SERVETHELOOP DUMP (FOR THE RLOOP)

ERNEST YEUNG ERNESTYALUMNI@GMAIL.COM

gmail : ernestyalumni
linkedin : ernestyalumni
twitter : ernestyalumni

Contents

Abstract. servetheloop notes "dump" - I dump all my notes, including things that I tried but are wrong, here.

Part 1. Eddy Currents, Eddy Current Braking

1. Eddy Currents

Keywords: Eddy currents;

cf. Smythe (1968), Ch. X (his Ch. 10) [2]

Assume Maxwell's "displacement current" is negligible; this is ok if frequencies are such that wavelength λ large compared to dimensions of apparatus L. $\lambda \gg L$ or $\frac{c}{\nu} \gg L$.

I will write down the "vector calculus" formulation of electrodynamics, along side Maxwell's equations, or electrodynamics, over spacetime manifold M. The latter formulation should specialize to the "vector calculus" formulation.

From

$$\text{curl}\mathbf{E} = -\frac{d\mathbf{B}}{dt}\,(SI) \qquad \text{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}\,(cgs) \text{ or } \frac{\partial B}{\partial t} + \mathbf{d}E = 0$$

Suppose $B = \text{curl}A$ or $B = \mathbf{d}A$ (EY 20170528: is this where the assumption above about $\lambda \gg L$ comes in?), then

$$-\frac{\partial B}{\partial t} = \mathbf{d}E \xrightarrow{\int_S} \int_S \mathbf{d}E = \int_{\partial S} E = -\int_S \frac{\partial B}{\partial t} \xrightarrow{B=\mathbf{d}A} -\int_S \frac{\partial}{\partial t}\mathbf{d}A \xrightarrow{\text{flat space}} \int_S \mathbf{d}E = -\int_S \mathbf{d}\frac{\partial A}{\partial t}$$

and so

$$\mathbf{E} = \frac{-\partial \mathbf{A}}{\partial t} \tag{1}$$

up to gauge transformation, if $B = \mathbf{dA} = \text{curl}\mathbf{A}$

Since this $\mathbf{E}$ field is formed in a conductor, Ohm's law applies. Let's review Ohm's law. Smythe (1968) refers to its 6.02 Ohm's Law - Resistivity section [2]. Indeed, in a lab, the definition of resistance can be defined as this ratio:

$$R_{AB} := \frac{-\int_A^B \mathbf{E}}{I_{AB}} = \frac{V_A - V_B}{I_{AB}} = \frac{\varepsilon_{AB}}{I_{AB}} \tag{2}$$

Moving along, the right way to think about resistivity ρ is to consider conductivity.

Assume current density is linear to $\mathbf{E}$ field (as $\mathbf{E}$ field pushes charges along). This linear response is reasonable.

Also, assume current density $\mathbf{J}$ is uniform over dA (i.e. surface S).

Define

$$\sigma \equiv \quad \textbf{conductivity} \tag{3}$$

Then the empirical relation/equation that underpins *Ohm's law* is

$$\mathbf{J} = \sigma \mathbf{E} \tag{4}$$

and define *resistivity* from there:

$$\sigma := \frac{1}{\rho} \tag{5}$$

where ρ is the *resistivity*.

Thus

$$\sigma \int_A^B -\mathbf{E} = \sigma V_{AB} = \int_A^B \mathbf{J} = I_{AB}\frac{l}{A}$$
$$\frac{1}{\rho}R = \frac{l}{A} \text{ or } \boxed{R = \rho\frac{l}{A}}$$

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Key words and phrases. Eddy current brakes.

References

[1] Scott B. Hughes. **Magnetic braking: Finding the effective length over which the eddy currents form**. Magnetic braking: Finding the effective length over which the eddy currents form
[2] William R. Smythe, **Static and Dynamic Electricity**. 3rd ed. (McGraw-Hill, New York, 1968).