#SERVETHELOOP DUMP (FOR THE RLOOP)

ERNEST YEUNG ERNESTYALUMNI@GMAIL.COM

gmail : ernestyalumni
linkedin : ernestyalumni
twitter : ernestyalumni

Contents

Abstract. servetheloop notes "dump" - I dump all my notes, including things that I tried but are wrong, here.

Part 1. Eddy Currents, Eddy Current Braking

1. Eddy Currents

Keywords: Eddy currents;

cf. Smythe (1968), Ch. X (his Ch. 10) [2]

Assume Maxwell's "displacement current" is negligible; this is ok if frequencies are such that wavelength λ large compared to dimensions of apparatus L. $\lambda \gg L$ or $\frac{c}{\nu} \gg L$.

I will write down the "vector calculus" formulation of electrodynamics, along side Maxwell's equations, or electrodynamics, over spacetime manifold M. The latter formulation should specialize to the "vector calculus" formulation.

From

$$\text{curl}\mathbf{E} = -\frac{d\mathbf{B}}{dt}\,(SI) \qquad \text{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}\,(cgs) \text{ or } \frac{\partial B}{\partial t} + \mathbf{d}E = 0$$

Suppose $B = \text{curl}A$ or $B = \mathbf{d}A$ (EY 20170528: is this where the assumption above about $\lambda \gg L$ comes in?), then

$$-\frac{\partial B}{\partial t} = \mathbf{d}E \xrightarrow{\int_S} \int_S \mathbf{d}E = \int_{\partial S} E = -\int_S \frac{\partial B}{\partial t} \xrightarrow{B=\mathbf{d}A} -\int_S \frac{\partial}{\partial t}\mathbf{d}A \xrightarrow{\text{flat space}} \int_S \mathbf{d}E = -\int_S \mathbf{d}\frac{\partial A}{\partial t}$$

and so

$$\mathbf{E} = \frac{-\partial \mathbf{A}}{\partial t} \tag{1}$$

up to gauge transformation, if $B = \mathbf{dA} = \text{curl}\mathbf{A}$

Since this $\mathbf{E}$ field is formed in a conductor, Ohm's law applies. Let's review Ohm's law. Smythe (1968) refers to its 6.02 Ohm's Law - Resistivity section [2]. Indeed, in a lab, the definition of resistance can be defined as this ratio:

$$R_{AB} := \frac{-\int_A^B \mathbf{E}}{I_{AB}} = \frac{V_A - V_B}{I_{AB}} = \frac{\varepsilon_{AB}}{I_{AB}} \tag{2}$$

Moving along, the right way to think about resistivity ρ is to consider conductivity.

Assume current density is linear to $\mathbf{E}$ field (as $\mathbf{E}$ field pushes charges along). This linear response is reasonable.

Also, assume current density $\mathbf{J}$ is uniform over infinitesimal surface area dA (i.e. surface S).

Define

$$\sigma \equiv \quad \textbf{conductivity} \tag{3}$$

Then the empirical relation/equation that underpins *Ohm's law* is

$$\mathbf{J} = \sigma\mathbf{E} \tag{4}$$

and define *resistivity* from there:

$$\sigma := \frac{1}{\rho} \tag{5}$$

where ρ is the *resistivity*.

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Key words and phrases. Eddy current brakes.

Thus

$$\sigma \int_A^B -\mathbf{E} = \sigma V_{AB} = \int_A^B \mathbf{J} = I_{AB}\frac{l}{A}$$
$$\frac{1}{\rho}R = \frac{l}{A} \text{ or } \boxed{R = \rho \frac{l}{A}}$$

From Maxwell's Equations,

$$\delta(B - 4\pi \mathbf{M}) = \frac{4\pi}{c} J_{\text{free}} \tag{6}$$

If $B = H + 4\pi M = (1 + 4\pi\chi_m)H = \mu H$, then

$$\delta H = \delta \frac{B}{\mu} = \frac{4\pi}{c}J_{\text{free}} \text{ or } \delta B = \mu \frac{4\pi}{c}J_{\text{free}} \Longleftrightarrow \text{curl}B = \mu J_{\text{free}} \quad (SI) \tag{7}$$

and if $B = \mathbf{d}A$ and $\mathbf{d}\delta A = 0$.

I build upon the physical setup proposed by Jackson (1998) [3] in Section 5.18 "Quasi-Static Magnetic Fields in Conductors; Eddy Currents; Magnetic Diffusion."

For a system (with characteristic) length L, L being small,
compared to electromagnetic wavelength associated with dominant time scale of problem T,

$$f := \frac{1}{T}; \quad \omega = 2\pi f; \quad \omega\lambda = c \Longrightarrow \lambda = \frac{c}{\omega} = \frac{c}{2\pi f} = \frac{Tc}{2\pi}$$
$$\frac{L}{\lambda} = \frac{LTc}{2\pi} \gg 1$$

From Maxwell's equations, in particular, Faraday's Law, and in its integral form (over 2-dim. *closed* surface S),

$$\mathbf{d}E + \frac{\partial}{\partial t}B = 0 \text{ or } -\mathbf{d}E = \frac{\partial B}{\partial t} \xrightarrow{\int_S} \int_S \frac{\partial B}{\partial t} = -\int_S \mathbf{d}E = -\int_{\partial S} E \tag{8}$$

So on S, changing magnetic flux $\int \frac{\partial B}{\partial t}$ results in E field, circulating around boundary of S, ∂S.

We know that in a conductor, free conducting electrons get pushed around by E fields, result in a current density J.

J is related to E, *empirically* (by Ohm's Law)

$$J = \sigma E$$

where σ is the resistivity.

Then use the force law on this induced current J from the B field set up:

$$F_{\text{net}} = \frac{1}{c}\int_S J \times B dA$$

By working through the right-hand rule, F_{net} the force on those currents induced in the conductor due to the B that's there, is in the direction to help oppose changing (increasing or decreasing $\frac{\partial B}{\partial t}$).

To find B, suppose $B = dA$, i.e. $B \in H^2_{\text{deRham}}(M)$, i.e. $B = \text{curl}A$.

For sure,

$$\delta(B - 4\pi c\mathbf{M}) = 4\pi J \Longleftrightarrow \text{curl}(B - 4\pi c\mathbf{M}) = \text{curl}H = 4\pi J$$

Be warned now that the relation $B = \mu H$ may not be valid on all domains of interest; μ could even be a tensor! (e.g. $B_{ij} = \mu_{ij}^{kl}H_{kl}$). However, both Jackson (1998) [3] in Sec. 5.18 Quasi-Static Magnetic Fields in Conductors; Eddy Currents; Magnetic Diffusion, pp. 219, and Smythe (1968), Ch. X (his Ch. 10), pp. 368 [2], continues on *as if* this relation is linear: $B = \mu H$.

Nevertheless, as we want to find B by finding its "vector potential" A, we obtain a diffusion equation:

$$\begin{aligned} -\delta B = *\mathbf{d}*\mathbf{d}A = (-1)\delta\mathbf{d}A = (-1)(\Delta - \mathbf{d}\delta)A \xrightarrow{\mathbf{d}\delta A = 0} -\Delta A = \\ = 4\pi\mu J = 4\pi\mu\sigma E = 4\pi\mu\sigma\left(-\frac{\partial A}{\partial t}\right) \\ \Longrightarrow \boxed{\Delta A = 4\pi\mu\sigma\frac{\partial A}{\partial t}} \end{aligned} \tag{9}$$

where in the first 2 steps (equalities), $-\delta B = *\mathbf{d}*\mathbf{d}A = (-1)\delta\mathbf{d}A$ it's interesting to note that the codifferential δ for the 2 form B had to be written out explicitly, and then the codifferential for the 1-form A is *different* from the δ for B by a(n important) factor of (-1); where $\mathbf{d}\delta A = 0$ must be satisfied by the form A takes; and where, since $B = \mathbf{d}A$,

$$\mathbf{d}E + \frac{\partial B}{\partial t} = \mathbf{d}E + \frac{\partial}{\partial t}\mathbf{d}A = \mathbf{d}\left(E + \frac{\partial A}{\partial t}\right) = 0 \Longrightarrow E = -\frac{\partial A}{\partial t} + \text{grad}\Phi \xrightarrow{\Phi = \text{ constant}} E = -\frac{\partial A}{\partial t} \tag{10}$$

whereas a choice of gauge for E was chosen so that $\Phi =$ constant (and so a particular form for E was chosen, amongst those in the *same* equivalence class of $H^1_{\text{deRham}}(M)$.

To ensure that the differential geometry formulation is in agreement with the practical vector calculus formulation, compare Eq. 9 with Eq. (5.160) of Jackson (1998) [3] and Eq. (10) in Sec. 10.00 of Smythe (1968) [2].

To summarize what's going on, I think one should at least understand in one's head how Maxwell's Equations apply, (and I will try to write in SI here)

$$\boxed{\begin{gathered} \int_S \frac{\partial \mathbf{B}}{\partial t} dA = -\oint \mathbf{E}\cdot d\mathbf{s} \Longrightarrow \mathbf{J} = \sigma\mathbf{E} \Longrightarrow \mathbf{F}_{\text{net}} = \int_S \mathbf{J}\times\mathbf{B}dA \\ \text{find } \mathbf{B} = ? \qquad \text{using form } \mathbf{B} = \nabla\times\mathbf{A}, \\ \nabla^2\mathbf{A} = \mu\sigma\frac{\partial \mathbf{A}}{\partial t} \qquad (SI) \end{gathered}} \tag{11}$$

where, a change in magnetic flux over a surface S over the conductor, $\int_S \frac{\partial \mathbf{B}}{\partial t}dA$ induces a circulation of E field around S, $-\oint \mathbf{E}\cdot d\mathbf{s}$, and this E field is pushing around *free conducting charges* according to Ohm's law, $\mathbf{J} = \sigma\mathbf{E}$, with σ being the conductivity of the conducting material, and this current density $\mathbf{J}$ is then acted upon by the prevailing B field, according to the usual force law. To find $\mathbf{B}$, one can find $\mathbf{A}$ and *try* to find $\mathbf{A}$ analytically.

Keep in mind that for $\nabla^2\mathbf{A} = \mu\sigma\frac{\partial\mathbf{A}}{\partial t}$, we had used, critically, the Maxwell equation $\nabla\times\mathbf{H} = \mathbf{J}$, with $\mathbf{J}$ being the *induced current of free conducting charges on the conductor.* This $\mathbf{H}$ will contribute (through linear superposition) to the $\mathbf{B}$ that could already be there due to the permanent magnet.

What can we measure quantitatively?

- Can we measure $\mathbf{J}$ inside (on) the conductor?
- Can we separate magnetization $\mathbf{M}$ of material from $\mathbf{B}$, to obtain the actual $\mathbf{B}$ (and then use linear superposition, $\mathbf{B}_{\text{total}} = \mathbf{B}_{\text{permanent magnet}} + \mathbf{B}_{\text{induced currents}}$?

Also, keep in mind the context that the conductor at hand is the long, almost semi-infinite rectangle of a conductor, aluminum sub-rail, specified by the SpaceX Hyperloop. Force on that will cause an equal and opposite force on the pod, with its permanent magnets attached, and thus braking the pod.

Part 2. System Theory, including Avionics

cf. Hermann (1974) [4].

For a given field $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$ (or some field, in general).

Let U, X, Y be either vector spaces (or more generally, R-modules, with $R = \mathbb{K}$), or manifolds. Let

$$\begin{aligned} U &\equiv \text{ input space} \\ Y &\equiv \text{ output space} \\ X &\equiv \text{ state space} \end{aligned}$$

Consider maps f, g, s.t.

$$\begin{aligned} f &: X \times U \to X \\ g &: x \times U \to Y \\ \frac{dx}{dt} &= f(x, u) \\ y(t) &= g(x(t), u(t)) \end{aligned} \tag{12}$$

Ordered 5-tuple $\sigma = (U, X, Y, f, g)$ is a **system**.
$t \mapsto (x(t), u(t), y(t)) \in X \times U \times Y =:$ **trajectory** of system, they're solutions to $\frac{dx}{dt} = f(x, u)$.

Definition 1. *Input-output pair: $t \mapsto (u(t), y(t)) \in U \times Y$ if $\exists$ curve $t \mapsto x(t) \in X$ s.t.*

$$t \mapsto (x(t), u(t), y(t)) \tag{13}$$

Choose input curve $t \mapsto u(t)$ so to achieve some desired output $t \mapsto y(t)$.
If we have linear maps f, g or at least "linearize" the system, then f, g take forms

$$\begin{aligned} f(x, u) &= Ax + Bu \\ g(x, u) &= Cx + Du \\ A &: X \to X \\ B &: U \to X \\ C &: X \to Y \\ D &: U \to Y \end{aligned} \tag{14}$$

And so, for a (linear) system $(X, U, Y, A, B, C, D,)$, s.t.

$$\begin{aligned} \frac{dx}{dt} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

Discretize t or replace differential with difference equations. And so for
$t \in$ additive semigroup $\mathbb{Z}^+$,

$$\begin{aligned} t &\mapsto u(t) \in \mathbb{Z}^+ \to U \\ t &\mapsto y(t) \int \mathbb{Z}^+ \to Y \\ \frac{du}{dt} &\xrightarrow{\text{discretize}} \Delta u(t) = u(t+1) - u(t) \\ \Delta u(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{15}$$

Equivalence of systems with same X, U, Y.
Let $GL(X) = \{g | g : X \to X\}$.
Given $t \mapsto (x(t), u(t), y(t))$ s.t. it's a solution to

$$\begin{aligned} \frac{dx}{dt} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \qquad (3.2 of Hermann (1974)) \tag{16}$$

cf. Hermann (1974) [4].
Set

$$x_1(t) = gx(t) \tag{17}$$

Then

$$\frac{dx_1(t)}{dt} = g\frac{dx}{dt} = g(Ax + Bu) = A_1 x_1 + B_1 u \quad \text{and so} \quad \begin{aligned} A_1 &= gAg^{-1} \\ B_1 &= gB \end{aligned} \quad y(t) = Cg^{-1}x_1 + Du = C_1 x_1 + D_1 u \quad \text{with} \quad \begin{aligned} C_1 &= Cg^{-1} \\ D_1 &= D \end{aligned}$$

$$\Longrightarrow \sigma_1 = (U, X, Y, A_1, B_1, C_1, D_1)$$

σ, σ_1 automatically have the same input-output pairs.

Definition 2.

$$\begin{aligned} \sigma &= (A, B, C, D) \\ \sigma_1 &= (A_1, B_1, C_1, D_1) \end{aligned} \tag{18}$$

*2 linear systems with same state, input, output space X, U, Y, σ, σ_1 **algebraically equivalent** iff $\exists g \in GL(V)$ s.t.*

$$\begin{aligned} A_1 &= gAg^{-1} \\ B_1 &= gB \\ C_1 &= Cg^{-1} \\ D_1 &= D \end{aligned} \tag{19}$$

i.e. Let $\Sigma = \{\sigma\} =$ collection of linear systems $\sigma = (A, B, C, D)$,
i.e. $\Sigma = L(X, X) \times L(U, X) \times L(X, Y) \times L(U, Y)$.
Let $GL(X) = \{g | g : X \to X, \forall g, \exists g^{-1}\}$.
If $g \in GL(X)$ and

$$\sigma = (A, B, C, D) \in \Sigma$$

then

$$g\sigma = (gAg^{-1}, gB, Cg, D)$$

Theorem 1 (3.2)**.** *equivalence classes in 1-to-1 correspondence with **orbit space** $GL(X)\backslash\Sigma$.*

1.0.1. *cf. 4. Impulse response and Transfer Functions: Observatibility and Controllability of Hermann (1974)* [4]. From Eq.

(3.2) of Hermann (1974) [4], i.e. $\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$, i.e. linear, time-invariant, "lumped parameter" systems.
"weighting parameter" = "impulse response".
It's easy to solve

$$\dot{x} = Ax + Bu$$

with solution

$$x(t) = \int_0^t \exp\left(A(t-s)\right)Bu(s)ds + \exp\left(At\right)x(0)$$

since

$$y = \int_0^t Ce^{A(t-s)}Bu(s)ds + Ce^{At}x(0) + Du$$

i.e. by variation of parameters method.
Consider

$$\begin{aligned} \dot{x}^1 - Ax^1 &= 0 \\ x^1 &= \exp\left(At\right) \end{aligned}$$

If $x_p(t) = c_1(t)x^1(t) = c_1 \exp{(At)}$,

$$\dot{x}_p = c_1 e^{At} + c_1 A e^{At}$$
$$\dot{x}_p = Ax_p - Bu = \dot{c}_1 + c_1 A - Ac_1 - Bu = 0$$
$$\dot{c}_1 = Bu \text{ or } c_1 = \int_0^t ds Bu(s)$$
$$c_1(0) = x(0) \qquad \text{(EY : 20170602 I'm not sure if this is the right derivation)}$$

Definition 3. *Function*

$$t \mapsto Ce^{At}B, \qquad R \to L(U,Y) \tag{20}$$

is called ***impulse response*** *for system* $\sigma = (A,B,C,D)$

Its Laplace transform

$$s = C(A\cdot s)^{-1}B \tag{21}$$

is called **frequency response** of system.

Remarks A) **Resolvants**. In functional analysis, operator-valued function $s \mapsto (A\cdot s)^{-1}$ is called resolvant of operator$A$.

Theorem 2 (4.1 Hermann (1974) [4])**.** *Let*

$$\sigma = (U,X,Y,A,B,C,D)$$
$$\sigma_1 = (U,X_1,Y,A_1,B_1,C_1,D_1)$$

Suppose σ,σ_1 *equivalent, same input-output pairs. Then*

$$C(A\cdot s)^{-1}B = C_1(A_1\cdot s)^{-1}B_1 \qquad s\in\mathbb{C} \tag{22}$$

i.e. they have the same frequency responses. Also $D = D_1$

Proof. Let $t\mapsto u(t)$ input curve, $x(0)$ initial state vector.
$t\mapsto y(t)$ output curve determined by $(t\mapsto u(t), x(0))$ in system σ.
Using Eq. (4.4) Hermann (1974) [4],

$$\begin{aligned} y &= \int_0^t Ce^{A(t-s)}Bu(s)ds + Ce^{At}x(0) + Du \\ y(t) &= \int_0^t Ce^{A(t-s)}Bu(s)ds + Ce^{At}x(0) + Du(t) \end{aligned} \tag{23}$$

By hypotheses, σ,σ_1 equivalent, so

state vector $x_1(0)\in X$, s.t.

$$(t\mapsto u(t), x_1(0)) \text{ determines } t\mapsto y(t), \text{ i.e.}$$

$$y(t) = \int_0^t C_1 e^{A_1(t-s)}B_1 u(s)ds + C_1 e^{A_1 t}x_1(0) + D_1 u(t)$$

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Part 3. Screws, Bolts, Fasteners (through Shigley's Mechanical Engineering Design)

cf. Budynas and Nisbett (2014) [5].

2. Screws, Fasteners, Design of Nonpermanent Joints

cf. Ch. 8 "Screws, Fasteners, and the Design of Nonpermanent Joints", Budynas and Nisbett (2014) [5].
cf. Section 8-1, "Thread Standards and Definitions", pp. 410, Budynas and Nisbett (2014) [5].
pitch ≡ distance between adjacent thread forms, measured parallel to thread axis.
$N :=$ number of thread forms per unit length (per inch U.S.); pitch $= 1/N$
$d \equiv$ major diameter := largest diameter of screw thread.
$d_r \equiv$ minor (or root) diameter = smallest diameter of screw thread.
$l \equiv$ lead = distance nut moves parallel to screw axis when nut given 1 turn for
single thread $= l = p$
multiple-threaded -2 or more threads out beside each other (imagine 2 or more strings wound side by side around a pencil).
double-threaded $\to l = 2p$
triple-threaded $\to l = 3p$
Imagine square-threaded power screw, single thread, unrolled or developed, for exactly single turn.
$d_m :=$ mean thread diameter circle. $\lambda :=$ lead angle of thread.
Consider sum of all forces **on** the single screw thread.
To raise load, force P_R acts to the right **on** single screw thread.
$\mathbf{P}_R = (P_R, 0)$; $P_R > 0$
$\mathbf{N} = N(-\sin\lambda, \cos\lambda)$
friction force $\mathbf{fN}$ (Shigley) $\equiv -\mu N(\cos\lambda, \sin\lambda)$
$\mathbf{F}$ = sum of all axial forces acting upon normal thread area $= (0, -F)$.

$$\mathbf{P}_R + \mathbf{N} + \mathbf{fN} + \mathbf{F} = (P_R - N\sin\lambda, -\mu N\cos\lambda, N\cos\lambda - \mu N \sin\lambda, -F) \tag{24}$$

Surely $(\mathbf{P}_R + \mathbf{N} + \mathbf{fN} + \mathbf{F})_x = 0$, so that $P_R = N(\sin\lambda + \mu\cos\lambda)$.
We can imagine $m_{\text{thread}}a_{\text{thread}} = N(\cos\lambda - \mu\sin\lambda) - F = \frac{P_R}{\sin\lambda + \mu\cos\lambda}(\cos\lambda - \mu\sin\lambda) - F \neq 0$
Lowering load, P_L acts to the left **on** simple screw thread.
$\mathbf{P}_L = (-P_L, 0)$; $\mathbf{fN} \equiv \mu N(\cos\lambda, \sin\lambda)$.

$$\sum_{\text{on single screw thread}} \mathbf{F}_i = \mathbf{P}_L + \mathbf{N} + \mathbf{fN} + \mathbf{F} = (-P_L - N\sin\lambda + \mu N\cos\lambda, N\cos\lambda + \mu N\sin\lambda - F) \tag{25}$$

Surely $(\mathbf{P}_L + \mathbf{N} + \mathbf{fN} + \mathbf{F})_x = 0$ so that $P_L = N(\mu\cos\lambda - \sin\lambda)$.
We can imagine

$$m_{\text{thread}}a_{\text{thread},y} = F_{\text{net},y} = N(\cos\lambda + \mu\sin\lambda) - F = \frac{P_L}{\mu\cos\lambda - \sin\lambda}(\cos\lambda + \mu\sin\lambda) - F$$

At equilibrium of forces,
For raising load,

$$\begin{aligned} \tau = \mathbf{r}\times\mathbf{P} = \mathbf{r}\times\mathbf{P}_R = \mathbf{r}\times\left(F\left(\frac{\sin\lambda + \mu\cos\lambda}{\cos\lambda - \mu\sin\lambda}\right)\right) = \mathbf{r}\times\left(F\left(\frac{\tan\lambda + \mu}{1-\mu\tan\lambda}\right)\right) = \\ = \mathbf{r}\times F\left(\frac{\frac{l}{\pi d_m} + \mu}{1 - \mu\frac{l}{\mu d_m}}\right) = \mathbf{r}\times F\left(\frac{l + \mu d_m \pi}{\mu d_m - \mu l}\right) \end{aligned}$$

For lowering load,

$$\begin{aligned} \tau = \mathbf{r}\times\mathbf{P} = \mathbf{r}\times\mathbf{P}_L = \mathbf{r}\times\left(F\left(\frac{\mu\cos\lambda - \sin\lambda}{\cos\lambda + \mu\sin\lambda}\right)\right) = \mathbf{r}\times\left(F\left(\frac{\mu - \tan\lambda}{1+\mu\tan\lambda}\right)\right) = \\ = \mathbf{r}\times F\left(\frac{\mu - \frac{l}{\pi d_m}}{1 + \mu\frac{l}{\mu d_m}}\right) = \mathbf{r}\times F\left(\frac{\pi\mu d_m - l}{\pi d_m + \mu l}\right) \end{aligned}$$

Load lowers itself, by causing screw to spin without external effort.

(26) $$\tau_L = \mathbf{r} \times F \left(\frac{\pi d_m \mu - l}{\pi d_m + \mu l} \right) \leq 0 \text{ or } \mu \leq \tan \lambda$$

If $\tau_L > 0$, screw is **self-locking**; in this case, $\mu > \tan \lambda$

Efficiency expression (for power screws)

(27) $$\tau_0(\mu = 0) = \frac{d_m}{2} F \left(\frac{l}{\pi d_m} \right) = \frac{Fl}{2\pi}$$

Since thread friction has been eliminated, torque required only to raise the load.

Then define efficiency ϵ_{load} as

(28) $$\epsilon_{\text{load}} = \frac{\tau_0}{\tau_R} = \frac{Fl}{2\pi\tau_R} = \frac{Fl}{2\pi \frac{d_m}{2} F \left(\frac{l+\mu d_m \pi}{\pi d_m - \mu l} \right)} = \frac{l}{\pi d_m} \left(\frac{\pi d_m - \mu l}{l + \mu d_m \pi} \right) = \frac{1 - \frac{\mu l}{\pi d_m}}{1 + \frac{\mu d_m \pi}{l}}$$

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EY : 20170606 - I was only able to obtain a copy of the 9th edition to use. If you'd like to help us keep up to date with the 10th edition or to help the rLoop in general, please donate to the www.rLoop.org!