Hall Effect in Semiconductors

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Abstract

In this experiment, hall effect created by a magnetic field passing through an n-doped germanium sample is investigated as well as the resistivity (ρ) of the sample. Passing a current I perpendicular to the magnetic field (B) , 10 samples with varying magnetic field and 10 samples with varying current are used to determine the hall constant R_h (m^3/C). It is found to be $R_h = -7.20 \pm 1.45 \times 10^{-3}$ and $R_h = -7.43 \pm 1.53 \times 10^{-3}$, respectively. Using R_h obtained from the constant I dataset and ρ, mobility of the charge carriers μ_H is found to be $\mu_H = 2.65 \pm 0.86 \times 10^{-1}$ $m^2/V.s$. Which is characteristic property of the sample with excepted value of $3.9 \pm 0.1 \times 10^{-1}$ $m^2/V.s$. [8]

I Introduction and Theory

History and Motivation: The Hall effect was discovered in 1879 by Edwin Hall [4]. While testing a theory on interactions of electric current with magnetic fields from James Clerk Maxwell [6], he measured a small, transverse voltage, now known as the Hall Voltage (V_H) when he passed a current through a thin gold foil and applied a perpendicular magnetic field. This discovery was proved that the magnetic field deflects the charge carriers themselves and not the carrying medium. Furthermore, the voltage's negative polarity correctly indicated that the charge carriers were negative, which is 18 years before the electron's formal discovery by J.J. Thomson [9].

Hall's observation is the primary motivation for this experiment. The Hall effect [2]is a powerful method for characterizing a semiconductor's fundamental electronic properties. By measuring the Hall voltage on our n-doped Germanium sample and combining it with a resistivity measurement ρ, we can definitively determine three key parameters:

- **Carrier Type:** Whether the material is n-type (electrons) or p-type (holes).
- **Carrier Concentration (n):** The number of charge carriers per unit volume.
- **Carrier Mobility (μ):** The ease with which carriers move through the crystal.

Theory: Experiment is based on the principle that when a current (I) and magnetic field (B) is present, charge carriers deflect in a direction perpendicular to both the current and the magnetic field. This deflection creates an accumulation at the one side of the conductor material and thus a potential difference called **Hall Voltage** is created. It can be derived from the equation for the hall effect:

$$E_h = R_h j_\perp B \tag{1}$$

Where E_h is the electric field generated by the hall effect and $j_\perp$ is the perpendicular component of the current density to the magnetic field. For a material with with w and a thickness t, we can do some substitutions for E_h and $j_\perp$, we can derive the **relation for hall coefficient**[5] as:

$$E_h w = V_h$$

$$j_\perp = \frac{I}{wt}$$

$$V_h = R_h \frac{I}{t} B$$

or;

$$R_h = \frac{V_h t}{IB} \tag{2}$$

Hall coefficient R_h is a material specific property. For an n-type semiconductor, where $q = -e$, R_H is negative thus it can be used to determine the type of charge carriers in a semiconductor. Moreover, one can relate R_h to the concentration and mobility of the charge carriers in a semiconductor as:

$$n = \frac{1}{|R_h| e} \tag{3}$$

$$\mu_h = \frac{|R_h|}{\rho} \tag{4}$$

Where ρ is the resistivity of the semiconductor and e is the fundamental charge of the electron. Notice since $e < 0$ for the case of electron, carrier concentration for an n-doped semiconductor is also negative as expected.

II Setup and Method

Setup: The experimental setup consists of

- PHYWE 11802.00 Hall Probe with holder and stand
- Large electromagnet with 24 V power supply
- Teslameter
- 2V power supply for the hall probe
- Digital multimeter

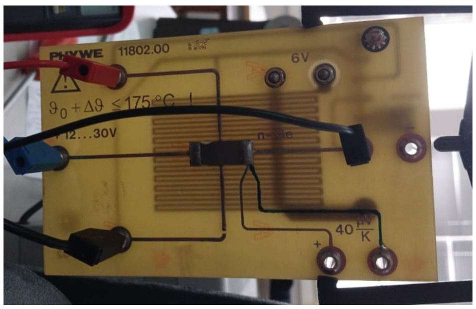

Figure 1: Hall probe with power supply terminal (red and hall voltage measurement terminal (blue)

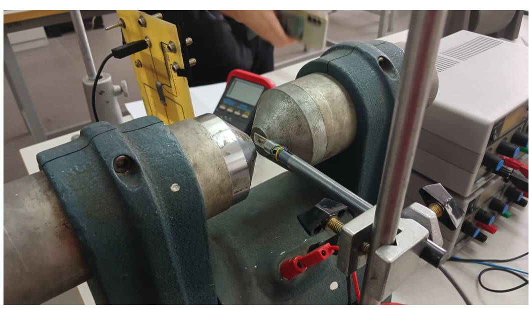

Figure 2: Electromagnet and teslameter's probe

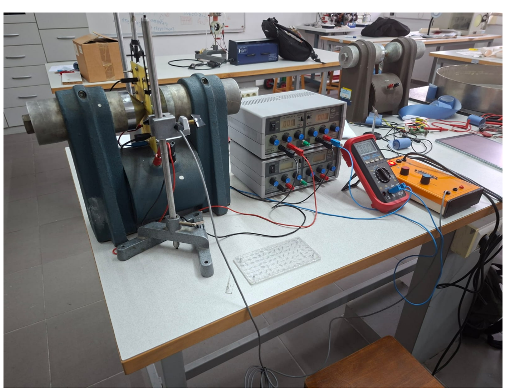

Figure 3: Full setup with hall prob inserted in the electromagnet

Method: The procedure for this experiment using the PHYWE Hall Probe with n-doped germanium is as follows:

1. **Resistivity:** Power supply for the hall probe is turned on and set to current mode. At least 10 pairs of currents ranging from 0 to 100 mA were recorded, noting the current passing through the probe and the corresponding voltage drop across terminals C and D. 10 measurements then plotted to obtain **resistance** of the probe. Measuring the dimensions of the crystal resistivity can be found using the formula:

$$\rho = R \times \frac{\text{cross sectional area}}{length}$$

2. **Electro magnet and Teslameter Calibration:** Electromagnet is momentarily turned on and teslameter probe is positioned in the gap to measure the polarity of the field. Once it is positioned in a way that it reads positive values in the direction of the field, electromagnet is turned off and teslameter is calibrated to 0 reading to eleminate any ambient magnetic field.

3. **Constant Current Measurements:** The Hall probe was positioned within the magnet gap. The current (I) supplied to the probe was set to a value between 100-150 mA. The orientation of the probe and the polarity of the current were arranged so that current flowed from terminal C to D, and the magnetic field was directed out of the semiconductor side of the probe. The Hall voltage (V_h) was measured between terminals A and B. At least 10 data pairs of the Hall voltage versus the magnetic field intensity were recorded. **Polarity** of this voltage requires close attention since the majority charge carriers and the type of semiconductor (n-type or p-type) can be determined based on the polarity.

4. **Constant Magnetic Field Measurements:** Above step is repeated for fixed magnetic field of around 200 mT and varying current. 10 data pairs are recorded for Hall voltage (V_h) vs. Magnetic field (B). All data pairs are then plotted to obtain **Hall constant** R_h using the formula:

$$R_h = \frac{V_h t}{IB}$$

III Data

The raw experimental datasets consist of 10 Hall voltage (V_h) vs. Magnetic Field (B) data pairs and 10

Hall voltage (V_h) vs. Current (I) data pairs that are used in calculation of the hall constant. There is also 10 additional data pair of V vs. I from the resistance measurement. Below are the table for the Hall Constant datasets and resistance data set can be found in resistance data. For the raw data of this experiment, please refer to [1].

Uncertainties in Magnetic field measurement is taken as **%5** according to [3]. Uncertainties are **0.1 mV** and **0.1 mA** for the voltage and current measurements, respectively.

B (mT) $\pm\%5$	V_h (mV) ± 0.1
20.2 ± 1.0	-90.8
40.9 ± 2.0	-105.8
64.2 ± 3.2	-119.7
80.4 ± 4.0	-134.4
100.7 ± 5.0	-149.2
120.6 ± 6.0	-163.7
140.3 ± 7.0	-177.9
160.1 ± 8.0	-192.2
180.3 ± 9.0	-206.71
199.8 ± 10.0	-220.7

Table 1: Voltage vs. B with constant $I = 100.1$ mA

I (mA) ± 0.1	V_H (mV) ± 0.1
10.1	-14.6
20.8	-30.3
30.0	-43.7
40.4	-58.8
50.2	-73.3
60.2	-88.2
70.2	-103.1
80.0	-117.7
90.5	-133.8
100.0	-148.4

Table 2: Voltage vs. Current with constant $B = 200.0$ mT

IV Analysis

Resistivity:

Resistivity of a conductor material with cross sectional are A and length L is given as:

$$R = \rho \frac{L}{A}$$

where ρ is resistivity. We can arrange this equation to calculate ρ from a known resistance:

$$\rho = R\frac{wt}{L}$$

with σ_ρ being:

$$\sigma\rho = \rho \cdot \sqrt{\left(\frac{\sigma R}{R}\right)^2 + \left(\frac{\sigma w}{w}\right)^2 + \left(\frac{\sigma t}{t}\right)^2 + \left(\frac{\sigma L}{L}\right)^2}$$

Derivation can be found in Appendix on error propagation. Here w is width and t is the thickness of the material. For the hall probe those dimensions [7] for the n-doped germanium crystal are given as:

$$w = 10 \pm 1mm$$
$$t = 1 \pm 0.2mm$$
$$L = 20 \pm 1mm$$

Resistance of the probe can also be found as $54.3 \pm 6.2\Omega$ by applying a linear fit to the resistance V vs. I dataset:

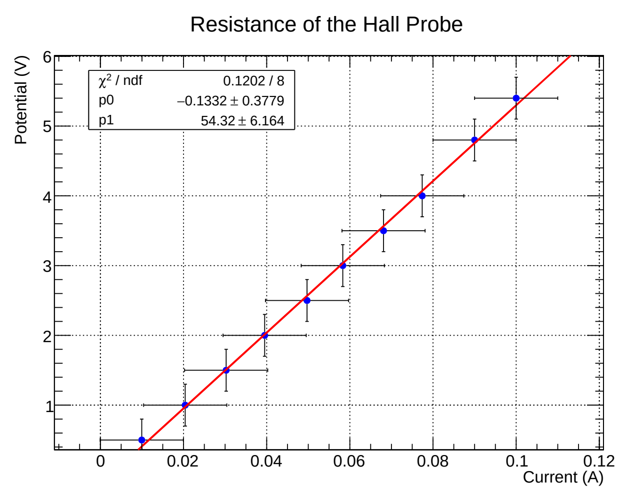

Figure 4: Resistance as the slope of V/I line. Uncertainties in V is multiplied by 3 to improve visibility. Real values are used in fit process.

According to this values resistivity of the hall probe is calculated as:

$$\rho = 54.3\frac{10 \times 1}{20} = 27.2\Omega \times \text{mm}$$

and:

$$\sigma_\rho = 6.9\Omega \times \text{mm}$$

Hall coefficient for constant I:

Using the constant current varying magnetic field dataset, Hall constant of the material is extracted from the slope of V_h vs. B graph.

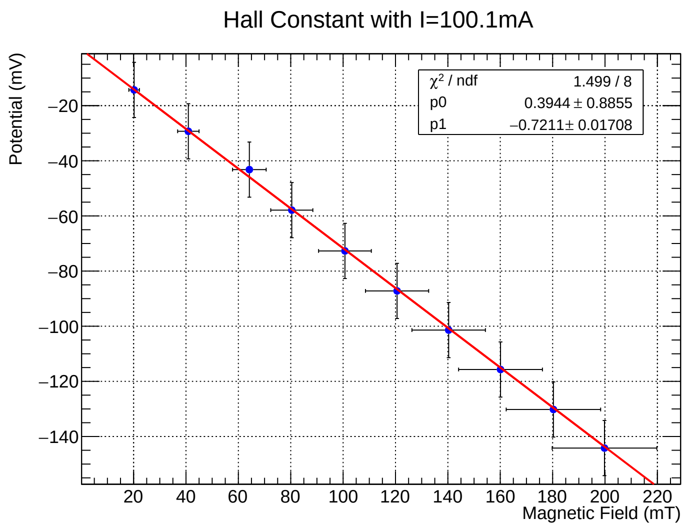

Figure 5: Hall Voltage vs. Magnetic field. Here the p1 represents the $R_h \times j_\perp$ and uncertainties in V_h and B are extrapolated by a factor of 100 and 2, respectively to improve visibility.

From (2), Hall coefficient from this fit is:

$$R_h = \frac{V_h}{B}\frac{t}{I_{\text{const}}} = p1\frac{1mm}{100.1mA} = -7.20 \times 10^{-3}\ m^3/C$$

Uncertainty in R_h (similarly to the uncertainty in ρ) is given as:

$$\sigma_{R_h} = R_h \cdot \sqrt{\left(\frac{\sigma_{p1}}{p1}\right)^2 + \left(\frac{\sigma_t}{t}\right)^2 + \left(\frac{\sigma_I}{I}\right)^2}$$

and found to be:

$$\sigma_{R_h} = 1.45 \times 10^{-3}\ m^3/C$$

calculating n and μ_h:

According to (3), majority charge carrier concentration (neglecting the minority charge carrier concentration) in the semiconductor can be found as:

$$n = \frac{1}{|R_h|e} = \frac{1}{7.20 \times 10^{-3} \times 1.60 \times 10^{-19}} = 8.68 \times 10^{20}$$

Which is an exceptable value for these types of semiconductors. Mobility of the charge carriers is, according to (4):

$$\mu_h = \frac{|R_h|}{\rho}$$

with:

$$\sigma_{\mu_h} = \mu_h \cdot \sqrt{\left(\frac{\sigma_{R_h}}{|R_h|}\right)^2 + \left(\frac{\sigma_\rho}{\rho}\right)^2}$$

As mentioned in the theory, this value is a material specific property and known to be $\mu_e \approx 0.39 \pm 0.1\ \mathrm{m}^2/(\mathrm{V}\cdot\mathrm{s})$ for n-doped germanium. Experimental value for μ_h is calculated to be: $2.65 \pm 0.86 \times 10^{-1}\mathrm{m}^2/(\mathrm{V}\cdot\mathrm{s})$.

Hall constant for constant B dataset:

As before, using the constant magnetic field and varying current dataset, Hall constant is extracted from the slope of V_h vs. B graph .

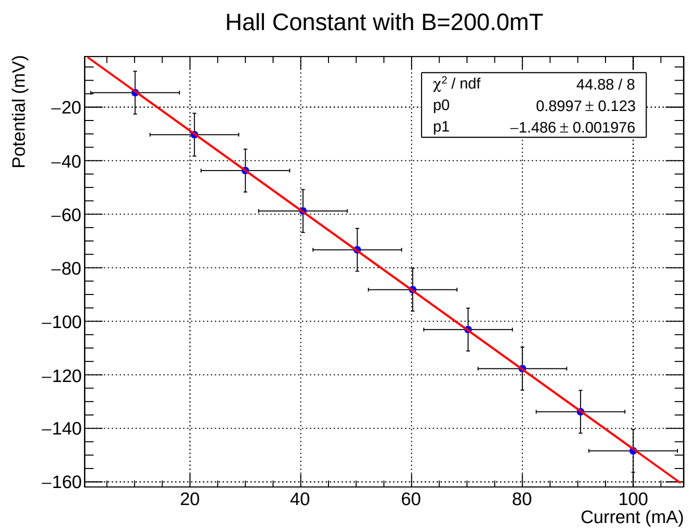

Figure 6: Hall Voltage vs. Current. Here the p1 represents the $R_h \times B$ and uncertainties in V_h and B are extrapolated by a factor of 80 to improve visibility.

From (2), Hall coefficient from this fit is:

$$R_h = \frac{V_h}{I}\frac{t}{B_{\text{const}}} = p1\frac{1mm}{200.0mT} = -7.43 \times 10^{-3}\ m^3/C$$

Which is concistent with the previous measurement. This is expected since hall coefficients realtion with both current and magnetic filed is linear. Uncertainty in R_h is given as:

$$\sigma_{R_h} = R_h \cdot \sqrt{\left(\frac{\sigma_{p1}}{p1}\right)^2 + \left(\frac{\sigma_t}{t}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2}$$

and found to be: $\sigma_{R_h} = 1.53 \times 10^{-3}\ m^3/C$

V Results and Conclusion

It is at this point noteworthy to make a remark the non-zero intercept in both plots for Hall Constant. If one remembers the calibration step for teslameter, they can assume an ignored ambiant magnetic field

B_0. Then instead of (1) the correct formula for hall constant is:

$$\begin{aligned} V_h &= R_h \frac{I}{t}(B + B_0) = R_h \frac{I}{t} B + R_h \frac{I}{t} B_0 \\ &= R_h \frac{I}{t} B + V_0 (\text{a constant}) \end{aligned}$$

Luckily, constant V_0 does not depend on I or B so we can ignore it in fitting process and interpret the slope directly as in (2).

After that remark it can be concluded that both datasets allign with each other in terms of hall constant R_h with:

- $R_h = -7.20 \pm 1.45 \times 10^{-3}$ for constant I dataset
- $R_h = -7.43 \pm 1.53 \times 10^{-3}$ for constant B dataset

Although uncertainty being relatively high in both values mainly due to high uncertainty in dimensions of the material thickness t ($\sigma_t = \%20 \times t$), it predicts a charge carrier mobility of:

- $2.65 \pm 0.86 \times 10^{-1} \mathrm{m}^2/(\mathrm{V \cdot s})$ for constant I dataset
- $2.74 \pm 0.90 \times 10^{-1} \mathrm{m}^2/(\mathrm{V \cdot s})$ for constant B dataset

which gives $\mu_h = 2.69 \pm 0.88 \times 10^{-1} \mathrm{m}^2/(\mathrm{V \cdot s})$ when combined. The difference from theoretical value $\mu_e \approx 0.39 \pm 0.1\ \mathrm{m}^2/(\mathrm{V \cdot s})$ is:

$$\sigma = \frac{|\mu_{\text{real}} - \mu_{\text{h}}|}{\sigma_{\text{h}}}$$

$$\begin{aligned} \sigma &= \frac{|0.39 - 0.269|}{0.088} \\ &= \frac{0.121}{0.088} \approx 1.375 \end{aligned}$$

Which is in reasonable range mainly due to high uncertainty of 0.88×10^{-1} in the measurement. A future improvement might be a higher precision measurement of the dimensions of the material to have a clearer idea on the resistivity ρ and the current density $j_\perp$.

References

[1] capta1Nemo. *PHYS443 Experiments GitHub Repository*. https://github.com/capta1Nemo/443experiments.git.

[2] Encyclopædia Britannica. *BHall Effect*. https://www.britannica.com/science/Hall-effect. 2024.

[3] E. Gülmez. *Advanced Physics Experiments*. Boğaziçi University, 1997.

[4] Edwin H. Hall. *On a New Action of the Magnet on Electric Currents*. Vol. 2. 3. 1879, pp. 287–292.

[5] Charles Kittel. *Introduction to Solid State Physics*. 8th. Hoboken, NJ: John Wiley & Sons, 2005.

[6] James Clerk Maxwell. *A Treatise on Electricity and Magnetism*. Vol. 2, Part IV, Ch. 7, which discussed the force on the conductor. Oxford: Clarendon Press, 1873.

[7] *Operating Instructions: Hall Effect, n-germanium (11802.00) and Hall Effect, p-germanium (11805.00)*. PHYWE. URL: http://www.hep.fsu.edu/~wahl/phy4822/expinfo/hall/phywe.pdf.

[8] M. B. Prince. "Drift Mobilities in Semiconductors. I. Germanium". In: *Physical Review* 92 (3 1953), pp. 681–687. DOI: 10.1103/PhysRev.92.681. URL: https://doi.org/10.1103/PhysRev.92.681.

[9] J. J. Thomson. "Cathode Rays". In: *Philosophical Magazine* 44.269 (1897), pp. 293–316.

Appendix

A.1 Error propagation formulas

To calculate the uncertainty on that function based on the uncertainty of its variables, we can refer to general error propagation equation for multi variable functions. For a function of $f(x_1, x_2, ...)$ the uncertainty is:

$$\sigma_f^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \sigma_{x_i} \right)^2 \tag{5}$$

Derivation of error for linear functions:

If the formula for Q only involves multiplication and division:

$$Q = \frac{x \cdot y}{z}$$

Then uncertainty from above sum is:

$$\begin{aligned} \sigma_Q^2 &= \left(\frac{\partial f}{\partial x} \sigma_x \right)^2 + \left(\frac{\partial f}{\partial y} \sigma_y \right)^2 + \left(\frac{\partial f}{\partial z} \sigma_z \right)^2 \\ &= \left(\frac{y}{z} \sigma_x \right)^2 + \left(\frac{x}{z} \sigma_y \right)^2 + (xy\sigma_z)^2 \\ &= Q^2 \ \left(\frac{\sigma_x}{x} \right)^2 + \left(\frac{\sigma_y}{y} \right)^2 + \left(\frac{\sigma_z}{z} \right)^2 \end{aligned}$$

The **relative** uncertainties of each variable add in quadrature to give the total **relative** uncertainty of the result.

General Formula:

$$\left(\frac{\sigma_Q}{Q} \right)^2 = \left(\frac{\sigma_x}{x} \right)^2 + \left(\frac{\sigma_y}{y} \right)^2 + \left(\frac{\sigma_z}{z} \right)^2$$

To find the final **absolute uncertainty (**σ_Q**)**, you rearrange this formula:

$$\sigma_Q = Q \cdot \sqrt{\left(\frac{\sigma_x}{x} \right)^2 + \left(\frac{\sigma_y}{y} \right)^2 + \left(\frac{\sigma_z}{z} \right)^2}$$

Additional Material

Resistance dataset

The datasets in text format can also be found in [1].

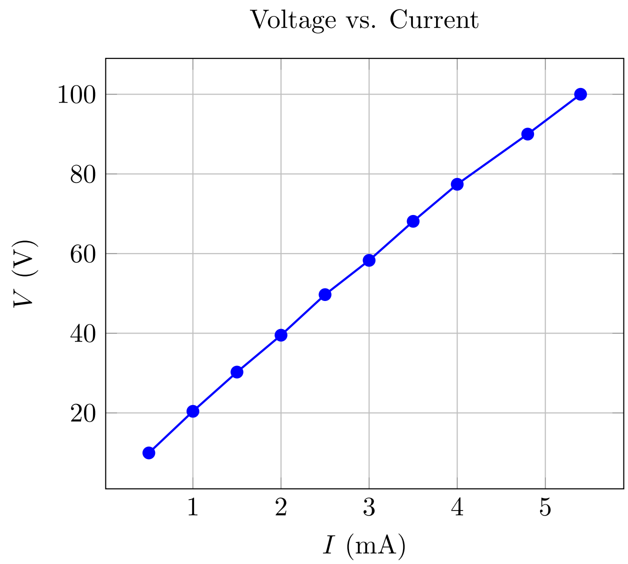

Table 3: Voltage vs. Current Data

V (V) ±0.1	I (mA)±0.1
0.5	9.95
1.0	20.40
1.5	30.25
2.0	39.51
2.5	49.70
3.0	58.30
3.5	68.10
4.0	77.40
4.8	90.00
5.4	100.00

Codes

All the codes here and more can be found in the GitHub repository [1].

```python

def resistivity(r):
    return r[1][0]*(t*a)/l
def sigma_r(res,r):
    # --- Calculate squared relative uncertainties ---
    rel_R_sq = math.pow(r[1][1]/ r[1][0], 2)
    rel_w_sq = math.pow(sa / a, 2)
    rel_t_sq = math.pow(st / t, 2)
    rel_L_sq = math.pow(sl / l, 2)

    total_rel_uncertainty = math.sqrt(rel_R_sq + rel_w_sq + rel_t_sq + rel_L_sq)
    return res * total_rel_uncertainty

r=resistivity(r_parameters)*1e-3
sr=sigma_r(r,r_parameters)
print(f"resistivity is {r:.3g}+-{sr:.3g} ohm X meters")

def hall_coef(r,const):
    hall=r[1][0]*t/const[0]
    rel_R_sq = math.pow(r[1][1]/ r[1][0], 2)
    rel_t_sq = math.pow(st / t, 2)
    rel_C_sq = math.pow(const[1]/const[0], 2)

    total_rel_uncertainty = math.sqrt(rel_R_sq +  rel_t_sq + rel_C_sq)
    return [hall,hall * total_rel_uncertainty]

hall_coefI=hall_coef(I_parameters,constI)
print(f"Hall coef for consant I is {hall_coefI[0]:.3g}+-{-hall_coefI[1]:.3g} m^3/C ")

def density(r,sr,hall):
    mu=hall[0]/r
    rel_r_sq = math.pow(sr/ r, 2)
    rel_h_sq = math.pow(hall[1] /hall[0], 2)
    total_rel_uncertainty = math.sqrt(rel_r_sq +  rel_h_sq )
    return [mu,mu * total_rel_uncertainty]
mu1=density(r,sr,hall_coefI)
print(f"Mobility for fixed I is {mu1[0]:.2e}+-{-mu1[1]:.1e}")
mu2=density(r,sr,hall_coefB)
print(f"Mobility for fixed B is {mu2[0]:.2e}+-{-mu2[1]:.1e}")
hall_avg=[(hall_coefB[i]+hall_coefI[i])/2 for i in range(2)]
mu3=density(r,sr,hall_avg)
print(f"Mobility for combined result is {mu3[0]:.2e}+-{-mu3[1]:.1e}")
```

Code 1: Various functions used in the analysis with some error propogation formulas

```python
ROOT.gStyle.SetOptFit(1)

data_file_path = os.path.join('data', 'AkimSabit.csv')
try:
    data = np.loadtxt(data_file_path, delimiter=',',skiprows=1)
    print(f"consant I is {float(data[1,0])}mA")
    constI=[float(data[1,0]),0.01]
    x = data[:, 1]
    y = data[:, 2]+76.5
    sx =x*0.05
    sy = np.full(len(y), 0.1)
    n_points = len(x)
    print(f"Successfully loaded {n_points} points from '{data_file_path}'.")
except (IOError, IndexError) as e:
    print(f"Error: Could not read or parse '{data_file_path}'.")
    exit()

graph = ROOT.TGraphErrors(len(x))
for i in range(len(x)):
```

```
    graph.SetPoint(i, x[i], y[i])
    graph.SetPointError(i, sx[i], sy[i])

fit_func = ROOT.TF1("fit_func", "[0] + [1]*x", min(x), max(x))
fit_func.SetParameters(0, 0)  # initial guesses

graph.Fit(fit_func)
I_parameters=[(fit_func.GetParameter(i),fit_func.GetParError(i)) for i in range(2)]

visual_scale_factor =100

print(f"Applying a visual scaling factor of {visual_scale_factor}.")

for i in range(len(x)):
    graph.SetPointError(i, sx[i]*2 , sy[i]* visual_scale_factor)
graph.SetMarkerStyle(20)
graph.SetMarkerColor(ROOT.kBlue)
graph.SetTitle(f"Hall Constant with I={constI[0]}mA ;Magnetic Field (mT);Potential (mV)")

c = ROOT.TCanvas("c", "Line Fit", 800, 600)
graph.Draw("AP")
fit_func.Draw("same")
c.Update()
stats = graph.GetListOfFunctions().FindObject("stats")
if stats:
    stats.SetX1NDC(0.55)  # lower-left x (0 to 1)
    stats.SetY1NDC(0.75)  # lower-left y
    stats.SetX2NDC(0.85)  # upper-right x
    stats.SetY2NDC(0.87)   # upper-right y
    stats.SetTextSize(0.03)
    c.Modified()

ROOT.gPad.SetTicks(1, 1)
ROOT.gPad.SetGrid(1, 1)
c.Update()
c.SaveAs("const I.pdf")
del c, graph, fit_func
```

Code 2: PyROOT code to fit linear function to data

This is a general macro and used for all 3 datasets with different visual scale factors.