A peculiar continued fraction

$$f(x) = \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \ddots}}}$$

now consider f(x) - 1, in other words

$$f(x) = 1 + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \ddots}}} \quad \text{for x = 1 we have ...}$$

$$f(1) = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}$$

I will show that this continued fraction "converges" and surprisingly "converges" to the golden ratio

the "compact" form of the peculiar continued fraction, is found by...

$$f(x) = \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \ddots}}}, \quad y = f(x) \implies y = \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \ddots}}}$$

Therefore...

$$y = \frac{1}{x + y} \equiv y = \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \ddots}}}$$

$$y = \frac{1}{x + y} \implies y(x + y) = 1 \implies y^2 + xy = 1$$

A nice compact quadratic polynomial that can be expanded to the pecuilar continued fraction

Following the same algebraic manipulation it is easy to see that...

$$f(x) = 1 + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \ddots}}}$$

when

$$f(x) = y \quad \text{and} \quad x = 1$$

$$\implies y = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}} \quad \text{is equivalent to} \quad y = 1 + \frac{1}{y}$$

Then be the same algebraic manipulation we arrive at a very similar quadratic equation:

$$0 = y^2 - y - 1$$

From the quadratic equation we can find the roots:

$$y = \frac{1 \pm \sqrt{5}}{2}$$

and strangely ϕ, the golden ratio, is the positive root of the polynomial..

$$\phi = \frac{1 + \sqrt{5}}{2}$$