Computer-Aided Joint Error Analysis of Robots

B. BENHABIB, R. G. FENTON, AND A. A. GOLDENBERG

Abstract—**Various definitions of task-space tolerances, using geometric pseudoenvelopes, are introduced, and new approaches to "direct" and "inverse" joint-error analysis of robots are formulated. These error analyses lead to the development of a "feasible joint-tolerance domain" concept for use in computer-aided design of robots.**

I. Introduction

ACCURACY of a robot, the measure of its ability to reach a specified task space and orientation [1], is an important parameter, especially if the robot is to be used for complex manufacturing operations requiring stringent tolerances. The inaccuracy of a robot is generally caused by a combination of joint errors, manufacturing errors, and arm deflections. Joint errors are caused by a combination of computer round-offs, encoder resolutions, calibration errors, and transmission errors. Manufacturing errors are attributed to errors in link lengths, distances, and twist angles [2], [3], [5]. Deflections are normally insignificant, although load-induced deflections may be important for fast-moving and light-weight robots, and thermal deflections may be the dominant cause of inaccuracy if the robot works in an environment of widely fluctuating temperature [4].

The effects of manufacturing errors, which are constant values, may be eliminated by a complex calibration procedure of the robot [3]. Joint errors, on the other hand, are random values within specified tolerance bounds and cannot be easily controlled during the operation of the robot. Furthermore, joint errors significantly contribute to the Cartesian positional and orientational errors of the robot [3].

The maximum allowable positional and orientational errors in task space can be defined by task-space tolerances using geometric pseudoenvelopes. A similar approach can be used to define maximum allowable joint errors using the "feasible joint-tolerance domain" developed in this paper. Based on the kinematic model of the manipulator, the robot designer can relate the task-space tolerances to the joint tolerances. The "direct" error analysis determines task-space tolerances corresponding to given joint tolerances, considering that the effect of all other errors are negligible. The "inverse" error analysis determines joint tolerances corresponding to given task-space tolerances, again neglecting the effects of manufacturing errors and arm deflections. In this paper various definitions of task-space tolerances are introduced, and methods for direct and inverse error analyses are developed and illustrated by numerical examples.

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II. Task-Space Tolerance

In general usage of robots, an object (tool) is usually attached to the end effector. Generally, both the position and orientation of this object (tool) are important, and accordingly, the maximum allowable variation of the position and orientation of this object from its desired values may be specified by positional and orientational tolerances.

A. Tolerance on Positional Error

Tolerance on positional error in task space can be defined by specifying tolerance envelopes (geometric pseudovolumes) [6]. These envelopes represent the feasible region of locating a specified point "D," generally, the origin of the tool frame attached to the end effector. The end effector has to be positioned in such a way that point D is located inside the specified tolerance envelope in order to satisfy the tolerance on positional error. The tolerance envelope of point D can be defined as a pseudosphere or as a pseudoprism.

Pseudospherical Positional Tolerance: The positional tolerance may be defined by a pseudosphere of radius R with the center at the origin of the tool frame as shown in Fig. 1. The inaccuracy in positioning, resulting from joint errors, causes point D to occupy position D_a instead of its desired position D_d. The inaccuracy in positioning is defined as

$$^{0}\boldsymbol{e} = (^{0}\boldsymbol{D})_a - (^{0}\boldsymbol{D})_d \tag{1}$$

or

$$^{t}\boldsymbol{e} = (^{t}\boldsymbol{D})_a \tag{2}$$

where $(^{0}\boldsymbol{D})_d$ is the desired (correct) position of point D_d with respect to frame F_0; $(^{0}\boldsymbol{D})_a$ is the actual (erroneous) position of point D_a with respect to frame F_0; $(^{t}\boldsymbol{D})_a$ is the actual (erroneous) position of point D_a with respect to frame $(F_t)_d$; $^{t}\boldsymbol{e}$ is the error vector resulting from erroneous positioning of point D_a with respect to frame $(F_t)_d$; $^{0}\boldsymbol{e}$ is the error vector resulting from erroneous positioning of the tool frame (point D_a) with respect to the base frame F_0. The following condition must be satisfied to ensure that the actual position, point D_a, is located inside the specified pseudospherical envelope

$$\| ^{0}\boldsymbol{e} \| = \| ^{t}\boldsymbol{e} \| \le R \tag{3}$$

where $\| \cdot \|$ represents the Euclidean norm operator.

Pseudoprismatic Positional Tolerance: The positional tolerance can also be defined by a pseudoprismatic envelope of dimensions $(2c_x, 2c_y, 2c_z)$ centered at the origin of frame $(F_t)_d$, as shown in Fig. 2. The inaccuracy in positioning is defined as follows. If the pseudoprismatic envelope is built

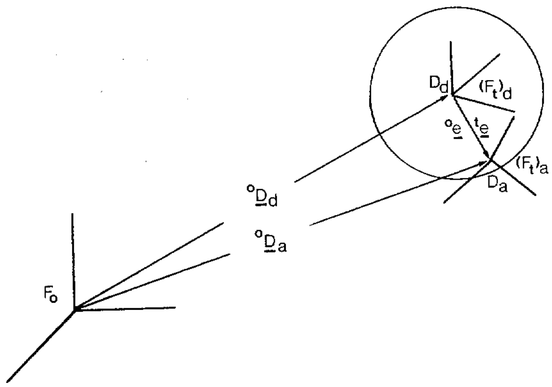

Fig. 1. Pseudospherical positional tolerance.

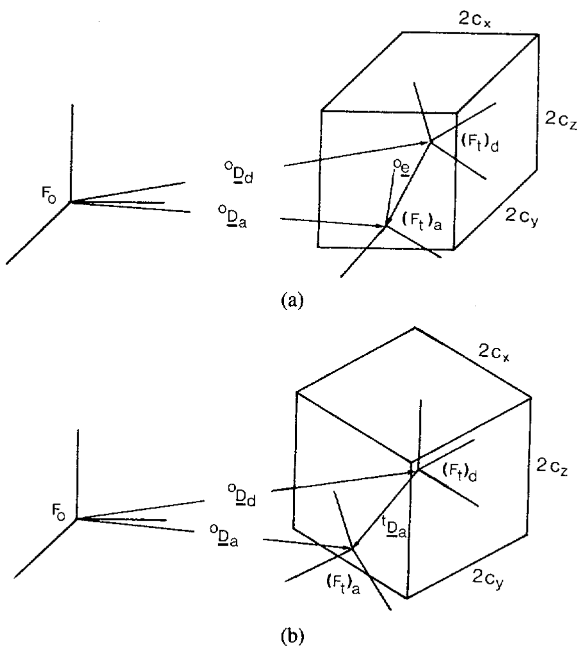

Fig. 2. Pseudoprismatic positional tolerance envelopes. (a) Built with respect to base frame. (b) Built with respect to tool frame.

with respect to frame F_0 (Fig. 2(a)),

$$^0\boldsymbol{e}=(^0\boldsymbol{D})_e-(^0\boldsymbol{D})_d. \tag{1}$$

If the pseudoprismatic envelope is built with respect to frame $(F_t)_d$ (Fig. 2(b)),

$$^t\boldsymbol{e}=(^t\boldsymbol{D})_a. \tag{2}$$

To ensure that point D_a is within the specified pseudoprism, the following conditions must be satisfied. If the tolerance envelope is specified with respect to frame F_0,

$$|^0e_x|\le c_x \qquad |^0e_y|\le c_y \qquad |^0e_z|\le c_z. \tag{4}$$

If the tolerance envelope is specified with respect to frame $(F_t)_d$,

$$|^te_x|\le c_x \qquad |^te_y|\le c_y \qquad |^te_z|\le c_z. \tag{5}$$

B. Tolerance on Orientational Error

The orientation of an object (tool) attached to the end effector can be defined by specifying two unit vectors denoted

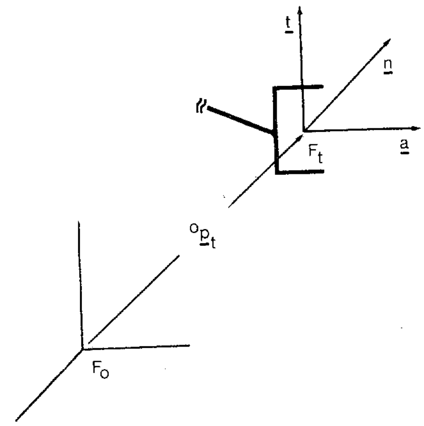

Fig. 3. Tool frame unit orientation vectors $(\boldsymbol{n}, \boldsymbol{t}, \boldsymbol{a})$.

as *approach* and *tangent* vectors, as shown in Fig. 3. The tool frame position and orientation are given as

$$T_{0t}=\left[\begin{array}{ccc|c} n & t & a & p \\ \hline 0 & 0 & 0 & 1\end{array}\right]. \tag{6}$$

T_{0t} is the transformation matrix between the robot's base frame F_0 and its tool frame F_t; $\boldsymbol{a}$ is the unit approach vector parallel to the tool frame axis z_t; $\boldsymbol{t}$ is the unit tangent vector parallel to the tool frame axis y_t. Tolerances on orientational error can be defined by a geometric pseudovolume (pyramid or cone) with respect to one of the axes (x_t, y_t, z_t) and a pseudosurface (triangle) around one of the other two axes.

Pseudoconical Orientational Tolerance: The orientational tolerance can be defined by a pseudoconical envelope of angle α_e, centered around the z_t (or x_t, or y_t) axis and a pseudotriangular envelope around the y_t (or x_t, or z_t) axis as shown in Fig. 4. The inaccuracy in orientation of the tool frame y_t and z_t axes, due to joint errors, is defined for the *approach* vector (for small angles) as

$$r_1=d\cdot(\delta x_t^2+\delta y_t^2)^{1/2} \tag{7}$$

and for the *tangent* vector as

$$r_2=d\cdot|\delta z_t| \tag{8}$$

where d is the height of the triangle and the cone.

To ensure that the actual $(z_t)_a$ axis (actual approach vector) is within the pseudoconical envelope and the actual $(y_t)_a$ axis (actual tangent vector) is within the pseudotriangular envelope, the following conditions must be satisfied:

$$r_1\le R_1 \qquad r_2\le R_2. \tag{9}$$

Considering that $R_1=\alpha_e\cdot d$ and $R_2=\beta\cdot d$, for small values of α_e and β, (9) can be rewritten in terms of rotation angles, using (7) and (8), as follows:

$$(\delta x_t^2+\delta y_t^2)^{1/2}\le\alpha_e \tag{10}$$

$$|\delta z_t|\le\beta. \tag{11}$$

Pseudopyramidical Orientational Tolerance: The orientational tolerance may be defined by a pseudopyramidical envelope of angle α, centered around the z_t (or x_t, or y_t) axis and a pseudotriangular envelope around the y_t (or x_t, or z_t)

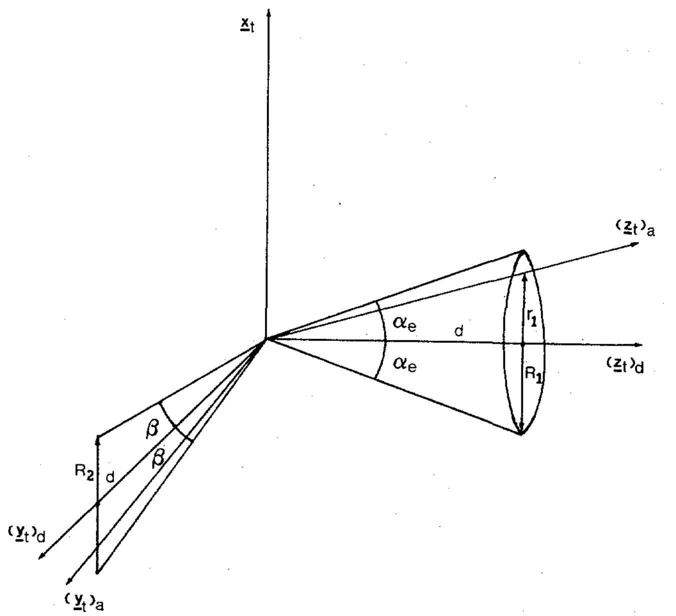

Fig. 4. Pseudoconical and pseudotriangular orientation tolerance envelopes.

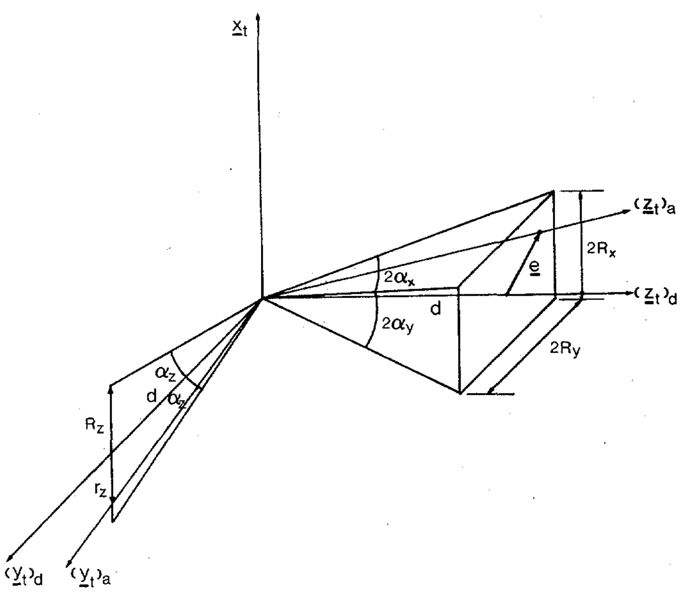

Fig. 5. Pseudopyramidical and pseudotriangular orientational tolerance envelopes.

axis as shown in Fig. 5. The inaccuracy in orientation of the tool frame is defined as follows: for the *approach* vector (for small angles) as

$$^{t}e=(d \cdot \delta x_t,\ d \cdot \delta y_t,\ 0) \tag{12}$$

and for the *tangent* vector as

$$r_z=d \cdot |\delta z_t|. \tag{13}$$

To ensure that the actual $(z_t)_a$ axis (actual approach vector) is within the pseudopyramidical envelope and the actual $(y_t)_a$ axis (actual tangent vector) is within the pseudotriangular envelope, the following conditions must be satisfied:

$$|^{t}e_x| \leq R_x \qquad |^{t}e_y| \leq R_y \qquad r_z \leq R_z. \tag{14}$$

Considering that $R_x = \alpha_y \cdot d$, $R_y = \alpha_x \cdot d$, and $R_z = \alpha_z \cdot d$ for small values of α_x, α_y, and α_z, (14) can be rewritten in terms of rotation angles, using (12) and (13), as follows:

$$|\delta x| \leq \alpha_x \qquad |\delta y| \leq \alpha_y \qquad |\delta z| \leq \alpha_z. \tag{15}$$

III. Direct Error Analysis

The purpose of the *direct error analysis* is to determine the maximum task-space errors in terms of the maximum joint errors. If the computed Cartesian task-space error values are unacceptable, the kinematic parameters can be altered and/or the joints have to be redesigned. Hence the robot's performance to meet given accuracy requirements can be evaluated using the direct error analysis. The relationship between task-space and joint errors, not considering the effects of manufacturing errors and deflections, can be expressed as follows [7]:

$$\Delta E=[J(q)]\delta q \tag{16}$$

where $\Delta E = (\Delta x, \Delta y, \Delta z, \delta x, \delta y, \delta z)$ is the positional and orientational error vector of the tool frame with respect to the base frame F_0, $\delta q = (\delta q_1, \cdots, \delta q_N)$ is the joint-error vector of the N degrees of freedom (DOF) robot, and $J(q)$ is the $(6 \times N)$ Jacobian matrix of the robot evaluated at the specified joint configuration.

The maximum task-space errors at a specified robot configuration can be determined by considering the absolute values of the elements of the Jacobian matrix and the maximum values of the joint error vector δq_j^* as follows:

$$\Delta E_i^+ = \left| \sum_{j=1}^{N} J_{ij} \delta q_j^* \right|, \qquad i=1, 2, \cdots, 6. \tag{17}$$

Task-space errors are robot-configuration-dependent, and they can be obtained covering the complete workspace of the robot. The workspace is defined by physical limits imposed on the N joints of the robot, (i.e., N-dimensional space in joint coordinates). The entire workspace can be scanned to determine the global maximum task-space error by using a numerical search technique, or alternatively, by using the Monte Carlo simulation technique where a sufficiently large set of randomly selected configurations is considered to represent the workspace in the following relation:

$$\Delta E^* = \max\ [\Delta E_k^+;\ k=1, 2, \cdots, M] \tag{18}$$

where ΔE^* is the global maximum task space vector; ΔE_k^+ is the maximum task-space error vector at the kth robot configuration; M is the number of randomly selected task-space locations. The elements of (18) can be used to determine the values of task-space positional and orientational tolerances as formulated in Section II.

IV. Inverse Error Analysis

The new *inverse error analysis* method, outlined in this section, is used to determine the required joint tolerances as functions of the specified task-space tolerances. This approach yields a new concept of feasible joint-tolerance domain by considering the coupling effect of joint tolerances. By defining the maximum allowable Cartesian task-space errors, a joint-tolerance envelope is determined for the robot. This envelope can be applied as a CAD tool to assign different tolerance values for different joints.

For this analysis the joint tolerances are expressed in terms of one *effective joint tolerance* δq_e as follows:

$$\delta q_j^* = k_j \delta q_e, \qquad j = 1, N \tag{19}$$

where k_j are positive constant coefficients. Using (19), the following relation is obtained between task-space errors and corresponding joint tolerances:

$$\Delta E_i^+ = \left[\left| \sum_{j=1}^{N} J_{ij} k_j \right| \right] \delta q_e, \qquad i = 1, 2, \cdots, 6. \tag{20}$$

The elements of ΔE^+ can also be expressed as follows:

$$\Delta E_i^+ = B_i \delta q_e \tag{21}$$

where

$$B_i = \left| \sum_{j=1}^{N} J_{ij} k_j \right| . \tag{22}$$

For a set of k_j values the required joint tolerances can be determined for specified task-space tolerances using (20). To solve (20) for the global values of joint tolerances, the entire workspace of the robot has to be considered, as discussed in Section III. The computation of joint tolerances for two important special cases and for the general case are as follows.

Case 1: All joints have the same tolerance, i.e.,

$$k_j = 1, \qquad j = 1, N.$$

If a pseudospherical tolerance envelope of radius $R_{\max}$ is specified for the positional error of the tool frame, then the following condition must be satisfied:

$$(\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2} \le R_{\max}. \tag{23}$$

If a pseudoconical tolerance envelope of effective angle α_e is specified for the orientation of the "approach" vector (the z_t axis), then the following condition must be satisfied:

$$(\delta x^2 + \delta y^2)^{1/2} \le \alpha_e. \tag{24}$$

If a pseudotriangular tolerance envelope of angle β is specified for the orientation of the "tangent" vector (the y_t axis), then the following condition must be satisfied:

$$|\delta z| \le \beta. \tag{25}$$

The effective joint tolerance can now be determined using (19)–(24) as follows:

$$\delta q_e = \min\, [R_{\max}/(B_1^2 + B_2^2 + B_3^2)^{1/2},\ \alpha_e/(B_4^2 + B_5^2)^{1/2},\ \beta/B_6)]. \tag{26}$$

If the positional tolerance is defined by a pseudoprismatic envelope of dimensions $(2c_x, 2c_y, 2c_z)$, the first term in (26) is replaced by

$$K_1 = \min\, [c_x/B_1,\ c_y/B_2,\ c_z/B_3]. \tag{27}$$

If the orientational tolerance of the "approach" vector is defined by a pseudopyramidical envelope, the second term in (26) is replaced by

$$K_2 = \min\, [\alpha_x/B_4,\ \alpha_y/B_5]. \tag{28}$$

Solving (26) provides the following results:

$$\delta q_j^* = \delta q_e, \qquad j = 1, N. \tag{29}$$

Case 2: The joints are divided into two groups, joints (1–M) and joints (M + 1–N). The two group of joints have different tolerances:

$$k_j = \begin{cases} k_1, & j = 1, M \\ k_2, & j = M+1, N. \end{cases} \tag{30}$$

In this case (22) becomes

$$B_i = \left[\left| \sum_{j=1}^{M} J_{ij} \right| k_1 + \left| \sum_{j=M+1}^{N} J_{ij} \right| k_2 \right] . \tag{31}$$

Thus the effective joint-tolerance is defined as in case 1 using (26). Equation (26) provides a two-dimensional feasible joint-tolerance domain for the two groups of joints as:

$$\delta q_j^* = \begin{cases} k_1 \delta q_e, & j = 1, M \\ k_2 \delta q_e, & j = M+1, N. \end{cases} \tag{32}$$

Varying the combinations of the k_1/k_2 values and repeatedly solving (26), an approximate two-dimensional feasible joint-tolerance domain can be determined as shown in Fig. 6. Joint-tolerance combinations represented by points on this diagram inside the feasible region are acceptable; joint-tolerance combinations represented by points in the nonfeasible region would result in excessive task-space tolerances.

Case 3: In the general case the joint tolerances of each joint are different. Thus the user specifies a set of arbitrary k_j values. A desired set of joint tolerances can either be obtained by repeatedly changing the k_j values or specifying an objective function Z to be minimized as follows:

$$\min\, Z = f(k_j;\ j = 1, N) \tag{33}$$

where Z is a function of joint errors (for example, Z may be the total cost of the robot which is a function of the required joint tolerances).

V. Example

To illustrate the method outlined here, the joint tolerances of a six degrees-of-freedom (DOF) robot were calculated for specified task-space tolerances. The Monte Carlo technique was utilized to represent the workspace of the robot by using a large number of locations. In this example, 500 robot configurations were randomly selected in the permissible workspace of the robot and mapped onto the task-space locations. The six DOF robot has three prismatic and three revolute joints, and its prismatic workspace is of size 1 × 1 × 1 m, Fig. 7. The joints of this robot form two groups:

a) q_P, prismatic joints (q_1, q_2, q_3); and
b) q_R, revolute joints (q_4, q_5, q_6).

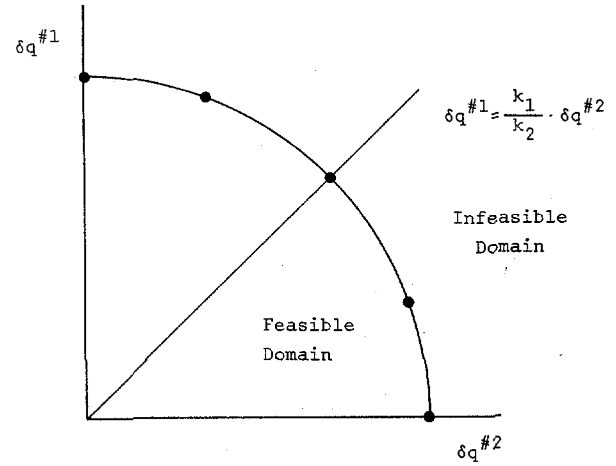

Fig. 6. Two-dimensional feasible joint-tolerance domain.

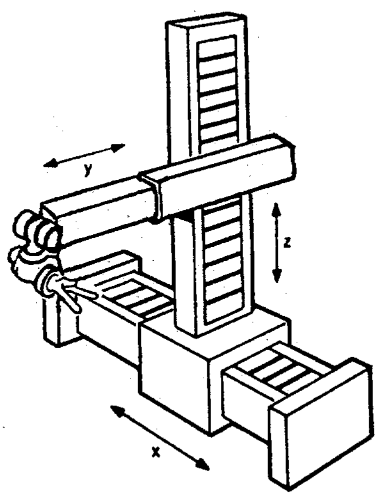

Fig. 7. Six degrees-of-freedom robot.

All prismatic and all revolute joints are assumed each to have the same tolerance so that

$$\delta q_P^* = \delta q_1^* = \delta q_2^* = \delta q_3^* = k_1 \delta q_e$$

and

$$\delta q_R^* = \delta q_4^* = \delta q_5^* = \delta q_6^* = k_2 \delta q_e.$$

The task-space tolerances are specified by a pseudocubical envelope of size $2c$ built with respect to a tool frame located at 0.1 m from the end effector frame along the z_e axis, Fig. 8. Computed joint-tolerance requirements to maintain the specified task-space positional tolerance are listed in Table I.

The required joint tolerances to maintain the orientational tolerance in task space have also been determined, and the results are listed in Table II. The first three joints of this robot are prismatic and do not affect the orientation of the tool frame; therefore, the computations based on the required orientational tolerance define only δq_R. Computed values of δq_R listed in Table II correspond to three different values of the specified pseudoconical tolerance α_e for the approach vector z_t of the tool frame.

The results listed in Tables I and II, together with the results computed for positional tolerances of $c = \pm 0.5$ mm, are

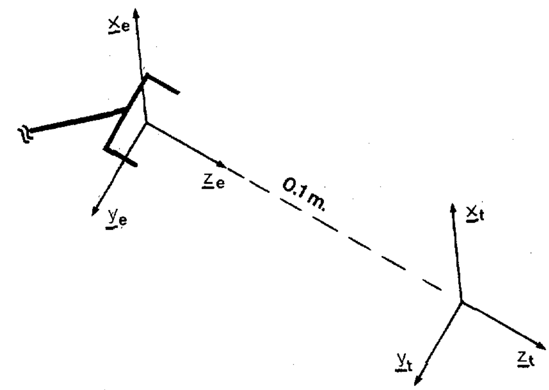

Fig. 8. Robot's end effector and tool frames.

TABLE I
JOINT TOLERANCES FOR $c = \pm 1.0$ mm

Coefficient ($k = k_1/k_2$)	δq_P^* (m)	δq_R^* (rad)
100	0.00058	0.00001
10	0.00058	0.00006
1	0.00057	0.00057
0.1	0.00042	0.00420
0.01	0.00007	0.00694
0.001	0.00001	0.00707

TABLE II
JOINT TOLERANCES FOR SPECIFIED TASK-SPACE ORIENTATIONAL TOLERANCES

α_e (rad)	0.001745	0.003490	0.008725
δq_R^* (rad)	0.001234	0.002468	0.006170

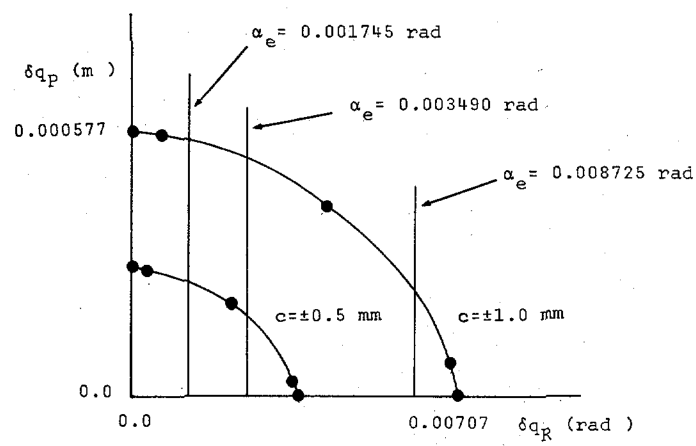

Fig. 9. Feasible joint-tolerances domains.

plotted in Fig. 9. The feasible joint-tolerance domain for $c = \pm 1.0$ mm and $\alpha_e = 0.008725$ rad is illustrated in Fig. 10. Optimal selection of joint tolerances for both revolute and prismatic joints can easily be performed with the help of this diagram.

VI. Conclusion

The methods presented in this paper for defining task-space tolerances as geometric pseudovolumes are practical and easily adaptable to any industrial robot. The algorithms

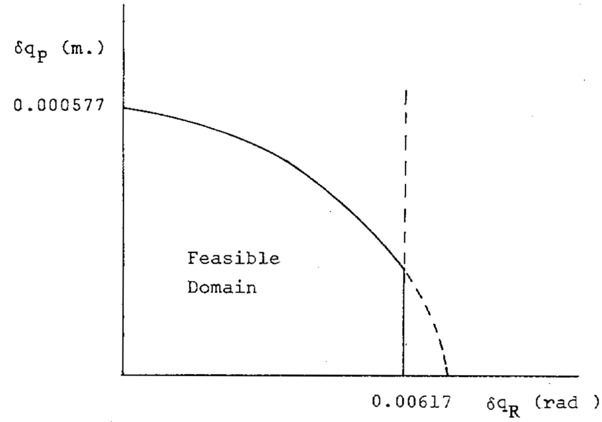

Fig. 10. Feasible joint-tolerances domain.

developed here for the "direct" and "inverse" error analysis can be used as independent software modules in any computer-aided design package for robots. The feasible joint-tolerance domain concept introduced herein provides the designer with the flexibility of specifying different tolerance requirements for the joints and selecting an optimal set of tolerances.

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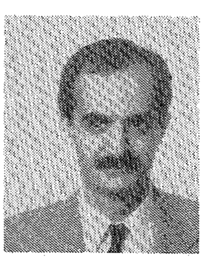

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