KINEMATIC MODELLING FOR ROBOT CALIBRATION

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Abstract

There is a considerable amount of literature dealing with robot calibration. A classification hierarchy of calibration procedures is discussed which groups robot calibration by objective. The state of the art for each classification is discussed with citations from recent literature. Several proposed models are described.

Based upon the common assumption of all lower pair joints, three concepts are discussed which may be applied for comparing most of the calibration models proposed in the literature. These concepts are completeness, equivalence, and proportionality. Each of these are discussed in detail in the text.

Attention is also paid to the possibility that joints may actually be higher pairs. Although this possibility is commonly accepted in coordinate measuring machine calibration, it is not widely recognized in the robot literature. The paper discusses possible causes and effects of higher order joint pairs and demonstrates how they might be modeled.

Introduction

In the recent past, much attention has been paid to all forms of robot calibration. Because of the wide variety of approaches to this problem, Roth, Mooring, and Ravani [15] have proposed a hierarchy of calibration techniques. According to this scheme, robot calibration may be divided into three levels. Level 1 is defined as "Joint Level Calibration". At this level, the primary goal is to determine the correct relationship between the joint transducer signal and the actual joint displacement. Level 2 is defined as "Kinematic Model Calibration". At Level 2, the goal is to improve the accuracy of the kinematic model of the manipulator. With this type of calibration, the assumption is usually made that the links of the manipulator are rigid and the joints are either revolute or prismatic. Some of the approaches to calibration at Level 2 include effects due to backlash or eccentricity in the joint gearing. These "non-geometric" errors most often effect the relationship between the joint transducer signal and the actual joint displacement and do not invalidate the assumption of rigid links and perfect joints. Level 3 calibration is defined as "Dynamic Model Calibration". At this level the goal is to determine inertial properties of the various links in the robot as well as the kinematic constraints.

The state of the art in calibration varies from level to level. For example, techniques for Level 1 calibration are well known. Since many robots use incremental encoders as a joint transducer, a Level 1 calibration must be done every time the robot is powered up in order to establish a reference or home position. Level 2, on the other hand, is not so well understood. At this time, a number of Level 2 calibration techniques have been proposed and a few have been tested and verified in the laboratory. While progress has been made, a number of issues still need to be resolved before Level 2 calibration will become standard. For example, almost every Level 2 calibration technique that has been reported utilizes a slightly different modelling procedure. While similar in basic form, these models vary in the number and form of the parameters that are to be determined. The purpose of this work, therefore, is to examine the various models that have been proposed for Level 2 calibration and to identify the basic requirements for such a model.

Literature Survey

The most popular method of developing a kinematic model for a manipulator is the procedure proposed by Denavit and Hartenberg [3] as a general approach to modeling the motion of a closed loop, three dimensional mechanism. Paul [14] demonstrated the use of the Denavit-Hartenberg technique for modeling a serial link manipulator. The procedure consists of assigning a coordinate system to each link at the joint axis and then expressing the relationship between consecutive coordinate systems with homogeneous transformation matrices. All of the individual link transformation matrices may then be multiplied together to produce one transformation relating the coordinate system in the end effector to the base coordinate frame. The resulting matrix is a function of the joint displacement variables and the various parameters that describe the geometry of the manipulator. To simplify the derivation of each link transformation matrix, Denavit and Hartenberg developed a set of rules for locating the various coordinate systems. Following these rules insures that every transformation matrix has the same functional form and that the geometric parameters are well defined.

The Denavit-Hartenberg model has been used for the calibration problem by Wu [22,23]. In this work, Wu treated the geometric parameters as variables in addition to the joint displacement variable. He assumed that the actual geometric parameters were very near to the nominal or design values and expanded the total transformation as a Taylor series about the nominal values. Keeping only the first order terms resulted in a linear expression for the differential deviations of the Denavit-Hartenberg parameters. A similar technique was reported by Hayati [5]. This work was later expanded by Veitschegger and Wu [19] to include second order terms. Ibarra and Perreira [9] have also presented a technique whereby small variations in the end effector pose are related to differential changes in both the geometric parameters and joint variables in the Denavit-Hartenberg model. Kumar and Prakash [1983] have presented a similar model. This model was used in other work [11,20] to map the effects of joint errors rather than as a calibration tool.

One limitation of these approaches arises out of the particular set of geometric parameters that are used in the Denavit-Hartenberg model. Each coordinate system must be located at the intersection of the joint axis and the common normal between the axis and the next consecutive joint axis. One of the geometric parameters in each joint transformation matrix is the length of the common normal. Another of the geometric parameters is the distance along the joint axis from the origin of the axis coordinate system to the intersection of the joint axis and the common normal with the previous axis [3]. If two consecutive joint axes are parallel, there are an infinite number of common normals of the same length and the location of the axis coordinate system may be made arbitrarily. If these axes are very slightly misaligned and also intersect, the length of the common normal is not continuous and jumps to zero. More importantly, the location of the axis coordinate system jumps to a large distance out along the joint axis. For slight misalignments, this distance and the associated geometric parameter can approach infinity. The resulting kinematic parameters are not continuous and small variations in axis alignment may produce large changes in the geometric parameters. This invalidates the assumptions on which the linear model was based and may lead to severe numerical difficulties.

Others have recognized this limitation in the Denavit-Hartenberg formalism and proposed changes. Hayati and Mirmirani [6,7] use the Denavit-Hartenberg approach for all axis configurations except for near parallel axes. In this case, the origin of the coordinate frame is defined to be in the X-Y plane of the previous coordinate frame. This insures that small misalignments are represented by small variations in the kinematic parameters. It is interesting to note that in either case, three geometric parameters and one joint variable are required to define the geometry for any one link. Hsu and Everett [8] approach the problem by modifying the Denavit-Hartenberg formalism for all joints. In their model, the location of the coordinate system along any coordinate axis is treated as a variable rather than being implicitly specified. This results in four geometric parameters and one joint variable for each link. Introduction of this extra parameter for each joint leads to the question of independence. Since the techniques mentioned earlier completely describe the robot geometry with four variables per joint, it seems reasonable to assume that additional parameters would not be independent. Unfortunately, there are no examples given in the work by Hsu and Everett and this question is not addressed.

A different modeling approach has been proposed in work by Mooring [12] and Mooring and Tang [13]. This model is based on the general spatial rigid body displacement equation which is sometimes referred to as Rodrigues equation [1]. With this approach each joint axis is defined by a point through which the axis passes and a unit vector along the axis. The model may then be expanded so that small changes in axis position and orientation are represented by small variations in the chosen parameter set. This approach employs a total of 4 parameters for each joint in addition to the joint variable. While this method avoids the problems with near parallel axes, it suffers from the disadvantage of using parameters that may be quite different from those used in the Denavit-Hartenberg approach. It is interesting to note that this approach also makes use of one more geometric parameter than the Denavit-Hartenberg convention.

Yet another technique has been developed by Stone, Sanderson and Neuman [17]. Their model, termed the "S-model", relates coordinate frames on consecutive joint axes but there are five geometric parameters and one joint variable in the expression. This construction allows several of the six parameters to be specified and thus avoids the parallel axis problem. In this work, the authors have developed a set of equations that allow the Denavit-Hartenberg parameters to be extracted from the S-model parameters.

All of the models presented thus far have been based on the assumption that the only significant errors in the robot are those that may be expressed in the kinematic model. Whitney, Lozinski, and Rourke [21] demonstrated the significance of such errors as joint compliance, gear eccentricity and joint backlash. These errors were termed "non-geometric" errors and included in the modeling process. The geometric errors were modeled by placing coordinate frames on each joint axis and then expressing the relationship between consecutive axes with a sequence of rotations and displacements. The parallel axis problem was avoided by not adhering to the Denavit-Hartenberg procedure. The non-geometric errors were modeled by either testing for or assuming the functional form of the error and including the coefficients in the identification process. Later work by Chen and Chao [2] also included the effect of non-geometric errors. This work also presented a novel approach to the modeling of geometric errors which is based on work by Sheth and Uicker [16]. With this approach, the nominal relationship between axes is represented in a transformation which may be obtained with any modeling technique. The geometric errors are then included in a differential displacement after the nominal transformation. The joint displacement is then expressed in a third matrix. This approach allows the nominal model to be developed through any procedure. The axis variations are then included in the differential displacement matrix and are, therefore, proportional to the degree of axis variation.

The problem of kinematic modeling and calibration is also found in the area of machine tools, and in particular, the context of coordinate measuring machines. A coordinate measuring machine consists of three rigid components with motions along straight, mutually perpendicular axes and is similar to a three link prismatic robot with no wrist joints. The probe at the end of the third link can be moved to any point in the workspace while the controlled readout shows the coordinates of that point.

Zang, Veale, Charlton, Borchardt, and Hocken [24] have developed methods for applying software error correction to a commercial three axis coordinate measuring machine. The technique incorporates compensation for geometric positioning errors and some thermal effects. Geometric error compensation is based on a rigid body model of workpiece motion in the machine coordinate frame. Each axis was treated as a cam surface and not constrained to be a perfect prismatic axis. In this way, variations in the linearity or orientation of the axis may be included in the model as functions of the joint displacement. The dominant thermal effects in the machine are removed by introducing the concept of an "effective" nominal differential expansion coefficient. This modeling technique was incorporated into an automatic calibration scheme for the machine and the performance was improved by a factor of ten. Later work by Driels and Pathre [4] extended this concept to the modeling of robots. Their model addresses both revolute and prismatic axes which allows for the modeling of eccentricity in the bearings that approximate a revolute axis. This approach to kinematic modeling is quite appealing in that it may account for several of the "non-geometric" errors that have been addressed in the conventional modeling techniques.

As illustrated by the literature review, the problem of "Level 2" calibration has been addressed by a number of investigators. Because of difficulties with the Denavit-Hartenberg modelling approach, a number of alternative models have been proposed. Many of these models contain different numbers of geometric parameters and some are based on the assumption that the joints are higher pairs. In the following sections, we will consider the fundamental properties that a kinematic model must possess if it is to be suitable for a "Level 2" calibration procedure. After a brief look at some basic motion considerations, we will examine models based on the assumption that the joints are either revolute or prismatic. This will be followed by an examination of an approach to modelling joints that were designed to be revolute or prismatic joints but are actually higher pairs.

Basic Considerations

Before examining kinematic models for the purpose of calibration, we will consider some basic properties of rigid body motion. Consider, for example, a body **A** containing a body centered coordinate system η_x, η_y, η_z and moving in reference to a fixed coordinate system x, y, z. If ${}^A\vec{p}$ represents the location of point **p** in the body centered coordinate system and $\vec{p}$ defines the location of point **p** in the fixed system, then the following relationship holds.

$$\vec{p} = [T_0]^A\vec{p} \tag{1}$$

where the matrix $[T_0]$ is a 4x4 homogeneous transformation matrix and the vectors $\vec{p}$ are of the form

$$\vec{p} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \tag{2}$$

It is well known [1] that the homogeneous transformation matrix $[T_0]$ contains 6 independent parameters and that specification of these parameters will completely define the position and orientation of the rigid body with respect to the base coordinate frame. If we now assume that the body **A** is displaced through some finite distance, as illustrated in Figure 1, the new position, $\vec{p'}$, of the point **p** in the fixed coordinate system may be expressed as

$$\vec{p'} = [S][T_0]^A\vec{p} \tag{3}$$

where the matrix $[S]$ describes the displacement. While $[S]$ may take on a number of functional forms, we shall use a screw matrix which has the following form [18].

$$[S] = \begin{bmatrix} & [R_{\phi,\vec{u}}] & & (\vec{r} + s\vec{u} - [R_{\phi,\vec{u}}]\vec{r}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

where $\vec{u}$ is a unit vector along the screw axis, ϕ is the angle of rotation about the screw axis, $\vec{r}$ is a vector locating any point on the screw axis and $[R_{\phi,\vec{u}}]$ is defined as follows.

$$[R_{\phi,\vec{u}}] = \begin{bmatrix} u_x^2V(\phi) + C(\phi) & u_xu_yV(\phi) - u_zS(\phi) & u_xu_zV(\phi) + u_yS(\phi) \\ u_xu_yV(\phi) + u_zS(\phi) & u_y^2V(\phi) + C(\phi) & u_yu_zV(\phi) - u_xS(\phi) \\ u_xu_zV(\phi) - u_yS(\phi) & u_yu_zV(\phi) + u_xS(\phi) & u_z^2V(\phi) + C(\phi) \end{bmatrix} \tag{5}$$

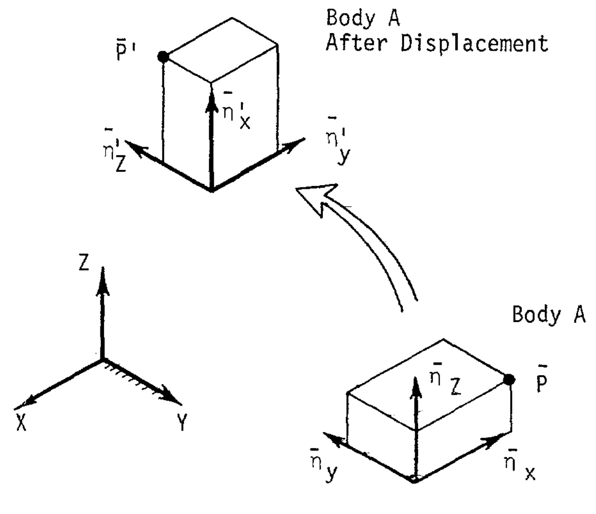

Figure 1 - Rigid Body Displacement of Body A

where $S(\phi)$ implies $\sin(\phi)$, $C(\phi)$ implies $\cos(\phi)$, and $V(\phi)$ implies 1 - $\cos(\phi)$.

As mentioned earlier, 6 parameters must be specified to define the displacement. In this particular formulation, the independent parameters are the rotation angle ϕ, the displacement s, two components of the unit vector $\vec{u}$, and two components of the vector $\vec{r}$. Only two components of $\vec{u}$ are independent because $\vec{u}$ has unit length and only two components of $\vec{r}$ are independent because $\vec{r}$ can define any point along the screw axis.

Using this formulation, the location of any point on the body **A** may be determined after an arbitrary displacement by specifying the initial position of the body, $[T_0]$, and the six parameters that comprise the screw displacement. Any number of displacements may be described by simply expressing one displacement after the other in the order that they occur. For example, "n" consecutive displacements would be expressed as

$$\vec{p}' = [S_n][S_{n-1}]...[S_1][T_0]^A\vec{p} \tag{6}$$

It should be noted that no restrictions have been placed on the motion of the body and that the parameters in the screw matrix will describe any rigid body displacement.

Now that we have reviewed a means to describe the motion of a body, we will turn our attention to the problem of developing a kinematic model that will be suitable for calibration. As mentioned in the literature survey, one may proceed under the assumption that all of the joints in the manipulator are "perfect" revolute or prismatic joints or it may be assumed that the joints produce well defined, but irregular, motion. The approach toward modelling will be significantly affected by this assumption and, therefore, will be treated separately.

Revolute and Prismatic Axes

Given the assumption that all of the joints in a manipulator are either revolute or prismatic, a number of approaches to kinematic modelling for calibration have been proposed. Any such model must possess three properties if it is to be useful. First, the model must contain a sufficient number of parameters to completely specify the motion of the robot under study. In this work, we will refer to this concept as completeness. Most calibration procedures assume that a perfect or nominal robot model is known. The purpose of the calibration process is to determine the deviation of a particular robot from the nominal model. In order to accomplish this, the model must contain a sufficient number of independent coefficients to express any possible variation in the kinematic structure of the robot. The second desirable property of a model is the ability to establish a relationship between the functional form of the model and that of any other acceptable model. We shall refer to this property as equivalence. The third property that the model should possess is that small changes in the geometry of the robot should be reflected by small variations in the model parameters. We shall refer to this property as proportionality. Since most of the calibration procedures rely on numerical algorithms for identification of the unknown parameters, a model must exhibit proportionality so that numerical stability may be assured.

As we consider these three properties, it becomes apparent that proportionality has received the most attention. Many of the modified models for calibration have been developed simply because the Denavit-Hartenberg approach lacks proportionality when consecutive axes are very nearly parallel [19,9,6,7,8,12,13,17]. Unfortunately, model completeness and equivalence have not always been given the same importance. For this reason, we will now turn our attention to these two properties.

For a model to be complete, it must be capable of expressing any possible variation in the geometry of a robot. Consider the motion described by Equation 6. This represents the motion of a rigid body through "n" displacements. We may assume that the rigid body is the robot end effector and that motion about each joint axis represents one displacement. Since the assumption has been made that all joints are either revolute or prismatic, some portion of each joint displacement is specified. For example, a revolute joint produces a rotation about the joint axis and no displacement along it. The parameter s in the screw matrix becomes zero. If we consider the rotation ϕ to be the joint variable, we are left with four parameters to position and orient the rotation axis in space. This implies that four parameters are required to locate any revolute axis and one joint variable is necessary to express motion about the axis. In the case of a prismatic axis, the screw parameter s becomes the joint variable and ϕ becomes zero. It is easily shown that if ϕ is set to zero, the matrix $[R_{\phi,\vec{u}}]$ becomes the identity matrix. The screw matrix, Equation 4, then becomes

$$[S] = \begin{bmatrix} & [I] & & s\vec{u} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

It is now clear that, for a prismatic joint, only two parameters orienting the joint axis and one joint variable are necessary to define the motion of the joint. Recalling that 6 parameters were required to specify the initial position of the end effector, we may now write the following equation

$$C = 4R - 2P + 6 \tag{8}$$

where R is the number of revolute joints, P is the number of prismatic joints, and C is the total number of parameters that must appear in the model for it to be complete. For example, a model of a robot with six revolute axes such as a PUMA 560 must have 30 parameters in addition to the six joint variables to be complete. To examine this idea more closely, we will consider the 3 revolute axis manipulator illustrated in Figure 2. If the Denavit-Hartenberg approach is applied, one coordinate system will be located on each of the 3 joint axes and one coordinate system will be located at the end effector as illustrated. Following the Denavit-Hartenberg formalism, 3 parameters and 1 joint variable are required between coordinate systems 1-2, 2-3, and 3-4. The relationship between the reference coordinate system, 0, and the robot, however, is more complicated. If we follow the convention, the x axis of the reference system must lie along the common normal between the z axis of the reference system and the axis for joint 1. Also, the origin of the reference system must be located on the common normal. This means that the location and orientation of the reference system will become a function of the robot geometry, which is not acceptable for calibration purposes. For this reason, we must abandon the Denavit-Hartenberg convention and use 6 parameters to describe the relationship between the reference system and the first axis. This leaves us with a total of 3 joint variables and 15 parameters to define the motion of the robot. This number, however, does not agree with Equation 8 which indicates that 18 parameters are required in addition to the 3 joint variables. When following the Denavit-Hartenberg convention, one is making several implicit assumptions. One of these is that the position of the robot when all of the joint variables are zero is a function of the geometry of the robot. This implies that two slightly dissimilar robots will have their end effectors in different locations when in the "zero" position. If we recall that the purpose of calibration is to make two slightly different robots perform as closely as possible to a nominal or design machine, it is clear that the zero position must be unique and not a function of individual robot geometry. This problem may be corrected by simply adding an offset angle to each joint variable. These offsets add 3 additional parameters which brings the total in our example to the expected value of 18.

The addition of the joint offsets in the example raises an interesting point. There are a number of kinematic models that have been presented whose number of parameters differ from that predicted by Equation 8. Basically, it is our contention that any proposed model must have a reference coordinate system that is independent of individual robot geometry and must yield a unique "zero" position for any number of dissimilar robots if it is to be suitable for calibration. If these two criteria are met in addition to completely modelling the motion of each joint, the model will contain the number of independent parameters indicated

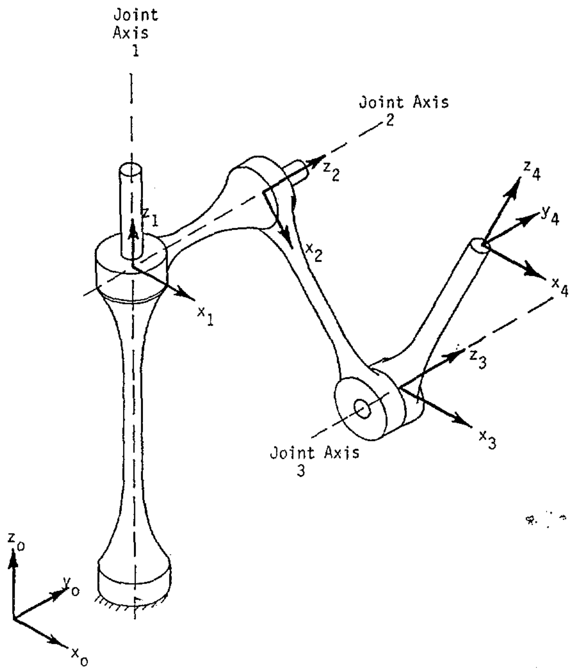

Figure 2 - Three Axis Manipulator

in Equation 8 and, therefore, will be complete. Furthermore, any two complete models will be equivalent. Since any complete model has the correct number of independent parameters, relationships must exist between these parameters and those of any other complete model.

Joints Comprised of Higher Pairs

Although most manipulators are designed with the intention that all of the joints will be either revolute or prismatic, it is physically impossible to construct a joint that will perfectly generate this type of motion. For example, most "prismatic" joints consist of a carriage constrained to move along a bar. Since the bar will be subject to some slight curvature or irregularities along its surface, the generated motion will not be purely prismatic. This phenomenon has been recognized in the area of calibration of coordinate measuring machines. In his report on software error compensation, Zhang [Zhang1] reported the use of a rigid body model with six degrees of freedom per axis for a coordinate measuring machine to allow for imperfections in the machine axes.

Figure 3 is an illustration of a joint that is intended to be prismatic but is subject to some error. As illustrated in the figure, the curvature in the bar causes the carriage to deviate from the desired path. This deviation induces an error in orientation as well as position. Since a prismatic joint will only allow for translation along a straight line and does not allow for a change in orientation during a displacement, these errors cannot be accounted for with a simple prismatic joint model. While the axis of the nominal prismatic joint may be varied so as to minimize the effect of these errors over a given range, the errors may not be eliminated. To account for these types of errors, the joint model may be modified so as to represent higher pair motion. Since most robots have joints that approximate lower pairs, the modelling process can be simplified. For example, we will consider the joint illustrated in Figure 3. The predominant motion of the carriage is along the bar. This implies that we may model the total motion as the combination of a prismatic displacement and a small additional motion to correct for the error. This may be expressed as

$$\vec{p'} = [\delta S][S][T_0]^A\vec{p} \tag{9}$$

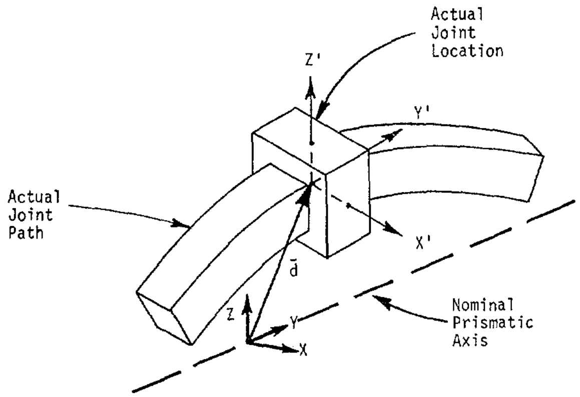

Figure 3

where $[S]$ represents the motion of a prismatic joint and $[\delta S]$ may be expressed as

$$[\delta S] = \begin{bmatrix} 0 & -\delta_z & \delta_y & d_x \\ \delta_z & 0 & -\delta_x & d_y \\ -\delta_y & \delta_x & 0 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

where d_x, d_y, and d_z represent small displacements in the indicated directions and δ_x, δ_y, and δ_z represent small rotations about the coordinate axes. We shall refer to $[\delta S]$ as the correction matrix.

Given this formulation, the process of calibration becomes one of determining the correct values of the correction matrix for each displacement of the joint. There are several important points to note about this process. First, the values in the correction matrix will depend on the orientation of the prismatic axis. Since the correction matrix will account for errors about the prismatic axis, the orientation of the prismatic axis may be specified to be some nominal value and omitted from the calibration procedure. If both the correction matrix and the prismatic axis are included in the calibration, their dependence on each other may result in numerical difficulties.

A second point to note is that the values in the correction matrix will be different for every possible displacement of the carriage. Since the shape of the bar is constant, however, a given displacement will repeatably result in a given correction matrix. In other words, the correction matrix is a time independent function of the joint displacement. This implies that while the joint is a higher pair, it still has only one degree of freedom. To simplify the calibration process, it is desirable to choose a functional form for the terms in the correction matrix. Since the joint is intended to be prismatic, it is usually acceptable to assume that any term, δ_i, in the correction matrix will be a smooth, continuous, slowly varying function of the joint displacement. The number of parameters to be identified, therefore, depends on the particular function chosen for each element of the correction matrix. For example if each term is modelled with a quadratic function such as

$$\delta_i = K_1 s^2 + K_2 s + K_3 \tag{11}$$

then each correction matrix will contain 18 parameters to be determined. The particular form of the function will be highly dependent on the expected level of deviation from true prismatic or revolute motion. It should be noted that if irregularities exist in the axis, the functions in the correction matrix may not be continuous. This may result when two bars forming the axis are not joined evenly and a "jump" results at the interface.

Since the number of parameters is dependent on the function chosen for the terms in the correction matrix, the concepts of completeness and equivalence do not apply for this type of model. For example, one manipulator may require that all terms in the correction matrix be approximated with harmonic functions where as another may only require a few terms consistent with deflection in a given direction. While both may be adequate for their specific situation, they would not have the same number of parameters and they would not be equivalent.

Conclusions

Kinematic models intended for calibration may be classed into two categories. The first category includes models that are based on the assumption that all joints in the manipulator are either revolute or prismatic. In this case, the model should exhibit three qualities which we have referred to as completeness, equivalence, and proportionality. Since most calibration schemes consist of a minimization of some performance index through a numerical technique, proportionality is important to insure numerical stability. Furthermore, any model must contain a sufficient number of parameters to include all necessary aspects of the manipulator motion while not containing additional dependent parameters that may cause numerical difficulties. We have attempted to demonstrate that such a complete model will have a certain number of independent parameters and that all complete models will be equivalent.

The second category of models includes those that are based on the assumption that some of the joints may be higher pairs. In this category, it may be assumed that the joint has a predominant motion that is similar to that of a prismatic or revolute pair but some additional forms of motion exist. In this case the model is modified to allow for this extra motion which may be expressed as a function of the joint variable. Because of the wide range of allowable functional forms, the concepts of completeness and equivalence do not apply to this category.

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