ROBOT GEOMETRY CALIBRATION

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ABSTRACT

Autonomous robot task execution requires that the end-effector of the robot be positioned accurately relative to a reference world coordinate frame. This paper presents a complete formulation to identify the actual robot geometric parameters. The method applies to any serial link manipulator with arbitrary order and combination of revolute and prismatic joints. A method is also presented to solve the inverse kinematic of the actual robot model which usually is not a so-called simple robot. This paper presents experimental results performed by utilizing a PUMA 560 with simple measurement hardware. As a result of this calibration a precision move command is designed and integrated into a robot language, RCCL, and used in the NASA Telerobot Testbed at Jet Propulsion Laboratory.

INTRODUCTION

In order to compensate for robot kinematic inaccuracies, one must first establish the error sources that cause the end-point positioning errors. Positioning inaccuracies are caused by many factors, such as link parameter errors, clearances in the mechanism's connections, wear, thermal effects, flexibility of the links and the gear train, gear backlash, encoder resolution errors, errors associated with relating the theoretical robot coordinate frame to the world coordinate frame, and control errors. Although in principle it might be possible to control the robot to compensate for these effects (e.g., see reference [1]) practical techniques are not yet developed. On the other hand, part of the positioning error is due to some biases that could be compensated. For example, the link twist error will result in a substantial position error at the end effector whose magnitude is equal to the error in the angle multiplied by an equivalent link length.

In the last five years, several researchers have devoted their attention to the robot accuracy problem. These efforts have mainly been directed toward the identification of the robot kinematic parameters. A partial list of these works is given in references [2]-[14]. The problem can be divided into three distinct parts. The first part is concerned with a particular mathematical formulation that results in an observation equation from which the error sources can be solved for. The second part is concerned with actual identification of error sources. The third part is concerned with solving the inverse kinematic problem for the calibrated robot which now most probably cannot be solved analytically due to the fact that the robot no longer is modeled as a so-called "simple" robot.

This paper will address all three elements outlined above. First a concise formulation is developed to obtain an observation equation for a general serial link robot. This formulation is based on the modified Denavit-Hartenberg parameters as defined by Craig [13] for non-parallel joints and by Hayati [6] for parallel links. Second, a simple inverse kinematics solution based on the nominal robot geometry and the Jacobian is developed to solve the inverse kinematics of the calibrated robot. Lastly, an experimental set-up is described to perform the robot calibration. It is shown that the absolute positioning error can be reduced from 6 mm to 2 mm.

OBSERVATION EQUATION

In this section we will develop a formulation to relate the link errors to the discrepancy between the nominal and actual end-effector measurement errors. To start with, we will assume that the robot has n links. Figure 1 depicts a schematic picture of a general serial link robot with a world and a tool frame.

Note that an extra frame (frame o) is introduced between the world and the first frames. This redundant frame definition is necessary so that the first frame can be defined utilizing the modified Denavit-Hartenberg parameters in a preferred location by the user. In the development of the error model we will assume that all the transformations are defined by the modified D-H parameters except for two transformations. These are ${}^{w}_{0}\mathbf{T}$ and ${}^{n}_{t}\mathbf{T}$. More flexibility in frame definitions is obtained if one defines these two transformations in the general form of

$$\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{p} \\ 0 & 1 \end{bmatrix}$$

The D-H link transformation is obtained from [13]

$$\begin{aligned} &{}^{i-1}_{i}\mathbf{T}(\alpha_{i-1}, a_{i-1}, \theta_i, d_i) \\ &\quad = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{1}$$

where ${}^{i-1}_{i}\mathbf{T}$ specifies a transformation to obtain the i-frame relative to the $i-1$-frame.

Let us denote the link parameter errors as $\delta\alpha_{i-1}$, δa_{i-1}, $\delta\theta_i$, and δd_i. If a link has a revolute joint, then $\delta\alpha_{i-1}$, δa_{i-1}, and δd_i represent fixed biases, where $\delta\theta_i$ is an error model for encoder null offset. Since we are assuming that these errors are small, one can model the net effect on the link by transforming them to the "end" of the link. Let $\mathbf{T}^N$ and $\mathbf{T}^A$ denote the nominal and actual link transformations for each link, then the error model for each link is given from

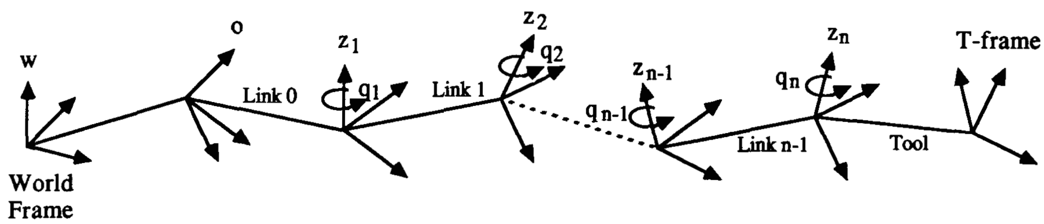

Figure 1. Schematic drawing of link frames.

$$ {}^{i-1}_{i}\mathbf{T}^A = {}^{i-1}_{i}\mathbf{T}^N + d\left({}^{i-1}_{i}\mathbf{T}\right) = {}^{i-1}_{i}\mathbf{T}^N \Delta \left({}^{i-1}_{i}\mathbf{T}\right) \tag{2} $$

From equation (2) one can solve for $\Delta\left({}^{i-1}_{i}\mathbf{T}\right)$ as

$$ \begin{aligned} \Delta\left({}^{i-1}_{i}\mathbf{T}\right) &= \left({}^{i-1}_{i}\mathbf{T}^N\right)^{-1}\left[{}^{i-1}_{i}\mathbf{T}^N + d\left({}^{i-1}_{i}\mathbf{T}\right)\right] \\ &= \mathbf{I} + \left({}^{i-1}_{i}\mathbf{T}^N\right)^{-1} d\left({}^{i-1}_{i}\mathbf{T}\right) \end{aligned} \tag{3} $$

The term $d({}^{i-1}_{i}\mathbf{T})$ may now be computed from Eqn. 1 as

$$ \begin{aligned} d\left({}^{i}_{i-1}\mathbf{T}\right) &= \frac{\partial^{i}_{i-1}\mathbf{T}}{\partial\alpha_{i-1}}\delta\alpha_{i-1} + \frac{\partial^{i}_{i-1}\mathbf{T}}{\partial a_{i-1}}\delta a_{i-1} \\ &+ \frac{\partial^{i}_{i-1}\mathbf{T}}{\partial\theta_i}\delta\theta_i + \frac{\partial^{i}_{i-1}\mathbf{T}}{\partial d_i}\delta d_i \end{aligned} \tag{4} $$

$\Delta\left({}^{i-1}_{i}T\right)$ can now be obtained by expanding equation (4) and substituting the results into equation (3). The result is

$$ \Delta\left({}^{i-1}_{i}\mathbf{T}\right) = \tag{5} $$
$$ \begin{bmatrix} 1 & -\delta\theta_i & -s\theta_i\delta\alpha_{i-1} & c\theta_i\delta a_{i-1} - s\theta_i d_i \delta\alpha_{i-1} \\ \delta\theta_i & 1 & -c\theta_i\delta\alpha_{i-1} & -s\theta_i\delta a_{i-1} - c\theta_i d_i \delta\alpha_{i-1} \\ s\theta_i\delta\alpha_{i-1} & c\theta_i\delta\alpha_{i-1} & 1 & \delta d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Equation (5) represents the error transformation, accurate to the first order, due to link $i-1$ error parameters. Since the actual measurements will be done in the tool-frame of the arm, one must transform these errors to the tool-frame. This can be accomplished by simply transforming the error term given by equation (5) using the differential error transformation between the i-frame and the tool- frame. The errors at the tool-frame are then

$$ {}^t_i\mathbf{e} = {}^t_i\mathbf{J}\ \mathbf{e}_i \tag{6} $$

where ${}^t_i\mathbf{e}$ is a six vector whose first three elements define the positional error cause by the $i-1$ link parameters errors and the last three elements correspond to the rotational errors. The term ${}^t_i\mathbf{J}$ represents a 6×6 differential error transformation matrix [13, page 154]

$$ {}^t_i\mathbf{J} = \begin{bmatrix} {}^t_i\mathbf{R} & {}^t\mathbf{p}_{i_{ORG}} \times {}^t_i\mathbf{R} \\ 0 & {}^t_i\mathbf{R} \end{bmatrix} \tag{7} $$

where ${}^t_i\mathbf{T} = {}^t_n\mathbf{T}\cdots{}^{i+2}_{i+1}\mathbf{T}\,{}^{i+1}_{i}\mathbf{T}$

The vector $\mathbf{e}_i$ represents the error terms in Eqn. (5) in a vector notation and is given by

$$ \mathbf{e}_i = \begin{bmatrix} c\theta_i\delta a_{i-1} - s\theta_i d_i\delta\alpha_{i-1} \\ -s\theta_i\delta a_{i-1} - c\theta_i d_i\delta\alpha_{i-1} \\ \delta d_i \\ c\theta_i\delta\alpha_{i-1} \\ -s\theta_i\delta\alpha_{i-1} \\ \delta\theta_i \end{bmatrix} \tag{8} $$

The error modeling of the ${}^w_0\mathbf{T}$ and ${}^n_t\mathbf{T}$ can be obtained by assuming that the actual and nominal transformations are related by

$$ \mathbf{T}^A = \mathbf{T}^N + d\mathbf{T} = \mathbf{T}^N\Delta\mathbf{T} $$

where $\Delta\mathbf{T}$ is a small transformation

$$ \Delta\mathbf{T} = \begin{bmatrix} 1 & -\delta\theta_z & \delta\theta_y & \delta x \\ \delta\theta_z & 1 & -\delta\theta_x & \delta y \\ -\delta\theta_y & \delta\theta_x & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9} $$

With this definition, one can define $\mathbf{e}_0$ and $\mathbf{e}_t$ as

$$ \mathbf{e}_0 = \begin{bmatrix} {}^0\delta x \\ {}^0\delta y \\ {}^0\delta z \\ {}^0\delta\theta x \\ {}^0\delta\theta y \\ {}^0\delta\theta z \end{bmatrix} \quad , \quad \mathbf{e}_t = \begin{bmatrix} {}^t\delta x \\ {}^t\delta y \\ {}^t\delta z \\ {}^t\delta\theta x \\ {}^t\delta\theta y \\ {}^t\delta\theta z \end{bmatrix} \tag{10} $$

The total error represented at the t frame can now be obtained by transforming all the individual $\mathbf{e}$'s to the t-frame as

$$ \mathbf{e} = \sum_{i=0}^{n+1} {}^{n+1}_{i}\mathbf{J}\ \mathbf{e}_i \tag{11} $$

where $n+1$ is used to denote the t frame. Note that ${}^{n+1}_{n+1}\mathbf{J}$ is simply the unity matrix. Eqn. (11) can also be written explicitly in terms of the actual link errors by writing Eqn. (8) in the form of

$$ \mathbf{e}_i = \begin{bmatrix} -s\theta_i d_i & c\theta_i & 0 & 0 \\ -c\theta_i d_i & -s\theta_i & 0 & 0 \\ 0 & 0 & 0 & 1 \\ c\theta_i & 0 & 0 & 0 \\ -s\theta_i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta\alpha_{i-1} \\ \delta a_{i-1} \\ \delta\theta_i \\ \delta d_i \end{bmatrix} = \mathbf{G}_i\mathbf{x}_i \tag{12} $$

By direct substitution of (12) in (11) one obtains

$$ \mathbf{e} = \sum_{i=0}^{n+1} {}^{n+1}_{i}\mathbf{J}\ \mathbf{G}_i\ \mathbf{x}_i \tag{13} $$

where $\mathbf{G}_0 = \mathbf{G}_{n+1} = \mathbf{I}_{6\times 6}$. Eqn. (13) can be written in the form of a matrix error equation as

$$\mathbf{e} = [{}^{n+1}_{0}\mathbf{J},\ {}^{n+1}_{1}\mathbf{J}\mathbf{G}_1, \ldots, {}^{n+1}_{n}\mathbf{J}\mathbf{G}_n, I_{6\times 6}] \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_{n-1} \\ \mathbf{x}_n \end{bmatrix}$$

or

$$\mathbf{e} = \mathbf{H}\ \mathbf{x} \tag{14}$$

where $\mathbf{H}$ is a $6 \times (4n+12)$ observation matrix and $\mathbf{x}$ is a $4n+12$ error vector. For a six link manipulator, one has to solve for 36 errors. Some of these errors are not independent from each other due to the fact that a redundant frame (frame 0) was introduced to accommodate the selection of a preferred location for the first frame. Thus, in reality, one can remove $\mathbf{x}_1$ frame Eqn. (14) reducing the total error terms to 32 for a six link arm.

LINK PARAMETER ESTIMATION

In practice, $\mathbf{e}$ can be computed by measuring the position of the t-frame by an accurate measurement technique and comparing it to the nominal t-frame transformation. Let ${}^{w}_{t}\mathbf{T}^{M}$, ${}^{w}_{t}\mathbf{T}^{A}$, and ${}^{w}_{t}\mathbf{T}^{N}$ represent the measured, actual, and nominal t-frame transformations, then they may be related to each other by the following relations:

$${}^{w}_{t}\mathbf{T}^{A} = {}^{w}_{t}\mathbf{T}^{M}(-\Delta\mathbf{T}_{\text{noise}}) \tag{15}$$

where the subscript "noise" refers to the measurement noise. The discrepancy between the actual and nominal ${}^{w}_{t}\mathbf{T}$ is the result of link errors

$${}^{w}_{t}\mathbf{T}^{A} = {}^{w}_{t}\mathbf{T}^{N}(\Delta\mathbf{T}) \tag{16}$$

or

$$(\Delta\mathbf{T}) = \left({}^{w}_{t}\mathbf{T}^{N}\right)^{-1}\left[{}^{w}_{t}\mathbf{T}^{M}(-\Delta\mathbf{T}_{\text{noise}})\right] \tag{17}$$

Again, assuming first order approximations and the fact that the link errors are small, one can write

$$\left({}^{w}_{t}\mathbf{T}^{N}\right)^{-1}\left({}^{w}_{t}\mathbf{T}^{M}\right) = \mathbf{I} + \mathbf{E}_t \tag{18}$$

where E_t has the off-diagonal terms of a small transformation such as the one given in Eqn. (9). Similarly $(-\Delta T_{\text{noise}})$ can be written as $\mathbf{I} - \mathbf{E}_n$. Thus from (18) and (17) we have

$$\mathbf{I} + \mathbf{E} = (\mathbf{I} + \mathbf{E}_t)(\mathbf{I} - \mathbf{E}_n) \overset{\text{First Order}}{=} \mathbf{I} + \mathbf{E}_t - \mathbf{E}_n$$

or

$$\mathbf{E} = \mathbf{E}_t - \mathbf{E}_n$$

The above equation written in a vector form results in

$$\mathbf{e} = \mathbf{e}_t - \mathbf{e}_n$$

which can be used in equation (14). The final form of the observation equation is therefore

$$\mathbf{e}_t = \mathbf{H}\ \mathbf{x} + \mathbf{e}_n \tag{19}$$

This equation then can be used to solve for the error vector $\mathbf{x}$ by means of a minimum variance least squares algorithm.

PARALLEL JOINTS

The formulation given in the preceeding pages applies only to the case when the two revolute joint axes are not nominally parallel or near parallel [6]. If two consecutive joint axes are parallel, small joint axis misalignments can cause large link parameter variations, thus invalidating small variation assumptions. In the following a slight modification to the procedure defined by Craig [13] will be given to define a transformation between two consecutive parallel or near parallel joint axes which does satisfy the small variation assumption.

Let us assume that the joint axes $i-1$ and i are nominally parallel (see Figure 2). Following the frame assignment rules outlined in [13], one can obtain α_{i-2}, a_{i-2} for the $i-2$ link. In order to obtain θ_{i-1} and d_{i-1}, one needs to obtain the common normal between the $i-1$ and i-joint axes. But since the $i-1$ link is between two parallel joint axes, instead of finding the common normal between the $i-1$ and i joint axes, we *select* a point on the $i-1$ axis and pass a plane normal to the $i-1$ axis through this point. The $\mathbf{x}_{i-1}$ axis is defined by a line drawn from this point to a point on the i axis which is the intersection of the plane with the i axis. Now that $\mathbf{x}_{i-1}$ axis has been defined, one obtains θ_{i-1} and d_{i-1}. The i-frame is then described by the following equation.

$${}^{i-1}_{i}\mathbf{T}^{P} = \text{Rot}\ (x_{i-1}, \alpha_{i-1})\ \text{Trans}\ (x_{i-1}, a_{i-1})$$
$$\text{Rot}\ (y'_{i-1}, \beta_{i-1})\ \text{Rot}\ (x_i, \theta_i)\ \text{Trans}\ (z_i, d_i) \tag{21}$$

Following the same procedure for non-parallel links, the error terms, i.e. $\mathbf{G}$ and $\mathbf{e}$, for parallel or near-parallel links are given by

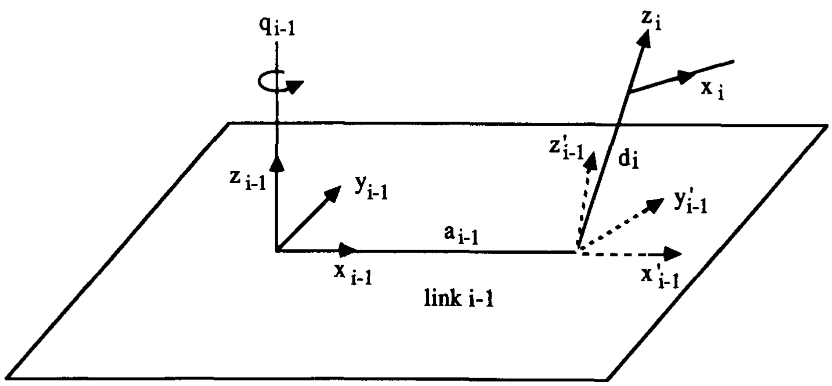

Figure 2.

$$
\mathbf{G}_i^p = \begin{bmatrix} -s\theta_i c\beta_{i-1} d_i & c\theta_i c\beta_{i-1} & c\theta_i d_i & 0 & 0 \\ -c\theta_i c\beta_{i-1} d_i & -s\theta_i c\beta_{i-1} & -s\theta_i d_i & 0 & 0 \\ 0 & s\beta_{i-1} & 0 & 0 & 1 \\ c\theta_i c\beta_{i-1} & 0 & s\theta_i & 0 & 0 \\ -s\theta_i c\beta_{i-1} & 0 & c\theta_i & 0 & 0 \\ s\beta_{i-1} & 0 & 0 & 1 & 0 \end{bmatrix} \tag{22}
$$

$$
\mathbf{x}_i^p = \begin{bmatrix} \delta\alpha_{i-1} \\ \delta a_{i-1} \\ \delta\beta_{i-1} \\ \delta\theta_i \\ \delta d_i \end{bmatrix} \tag{23}
$$

INVERSE KINEMATICS OF NEAR SIMPLE ROBOTS

The inverse kinematic problem arises after a calibration has been performed on a robot and the link parameters have been updated. It has been shown by Pieper [14] that robots with three consecutive joint axes intersecting at one point have an analytical inverse kinematics solution. Even if a robot is a "simple" robot, after the calibration there is no guarantee that it will remain simple. In this section a method is presented to obtain the inverse kinematics of such robots.

Let us assume that the nominal robot has an analytical inverse kinematics solution given by

$$
\mathbf{q}_N = f_N^{-1}(\mathbf{T}) \tag{24}
$$

where $\mathbf{q}$ is the joint angle solution vector and $\mathbf{T}$ is the end-frame homogeneous transformation described in the robot's base frame. We are, however, interested in the actual inverse kinematics solution symbolically shown by

$$
\mathbf{q} = f^{-1}(\mathbf{T}) \tag{25}
$$

If we use $\mathbf{q}_N$ from equation (24) to position the robot, the end-point will move close to the desired position $\mathbf{T}$. Let us call this position $\mathbf{T}_N$. We then have

$$
\mathbf{T}_N = f(\mathbf{q}_N) = f\left[f_N^{-1}(\mathbf{T})\right] \tag{26}
$$

where $f(\cdot)$ represents the actual forward kinematics which we can compute easily after the robot has been calibrated and the link error parameters have been obtained. The difference between $\mathbf{T}_N$ and $\mathbf{T}$ can be assumed to be a small transformation given from

$$
\mathbf{T} = \mathbf{T}_N + d\mathbf{T} = \mathbf{T}_N \Delta\mathbf{T} \tag{27}
$$

from which $\Delta\mathbf{T}$ may be computed as

$$
\Delta\mathbf{T} = \mathbf{T}_N^{-1}\mathbf{T} \tag{28}
$$

A direct substitution from (26) results in

$$
\Delta\mathbf{T} = \left\{f\left[f_N^{-1}(\mathbf{T})\right]\right\}^{-1}\mathbf{T} \tag{29}
$$

If we denote by $\delta\mathbf{T}$ the vector equivalent of $\Delta\mathbf{T}$ (see Eqns. 9 and 10), then the manipulator Jacobian can be used to find small joint angles which would move the end- effector from T_N to T.

$$
\delta\mathbf{q} = \mathbf{J}^{-1}\delta\mathbf{T} \tag{30}
$$

Thus the inverse kinematics solution is given by

$$
\mathbf{q} = \mathbf{q}_N + \delta\mathbf{q} \tag{31}
$$

This process breaks down when the manipulator is at or close to one of its singularities in which case one might just solve the inverse kinematics equations based on the nominal link parameters and not compensate for the link errors.

EXPERIMENTAL SET-UP AND RESULTS

Simulation studies have shown [15,21] that the proposed calibration technique works well as long as one can provide accurate measurements of the end-effector. We designed a simple calibration fixture to obtain the end-point measurements. The fixture consists of a simple peg with a pointed tip and a calibration plate. The peg is attached to a wrist force/torque sensor which indicates when the robot makes a contact with the calibration plate. The calibration plate has a matrix of index marks to provide accurate positions of the tip of the pointed peg. Note that with this method one can only measure the position of the end-effector and no information is available on the orientation. The measurement accuracy, however, is very good (.1 mm standard deviation). Since in general all the link errors will cause errors in the end-effector, the top three rows of the observation equation can be utilized for parameter estimation. Table 1 lists the results of the parameter estimation for a PUMA 560 with 50 end-point measurements. To test the validity of the estimated link parameter errors the inverse kinematic solution outlined in this paper was used to position the robot on the index marks on the calibration plate. Two sets of experiments were conducted. In the first case only the nominal robot link parameters were used for positioning the robot. The positioning error was as much as 6 mm. In the second set of experiments, the estimated errors of Table 1 were added to the nominal link parameters. The positioning error was consistently less than 2 mm. The accuracy could only be tested on the calibration plate, since no other fixture was available at this time.

Link no.		Nominal	Estimated	Post Calibration Link Parameters
1	α_0	-90.000	0.011	-89.989
	a_0	0.00	-0.67	-0.67
	θ_1	0.000	0.180	0.180
	d_1	0.00	2.10	2.10
2	α_1	-90.000	-0.079	-90.079
	a_1	0.00	0.05	0.05
	θ_2	0.000	-0.137	-0.137
	d_2	149.09	0.42	149.51
3 parallel link	α_2	0.000	-0.100	-0.100
	a_2	431.80	0.66	432.46
	β_2	0.000	-0.072	-0.072
	θ_3	0.000	0.204	0.204
	d_3	0.00	0.42	0.42
4	α_3	90.000	0.066	90.066
	a_3	-20.32	0.31	-20.01
	θ_4	0.000	-0.109	-0.109
	d_4	433.07	0.52	433.59
5	α_4	-90.000	0.124	-89.876
	a_4	0.00	-0.25	-0.25
	θ_5	0.000	-0.393	-0.393
	d_5	0.00	-0.03	-0.03
6	α_5	90.000	0.018	90.018
	a_5	0.00	-0.72	-0.72
	θ_6	0.000	0.000	0.000
	d_6	0.00	-0.08	-0.08

Table 1. Results of Calibration for a PUMA 560 (lengths, mm, angles, rad)

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