Charge to Mass Ratio of the Electron

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Abstract:
In this experiment, the charge-to-mass ratio (q/m_0) of the electron was determined by investigating the motion of electrons accelerated through an electric field and deflected by a magnetic field. Electrons were accelerated using potential differences of $\mathbf{V} = (147 \pm 1V, 125 \pm 1V)$ and subjected to a magnetic field generated by Helmholtz coils, causing them to follow a circular path. Measurements of the path's radius were taken for varying magnetic field strengths, allowing for the calculation of q/m_0 using the relationship derived from the balance of electric and magnetic forces. The same method is applied for constant magnetic field strengths $\mathbf{B} = ((1.27 \pm 0.0069) \times 10^{-3}T, (1.05 \pm 0.0069) \times 10^{-3}T)$ and varying electric field. The results of multiple trials were analyzed using weighted averages, which yielded a final value of $q/m_0 = (1.65 \pm 0.096) \times 10^{11}C/kg$, which is 1.15 sigmas away and consistent with the accepted value of $1.76 \times 10^{11}C/kg$.

I. History and Motivation

The study of the charge-to-mass ratio of the electron has been a cornerstone of modern physics, dating back to the late 19th and early 20th centuries. The first accurate measurement of q/m_0 was made by J.J. Thomson in 1897 during his famous cathode ray tube experiments, which led to the discovery of the electron. Thomson's work demonstrated that cathode rays were composed of negatively charged particles, and he was able to determine the ratio of their charge to mass. This discovery was pivotal in the development of atomic theory and quantum mechanics.

Later, Robert Millikan's oil-drop experiment (1909) provided an accurate measurement of the electron's charge q, which, when combined with Thomson's results, allowed for the determination of the electron's mass m_0. These experiments laid the foundation for our understanding of the electron as a fundamental particle and its role in atomic structure.

The motivation for this experiment is to replicate and verify these historical measurements using a rather more modern technique. By accelerating electrons through an electric field and observing their deflection in a magnetic field, we can determine q/m_0 and compare it to the accepted value of approximately $1.76 \times 10^{11}C/kg$.

II. Theory

electrons can be accelerated under an electric field with their kinetic energy is given by the formula:

$$K = \frac{1}{2}m_0 v^2 = q\mathbf{V} \tag{1}$$

where v is the velocity of the electron. Which we can interpret as:

$$v^2 = \frac{2q\mathbf{V}}{m_0} \tag{2}$$

Finally, magnetic field applies a Lorentz force to a moving, charged object by:

$$F = qv \times \mathbf{B} \tag{3}$$

Assuming the magnetic field's direction is perpendicular to electrons speed, it follows from (2) and (3) for an electron to have a circular path with radius r, relation between magnetic and electric field strengths should be:

$$\frac{m_0 v^2}{r} = qv \times \mathbf{B}$$

$$v = \frac{q}{m_0}\mathbf{B}r$$

$$\frac{2q\mathbf{V}}{m_0} = \left(\frac{q}{m_0}\mathbf{B}r\right)^2$$

We can manipulate this equation to obtain mass to charge ratio of the electron based on electric and magnetic field strengths and radius of the circular path.

$$\frac{q}{m_0} = \frac{2\mathbf{V}}{\mathbf{B}^2 r^2} \tag{4}$$

We can directly measure the electric field $\mathbf{V}$ but we have to compute magnetic field $\mathbf{B}$. To find the magnetic field at the z axis of two coils that has a distance a between them, we can use the equation:

$$B_z = \frac{\mu_0 I R^2}{\left(R^2 + \left(z - \frac{a}{2}\right)^2\right)^{3/2} + \left(R^2 + \left(z + \frac{a}{2}\right)^2\right)^{3/2}}$$

where I is the current going through the coils, R is the radius of the coils. In our case, we have z = 0, a = R and n turns at each coil. Which yields:

$$\mathbf{B} = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 I n}{R} \tag{5}$$

III. Setup

The experimental setup consists of:

- A narrow beam tube with two Helmholtz coils
- A DC power supply for the beam (0–300 V) and a DC voltmeter to measure V
- An AC power supply for the filament (6.3 V, 5 A) and an AC ammeter to check its output
- A DC power supply (0–5 A) for the coils and a DC ammeter to measure I

The narrow beam tube apparatus also includes a closure where the electrons travel and a measurement ladder to determine the radius r.

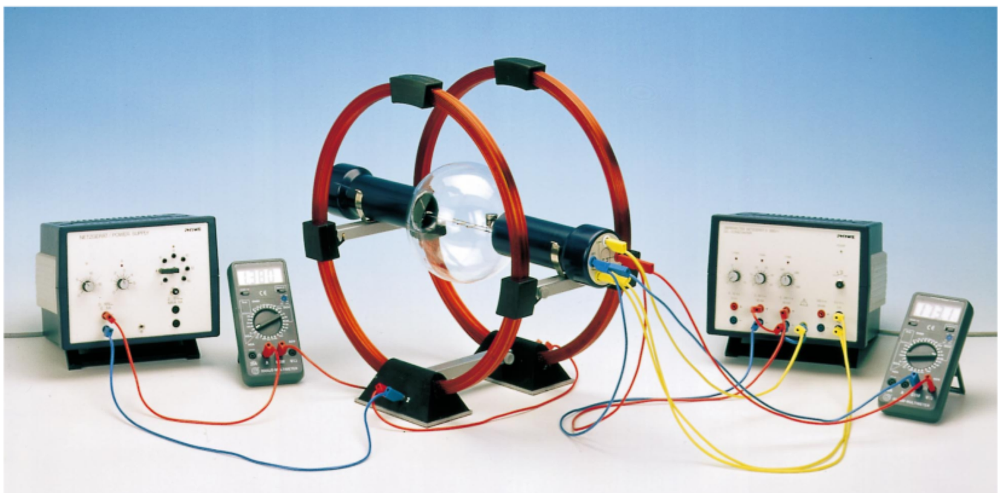


Figure 1: Experimental Setup with Narrow Beam Tube and Helmholtz Coils

IV. Method

The experiment was conducted following these steps:

1. Electric field is activated from its power supply and path of the electrons are observed as straight lines going vertically upward from the electron gun.
2. Power supply for the magnetic field is then turned on and path of the electrons are observed again.
3. The field strength is adjusted to achieve a circular path inside the bulb. In addition, the electron gun is rotated so that it is aimed exactly perpendicular to the direction of magnetic field. This is necessary so that electrons do not spiral to the left or right as they complete a circle.

4. A constant current between 1.5-2 A is chosen and the voltage is varied to achieve paths with different radius, measuring the voltage at each step of the ladder. This procedure is repeated for one more current value.

5. A constant voltage is chosen around 150-100 V and the current is varied to achieve paths with different radius, measuring the voltage at each step along the ladder. This process is repeated for one more voltage value.

V. Data

Measurements are taken for 4 r values under 2 constant currents and 2 constant voltages are below.

I (A)	**Voltages (V)**			
	2 cm	**3 cm**	**4 cm**	**5 cm**
1.83 ± 0.01	89 ± 1	135 ± 1	228 ± 1	354 ± 1
1.52 ± 0.01	73 ± 1	105 ± 1	169 ± 1	246 ± 1

Table 1: Voltage values for fixed currents

V (V)	**Currents (A)**			
	2 cm	**3 cm**	**4 cm**	**5 cm**
147 ± 1	3.03 ± 0.01	1.92 ± 0.01	1.39 ± 0.01	1.09 ± 0.01
125 ± 1	2.72 ± 0.01	1.65 ± 0.01	1.20 ± 0.01	0.94 ± 0.01

Table 2: Current values for fixed voltages

VI. Analysis

0.1 Magnetic field calculation:

as shown in (5), magnetic field from current I is calculated as:

$$\left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n}{R} = \alpha, \quad \mathbf{B} = \alpha I \tag{6}$$

for R=0.2m and n=154, α is found to be:

$$\alpha = \left(\frac{4}{5}\right)^{3/2} \frac{(4\pi \times 10^{-6}) \times 154}{0.2} = 6.92 \times 10^{-4}$$

and

$$\mathbf{B} = (6.92 \times 10^{-4}) \times I$$

Notice, since this relation is linear we can directly propagate error for I ($\pm 0.01A$ to error for $\mathbf{B}$:

$$\alpha \cdot \sigma_I = \sigma_B \tag{7}$$

$$\alpha \cdot 0.01 = \pm 6.92 \times 10^{-6}$$

Magnetic field calculation script (Code 1), is provided in codes section. Magnetic field strengths of the data points are as follows:

V (V)	**Magnetic Field (T)** ($\pm 0.0069 \times 10^{-3}$)			
	2 cm	**3 cm**	**4 cm**	**5 cm**
147 ± 1	2.10×10^{-3}	1.33×10^{-3}	9.62×10^{-4}	7.55×10^{-4}
125 ± 1	1.88×10^{-3}	1.14×10^{-3}	8.31×10^{-4}	6.51×10^{-4}

Table 3: Magnetic field values for fixed voltages

Current (A)	**Magnetic Field (T)**
1.83 ± 0.01	$(1.27 \pm 0.0069) \times 10^{-3}\,T$
1.52 ± 0.01	$(1.05 \pm 0.0069) \times 10^{-3}\,T$

Table 4: Magnetic field values for fixed currents

0.2 Line-fit for constant voltage:

we can obtain a linear relationship $y = \mathit{slope} * x$ between r and $\mathbf{B}$ by manipulating (4) into the form:

$$\frac{2\mathbf{V}}{r^2} = \frac{q}{m_0}\mathbf{B}^2 \tag{8}$$

where r^2 and $\mathbf{B}^2$ are variables and $\mathbf{V}$ is a constant. Here q/m_0 appears as the slope of this linear equation. Moreover, to calculate the uncertainty σ_y and σ_x, we use the error propagation formula for a function of multiple variables. For a general function $f(u, v)$, the uncertainty σ_f is given by:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial u} \cdot \sigma_u\right)^2 + \left(\frac{\partial f}{\partial v} \cdot \sigma_v\right)^2} \tag{9}$$

derivation of σ_y:

The formula for y is:

$$y = \frac{2\mathbf{V}}{r^2}$$

The uncertainty σ_y is:

$$\sigma_y = y \cdot \sqrt{\left(\frac{\sigma_V}{\mathbf{V}}\right)^2 + \left(\frac{2\sigma_r}{r}\right)^2}$$

derivation of σ_x:

The formula for x is:

$$x = \mathbf{B}^2$$

The uncertainty σ_x is:

$$\sigma_x = 2B \cdot \sigma_B$$

For the detailed derivation, refer to **Appendix A.1**.

$\sigma_V, \sigma_B, \sigma_r = \pm 1, \pm 0.01, \pm\sqrt{2}$, respectively. Refer to Code 2, for numerical calculations of σ_x and σ_y. After this, our data set of x and y with their uncertainties is:

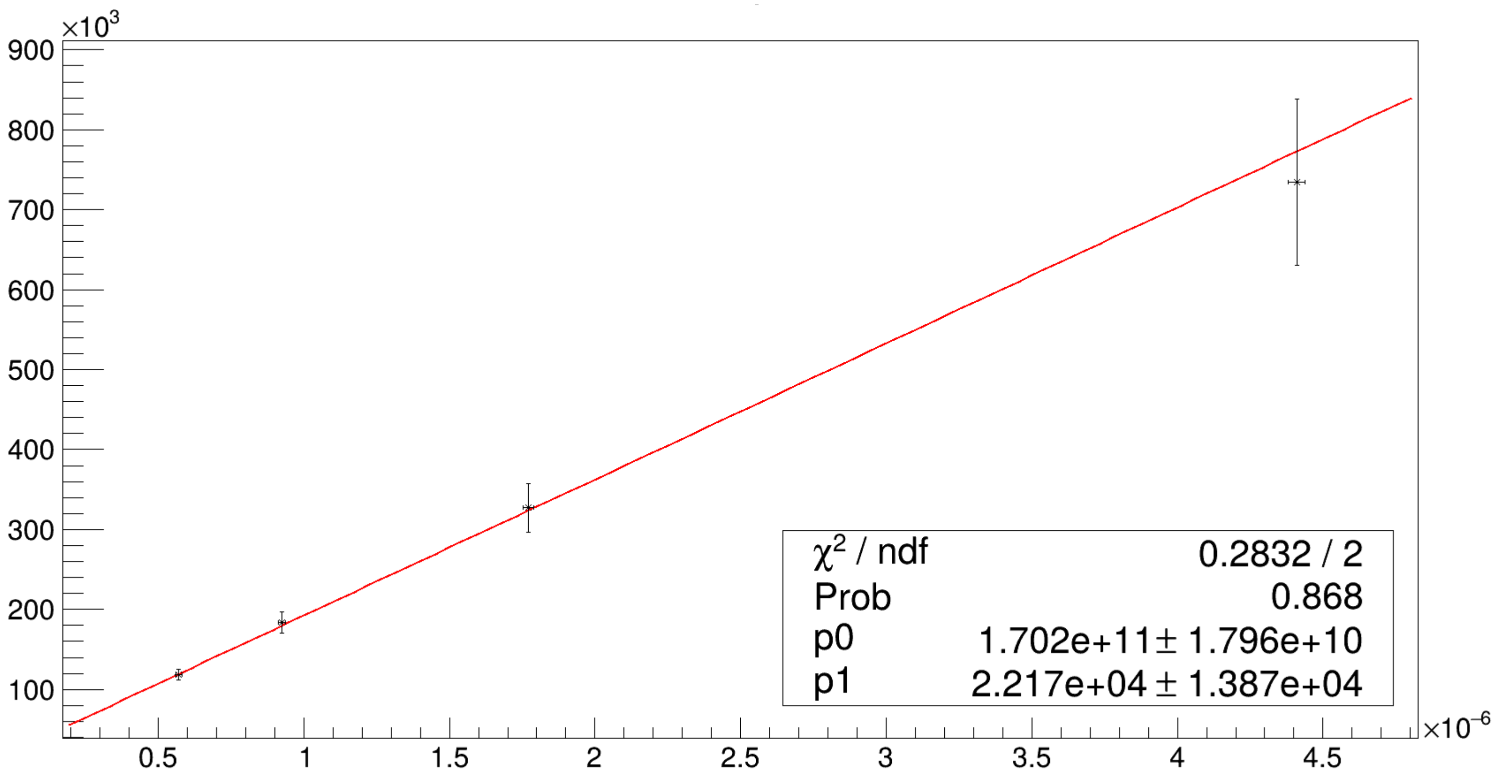
	Radius			
	2 cm	**3 cm**	**4 cm**	**5 cm**
$\frac{\mathbf{2V}}{r^2}$	$(7.35 \pm 1.04) \times 10^5$	$(3.27 \pm 0.309) \times 10^5$	$(1.84 \pm 0.131) \times 10^5$	$(1.18 \pm 0.067) \times 10^5$
$\mathbf{B}^2$	$(4.41 \pm 0.0291) \times 10^{-6}$	$(1.77 \pm 0.0184) \times 10^{-6}$	$(9.25 \pm 0.0133) \times 10^{-7}$	$(5.70 \pm 0.0104) \times 10^{-7}$

Table 5: Voltage values for $\mathbf{V}1 = 147 \pm 1$ Volts

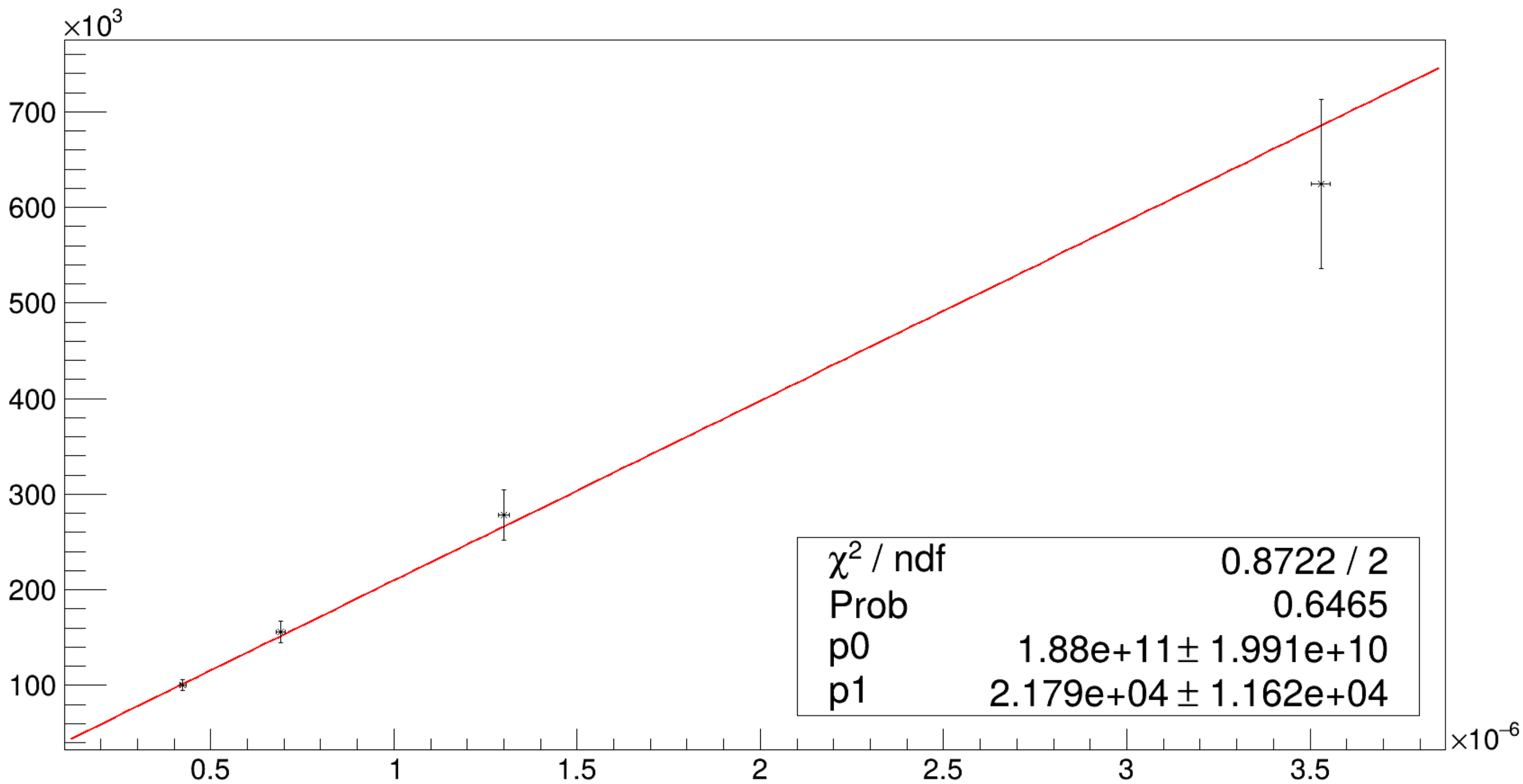
	Radius			
	2 cm	**3 cm**	**4 cm**	**5 cm**
$\frac{\mathbf{2V}}{r^2}$	$(6.25 \pm 0.885) \times 10^5$	$(2.78 \pm 0.263) \times 10^5$	$(1.56 \pm 0.111) \times 10^5$	$(1.00 \pm 0.0571) \times 10^5$
$\mathbf{B}^2$	$(3.53 \pm 0.0260) \times 10^{-6}$	$(1.30 \pm 0.0158) \times 10^{-6}$	$(6.91 \pm 0.0115) \times 10^{-7}$	$(4.24 \pm 0.00901) \times 10^{-7}$

Table 6: Voltage values for $\mathbf{V}2 = 125 \pm 1$ Volts

Finally we can use the root's fit function to fit a line to these data (Code 3). The slope of the fitted line is found to be $1.70 \times 10^{11} \pm 1.79 \times 10^{10}$ for $\mathbf{V}$=147 Volts. And $1.88 \times 10^{11} \pm 1.99 \times 10^{10}$ for $\mathbf{V}$=125 Volts



(a) Fit for $\mathbf{V}$=147 $\pm$ 1 Volts



(b) Fit for $\mathbf{V}$=125 $\pm$ 1 Volts

0.3 Line-fit for constant current:

This time we need a linear relation between $\mathbf{V}$ and r^2. Again, by manipulating (4) into the form:

$$2\mathbf{V} = \frac{q}{m_0}\mathbf{B}^2 r^2 \tag{10}$$

we obtain $y = 2\mathbf{V}$ and $x = \mathbf{B}^2 r^2$ for our linear relation. Now, the general formula for error propagation (9) yields:

The uncertainty σ_x:

$$\sigma_x = 2x \cdot \sqrt{\left(\frac{\sigma_B}{\mathbf{B}}\right)^2 + \left(\frac{\sigma_r}{r}\right)^2}$$

The uncertainty σ_y:

$$\sigma_y = 2\sigma_V$$

Derivation is once again, explained in **Appendix A.2**.
with the help of Code 2, x and y dataset is found to be:

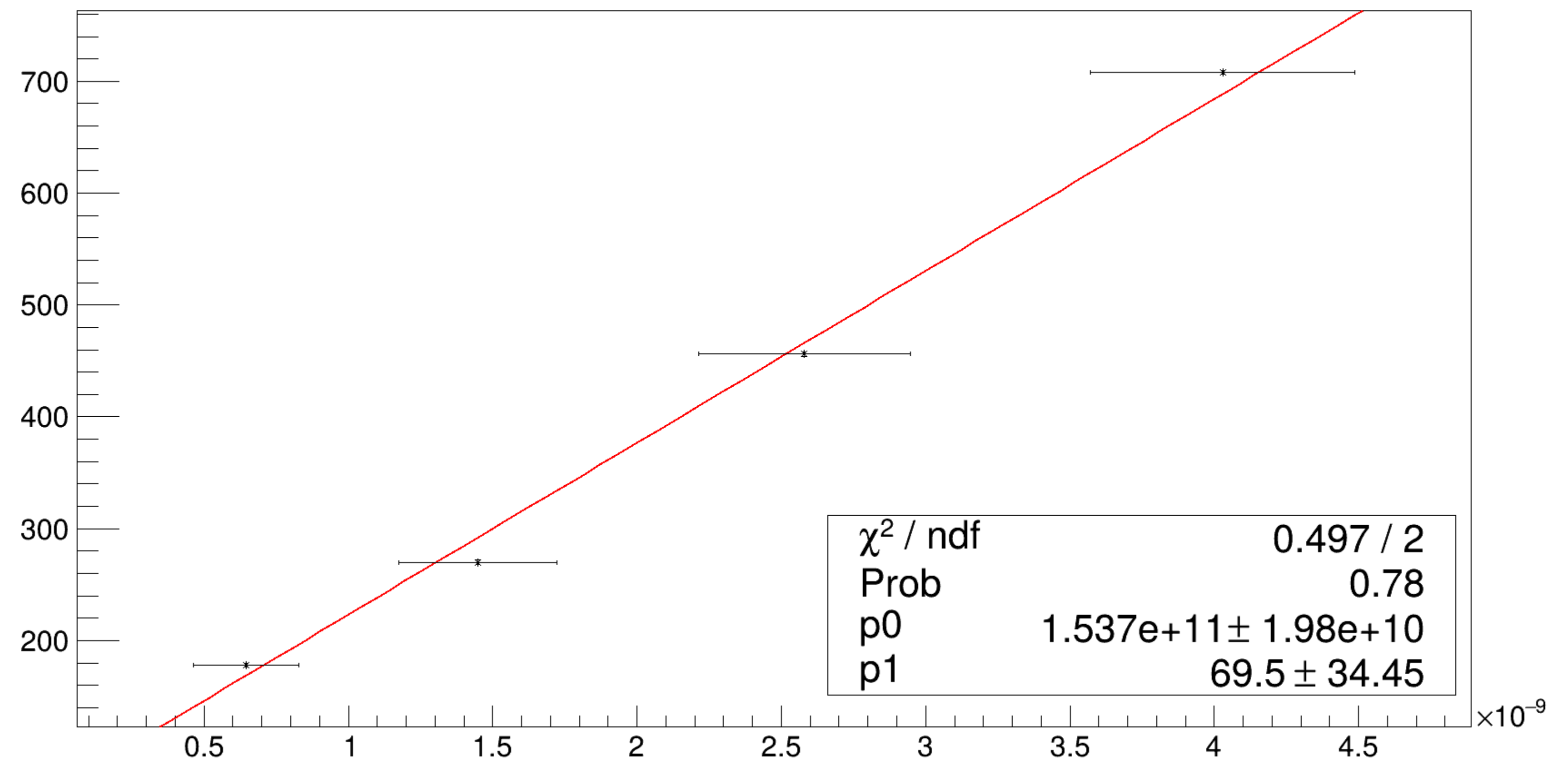
	Radius			
	2 cm	**3 cm**	**4 cm**	**5 cm**
2V	$(1.78 \pm 0.02) \times 10^2$	$(2.70 \pm 0.02) \times 10^2$	$(4.56 \pm 0.02) \times 10^2$	$(7.08 \pm 0.02) \times 10^2$
$\mathbf{B}^2 r^2$	$(6.45 \pm 1.83) \times 10^{-10}$	$(1.45 \pm 0.274) \times 10^{-9}$	$(2.58 \pm 0.366) \times 10^{-9}$	$(4.03 \pm 0.458) \times 10^{-9}$

Table 7: Values for $\mathbf{B}1 = 0.00127 \pm 6.92 \times 10^{-6}$ T

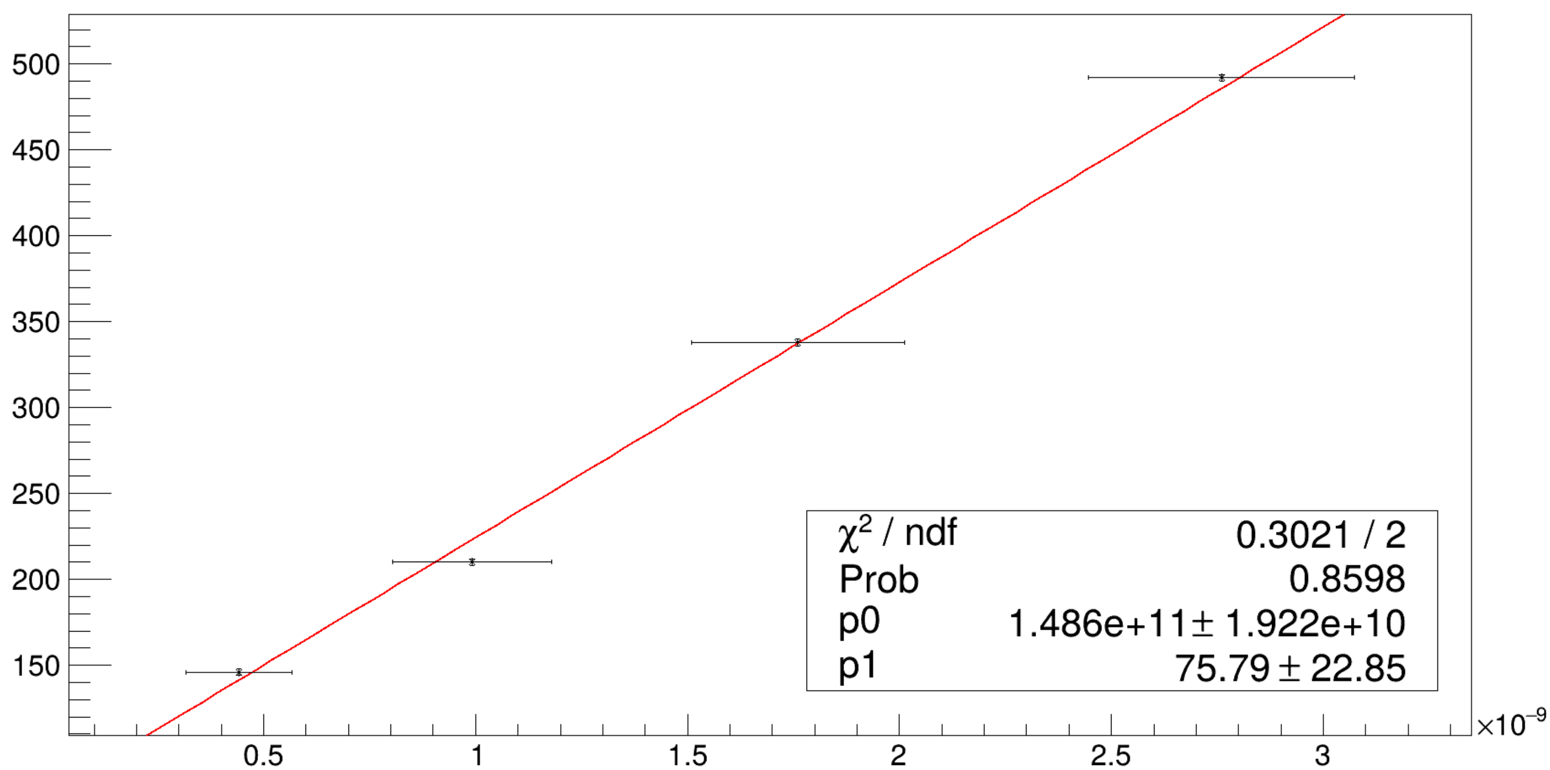
	Radius			
	2 cm	**3 cm**	**4 cm**	**5 cm**
2V	$(1.46 \pm 0.02) \times 10^2$	$(2.10 \pm 0.02) \times 10^2$	$(3.38 \pm 0.02) \times 10^2$	$(4.92 \pm 0.02) \times 10^2$
$\mathbf{B}^2 r^2$	$(4.41 \pm 1.25) \times 10^{-10}$	$(9.92 \pm 1.88) \times 10^{-10}$	$(1.76 \pm 0.251) \times 10^{-9}$	$(2.76 \pm 0.314) \times 10^{-9}$

Table 8: Values for $\mathbf{B}2 = 0.00105 \pm 6.92 \times 10^{-6}$ T

The slope of this fit again represents q/m_o and is found to be $1.54 \times 10^{11} \pm 1.98 \times 10^{10}$ for $\mathbf{B}$=$0.00127 \pm 6.92 \times 10^{-6}$T. And $1.49 \times 10^{11} \pm 1.92 \times 10^{10}$ for $\mathbf{B}$=$0.00105 \pm 6.92 \times 10^{-6}$T.



(a) Fit for $\mathbf{B}$=0.00127 $\pm$ 6.92 $\times$ 10^{-6}T



(b) Fit for $\mathbf{B}$=0.00105 $\pm$ 6.92 $\times$ 10^{-6}T

In both fits, χ^2 value is the statistical measure that quantifies the goodness of fit between the data and the fitted line. Its mathematical formula is given as:

$$\chi^2 = \sum_{i=1}^{N} \frac{(y_i - f(x_i))^2}{\sigma_i^2} \tag{11}$$

where y_i represents the data points $f(x_i)$ predicted value from the fitted line. Moreover, dividing by σ_i (the uncertainty at that point) adds weight to data points. That is;

- Data points with **small uncertainties** (σ_i is small) contribute more to the χ^2 because they are considered more reliable.
- Data points with **large uncertainties** (σ_i is large) contribute less to the χ^2 because they are considered less reliable.

It is desirable to have χ^2/ndf (number of degrees of freedom) to be around 1. Otherwise it implies overestimated or underestimated uncertainties. This is more clearly the case for constant voltage datasets which is also reflected in the probability that the fit accurately represents the data in both constant voltage fits.

0.4 Final value for q/m_0:

There are 4 different calculated values for q/m_0. Therefore, the final value is represented by the weighted average of those 4 values. The weighted average is calculated as:

$$\left(\frac{q}{m_0}\right)_{\text{final}} = \frac{\sum_{i=1}^{4} \left(\frac{q}{m_0}\right)^i / (\sigma^i)^2}{\sum_{i=1}^{4} 1/(\sigma^i)^2}$$

where $\left(\frac{q}{m_0}\right)^i$ are the individual values and σ^i are their respective uncertainties.

- $\left(\frac{q}{m_0}\right)^1 = 1.70 \times 10^{11} \pm 1.79 \times 10^{10}$
- $\left(\frac{q}{m_0}\right)^2 = 1.88 \times 10^{11} \pm 1.99 \times 10^{10}$
- $\left(\frac{q}{m_0}\right)^3 = 1.54 \times 10^{11} \pm 1.98 \times 10^{10}$
- $\left(\frac{q}{m_0}\right)^4 = 1.49 \times 10^{11} \pm 1.92 \times 10^{10}$

The weighted average is calculated as follows:

$$\left(\frac{q}{m_0}\right)_{\text{final}} = \frac{\frac{1.70\times10^{11}}{(1.79\times10^{10})^2} + \frac{1.88\times10^{11}}{(1.99\times10^{10})^2} + \frac{1.54\times10^{11}}{(1.98\times10^{10})^2} + \frac{1.49\times10^{11}}{(1.92\times10^{10})^2}}{\frac{1}{(1.79\times10^{10})^2} + \frac{1}{(1.99\times10^{10})^2} + \frac{1}{(1.98\times10^{10})^2} + \frac{1}{(1.92\times10^{10})^2}}$$

And the final uncertainty is:

$$\sigma_{\text{final}} = \sqrt{\frac{1}{(1.79 \times 10^{10})^2} + \frac{1}{(1.99 \times 10^{10})^2} + \frac{1}{(1.98 \times 10^{10})^2} + \frac{1}{(1.92 \times 10^{10})^2}}$$

After performing the calculations with Code 4, the final value for q/m_0 is:

$$\left(\frac{q}{m_0}\right)_{\text{final}} = 1.65 \times 10^{11} \pm 9.57 \times 10^9 = (1.65 \pm 0.096) \times 10^{11}$$

VII. Conclusion

In this experiment, the charge-to-mass ratio ($\frac{q}{m_0}$) of the electron was determined by analyzing the circular motion of electrons in a magnetic field. Two methods were employed to observe circular paths with radius of 2,3,4 and 5 cm: one with a constant voltage and varying magnetic field, and another with a constant magnetic field and varying voltage. The results from both methods were consistent, although, method with constant voltage and varying current yielded a closer result to the accepted value. Two methods overall yielded a weighted average value of:

$$\frac{q}{m_0} = (1.65 \pm 0.096) \times 10^{11}\,\text{C/kg}.$$

This value is in good agreement with the accepted value of 1.76×10^{11} C/kg. The uncertainties in the measurements were primarily due to the precision of the instruments used, such as the voltmeter and ammeter, as well as the manual measurement of the electron path radius. The discrepancies between the measured and accepted values could be attributed to systematic errors, such as misalignment of the electron gun or thickness of the measuring ladder.

To improve the accuracy of future experiments, the following suggestions are proposed:

- **Automated Radius Measurement**: Using a digital system to measure the radius of the electron path would reduce human error and improve precision.
- **Higher Precision Instruments**: Using more sensitive voltmeters and ammeters would reduce measurement uncertainties.
- **Multiple Trials**: Conducting more trials with varying parameters such as higher circular radii would provide a larger dataset for statistical analysis, improving the reliability of the results.

This experiment successfully demonstrated the principles of electron motion in electric and magnetic fields and provided a reasonable estimate of the charge-to-mass ratio of the electron.

References

[1] E. Gülmez, *Advanced Physics Experiments*. Boğaziçi University, 1997.

[2] GitHub repository for codes and scripts used in this experiment: https://github.com/capta1Nemo/PHYS442-experiments.git

Appendix

A.1 Derivation of σ_x and σ_y in constant voltage fit

1. **Derivation of σ_y:**
   The formula for y is:

$$y = \frac{2V}{r^2}$$

For a function $f(u, v)$, the uncertainty σ_f is given by:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial u} \cdot \sigma_u\right)^2 + \left(\frac{\partial f}{\partial v} \cdot \sigma_v\right)^2}$$

For $y = \frac{2V}{r^2}$, the uncertainty σ_y is:

$$\sigma_y = \sqrt{\left(\frac{\partial x}{\partial V} \cdot \sigma_V\right)^2 + \left(\frac{\partial x}{\partial r} \cdot \sigma_r\right)^2}$$

Partial derivative of y with respect to V is:

$$\frac{\partial y}{\partial V} = \frac{2}{r^2}$$

Partial derivative of y with respect to r

$$\frac{\partial y}{\partial r} = \frac{d}{dr}\left(\frac{2V}{r^2}\right) = -\frac{4V}{r^3}$$

Substitute the partial derivatives into the error propagation formula:

$$\sigma_y = \sqrt{\left(\frac{2}{r^2} \cdot \sigma_V\right)^2 + \left(-\frac{4V}{r^3} \cdot \sigma_r\right)^2}$$

Simplify the terms and factor out $\frac{2}{r^2}$:

$$\sigma_y = \sqrt{\left(\frac{2\sigma_V}{r^2}\right)^2 + \left(\frac{4V\sigma_r}{r^3}\right)^2}$$

$$\sigma_y = \frac{2}{r^2} \cdot \sqrt{\sigma_V^2 + \left(\frac{2V\sigma_r}{r}\right)^2}$$

Recall that $y = \frac{2V}{r^2}$. Multiply and divide by V to express σ_y in terms of y:

$$\sigma_y = \frac{2V}{r^2} \cdot \sqrt{\left(\frac{\sigma_V}{V}\right)^2 + \left(\frac{2\sigma_r}{r}\right)^2}$$

Since $y = \frac{2V}{r^2}$, we can write:

$$\sigma_y = y \cdot \sqrt{\left(\frac{\sigma_V}{V}\right)^2 + \left(\frac{2\sigma_r}{r}\right)^2}$$

2. **Derivation of σ_x**
   The formula for x is:

$$x = B^2$$

For a function $f(u)$, the uncertainty σ_f is given by:

$$\sigma_f = \left|\frac{\partial f}{\partial u}\right| \cdot \sigma_u$$

For $x = B^2$, the uncertainty σ_x is:

$$\sigma_x = \left|\frac{\partial x}{\partial B}\right| \cdot \sigma_B$$

The partial derivative of yx with respect to B is:

$$\frac{\partial x}{\partial B} = 2B$$

Substitute the partial derivative into the error propagation formula:

$$\sigma_x = |2B| \cdot \sigma_B$$

Since B is positive, we can drop the absolute value and find the uncertainty in x to be:

$$\sigma_x = 2B \cdot \sigma_B$$

A.2 Derivation of σ_x and σ_y in constant current fit

1. **Derivation of σ_x**
   The formula for x is:

$$x = \mathbf{B}^2 r^2$$

The error propagation formula for a function of multiple variables:

$$\sigma_x = \sqrt{\left(\frac{\partial x}{\partial B} \cdot \sigma_B\right)^2 + \left(\frac{\partial x}{\partial r} \cdot \sigma_r\right)^2}$$

First, compute the partial derivatives:

$$\frac{\partial x}{\partial B} = \frac{\partial}{\partial B}\left(\mathbf{B}^2 r^2\right) = 2\mathbf{B}r^2$$

$$\frac{\partial x}{\partial r} = \frac{\partial}{\partial r}\left(\mathbf{B}^2 r^2\right) = 2\mathbf{B}^2 r$$

Thus:

$$\sigma_x = \sqrt{\left(2\mathbf{B}r^2 \cdot \sigma_B\right)^2 + \left(2\mathbf{B}^2 r \cdot \sigma_r\right)^2}$$

Simplify the expression:

$$\sigma_x = 2\mathbf{B}^2 r^2 \cdot \sqrt{\left(\frac{\sigma_B}{\mathbf{B}}\right)^2 + \left(\frac{\sigma_r}{r}\right)^2}$$

Since $x = \mathbf{B}^2 r^2$, we can write:

$$\sigma_x = 2x \cdot \sqrt{\left(\frac{\sigma_B}{\mathbf{B}}\right)^2 + \left(\frac{\sigma_r}{r}\right)^2}$$

2. **Derivation of σ_y**

   The formula for y is:

$$y = 2\mathbf{V}$$

Using the error propagation formula for a function of multiple variables:

$$\sigma_y = \sqrt{\left(\frac{\partial y}{\partial V} \cdot \sigma_V\right)^2}$$

Thus:

$$\sigma_y = \sqrt{(2 \cdot \sigma_V)^2} = 2\sigma_V$$

Codes

```python

import numpy as np
mu_0 = 1.25663706127e-6

def B(I, sigma_I=0.01):
    alpha = np.sqrt((4/3)**3) * mu_0 * 154 / 20
    sigma_b = alpha * sigma_I
    return float(alpha * I), float(sigma_b)
```

Code 1: Magnetic Field Calculation Script

```

def constantV(data_set,magnetic_field):
eq_data=[]
sigma_r = np.sqrt(2) * 1e-3  # sqrt(2) mm converted to meters
for i in ["V1","V2"]:
    V=data_set[i].get("value")
    sigma_V = data_set[i].get("sigma_value", 0)
    B_set=magnetic_field[i].get("B_data")
    sigma_B = magnetic_field[i].get("B_errors", [0] * len(B_set))
    print(f"Processing{i}:V={V}+-{sigma_V},B_set={B_set}+-{sigma_B}")
    x=[]
    y=[]
    sigma_x = []
    sigma_y = []
    for j in range(len(B_set)):
        r=(j+2)*1e-2
        y_val=2*V/(r**2)
        y.append(float(f"{y_val:.3g}"))
        sigma_y_val = y_val * np.sqrt((sigma_V / V) ** 2 + (2 * sigma_r / r) ** 2)
        sigma_y.append(float(f"{sigma_y_val:.3g}"))
        x.append(float(f"{B_set[j]**2:.3g}"))
        sigma_x_val = 2 * B_set[j] * sigma_B[j]
        sigma_x.append(float(f"{sigma_x_val:.3g}"))
    eq_data.append([x,sigma_x,y,sigma_y])
return eq_data

def constantB(electric_field,magnetic_field):
eq_data=[]
sigma_r = np.sqrt(2) * 1e-3  # sqrt(2) mm converted to meters
for i in ["I1","I2"]:
    B=magnetic_field[i].get("B_data")
    sigma_B = magnetic_field[i].get("B_errors", 0)
    V_set=electric_field[i].get("V_data")
    sigma_V = electric_field[i].get("V_errors", [0] * len(V_set))
    print(f"Processing{i}:B={B}+-{sigma_B}T,V_set={V_set}+-{sigma_V}")
    x=[]
    y=[]
    sigma_x = []
    sigma_y = []
    for j in range(len(V_set)):
        r=(j+2)*1e-2
        x_val=B**2*(r**2)
        x.append(float(f"{x_val:.3g}"))
        sigma_x_val = 2*x_val*np.sqrt((sigma_B / B) ** 2 + (2 * sigma_r / r) ** 2)
        sigma_x.append(float(f"{sigma_x_val:.3g}"))
        y.append(float(f"{V_set[j]*2:.3g}"))
        sigma_y_val = 2 * sigma_V[j]
        sigma_y.append(float(f"{sigma_y_val:.3g}"))
    eq_data.append([x,sigma_x,y,sigma_y])
return eq_data
```

Code 2: dataset and sigma calculation script in python

```
{
    const int ndata = 4;

    float y[ndata] = {735000.0, 327000.0, 184000.0, 118000.0};
    float x[ndata] = {4.41e-06, 1.77e-06, 9.25e-07, 5.7e-07};
    float sy[ndata]= {104000.0, 30900.0, 13100.0, 6700.0};
    float sx[ndata]= {2.91e-08, 1.84e-08, 1.33e-08, 1.04e-08};

    TGraphErrors *mygraph = new TGraphErrors(ndata,x,y,sx,sy);
    mygraph->Draw("A*");
    gStyle->SetOptFit(1111);
    TF1 *fnew = new TF1("fnew","[0]*x+[1]",0,6);
    fnew->SetParameters(3000.1416,2.7182); // arbitrary starting
        parameters
    mygraph->Fit(fnew);
}
```

Code 3: Root linefit for 4 data points with sx and sy

```

values = np.array([1.70e11, 1.88e11, 1.54e11, 1.49e11])
uncertainties = np.array([1.79e10, 1.99e10, 1.98e10, 1.92e10])

weights = 1 / (uncertainties**2)
weighted_average = np.sum(values * weights) / np.sum(weights)

uncertainty_final = np.sqrt(1 / np.sum(weights))

print(f"Weighted average of q/m0:{weighted_average:.2e}")
print(f"Uncertainty of the weighted average:{uncertainty_final:.2e}")
```

Code 4: Mean value calculation script in python

Additional Material

original dataset in the form of python dictionary:

```
data_set = {
    "I1": {"data": [89, 135, 228, 354],"value": 1.83,"sigma_value":0.01},
    "I2": {"data": [73, 105, 169, 246],"value": 1.52,"sigma_value":0.01},
    "V1": {"data": [3.03, 1.92, 1.39, 1.09],"value": 147,"sigma_value":1},
    "V2": {"data": [2.72, 1.65, 1.20, 0.94],"value": 125,"sigma_value":1}}
```