

## A peculiar continued fraction

$$f(x) = \frac{1}{x + \frac{1}{x + \frac{1}{x + \ddots}}}$$

now consider  $f(x) - 1$ , in other words

$$f(x) = 1 + \frac{1}{x + \frac{1}{x + \frac{1}{x + \ddots}}} \quad \text{for } x = 1 \text{ we have ...}$$

$$f(1) = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \ddots}}}$$

I will show that this continued fraction "converges" and surprisingly "converges" to the golden ratio

the "compact" form of the peculiar continued fraction, is found by...

$$f(x) = \frac{1}{x + \frac{1}{x + \frac{1}{x + \ddots}}}, \quad y = f(x) \implies y = \frac{1}{x + \frac{1}{x + \frac{1}{x + \ddots}}}$$

Therefore...

$$y = \frac{1}{x + y} \equiv y = \frac{1}{x + \frac{1}{x + \frac{1}{x + \ddots}}}$$

$$y = \frac{1}{x + y} \implies y(x + y) = 1 \implies y^2 + xy = 1$$

A nice compact quadratic polynomial that can be expanded to the peculiar continued fraction

Following the same algebraic manipulation it is easy to see that...

$$f(x) = 1 + \frac{1}{x + \frac{1}{x + \frac{1}{x + \ddots}}}$$

when

$$f(x) = y \quad \text{and} \quad x = 1$$

$$\Rightarrow y = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \ddots}}} \quad \text{is equivalent to} \quad y = 1 + \frac{1}{y}$$

Then be the same algebraic manipulation we arrive at a very similar quadratic equation:

$$0 = y^2 - y - 1$$

From the quadratic equation we can find the roots:

$$y = 1 \pm \frac{\sqrt{5}}{2}$$