POSITIONING ERROR ANALYSIS FOR ROBOT MANIPULATORS WITH ALL ROTARY JOINTS

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ABSTRACT

Advanced industrial robots are commanded to accomplish different tasks with program sequences that are executed in digital computers. The operating software within these computers provide users with information on positions and orientations of the end effectors by computing them as functions of the joint variables. These functions are generally not exact enough such that differences between the computed and the actual positions can be significant. Error sources that contribute to these differences for robots with rotary joints are examined. The effects of these errors are parameterized and measurement data are fitted to obtain the values of these parameters. It is concluded that with sufficient but not exhaustive detail in the error modelling the differences can be reduced significantly.

Introduction

Industrial robots are generally placed in a structured environment to perform routine work. One attribute of industrial robots' that enable the accomplishment of tasks is good positioning repeatability. The repeatability error of a robot is a measure of the variation in the actual position of a point in the end effector when the robot's joint variables assume exactly the same values after maneuvers. Many commercial robot manipulators with reasonably large work volumes have their repeatability errors specified to be around 0.1 mm. With this repeatability, identical tasks can be repeated in a structured environment satisfactorily. Robot precision is, however, another story. Many industrial robots may be commanded via programs to have their end effectors reach numerically specified positions and orientations with respect to certain reference systems. The actual resulting positions may be quite different from what the programmer desires. Differences as high as 10 mm may be possible for the robots with roughly 0.1 mm repeatability. Because of this, programming a robot without teaching the desired locations with a pendant controller will not make the robot perform as desired in many applications. The difference between the desired position and the actual position is herein called the positioning error. Identifying and parameterizing the sources that contribute to these errors and estimating the values of the parameters are the subjects addressed by this paper.

Positioning errors are especially pronounced with robots linked together with rotary joints. Contributing to the errors are the less-than-perfect built-in kinematic functions and the inaccurate knowledge of the relation between the robot reference system and the reference system of the work piece. The built-in kinematic functions relate the end effector position and orientation to the joint variables. They provide the so-called "forward kinematic" and "inverse kinematic" relationships, and their uses are the keys to increased automation in robot operations. The nominal geometry of a robot as can be determined from the design drawings can be relied on for the development of the nominal kinematic functions. They are often used in the central controller for robots of the same model without taking into account the variations in the assembled robots. These variations owe much to the tolerances allowed in the manufacturing of the robot components. For the nominal functions to fit better in all of the robots of the same model, the tolerances specified in the design drawings have to be tightened. This is a potentially expensive approach to improving positioning accuracy. An alternative approach is to analyze the exact geometry of each individual robot to establish its unique kinematic functions, which some refer to as the "arm signature". This is most likely a more economical approach. In addition to the "signature" variations, gravitational loading also contributes to the positioning error through compliance, gear backlash, etc. The advantages of robots with improved positioning accuracy are numerous. At the basic level, it can reduce the pendant teaching efforts to a minimum. For instance, only a few reference positions have to be taught, all other positions can be specified as relative coordinates with respect to the taught positions. It also makes robot programs more readily transportable to machines of the same model or different models. At a more advanced level, tasks that require multi-robot coordination and more sophisticated external sensing can be more readily achieved if robots with high positioning accuracy are used. This is true because for these tasks the commanded positions have to be generated numerically.

General Approach

In order to understand the sources that contribute to the positioning errors, a general model of the robot that depicts sufficient detail of the kinematics is first established. All the mechanisms in

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the system suspected of having an effect on the kinematic relationship between the end effector and the base are included in the model. Mathematically, the kinematics is represented by algebraic functions of the joint variables, which are properly scaled from the joint encoder readings. These functions include parameters that characterize the error mechanisms contributing to the deviations of the exact robot geometries from the nominal robot geometries associated with various sets of joint variables. Once the values of these parameters are estimated from measurement data, the relative importance of the different errors sources in the model can be understood.

This approach seems straightforward conceptually. In practice, however, a few non-trivial points have to be seriously considered. A device or devices capable of providing accurate measurements for a sufficient variety of robot configurations should be used. It would be ideal if the measurement process could be automated and that type of device is rather inexpensive! By far, the most critical issue, however, is the modeling process, because without being represented as parameters an error source will not be identified. Without a sufficiently complete model, the estimated parameter values do not provide direct indications of physical errors. They are merely a set of values that best fit the data included in the estimation process, and may not fit other measurement data as well.

Literature Review

Published works in this same general area bear titles with key words such as "calibration", "accuracy improvement", or "parameter estimation", such as the works by Schefer[1], Barker[2], Hayati[3], Whitney, et al.[4], Foulloy and Kelley[5], Chen, et al.[6], and Vaishnav and Magrab[7]. Only systematic geometric errors have been considered as the error sources in all the papers with the exception of the work done at the Draper Laboratory by Whitney, et al. [4] which includes non-geometric errors as well. The models, however, are quite different as can be evidenced in the number of parameters introduced. The paper of Foulloy and Kelley[5] describes an approach to compensate for positioning errors in a local work space based on data collected with a calibration cube. All other papers address the precision issue in the overall work volume by introducing models that are universally valid. Among these, real measurements were reported only in Whitney, et al[4].

Kinematics

The estimation procedure for geometric errors introduced by Chen, et al.[6] was applied to a set of actual robot measurement data and the resulted fitting error was much greater than the robot repeatability error. This prompted a search for other sources of errors and non-geometric errors were then introduced. A description of these errors follows.

Geometric Errors

A two-link system as shown in Fig. 1 will be examined in detail to define the geometric errors. Rigid bodies A_1 and A_2 are connected by an ideal revolute joint whose axis is L. A point Q_2 of L can be chosen as the origin of a coordinate system R_2 fixed in A_2 for the convenient description of the positions of points in A_2. The coordinate axes can be arranged such that the z_2 axis is identical to L and the x_2 axis is to be aligned with a reference line on A_1 when the system assumes a particular configuration. At that configuration, the relative joint angle θ can be chosen to be zero and the configuration can be called the null configuration. The y_2 axis is uniquely determined with implied orthogonality and right-hand convention. If A_1 is an intermediate link of a robot arm, then a coordinate system R_1 fixed in A_1 can be similarly established at Q_1, a point fixed in A_1. If A_1 represents the robot base, then R_1 can be a coordinate system associated with a measurement device or a work piece. From the robot design data, nominal position of Q_2 and orientations of the x_2, y_2 and z_2 axes in R_1 when $\theta=0$ can be determined. This constitutes a nominal coordinate system R_2 fixed in A_1 consisting of axes x_2', y_2' and z_2' with origin at Q_2'. It is expected that R_2' does not coincide exactly with R_2 when $\theta=0$, and parameters have to be introduced to relate the two coordinate systems at that configuration. General origin translation and coordinate axis reorientation can be specified with three parameters each, and they can be a, b, c and α, β, γ, respectively. The parameters a, b, and c are the x, y, and z coordinates of Q_2 in R_2; α, β and γ are the Euler angles (in x-y-z sequence) that can bring the coordinate axes of R_2' parallel to the corresponding axes of R_2. Physical joint rotation of A_2 with respect to A_1 for an angle θ causes a point of interest P to change its position in R_1 which is to be determined.

Since P and R_2 are both fixed in A_2, the position of P in R_2 remains unchanged for any joint rotation. This position of P in R_2 can be specified with a "homogeneous coordinate" (see Paul[8]) ${}^2\mathbf{x}$, and the homogeneous coordinate ${}^1\mathbf{x}$ of P in R_1 becomes

$$ {}^1\mathbf{x} = \mathbf{N}\mathbf{H}\Theta\,{}^2\mathbf{x} \tag{1} $$

where **N** and **H** are the homogeneous transformation matrices of R_2' with respect to R_1 and R_2 with respect to R_2' at null configuration, respectively, and Θ is the transformation matrix characterizing the joint rotation. Matrix **N** can be determined from design drawings for the null configuration. Matrix **H** can be derived as:

$$ H = \begin{vmatrix} C_\beta C_\gamma & -C_\beta S_\gamma & S_\beta & a \\ C_\alpha S_\gamma + S_\alpha S_\beta C_\gamma & C_\alpha C_\gamma - S_\alpha S_\beta S_\gamma & -S_\alpha C_\beta & b \\ S_\alpha S_\gamma - C_\alpha S_\beta C_\gamma & S_\alpha C_\gamma + C_\alpha S_\beta S_\gamma & C_\alpha C_\beta & c \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{2} $$

and Θ is:

$$\Theta = \begin{vmatrix} C_\theta & -S_\theta & 0 & 0 \\ S_\theta & C_\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad (3)$$

where c_x denotes cos x and s_x denotes sin x, for x being α, β, γ, or θ. For an n-link robot, Eq. (1) can be expanded and the homogeneous coordinate ${}^0x$ of a point P on the end link in a base-fixed coordinate system R_0 becomes

$${}^0x = (N_1\ H_1\ \Theta_1)\ (N_2\ H_2\ \Theta_2)\ \ldots\ (N_n\ H_n\ \Theta_n)\ {}^nx \quad (4)$$

where ${}^nx$ is the homogeneous coordinate of P in R_n, the coordinate system fixed on the nth link established as described above. The subscripts 1, 2, ..., n, are used to denote coordinate transformations N, H and Θ between the systems fixed in successive links. The geometric parameters introduced are a_i, b_i, c_i, α_i, β_i and γ_i $(i = 1,\ldots,n)$, 6n in total. Since ${}^nx$ cannot be known exactly, 3 extra translational parameters may be introduced, namely, a_{n+1}, b_{n+1}, and c_{n+1} as in

$${}^nx = {}^n\bar{x} + [a_{n+1}\ b_{n+1}\ c_{n+1}\ 0]^T \quad (5)$$

where ${}^n\bar{x}$ is the approximate homogeneous coordinate of the target in R_n.

Non-geometric Errors

All error sources whose contributions to positioning errors cannot be characterized by geometric parameters are herein called non-geometric errors. The assumptions made in the definition of geometric parameters are that the links are rigid bodies, the joint rotations are ideal single axis rotations and the rotation angles θ_i's are exactly the angular displacements of the links from their null configurations. Though possible deviations of the real robot geometry from these assumptions are numerous, it is of interest to identify only those that contribute significantly to the overall error. Several possible error sources for Unimation PUMA robots (shown in Fig. 2) are mentioned by Whitney, et al.[4] and they include gear transmission error, joint drive compliance, backlash, base motion and cross coupling of joint rotations. Gear transmission error is due to the imperfection in the gears that transmit driving power from a motor to a joint. Joint drive compliance is the overall compliance between the motor rotor where the angular encoder is mounted (for PUMA) and the actual output shaft at the joint. This compliance has been pointed out[9] to be more significant in dynamic analysis than the structural flexibility for other robots as well. Gear backlash has been known to erode precision and cause other dynamical problems. All these error sources arise between the angular encoder and the output shaft at the joint, and the encoder reading need to be corrected for computing the actual angular displacement of the joint. The effect of base motion is that the direction of joint 1 axis with respect to a fixed coordinate system changes as robot configuration changes. but the base motion can be substantially reduced by reinforcing the base. Cross coupling of joint rotations appear in PUMA's wrist joints, and it has been compensated in the robot's central controller. The question is whether it has been exactly compensated.

Other error sources such as temperature variation, shaft wobbling and bending torsion of link structure are mentioned but neglected as insignificant in comparison with the accuracy of interest (0.1 mm for PUMA robots). Relative significance among all these error sources, however, is not a simple thing to decide, and it may vary from model to model. Experiments can be attempted to yield some measured values, but it may be difficult to distinguish among the contributing sources. Furthermore, it will be very time-consuming to perform measurements for individual robots to obtain the error values which will be accurate enough to achieve the desired accuracy in position computation. A more economical way is to analyze a generic robot of a particular model to understand the relative importance of various error sources and then to estimate the values of those parameters characterizing the more significant sources for each robot of the model. This estimation procedure should be done with both geometric and non-geometric parameters included. Estimation procedure can also be exploited for identifying the relative significance among error sources by testing if an added error mechanism in the model improves the fitting of data significantly or not. Little or no improvement in data fitting would indicate that the added error mechanism is not a significant one.

In the estimation process described in the following for PUMA robot, 4 parameters, h_1, h_2, h_3, and h_4, are employed to characterize the twist angles in joints 2 and 3 and the backlash in joint 3 drive system due to gravitational effect. These error sources are suspected to be the most significant non-geometric contributors to the overall positioning errors for PUMA robots. The parameters h_1, h_2 and h_3 are introduced in the process of formulating the weight-induced moments and the rotations due to them. The parameter h_4 is the amount of backlash in the joint 3 drive system. The true angles Θ_2 and Θ_3 become

$$\theta_2 = \bar{\theta}_2 + h_1\cos(\bar{\theta}_2+\bar{\theta}_3) + h_2\cos(\bar{\theta}_2) \quad (6)$$

$$\theta_3 = \bar{\theta}_3 + h_3\cos(\bar{\theta}_2+\bar{\theta}_3) + h_4\mathrm{SIGN}[\cos(\bar{\theta}_2+\bar{\theta}_3] \quad (7)$$

where $\bar{\theta}_2$ and $\bar{\theta}_3$ are the nominal angles derived by simply scaling the encoder readings at joints 2 and 3, respectively, and SIGN $[x] = x/|x|$. Here the encoder readings have been adjusted so that the null configuration happens when links 2 and 3 are fully stretched in a horizontal plane.

Linearized Estimation

All the parameters introduced are necessarily small; the angular parameters expressed in radians should be much smaller than 1 and the length parameters should be very small compared to the overall dimensions of the robot. It is assumed that the second order terms in these parameters can be neglected. With the 3-D position of a target point in the end link of a PUMA robot measured, a linear estimation problem can be established. For the ith configuration to be included in the process, a matrix equation

$$A_i\ \mathbf{p} - \mathbf{r}_i = \varepsilon_i \tag{8}$$

can be derived where **p** is a collection of all the 43 parameters, or

$$p = [a_1\ b_1\ c_1\ \alpha_1\ \beta_1\ \gamma_1\ a_2 \ldots\ldots$$
$$\gamma_6\ a_7\ b_7\ c_7\ h_1 \ldots h_4]^T \tag{9}$$

A_i is a 3 x 43 derivative matrix containing the first three rows of $\partial^0\mathbf{x}/\partial\mathbf{p}$ at $\mathbf{p} = \mathbf{0}$ or

$$\left[\frac{A_i}{0}\right] = \partial^0\mathbf{x}/\partial\mathbf{p}\Big|_{\mathbf{p}\ =\ \mathbf{0}} \tag{10}$$

and $\mathbf{r}_i$ is a 3 x 1 matrix denoting the difference between the nominally computed and the measured position coordinates, i.e., ${}^0\mathbf{x}_{nc}$ and ${}^0\mathbf{x}_{nm}$, or:

$$\left[\frac{\mathbf{r}_i}{0}\right] = {}^0\mathbf{x}_{nc} - {}^0\mathbf{x}_m \tag{11}$$

and ε_i is the lumped sum of the measurement error and the second and higher order terms in **p**. For m robot configurations used in the estimation process, Eq. (8) can be assembled to give

$$\mathbf{A}\ \mathbf{P} - \mathbf{r} = \varepsilon \tag{12}$$

where:

$$\mathbf{A} = [\mathbf{A}_1{}^T\ \mathbf{A}_2{}^T\ \ldots\ \mathbf{A}_m{}^T]^T \tag{13}$$

$$\mathbf{r} = [\mathbf{r}_1{}^T\ \mathbf{r}_2{}^T\ \ldots\ \mathbf{r}_m{}^T]^T \tag{14}$$

$$\varepsilon = [\varepsilon_1{}^T\ \varepsilon_2{}^T\ \ldots\ \varepsilon_m{}^T]^T \tag{15}$$

An estimation of the parameters can be obtained by minimizing $\varepsilon^T\varepsilon$ with respect to **p**. Since the number of geometric parameters introduced may be larger than is necessary to fully determine the forward kinematics of an idealized robot, an algorithm capable of handling singularity has to be used. It has been established with simulated data by Chen, et al. [6] that for a PUMA robot, out of the 39 geometric parameters, only 25 can be estimated. The other 14 parameters maintain their nominal values of zero.

Measurement

Measurements of a PUMA 760 at the National Bureau of Standards (NBS) have been performed with a stereotriangulation technique using three theodolites by Haight [10]. A ball-tip probe was attached at the end link and the ball center was treated as the target point. Theodolites were manually operated but the data were read and reduced with computer. Three-dimensional position coordinates of the target together with the associated joint readings were made available to the author for parameter estimation. The accuracy of the measurements is claimed to be +/- 0.02 mm per one meter of target travel, significantly better than the repeatability of PUMA 760.

The data available are configurations which were obtained from a series of tests including single-joint rotation tests and programmed line and circle tests. A total of approximately 200 different configurations have been recorded.

Estimation Results

Eighty (80) configurations were selected to be included in the estimation process. The choices were made to ensure significant variations in all the angular readings for each joint. Since the measurement coordinate system is unrelated to the robot's "world coordinate system," their nominal transformation matrix $\mathbf{N}_1$ has to be established before the estimation process begins. A robot's world coordinate system is the system in which position coordinates given by the central controller are referenced. The world system origin for PUMA 760 is defined to be the point where joint 1 and 2 axes intersect. This origin has been established by the NBS team at the mid-point of a line connecting the joint 1 and 2 axes at points of closest approach. With the position coordinates of this origin and the orientations of the joint 1 and 2 axes at robot's null state, $\mathbf{N}_1$ is established. All other nominal transformation matrices $\mathbf{N}_2, \ldots, \mathbf{N}_6$ are established based on the robot design data such that **x**, which is computed with the equation

$$\mathbf{x} = \Theta_1\ \mathbf{N}_2\ \Theta_2\ \ldots\ \mathbf{N}_6\ \Theta_6\ [0\ 0\ 0\ 1]^T \tag{16}$$

when no parameter corrections are made, exactly corresponds to the position coordinates given by the robot's central controller. The matrices used are based on the coordinate systems shown in Fig. 2 and are as follows:

$$N_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 164 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad N_3 = \begin{bmatrix} 1 & 0 & 0 & 650 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$N_4 = \begin{bmatrix} 0 & 0 & 1 & 550 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad N_5 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$N_6 = \begin{bmatrix} 0 & 0 & 1 & 125 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

With N_1 as described above, n being 6 and ${}^6\mathbf{x}$ being the known nominal positional coordinates of the target with respect to the coordinate system fixed in the last link, the position of the target with respect to the measurement coordinate system can be computed with Eq. (4). the distances between the points corresponding to the computed and the measured coordinates are called the error distances. The mean error distance in the 80 configuration ensemble for the nominally computed results is about 5.9 mm. The worst error is about 10 mm. Parameter estimation with only the 39 geometric parameters yields 25 non-zero parameters and this agrees with the simulated results done previously[6]. The parameter values are listed in Table 1 and the null values indicate that the associated parameters are not estimated. with the estimated parameters applied in Eq. (4), the resulted mean error distance is about 1 mm (worst one about 2 mm). This amounts to five times improvement. With non-geometric parameters h_3 and h_4 associated with joint 3 in Eq. (7) included, the resulted mean error distance is about 0.5 mm (worst one about 1 mm), and the parameter values can be found in Table 2. With parameters h_1, h_2, h_3 and h_4 in Eqs. (6) and (7) included, the mean error distance becomes about 0.27 mm (worst one about 0.6 mm). The parameter values for the last case are listed in Table 3.

Conclusion

Although the error sources contributing to the positioning inaccuracy of a robot may be numerous, it seems that they are not totally untractable. As can be seen from the estimation results, the mean error distance can be reduced to 0.27 mm for the configuration ensemble used in the process. Other non-geometric error sources can be parameterized and included in the estimation process, hopefully to further reduce the mean error distance. The universality of the parameters, however, has not been established as yet. From Tables 1, 2, and 3, it can easily be seen that the same geometric parameter assumes reasonably different values for different numbers of parameters used in the estimation process. This is so because in the **A** matrix, which is defined in Eq. (12), those columns associated with the geometrical parameters are far from being orthogonal to the columns associated with the geometrical parameters. Therefore, the effects of the non-geometric errors when they are not included in the estimation are distributed to geometric parameters. This implies that it is inappropriate to attribute the estimated parameter values to specific physical errors in the robot on which measurements are made. It is believed, however, that with more efforts, a set of parameters can be established such that some parameter values are reasonably indicative of physical errors.

Linearization of the estimation problem can be justified since the second or higher order terms of the estimated parameter values are negligible compared with the resulted mean error distance. The advantage of linearized estimation is that singularity condition can be more easily handled and therefore a parameter set which numbered more than the minimum required can be introduced. The minimum set can then be algorithmically determined. If there ever is a need to further improve the estimation process, another iteration of the linear procedure can be performed treating the already obtained parameter values as part of the nominal geometric relations. It is believed, however, that more comprehensive modeling may be more critical than the improved estimation process for industrial robots as far as error analysis and compensation is concerned.

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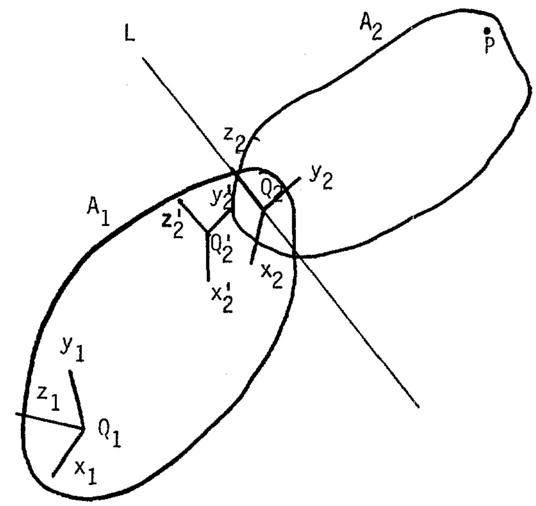

Fig. 1 Two-Link System

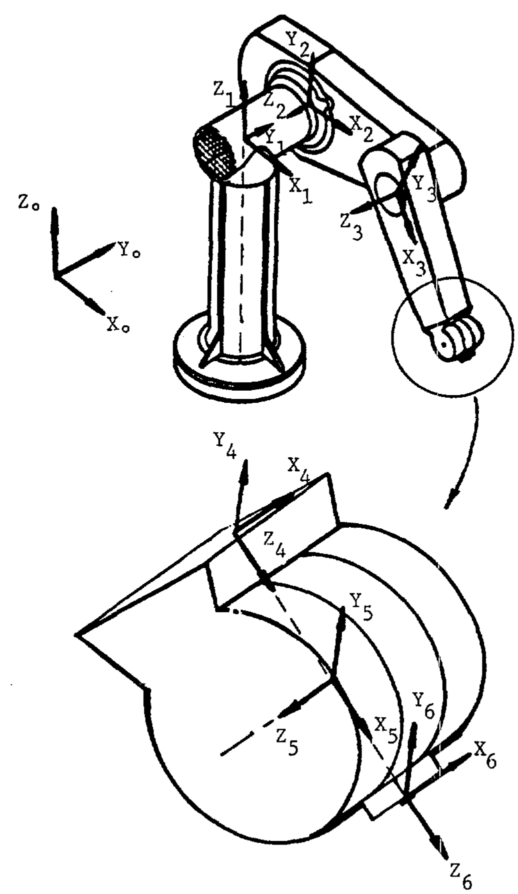

Fig. 2 PUMA Robot and Coordinate Systems

Table 1 Estimated Parameter Values (39 Parameters)

i =		1	2	3	4	5	6	7
a_i	(mm)	-0.346	-0.691	0.444	-0.150	0.951	0.383	-0.190
b_i	(mm)	.741	.232	1.712	-4.531	.637	0	-0.647
c_i	(mm)	-2.348	0	0	0	0	0	0
α_i	(10^{-3} rad)	0.206	-0.206	5.443	0	0	0	
β_i	(10^{-3} rad)	-0.399	0	0.612	2.173	-0.687	0	
γ_i	(10^{-3} rad)	-2.409	5.154	0	-14.131	-7.567	0	

Table 2 Estimated Parameter Values (41 Parameters)

i =		1	2	3	4	5	6	7
a_i	(mm)	-0.681	0	0.006	0.813	0	0	0.058
b_i	(mm)	0.705	-0.834	0	0	0.396	0	-0.152
c_i	(mm)	0	0	0	0.464	0	0	-0.105
α_i	(10^{-3} rad)	0.136	-0.233	5.265	-3.358	0	0	
β_i	(10^{-3} rad)	-0.326	-3.613	0.535	1.553	-2.823	8.477	
γ_i	(10^{-3} rad)	1.196	7.196	-8.859	-14.675	-6.264	0	

$$h_3 = -2.905 \times 10^{-3} \text{ rad}, \; h_4 = 0.111 \times 10^{-3} \text{ rad}$$

Table 3 Estimated Parameter Values (43 Parameters)

i =		1	2	3	4	5	6	7
a_i	(mm)	-0.748	0	-0.754	0.887	0	0	0.015
b_i	(mm)	0.705	-0.039	0	0	0.339	0	-0.177
c_i	(mm)	0	0	0	0.455	0	0	-0.0133
α_i	(10^{-3} rad)	0.140	-0.166	5.336	-1.376	0	0	
β_i	(10^{-3} rad)	-0.210	-0.530	0.497	1.320	-1.345	6.348	
γ_i	(10^{-3} rad)	-1.897	7.876	-8.942	-15.161	-5.458	0	

$$h_1 = -0.181 \times 10^{-3} \text{ rad}, \; h_2 = -1.437 \times 10^{-3} \text{ rad}$$
$$h_3 = -1.686 \times 10^{-3} \text{ rad}, \; h_4 = 0.235 \times 10^{-3} \text{ rad}$$

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