

# Sequence Models:

## ⊕ Recurrent Neural Networks (RNN)

### \* Seq. Models :

Speech recognition

Music generation

DNA Sequence analysis AGCCTCCAGCCTCC ...

Machine Translation

Video activity recognition

Name entity recognition...

### \* Notations

Ex:  $x$  : Harry Potter & Hermoine Granger invented a new spell.  
 $x^{(1)}$   $x^{(2)}$   $x^{(3)}$  ...  $x^{(t)}$  ...  $x^{(9)}$   $T_x = 9$   
 $y$  : 1 1 0 1 1 0 0 0 0  
 (persons name)  $y^{(1)}$   $y^{(2)}$  ...  $y^{(t)}$  ...  $y^{(9)}$   $T_y = 9$

$x^{(i)}(t)$  →  $t^{th}$  element of  $i^{th}$  Training ex

$T_x^{(i)}$  → length of  $i^{th}$  seq in  $i^{th}$  Training ex.

### Vocabulary

a	1
aaron	2
and	367
...	...
harry	4075
...	...
potter	6830
...	...
zulu	10,000

10,000 words :

$x^{(1)}$   
 $x^{(1)}$

0
0
...
1
0
0
...
0

→ 4075.

$x^{(2)} = a$ .

1
0
0
...
1
...
1
0

one hot vectors.

use one-hot representation

<unk> : for words not in vocabulary <unknown token>.

### \* Recurrent Neural N/w (RNN) model :

• why not a standard NN?

$x^{(1)}$  0

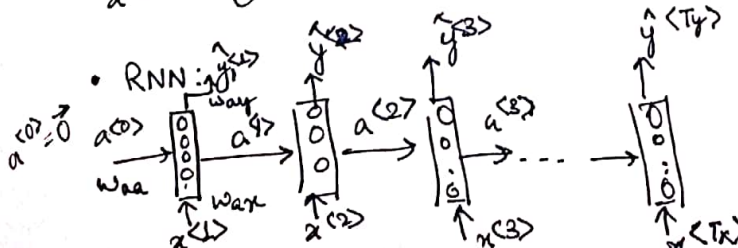
$x^{(2)}$  0

...

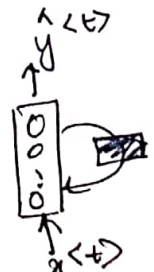
$x^{(T_x)}$  0

problems :

- i/p and o/p can be different lengths in diff examples
- doesn't share features learned across diff part of text.
- very large i/p layer.



notation :



- scans left to right

only earlier layers are used to predict : problematic as first few letters can't be names.  
 Ex: can confuse Ed & Jeddy & Roosevelt  $\rightarrow$  object name.  
 Sol<sup>n</sup>: bidirectional RNN.

$$a^{<t>} = g(w_{aa}a^{<t-1>} + w_{ax}x^{<t>} + b_a) \leftarrow \tanh/\text{Relu}$$

$$\hat{y}^{<t>} = g_2(w_{ya}a^{<t>} + b_y) \leftarrow \text{sigmoid}$$

$$\boxed{\begin{aligned} a^{<t>} &= g(w_{aa}a^{<t-1>} + w_{ax}x^{<t>} + b_a) \\ \hat{y}^{<t>} &= g(w_{ya}a^{<t>} + b_y) \end{aligned}}$$

$$\hat{y}^{<t>} = g(w_y a^{<t>} + b_y)$$

$$\boxed{\begin{aligned} a^{<t>} &= g(w_a [a^{<t-1>}, x^{<t>}] + b_a) \\ \hat{y}^{<t>} &= g(w_y a^{<t>} + b_y) \end{aligned}}$$

$$a^{<t>} = g(w_a [a^{<t-1>}, x^{<t>}] + b_a)$$

Ex:  $w_{aa}$   $100, 100$   $a^{<t-1>}$   $100$   $w_{ax}$   $(100, 10000)$

$$w_a = \begin{bmatrix} w_{aa} & w_{ax} \end{bmatrix} \begin{matrix} \downarrow 100 \\ \downarrow 10000 \end{matrix} \downarrow 100$$

$$\begin{matrix} 10 & 100 \end{matrix}$$

$$[a^{<t-1>}, x^{<t>}] = \begin{bmatrix} a^{<t-1>} \\ x^{<t>} \end{bmatrix} \begin{matrix} \uparrow 100 \\ \uparrow 10000 \end{matrix}$$

\* Backpropagation through time:

$$L^{<t>}(\hat{y}^{<t>}, y^{<t>}) = -y^{<t>} \log \hat{y}^{<t>} - (1 - y^{<t>}) \log (1 - \hat{y}^{<t>})$$

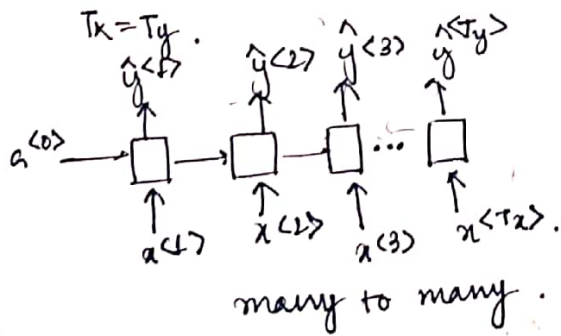
$$L(\hat{y}, y) = \sum_{t=1}^{T_y} L^{<t>}(\hat{y}^{<t>}, y^{<t>}) \leftarrow$$

Here, it's Backpropagation through time.

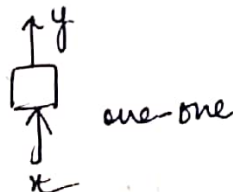
\* Different ~~types~~ types of RNN:

$T_x$  and  $T_y$  might not always be same.

• Examples of RNN architectures:

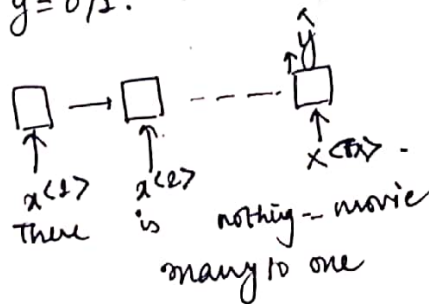


Ex:



Ex: sentiment classification:

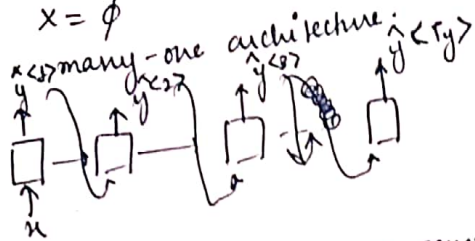
$x = \text{text}$   
 $y = 0/1$ .



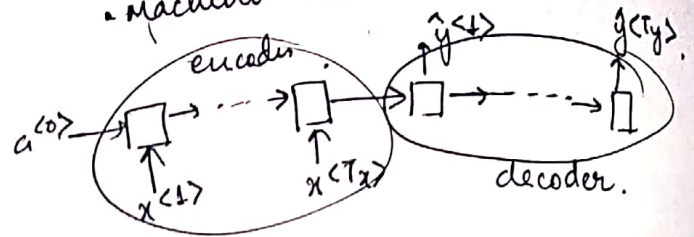
Music generation:

$$\rightarrow y^{(1)} y^{(2)} \dots y^{(T_y)}$$

$$x = \phi$$



Machine Translation:



\* Language model and sequence generation:

language modelling:

Speech recog:

- The apple & pair salad — (I)
- The apple & pear salad — (II)

$$P(1) = 3.2 \times 10^{-13}$$

$$P(2) = 5.7 \times 10^{-10}$$

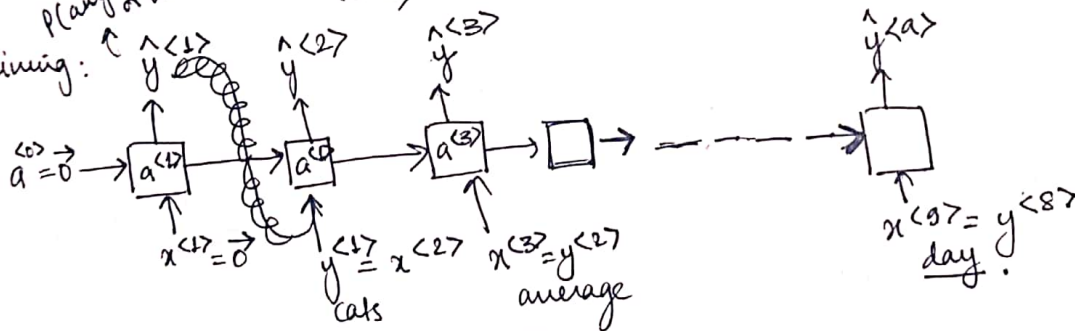
$$P(\text{sentence}) = ? \quad y^{(1)}, y^{(2)}, \dots, y^{(T_y)}$$

Training set: large corpus of eng text.

cats average is his cat day ~~cat~~ Tokenise this + 1 extra token  $\langle \text{EOS} \rangle$  end of sentence.

The Egyptian ~~Man~~ is beard of cat  $\langle \text{EOS} \rangle$

Training:



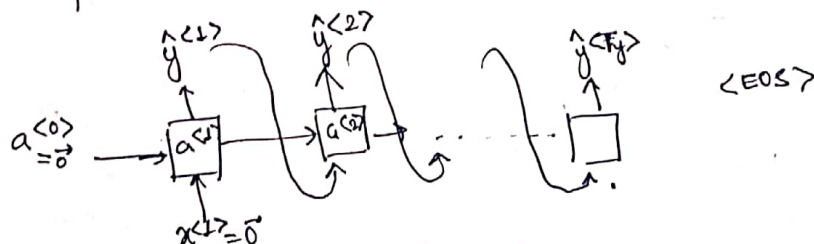
$$L(\hat{y}^{(t)}, y^{(t)}) = - \sum_i y_i^{(t)} \log \hat{y}_i^{(t)}$$

$$L = \sum_t L^{(t)}(\hat{y}^{(t)}, y^{(t)})$$

$P(\text{sec word, given first word intial})$

\* Sampling novel sequences:

Sampling



$$\rightarrow P(a)P(aaron)P(cat) \dots P(zulu) = \text{np.random.choice}$$

character level RNN:

Vocabulary [a, b, ..., z, 0, ..., 9, ., , ' , | , - , A , ..., Z]

has own adv & disadv

much longer sequences



# \* Vanishing gradients with RNNs :

the cat which already ate ..... , was full  
the cats ..... were full.

vanishing grads are problematic. can cause numerical overflow  
→ sol<sup>n</sup>: gradient clipping

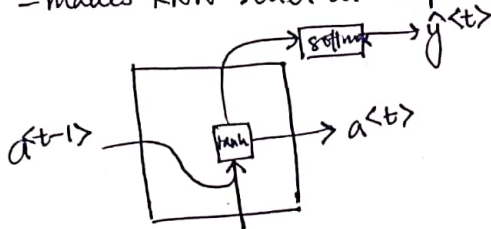
derivative can ↑ or  
↓ exponentially  
due to very deep FNN.

## \* Gated Recurrent Units (GRU)

- makes RNN better at capturing long range connections.

papers : cho et al, 2014 : on properties of  
neural n/c translat  
encodn-decodn  
approach.

chung-et-al, 2014 : Empirical Eval.  
of gated RNN in  
seq. modelling



### • GRU (simplified)

$c = \text{memory cell}$   
 $c^{<t>} = \frac{a^{<t>}}{c^{<t-1>}}$

$$\tilde{c}^{<t>} = \tanh(w_c [c^{<t-1>}, x^{<t>}] + b_c)$$

$$\Gamma_u = \sigma(w_u [c^{<t-1>}, x^{<t>}] + b_u)$$

$$c^{<t>} = \Gamma_u * \tilde{c}^{<t>} + (1 - \Gamma_u) * c^{<t-1>}$$

(elementwise multipl<sup>n</sup>)  
(as same dim<sup>n</sup>)

$c^{<t>} = \frac{1}{0}$  (singular)  
The cat, which ..... was full  
 $F \geq 0$   $F_u = 0$   
(when do you update).  
memorizes  
cat is  
similar.

### Full GRU:

$$\begin{aligned} \tilde{c}^{<t>} &= \tanh(w_c [\Gamma_r * c^{<t-1>}, x^{<t>}] + b_c) \\ \Gamma_u &= \sigma(w_u [c^{<t-1>}, x^{<t>}] + b_u) \\ \Gamma_r &= \sigma(w_r [c^{<t-1>}, x^{<t>}] + b_r) \\ c^{<t>} &= \Gamma_u * \tilde{c}^{<t>} + (1 - \Gamma_u) * c^{<t-1>} \end{aligned}$$

alternate notations.

## \* Long Short Term Memory: (LSTM)

- more powerful than GRU.

$$\tilde{c}^{<t>} = \tanh(w_c [a^{<t-1>}, x^{<t>}] + b_c)$$

$$\Gamma_u = \sigma(w_u [a^{<t-1>}, x^{<t>}] + b_u)$$

$$\Gamma_f = \sigma(w_f [a^{<t-1>}, x^{<t>}] + b_f)$$

$$\Gamma_o = \sigma(w_o [a^{<t-1>}, x^{<t>}] + b_o)$$

$$c^{<t>} = \Gamma_u * \tilde{c}^{<t>} + \Gamma_f * c^{<t-1>}$$

$$a^{<t>} = \Gamma_o * c^{<t>}$$

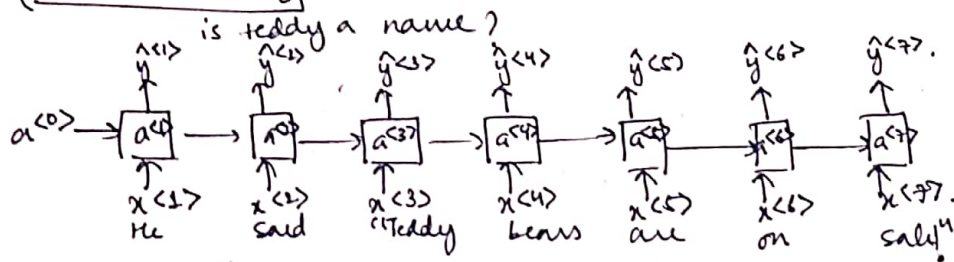
Paper : Hochreiter &  
Schmidhuber  
1997.  
LSTM.

variation →  $c^{<t-1>}$  replaces  $x^{<t>}$ .  
- peephole connection

## \* Bidirectional RNN

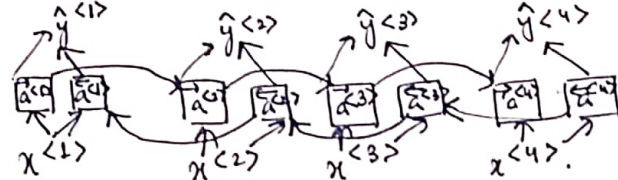
- Takes info from later & earlier inputs.

Ex: He said, "Teddy bears are on sale!"  
He said, "Teddy Roosevelt was a great Prez!"



\* BRNN.

also GRU / LSTM.

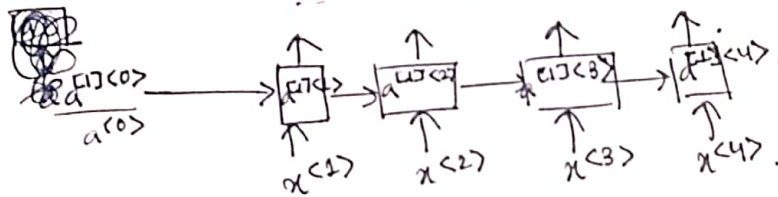


$$\hat{y}^{<t>} = g(w_y [\vec{a}^{<t>}, \overleftarrow{a}^{<t>}] + b_y)$$

## \* Deep RNNs :-

Ex:

multiple RNNs stacked together.



$$a^{[2]}_{<3>} = g(w_a [a^{[2]}_{<2>}, a^{[1]}_{<3>}] + b_a^{[2]})$$

## ⊕ Natural language processing & word Embeddings :-

# Word Embeddings.

\* Word Representation :

•  $V = [a, aaron, \dots, zulu, <UNK>]$   $|V| = 10,000$

1-hot representation

weakness:

- Treats the word as object into itself, and doesn't allow algo to easily generalise new words.

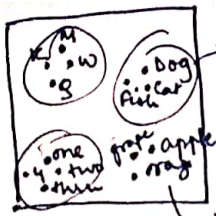
Featureised representation: word embedding

	Man (5391)	Woman (9553)	King (4014)	Queen (7157)	Apple (456)	Orange (6257)
gender	-1	1	-0.95	0.97	0.00	0.01
Royal	0.01	0.02	0.93	0.95	-0.01	0.00
Age	0.03	0.02	0.7	0.69	0.03	-0.02
Food	0.04	0.01	0.02	0.01	0.95	0.97
⋮						

300  
↑  
300D vector represents man  
5391

paper: Vans der  
Maaten  
and  
Hinton,  
2008.  
Visualizing  
data units  
in SVD.



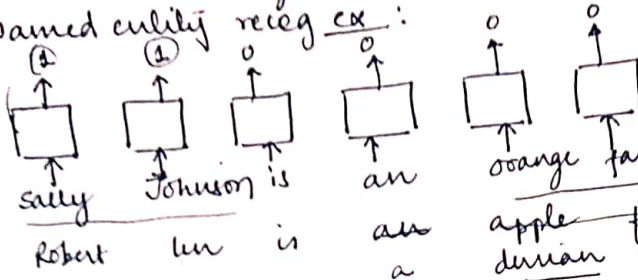


Similar objects grouped together.

t-SNE. objects are embeddings.

\* using word embeddings:

Ex: Named entity recog ex:



one hot encoding fails word embedding succeeds.

steps: Transfer learn a word embedding.

1. learn word embeddings from a large text corpus (1-100B words) (or download pre-trained embedding online)
2. transfer embedding to new task with smaller F.S. (300D dense feature vector)
3. optimal: continue to fine tune word embeddings with new data.

- word embeddings generalise algorithms much better.

\* Properties of word embeddings:

Analogies:

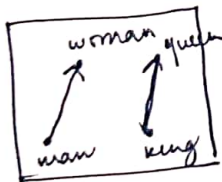
man  $\rightarrow$  woman, king  $\rightarrow$  ?

$e_{man}$   $e_{woman}$   
(embedding vectors).

$$\left. \begin{aligned} e_{man} - e_{woman} &\approx \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ e_{king} - e_{queen} &\approx \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned} \right\} \Rightarrow$$

Paper: Mikolov et al. 2013, linguistic Regularities in continuous space word representation

Analogies using word vectors:



300D

$$e_{man} - e_{woman} \approx e_{king} - e_{?}$$

Find word  $w$ :  $\arg \max_w \text{similarity}(e_w, e_{king} - e_{man} + e_{woman})$

t-SNE: 300D  $\rightarrow$  2D (parallelogram relationships may hold true (as above))

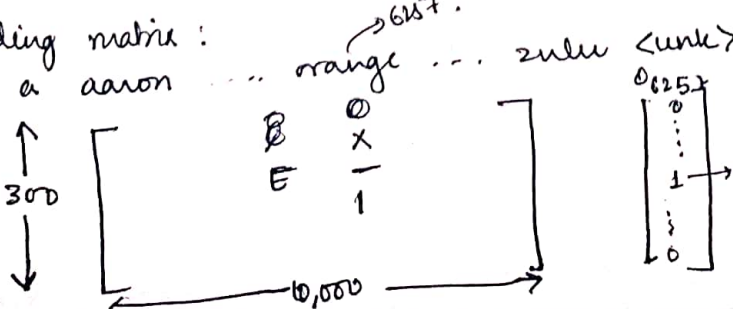
\* Cosine similarity:

$$\text{sim}(e_u, e_v) = \frac{u^T v}{\|u\|_2 \|v\|_2}$$



$\cos \phi = 1$   $\phi = 0^\circ$  similar  
 $\cos \phi = 0$   $\phi = 90^\circ$  different.

\* Embedding matrix:

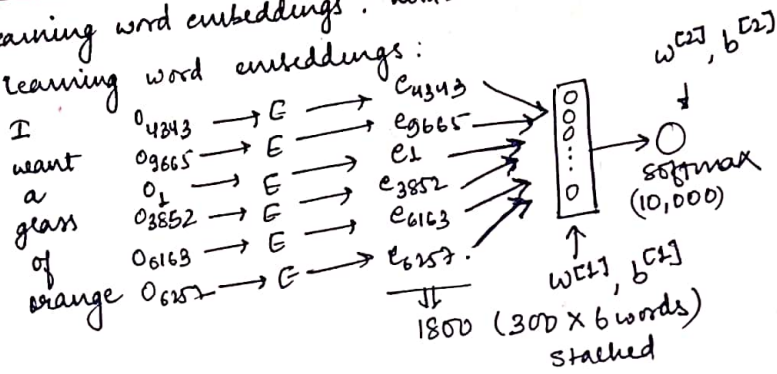


$$E_{(300, 10K)} \cdot q_{(10K, 1)} = e_{x=1}$$

$E \cdot q_j = e_j$   
= embedding for word  $j$

# learning word embeddings: word2vec & GloVe

\* learning word embeddings:



paper: Bengio 2003  
A neural probabilistic language model.

other context/target pairs:

I want a glass of orange juice to go along with my cereal.

Context: last 4 words or last 1 word or nearby one word  
Target: juice  
Skip-gram: glass?

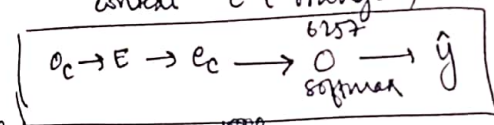
\* word2vec

• Skip grams (allows to create supervised learning task)  
I want a glass of orange juice to go along with my cereal.

paper: Mikolov 2013  
Efficient estimation of word representations in vector space

Model

Vocab = 10,000 words  
Context c ("orange") → Target t ("juice")



$$L(\hat{y}, y) = -\sum_{i=1}^{10000} y_i \log \hat{y}_i$$

y → one hot vector  
10,000

$$\text{softmax: } p(t|c) = \frac{e^{t^T e_c}}{\sum_{j=1}^{10000} e^{t^T e_j}}$$

θ → param associated with t.

problems:

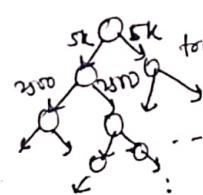
→ computational speed.

sol<sup>n</sup>: hierarchical softmax

How to sample context c?

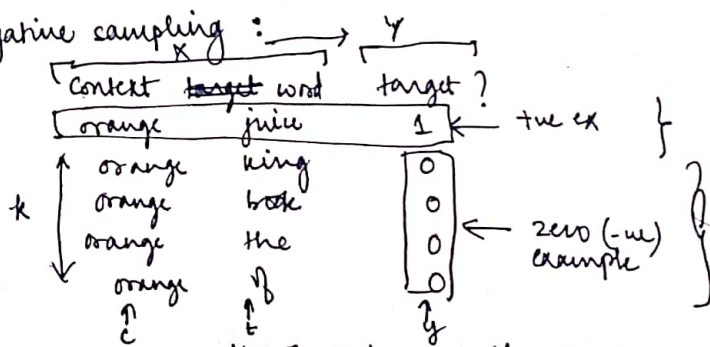
the, of, a, and, to, ...

orange, apple, banana ...



tree may be developed as non symmetrical, common words on top, whereas non common on lower level.

\* Negative sampling:



k = 5-20 for smaller datasets  
k = 2-5 for larger data sets.

paper: Mikolov 2013,  
Distributed representation of words & phrases & their composition



$$p(y=1|c,t) = \frac{\exp(\theta_1^T c_t)}{\sum_c \exp(\theta_c^T c_t)}$$

instead of 10000 softmax, treat as 10000 binary classification problem

\* glove word vectors: (global vectors for word representation)

I want a glass of orange juice to go along with my cereal

paper: pennington, 2014  
global vectors for word representation

$c, t$

$X_{ij} = \# \text{times } i \text{ appears in context of } j$

maybe:  $X_{ij} = X_{ji}$  (symmetric case)

model: minimise  $\sum_{i=1}^{10000} \sum_{j=1}^{10000} f(X_{ij}) (\theta_i^T e_j + b_i - b_j' - \log X_{ij})^2$

waiting term  
 $f(X_{ij}) = 0$  if  $X_{ij} = 0$

$\theta_i, e_j$  are symmetric.

$$e_w^{(final)} = \frac{e_w + \theta_w}{2}$$

A note on the featureisation view of word embeddings.

# Applications using word embeddings:

\* Sentiment classification:

$x \rightarrow y$   
The dessert is excellent  $\rightarrow$  4 stars.  
Service was slow  $\rightarrow$  2 stars  
Good for a quick meal, nothing special  $\rightarrow$  3 stars  
Completely lacking in good state, good ...  $\rightarrow$  1 star.

$x \rightarrow y \rightarrow$  can be used to monitor comments.

• Simple sentiment class. model:

The dessert is excellent  $\rightarrow$  4 stars.

8928 2468 4694 2180  
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $e_{8928} \quad e_{2468} \quad e_{4694} \quad e_{2180}$

Avg.  $\rightarrow$  300

softmax  
1-5

problem: if "good" appears many times in bad reviews, then classifier may think this is a good review

• RNN for sentiment classification (many to one)

Completely  $\rightarrow E \rightarrow e_{1852} \rightarrow a_{(1)}$   
lacking  $\rightarrow E \rightarrow e_{4966} \rightarrow a_{(2)}$   
in  $\rightarrow E \rightarrow e_{4424} \rightarrow a_{(3)}$   
good  $\rightarrow E \rightarrow e_{3882} \rightarrow$   
...  
ambience.  $\rightarrow E \rightarrow e_{300} \rightarrow a_{(10)}$

softmax  $\rightarrow y$



## \* Biasing word embeddings :

gender bias, etc. . .

man: woman as king: queen

man: computer programmer as woman: homemaker?

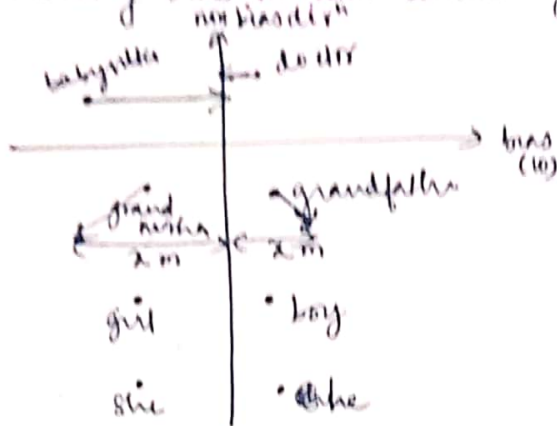
father: doctor as mother: nurse

is gender bias

gender bias

- word embeddings can ~~be used~~ reflect gender, ethnicity, age, sexual orientation, . . .

. addressing bias in word embeddings :



1. Identify bias direction

$\begin{cases} \text{male} - \text{female} \\ \vdots \\ \text{average} \end{cases}$

2. Neutralise : For every word that is definitional, project to get rid of bias

Ex: grandfather  
grandmother  
father  
mother

3. Equalisation of pairs

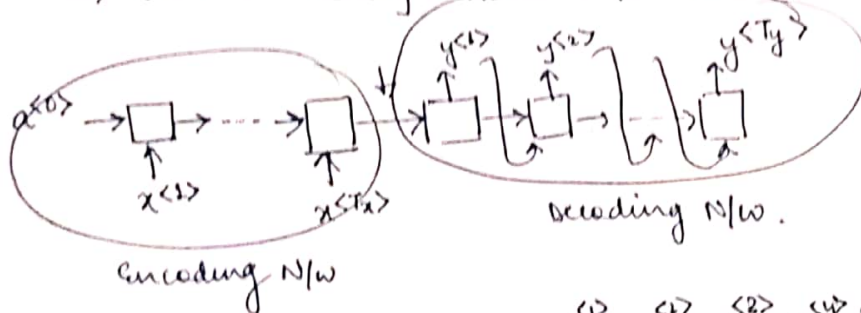
grandmother — grandfather . (Ex: equidistant from babysitter)

## ⊕ Sequence Models & Attention Mechanisms :

# Various sequence to sequence architecture :

\* Basic Models :

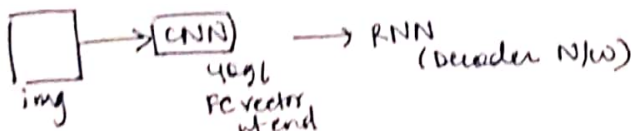
Translation:  $x^{(1)}$  Jane visits  $x^{(2)}$  l'Afrique en septembre  $x^{(3)}$   $x^{(4)}$   $x^{(5)}$   
 $y^{(1)}$  → Jane is visiting Africa in September  $y^{(2)}$   $y^{(3)}$   $y^{(4)}$   $y^{(5)}$



encoding N/w

decoding N/w

. img captioning :



$y^{(1)}$   $y^{(2)}$   $y^{(3)}$   $y^{(4)}$   $y^{(5)}$   
A cat sitting on a chair.

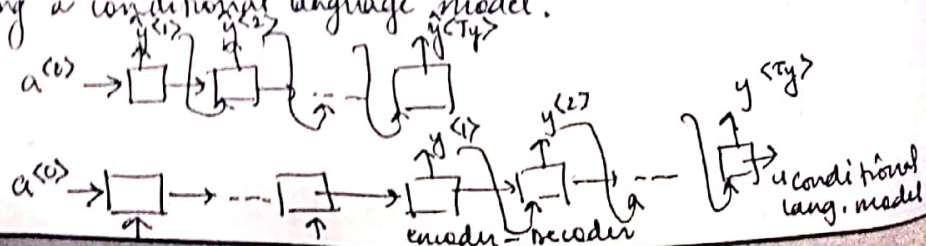
paper: Deep captioning with multilingual RNN.

\* Picking the most likely sentence :

. M/c translation as building a conditional language model.

language model

M/c translation:



$$P(y^{(1)}, \dots, y^{(T_y)} | x^{(1)}, \dots, x^{(T_x)})$$

ex:  $P(y^{(1)}, y^{(2)}, \dots, y^{(T_y)} | x)$

↓  
English possibility.

↘ french

$$\therefore \arg \max_{y^{(1)}, \dots, y^{(T_y)}} P(y^{(1)}, \dots, y^{(T_y)} | x)$$

↳ maximised by Beam search.

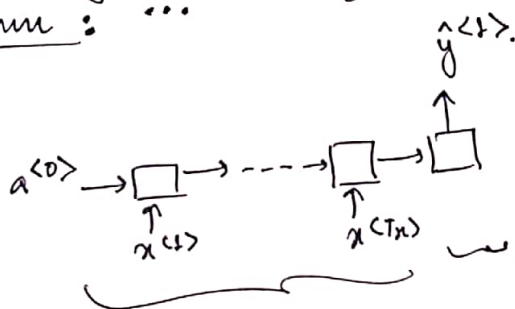
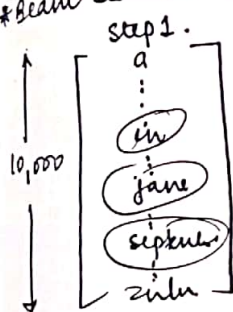
Why not greedy search?

Pick first words, then pick subsequently most likely words.

$$P(\hat{y}^{(1)} | x)$$

ex: Jane is — going to visit —  
going to be visiting.

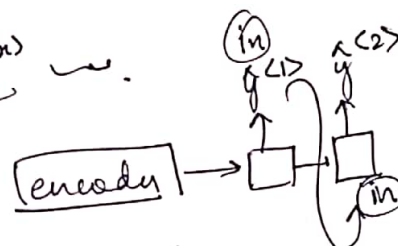
\* Beam Search Algorithm :



B = 3 (beam width)

$$\rightarrow P(y^{(1)} | x)$$

step 2 → evaluate 2nd word at first word.



$$P(y^{(2)} | x = \text{"in"})$$

$$\therefore P(y^{(1)}, y^{(2)} | x) = P(y^{(1)} | x) P(y^{(2)} | x, y^{(1)})$$

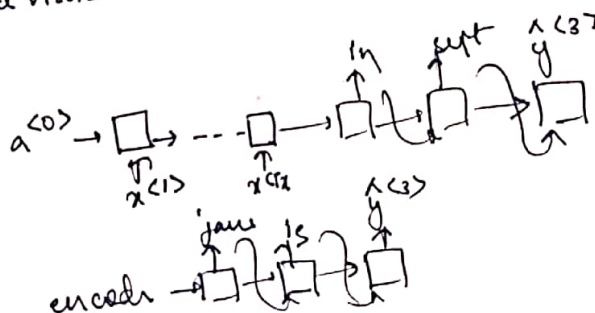
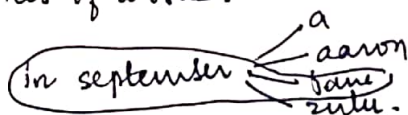
similarly with 'jane' & 'september'.

∴ 30,000 possibilities. (B × 10,000).

cut down possibilities

in sept ✓  
jane is ✓  
jane visits ✓ } → sept rejected with jane.

- 3 copies of Nhw for different choices of words.



\* Refinements to beam search :

• length normalisation

$$\arg \max_{y^{(1)}, \dots, y^{(T_y)}} \prod_{t=1}^{T_y} P(y^{(t)} | x, y^{(1)}, \dots, y^{(t-1)}) \Rightarrow X < 1$$

$$\arg \max_{y^{(1)}, \dots, y^{(T_y)}} \sum_{t=1}^{T_y} \log P(y^{(t)} | x, y^{(1)}, \dots, y^{(t-1)})$$

$\log P(y | x)$  rising  
 $P(y | x)$  rising



$$\frac{1}{T_y^\alpha} \sum_{t=1}^{T_y} \log P(y^{<t>} | x, y^{<1>}, \dots, y^{<t-1>}) \rightarrow \text{Normalisation:}$$

$T_y = 1, 2, 3, \dots, 30$

Take sentences & compute this, highest value is kept.

• Beam width  $B$ ?

large  $B$ : better result slower

small  $B$ : worse result faster.

on common system  $B=10, 100$

research sometimes uses  $B=1000, \dots$

→ unlike BFS or DFS, Beam search runs faster but not guaranteed to find exact maximum for  $\arg \max_y P(y|x)$

\* Error analysis in beam search:

ex: Jane visite l'Afrique en septembre

Human: Jane visits Africa in september ( $y^*$ )

Algorithm: Jane visited Africa last september ( $\hat{y}$ )

RNN computes  $P(y^*|x) \geq P(\hat{y}|x)$

Case 1:

$$P(y^*|x) > P(\hat{y}|x)$$

Beam search chose  $\hat{y}$ , but  $y^*$  higher val.

∴ Beam search is at fault.

Case 2:

$$P(y^*|x) < P(\hat{y}|x)$$

$y^*$  better trans than  $\hat{y}$ , but RNN predicted  $P(y^*|x) < P(\hat{y}|x)$   
RNN model at fault.

→ RNN

→ Beam search.  $B \uparrow$   
↓  
for more data.

• Process

Human	Algo	$P(y^* x)$ $2 \times 10^{-10}$	$P(\hat{y} x)$ $1 \times 10^{-10}$	At fault?
				B R R R ...

fraction of error due to beam search vs due to RNN

if Beam search resp, BRes  
else if RNN is at fault, train deeper N/w.

\* Attention model intuition:

• Problem of long sentences:

Bleu score



Sentence length.

Translates part of a long sentence at a time.

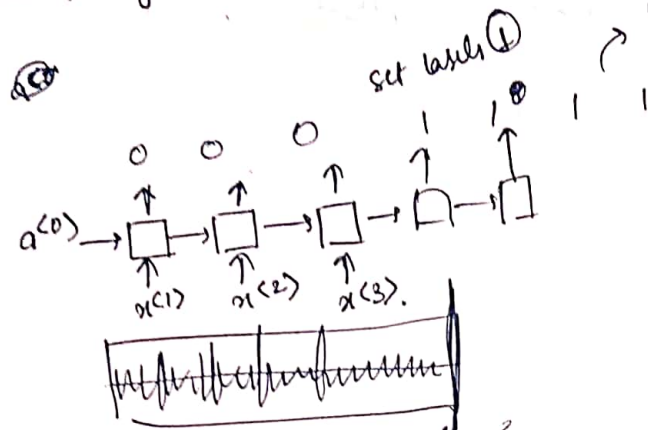
Paper: Bahdanau, 2014

Neural M/C translation by jointly learning to align & translate





\* Trigger word detection:  
 Ex: Alexa, Hey Siri, okay Google, ...



trigger word finished

multiple 1's slightly  
 exceeds out  
 ratio of 1's to 0's.