Modelling 1D Gyromagnetic Nonlinear Transmission Line using FDTD

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Abstract

In this project, we solve 1D telegrapher's equations coupling with 1D Landau-Lifshitz-Gilbert (LLG) equation for modelling gyromagnetic nonlinear transmission lines (GNLTLs) containing a nonlinear material ferrite as a part of dielectrics. The 1D telegrapher's equations and 1D LLG equation were solved using FDTD in MATLAB and the results are compared to a 3D coaxial GNLTL simulated in COMSOL Multiphysics.

1 Introduction

Nonlinear transmission lines (NLTLs) have been using for pulse sharping and high power microwave generation as solid-state RF sources. There are two main typologies of NLTLs: the lumped element nonlinear transmission lines and gyromagnetic nonlinear transmission lines in a coaxial geometry. The lumped element nonlinear transmission lines apply nonlinear capacitors and/or nonlinear inductors. The nonlinear capacitor has voltage dependent-capacitance and the nonlinear inductor has a current-dependent inductance.

Gyromagnetic nonlinear transmission lines (GNLTLs) use ferrites to provide the nonlinearity which induces the gyromagnetic precession when an external magnetic bias field is applied in the axial direction of the line [1]. Figure 1 shows the coaxial cross section of the GNLTLs. As a input voltage pulse propagates through the line the ferrite goes into saturation. This will compress the pulse to reduce its rise time and induce a damped precession of the magnetic field with high frequency eventually leading to RF generation at the output of the line. The coherent precession of the magnetization towards alignment with the effective magnetic field may be described by the Landau-Lifshitz (LL) equation given by

$$\frac{d\mathbf{M}}{dt} = \frac{\gamma}{1+\alpha^2}(\mathbf{M}\times\mathbf{H}) + \frac{\alpha\gamma}{(1+\alpha^2)M_s}(\mathbf{M}\times(\mathbf{M}\times\mathbf{H})) \tag{1}$$

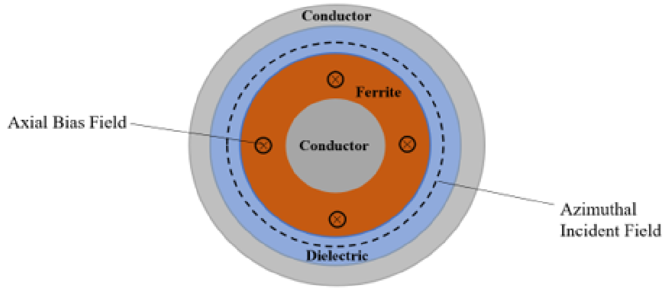

Figure 1: Cross section of a coaxial GNLTL.

where $\mathbf{M}$ is the magnetization, H is the effective magnetic field, and μ_0 is the vacuum permeability. Here, $\gamma = 1.76 \times 10^{11}[rads/s/T]$, $\alpha = 0.01 - 1$, and $M_s = 0.35T/\mu_0$ are the gyromagnetic ratio, the dimensionless damping factor, and saturation magnetization, respectively. The three parameters are all material dependent (or ferrite dependent). Gilbert presented an equivalent form of Eq. (1) as α is not small and it is called Landau-Lifshitz-Gilbert (LLG) equation which is given by

$$\frac{d\mathbf{M}}{dt} = \gamma \cdot \mu_0(\mathbf{M} \times \mathbf{H}) + \frac{\alpha}{M_s}(\mathbf{M} \times \frac{d\mathbf{M}}{dt}) \tag{2}$$

A 3D GNLTL simulation generally involves to solve 3D Maxwell's equation coupling with the 3D LLG/LL equation which can be decomposed into three directions.

2 Methods

2.1 1D LLG equation and telegrapher's equations

For a 1D simulation, the telegrapher's equation can be used coupling with 1D LLG equation for the principal TEM mode in a GNLTL. The 1D LLG equation is given by [2-3]

$$\frac{dM}{dt} = \alpha \cdot \frac{\gamma}{1+\alpha^2} \cdot \mu_0 H \cdot M_s(1 - \frac{M^2}{M_s^2}) \tag{3}$$

where H is the applied circumferential field defined in terms of the effective diameter of the ferrite given by

$$H = \frac{I}{2\pi \cdot d_e} \tag{4}$$

where I is the current and the radial thickness of the ferrite d_e is given by

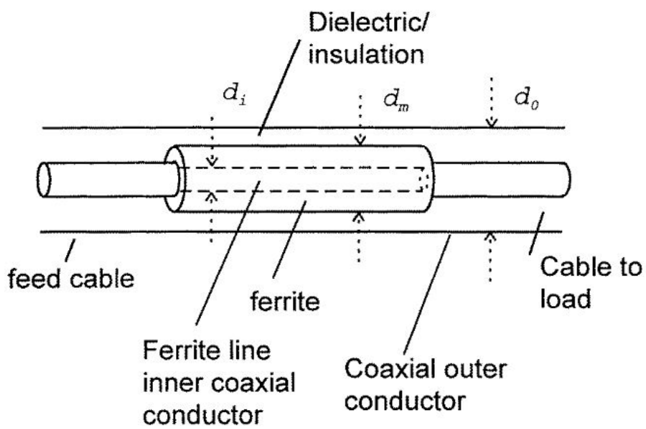

Figure 2: Ferrite loaded NLTL structure [3].

$$d_e = \frac{d_m - d_i}{ln(d_m/d_i))} \tag{5}$$

where d_m and d_i are the inner and outer diameters of the ferrite.

Considering a voltage pulse in the TEM mode, where the electric and magnetic fields do not have longitudinal components, one may use the telegrapher's equation for modelling a distributed gyromagnetic nonlinear transmission line (GNLTL). The telegrapher's equation for the GNLTL as an approximation can be given as [2-3]

$$\frac{dI}{dz} = -C_0 \frac{dV}{dt} \tag{6}$$

$$\frac{dV}{dz} = -\frac{d\phi}{dt} \tag{7}$$

where C_0 is the line capacitance and ϕ is the change rate of circumferential flux per unit length. For a ferrite loaded coaxial line ϕ can be given as [3]

$$\phi = L_0 I + \int_{dm/2}^{di/2} \mu_0 M(r) dr = L_0 I + \mu_0 M \frac{d_m - d_i}{2} \tag{8}$$

where M represents the circumferential component of magnetization. Substituting Eq.(8) into Eq.(7) would yield:

$$\frac{dV}{dz} = -L_0 \frac{dI}{dt} - C_V \frac{dM}{dt} \tag{9}$$

where the constant C_V is defined as $C_V = 0.5 \cdot \mu_0 (d_m - d_i)$.

Using the central differencing formula, Eq.(6) and Eq.(9) can be rewritten as discretized forms and rearranged to obtain $V^{k+1/2}(i)$ and $I^{k+1}(i+1/2)$, respectively, for 1D FDTD simulation (leap-frog method) as given by [4]

$$V^{k+1/2}(i) = V^{k-1/2}(i) - \frac{\Delta t}{\Delta z C_0}\left[I^k(i+1/2) - I^k(i-1/2))\right] \tag{10}$$

$$I^{k+1}(i+1/2) = I^k(i+1/2) - \frac{\Delta t}{\Delta z L_0}\left[V^{k+1/2}(i+1) - V^{k+1/2}(i)\right] - C_V\frac{\Delta t}{L_0}\frac{dM}{dt} \tag{11}$$

where k and i represent the time and space notation, respectively, while Δt and Δz stand for the time and space step, respectively. The term $\frac{dM}{dt}$ will be evaluated for all sections at each time step using Eq.(3) the 1D LLG equation which can be discretized and rearranged to yield:

$$M^{k+1}(i+1/2) = \Delta t \cdot C_M \frac{I^k(i+1/2)}{\pi d_e}\left[1 - (\frac{M^k(i+1/2)}{M_s})^2\right] + M^k(i+1/2) \tag{12}$$

where the constant C_M is defined as

$$C_M = \alpha\frac{\gamma}{1+\alpha^2}\mu_0 M_s \tag{13}$$

for simplification. Since $dt = \Delta t$, the last term in the right hand side (RHS) of Eq. (11) can be modified as

$$C_V\frac{\Delta t}{L_0}\frac{dM}{\Delta t} = C_V\left[M^{k+1/2}(i+1/2) - M^{k-1/2}(i+1/2)\right]/L_0 \tag{14}$$

Furthermore, the time step Δt satisfies the stability condition in the 1D FDTD simulation that is given by:

$$\Delta t \leq \Delta x\sqrt{L_0 C_0} \tag{15}$$

The absorbing boundary condition (ABC) was applied at one end and another end applied a input voltage pulse. Using the central differencing, the ABC is defined by [4]

$$V^{k+1/2}(R) = V^{k-1/2}(R-1) + \frac{\Delta z - v\Delta t}{\Delta z + v\Delta t}\left[V^{k-1/2}(R) - V^{k+1/2}(R-1)\right] \tag{16}$$

where $v = 1/\sqrt{L_0 C_0}$ is the phase velocity and R refers to the boundary section that uses the ABC. Inspired by the Fermi-Dirac distribution, the input voltage pulse is defined as

$$V(1) = V_p(1 - \frac{1}{e^{\frac{t-\mu}{a}} + 1}) \quad (17)$$

where a and μ are two fitting parameters and V_p is the magnitude of the input voltage. The two fitting parameters were selected to give a step pulse approximately with 0.5 ns rise time. The main parameters listed in ref. [1] were used in this 1D FDTD modelling in Matlab.

2.2 3D LLG and field simulation in COMSOL

The 3D simulation result using the finite element method in COMSOL Multiphysics was provided as a comparison and validation for the 1D FDTD model in MATLAB. In the 3D simulation, all the parameters such as line capacitance, line inductance, dimensions of each component, material-dependent parameters of the ferrite, etc. were assigned properly to represent the 1D FDTD simulation or vice versa.

It should be noted that two short linear regions were added at the two ends of the GNLTL for placing voltage probes since the lumped probes on the nonlinear region would disturb the fields [5]. The relative permittivity of the linear region was adjusted for impedance matching between the linear and nonlinear (ferrite-loaded) region to eliminate reflections. The diameter of the dielectric in the nonlinear/ferrite region was adjusted to obtain the same impedance as used in 1D simulation while keeping the inner and outer diameters of the ferrite the same.

Duo to the symmetry of the GNLTL, a 2D axi-symmetic model can be developed for reducing the simulation time [5]. This simulation uses the RF, AC/DC, and Mathematics modules in COMSOL Multiphysics. The main procedures for this modelling are given as follows, and more details can be found in ref. [5]. Eq. (1) the LL equation was decomposed down to three scalar ODEs in a cylindrical coordinate system and solved by using the Mathematics modules coupling with Maxwell's equations solved using the RF module (Electromagnetic Waves, Transient) for the transient field. The AC/DC module was used to generate a static external magnetic bias field in the axial direction. Two voltage lumped probes were placed at the two linear regions at the two ends to extract the input and output voltages as a function of time. A step function with smoothing was applied to generate an input voltage pulse at one end with 0.5 ns rise time. All the simulation parameters used in this model were listed in the last page of this report, while all the parameters used in the 1D FDTD model were listed in the Matlab code with full descriptions.

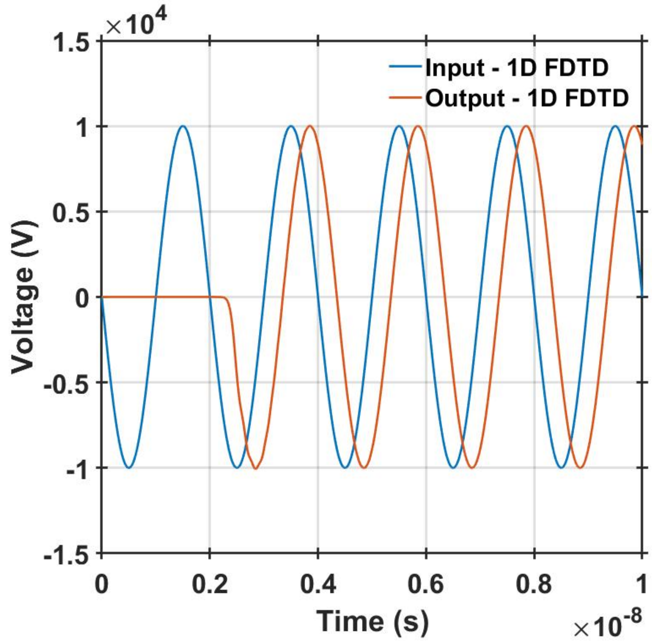

Figure 3: Input and output voltage pulses from a simulation of linear transmission line.

3 Results and Discussions

3.1 1D FDTD for a linear transmission line

To validate the 1D FDTD leap-frog method in Matlab, a linear transmission line was simulated first without considering the nonlinearity or the gyromagnetic precession. Here, Eq. (10) and Eq. (11) were numerically solved without the last term on the RHS of Eq. (11) which contains the $\frac{dM}{dt}$ term. A simple sinusoidal function was used for the input voltage. Figure 3 shows the input and output voltage as a function of time. The results validate the 1D FDTD leap-frog method and the applied ABC.

3.2 1D FDTD for GNLTL

With the nonlinearity term, Eq.(10)-(12) were solved for the GNLTL. Using the same simulation parameters from ref. [1], the oscillation at the output pulses should be expected if the FDTD method applied in Matlab was reliable when considering the nonlinearity. However, the oscillation occurs only under certain conditions suggesting it may be due to numerical error instead of the mechanism of the gyromagnetic precession or the nonlinearity of the ferrite.

Figure 4 shows the input and output voltage pulses from 1D FDTD for section number $n = 10$ ($\Delta z = 0.02m$, $\Delta t = 2.372 \times 10^{-12}s$) and the 3D results from COMSOL. The oscillation at the output pulse from 1D FDTD is similar to that from the 3D results using COMSOL. It seems to suggest that the 1D FDTD coupling with 1D LLG equation was solved numerically for modelling the RF or oscillation generation in a GNLTL. The comparison is similar to the results provided in ref.[1]. However, as the

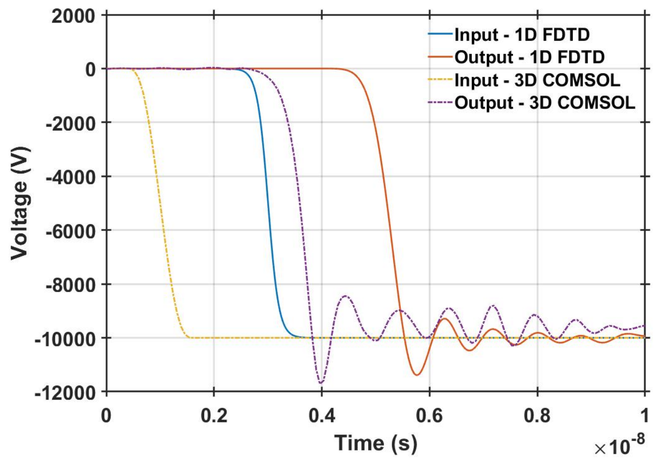

Figure 4: Input and output voltage pulses from a simulation of a GNLTL with $n = 10$, $\Delta z = 0.02m$, $\Delta t = 2.372 \times 10^{-12}s$.

number of section n changes, the oscillation behaves nonphysically or simply disappears as shown in Figure 5 and 6 for $n = 5$ and $n = 50$, respectively. The output pulse in Figure 5 with $n = 5$ is similar to the modelling result of "smaller n" from the previous work in ref. [1]. With $n = 5$, the rise time increases and the central frequency reduces. This should not happen since the central frequency of the oscillation should only depend on the material parameters, dimensions of each component, external bias magnetic field, and the transient field. An estimation of the central frequency is given by [6]

$$f_c \approx \frac{\gamma \mu_0 H_\theta}{4\pi} \sqrt{1 + \frac{\chi M_s}{\mu_0 \sqrt{H_\theta^2 + H_z^2}}} \tag{18}$$

where χ is the ferrite filling ratio, H_θ and H_z are the transient and bias magnetic fields, respectively. Though accurately determining the central frequency of these output pulses requires power spectrum analysis using fast Fourier transform, qualitative speaking, the central frequencies in both this work and in ref. [1] reduce as the simulation section is reduced. This suggests there might be numerical errors that have not been identified. Furthermore, the 3D result from COMSOL shows that the applied parameters should be physical and RF generation was obtained. The 3D models in COMSOL have been well benchmarked. This further suggests that the 1D FDTD coupling with the 1D LLG equation in both this work and in ref. [1] could be problematic due to numerical issues.

4 Conclusion

In this project, the 1D FDTD leap-frog method was applied to solve the telegrapher's equation coupling with 1D LLG equation for modelling the gyromagnetic nonlinear transmission line (GNLTL). The 1D FDTD method and an absorbing boundary condition were first validated by modelling a linear transmission line. A 3D GNLTL model applying the same parameters as used in the lumped element

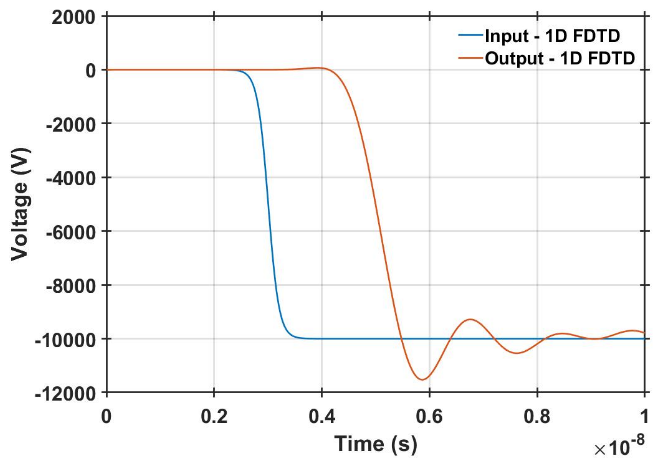

Figure 5: Input and output voltage pulses from a simulation of a GNLTL with $n = 5$, $\Delta z = 0.04m$, $\Delta t = 4.7439 \times 10^{-12}s$.

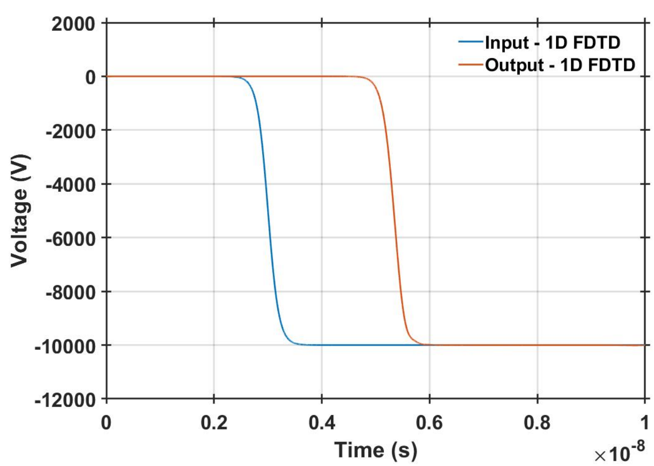

Figure 6: Input and output voltage pulses from a simulation of a GNLTL with $n = 50$, $\Delta z = 0.004m$, $\Delta t = 4.7439 \times 10^{-13}s$.

line was used as a comparison and validation for the 1D modelling. The oscillation at the output pulse only occurs under certain conditions. The output pulse's dependence on the simulation section is similar to the results reported in ref. [1] suggests numerical issues for the 1D method developed in Matlab.

A GNLTL would ideally be simulated in a 3D environment, or at least for the LLG equation to fully consider the movement of the magnetization under a transient voltage pulse and a bias field. In this work and previous work in ref. [1-3], the applied 1D LLG equation does not consider the effect of the external magnetic bias field. As shown in Eq.(4), the magnetic field is just dependent on the current from the transient field and the geometry of ferrite without considering the external magnetic bias field. This could be incomplete and inaccurate for RF modulation and the GNLTL simulation. Future work may include debugging the 1D FDTD method in Matlab and developing a 3D FDTD model using the 3D LLG equation for the simulation of GNLTL.

5 References

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Name	Expression	Value	Description
r_coax	0.6[mm]	6E-4 m	Inner radius of the ferrite
R_fout	1.75 [mm]	0.00175 m	Outer radius of the ferrite
R_coax	2.087[mm]	0.002087 m	Outer radius of the dielectrics
L_coax	200[mm]	0.2 m	Length of the nonlinear region
L_linear	50[mm]	0.05 m	Length of the linear region
Z_NLTL	59.95*((log(R_coax/R_fout)+mu_sat*log(R_fout/r_coax))*((1/eps_d)*log(R_coax/R_fout)+(1/eps_f)*log(R_fout/r_coax)))^0.5[ohm]	39.793 Ω	Impedance of the NLTL
eps_d	3	3	Permittivity of dielectric region
eps_f	15	15	Permittivity of ferrite
mu_sat	3	3	Saturation permeability of ferrite
Z_linear	59.95*log(R_coax/r_coax)/(eps_0/mu_0)^0.5[ohm^-1]	39.798 Ω	Impedance of the linear regions
V0	-10000[V]	-10000 V	Input pulse voltage
Ms	2.7852*10^5[A/m]	2.7852E5 A/m	Saturation magnetization
B_bias	25000[A/m]	25000 A/m	Biased magnetic field
alpha	0.1	0.1	Damping factor
gamma	2.2117*10^5[m/(A*s)]	2.2117E5 m/(s·A)	Gyromagnetic ratio
Cp	gamma/(1+alpha^2)	2.1898E5 m/(s·A)	Precession coefficient in the LL eq.
Cd	alpha*gamma/((1+alpha^2)*Ms)	0.078623 $m^2/(s\cdot A^2)$	Damping coefficient in the LL eq.
L_N	mu_v*(log(R_coax/R_fout)+mu_sat*log(R_fout/r_coax))/(2*pi)	6.7714E-7 H/m	Line inductance of the NLTL
C_N	2*pi*eps_v/(log(R_coax/R_fout)/eps_d+log(R_fout/r_coax)/eps_f)	4.2771E-10 F/m	Line capacitance of the NLTL
Z_N	(L_N/C_N)^0.5	39.789 Ω	Impedance of the NLTL (verified)
mu_v	1.256e-6[H/m]	1.256E-6 H/m	Vacuum peameability
eps_v	8.854e-12[F/m]	8.854E-12 F/m	Vacuum permittivity
eps_0	3.526	3.526	Permittivity of the linear regions
mu_0	1	1	Peameability of the linear regions
L_00	472*10^-9[H/m]	4.72E-7 H/m	Line inductance from ref. [1]
C_00	0.298*10^-9[F/m]	2.98E-10 F/m	Line capacitance from ref. [1]
Z_00	(L_00/C_00)^0.5	39.798 Ω	Impedance of the NLTL from ref. [1]

Figure 7: Simulation parameters used in COMSOL

(In 1D FDTD, the gyromagnetic ratio $\gamma = \frac{\mu_0 ge}{2m_e} = 1.76 \times 10^{11}[rads/s/T]$, while in COMSOL, $\gamma = \frac{ge}{2m_e} = 2.209 \times 10^5[\frac{m}{As}]$, where $g = 2$ for ferrite, e and m_e are the mass and charge of an electron)