build upon the physical setup proposed by Jackson (1998) [3] in Section 5.18 "Quasi-Static Magnetic Fields in Conductors; Eddy Currents; Magnetic Diffusion."

For a system (with characteristic) length L, L being small,
compared to electromagnetic wavelength associated with dominant time scale of problem T,

$$f := \frac{1}{T}; \quad \omega = 2\pi f; \quad \omega\lambda = c \Longrightarrow \lambda = \frac{c}{\omega} = \frac{c}{2\pi f} = \frac{Tc}{2\pi}$$
$$\frac{L}{\lambda} = \frac{LTc}{2\pi} \gg 1$$

From Maxwell's equations, in particular, Faraday's Law, and in its integral form (over 2-dim. *closed* surface S),

$$\mathbf{d}E + \frac{\partial}{\partial t} B = 0 \text{ or } -\mathbf{d}E = \frac{\partial B}{\partial t} \xrightarrow{\int_S} \int_S \frac{\partial B}{\partial t} = -\int_S \mathbf{d}E = -\int_{\partial S} E \tag{8}$$

So on S, changing magnetic flux $\int \frac{\partial B}{\partial t}$ results in E field, circulating around boundary of S, ∂S.

We know that in a conductor, free conducting electrons get pushed around by E fields, result in a current density J.

J is related to E, *empirically* (by Ohm's Law)

$$J = \sigma E$$

where σ is the resistivity.

Then use the force law on this induced current J from the B field set up:

$$F_{\text{net}} = \frac{1}{c} \int_S J \times B dA$$

By working through the right-hand rule, F_{net} the force on those currents induced in the conductor due to the B that's there, is in the direction to help oppose changing (increasing or decreasing $\frac{\partial B}{\partial t}$).

To find B, suppose $B = dA$, i.e. $B \in H^2_{\text{deRham}}(M)$, i.e. $B = \text{curl}A$.

For sure,

$$\delta(B - 4\pi c\mathbf{M}) = 4\pi J \Longleftrightarrow \text{curl}(B - 4\pi c\mathbf{M}) = \text{curl}H = 4\pi J$$

Be warned now that the relation $B = \mu H$ may not be valid on all domains of interest; μ could even be a tensor! (e.g. $B_{ij} = \mu^{kl}_{ij} H_{kl}$). However, both Jackson (1998) [3] in Sec. 5.18 Quasi-Static Magnetic Fields in Conductors; Eddy Currents; Magnetic Diffusion, pp. 219, and Smythe (1968), Ch. X (his Ch. 10), pp. 368 [2], continues on *as if* this relation is linear: $B = \mu H$.

Nevertheless, as we want to find B by finding its "vector potential" A, we obtain a diffusion equation:

$$\begin{aligned} -\delta B = *\mathbf{d} * \mathbf{d}A = (-1)\delta\mathbf{d}A = (-1)(\Delta - \mathbf{d}\delta)A \xrightarrow{\mathbf{d}\delta A = 0} -\Delta A = \\ = 4\pi\mu J = 4\pi\mu\sigma E = 4\pi\mu\sigma\left(-\frac{\partial A}{\partial t}\right) \\ \Longrightarrow \boxed{\Delta A = 4\pi\mu\sigma \frac{\partial A}{\partial t}} \end{aligned} \tag{9}$$

where in the first 2 steps (equalities), $-\delta B = *\mathbf{d} * \mathbf{d}A = (-1)\delta\mathbf{d}A$ it's interesting to note that the codifferential δ for the 2 form B had to be written out explicitly, and then the codifferential for the 1-form A is *different* from the δ for B by a(n important) factor of (-1); where $\mathbf{d}\delta A = 0$ must be satisfied by the form A takes; and where, since $B = \mathbf{d}A$,

$$\mathbf{d}E + \frac{\partial B}{\partial t} = \mathbf{d}E + \frac{\partial}{\partial t}\mathbf{d}A = \mathbf{d}\left(E + \frac{\partial A}{\partial t}\right) = 0 \Longrightarrow E = -\frac{\partial A}{\partial t} + \text{grad}\Phi \xrightarrow{\Phi = \text{ constant}} E = -\frac{\partial A}{\partial t} \tag{10}$$

whereas a choice of gauge for E was chosen so that $\Phi =$ constant (and so a particular form for E was chosen, amongst those in the *same* equivalence class of $H^1_{\text{deRham}}(M)$.

To ensure that the differential geometry formulation is in agreement with the practical vector calculus formulation, compare Eq. 9 with Eq. (5.160) of Jackson (1998) [3] and Eq. (10) in Sec. 10.00 of Smythe (1968) [2].

To summarize what's going on, I think one should at least understand in one's head how Maxwell's Equations apply, (and I will try to write in SI here)

$$\boxed{\begin{gathered} \int_S \frac{\partial \mathbf{B}}{\partial t} dA = -\oint \mathbf{E} \cdot d\mathbf{s} \Longrightarrow \mathbf{J} = \sigma\mathbf{E} \Longrightarrow \mathbf{F}_{\text{net}} = \int_S \mathbf{J} \times \mathbf{B} dA \\ \text{find } \mathbf{B} = ? \qquad \text{using form } \mathbf{B} = \nabla \times \mathbf{A}, \\ \nabla^2 \mathbf{A} = \mu\sigma \frac{\partial \mathbf{A}}{\partial t} \qquad (SI) \end{gathered}} \tag{11}$$

where, a change in magnetic flux over a surface S over the conductor, $\int_S \frac{\partial \mathbf{B}}{\partial t} dA$ induces a circulation of E field around S, $-\oint \mathbf{E} \cdot d\mathbf{s}$, and this E field is pushing around *free conducting charges* according to Ohm's law, $\mathbf{J} = \sigma\mathbf{E}$, with σ being the conductivity of the conducting material, and this current density $\mathbf{J}$ is then acted upon by the prevailing B field, according to the usual force law. To find $\mathbf{B}$, one can find $\mathbf{A}$ and *try* to find $\mathbf{A}$ analytically.

Keep in mind that for $\nabla^2 \mathbf{A} = \mu\sigma \frac{\partial \mathbf{A}}{\partial t}$, we had used, critically, the Maxwell equation $\nabla \times \mathbf{H} = \mathbf{J}$, with $\mathbf{J}$ being the *induced current of free conducting charges on the conductor*. This $\mathbf{H}$ will contribute (through linear superposition) to the $\mathbf{B}$ that could already be there due to the permanent magnet.

What can we measure quantitatively?

- Can we measure $\mathbf{J}$ inside (on) the conductor?
- Can we separate magnetization $\mathbf{M}$ of material from $\mathbf{B}$, to obtain the actual $\mathbf{B}$ (and then use linear superposition, $\mathbf{B}_{\text{total}} = \mathbf{B}_{\text{permanent magnet}} + \mathbf{B}_{\text{induced currents}}$?

Also, keep in mind the context that the conductor at hand is the long, almost semi-infinite rectangle of a conductor, aluminum sub-rail, specified by the SpaceX Hyperloop. Force on that will cause an equal and opposite force on the pod, with its permanent magnets attached, and thus braking the pod.

References

[1] Scott B. Hughes. **Magnetic braking: Finding the effective length over which the eddy currents form**. Magnetic braking: Finding the effective length over which the eddy currents form
[2] William R. Smythe, **Static and Dynamic Electricity**. 3rd ed. (McGraw-Hill, New York, 1968).
[3] J.D. Jackson. **Classical Electrodynamics** Third Edition. Wiley. 1998. ISBN-13: 978-0471309321