Modelling Fresnel diffraction with Python 3

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The goal of the code was to approximate the general formula for Fresnel diffraction using simpson;s rule for numerical integration. This was done for both 1-D and 2-D. The code was a success as it was able to show the changes in the diffraction pattern as the separation between the aperture and the incident wave changed.

I. INTRODUCTION

Fresnel diffraction

The Fresnel diffraction equation is an approximation of the kirchoff-Fresnel diffraction equation that can determine the diffraction pattern of waves passing through an aperture or around a obstacle in the near field [**?**]. Near field describes the region where the size aperture or obstacle is at a comparable distance to the screen and far field describes where the distance between the aperture and the screen is much larger than the aperture size or vice versa. The specific whereabouts for the effects of far and near field diffraction can be calculated from fresnel's number:

$$F = \frac{a^2}{z\lambda} \tag{1}$$

where a is the size of the aperture, λ is the wavelength of light passing through and z is the distance from the aperture to the screen. If $F \ll 1$ then the diffraction can be described accurately using the fraunhofer equation or fresnel equation. If $F > 1$ then it can be described only through the fresnel equation.

Simpson's rule

Simpsons rule is a numerical method that can approximate the value of a definite integral by using parabolic functions. This is more accurate than the trapezium rule as less will be 'missing' in the approximation. For smooth functions, the regular simpsons rule function works well however for this experiment, the diffraction patterns will not be smooth and the conventional simpsons rule will not be able to approximate accurately. Therefore a more accurate way to find the approximation will be to use to composite simpson's rule.

$$\int_{x_1}^{x_2} f(x)dx = \frac{\Delta x}{3}\sum_{i=1}^{N/2}(f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})) \tag{2}$$

The Composite simpsons rule breaks down the integral into sub integrals which are then evaluated using simpson's rule.

II. METHOD

In both the one dimensional case and two dimensional case, this equation had to be approximated.

$$E(x,z) = \frac{kE_0}{2\pi z}\int_{a_1}^{a_2} e^{\frac{ik}{2z}(x-a)^2} da \tag{3}$$

Where E is the electric field of the diffracted light at coordinates x from a distance z. the Intensity of the pattern will be proportional to $|E^2|$. The code by first asking the user if they wanted to display one dimensional of two dimensional diffraction, then asking for the wavelength, aperture width and screen distance.

Simpson's rule was defined using a function which had 4 variables, the function to integrate, the bottom limit of intergation, the top limit of intergation and the amount of pints to integrate over. For the 1 dimensional integration, a set of x values is generated and these values are then used in a for loop to create an array of intensity values. For 2 dimensional integration, a zero matrix is created with 100x100 points. Then then a set of x and y values in a similar way to the one dimensional setup. Then the simpson's calculation was used earlier for each point in the 2d matrix.

III. ANALYSIS: 1-DIMENSIONAL

From what was outlined earlier, as the distance to the aperture decreases or the size of the aperture increases, the Fresnel number will increase and therefore a fresnel pattern will occur.

Varying z

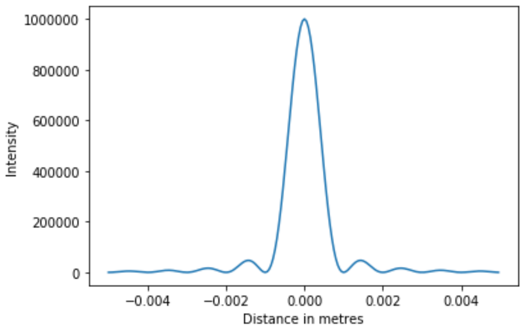
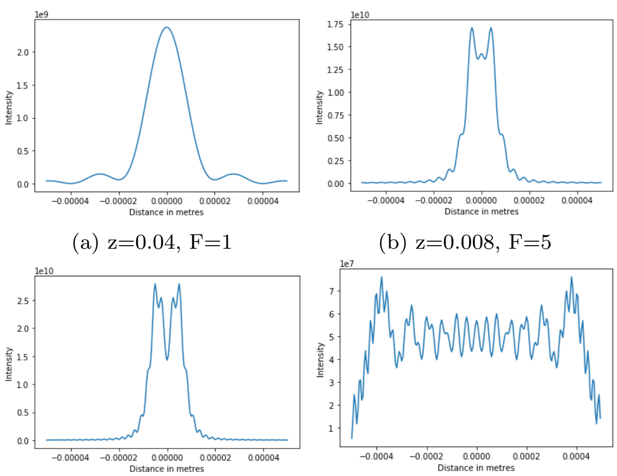


FIG. 1: An example of far field diffraction(z = 2cm, a = 20 micrometres and λ = 1000 nanometres

Initially with a large z compared to the aperture, the pattern resembled far field diffraction as expected (low fresnel number).

As the value of z was decreased to a value approaching a, the fresnel pattern begins to appear.



(a) z=0.04, F=1

(b) z=0.008, F=5

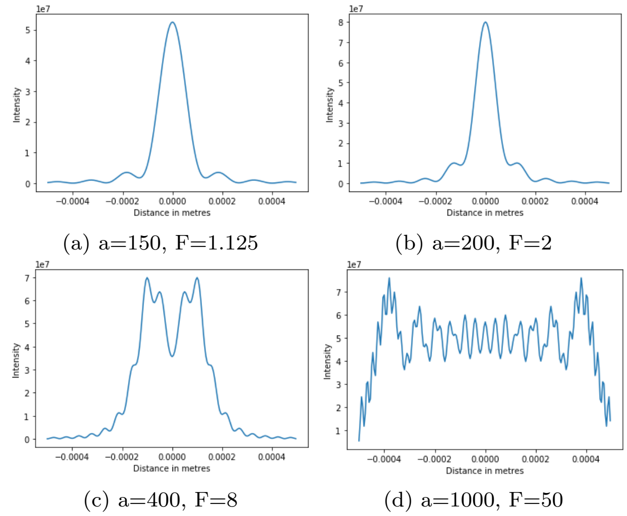
(c) z=0.005, F=8

(d) z=0.001, F=40

FIG. 2: Diagrams of the diffraction pattern as the z decreases. All z values are in cm

This shows that the fresnel number is inversely proportional to distance from the screen owhich is what we expected.

Varying a



(a) a=150, F=1.125

(b) a=200, F=2

(c) a=400, F=8

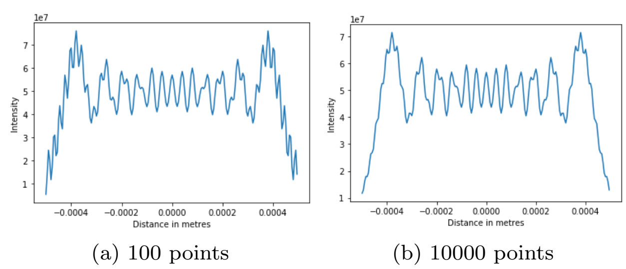
(d) a=1000, F=50

FIG. 3: Diagrams of the diffraction pattern as the value of the aperture width changes

As could be predicted from the fresnel equation, increasing the aperture increases the fresnel number, leading to fresnel patterns. Also because $F \propto a^2$, the increase in the aperture changes the value of fresnels number faster.

Varying N

By increasing N, the composite simpsons rule will be able to caluclate more sub intervals and therefore lead to a more accurate representation of the diffraction pattern. This is important for observing near field diffraction patterns as the patterns is less smooth and changes rapidly over the coordinates, so details can be easily missed or misrepresented.



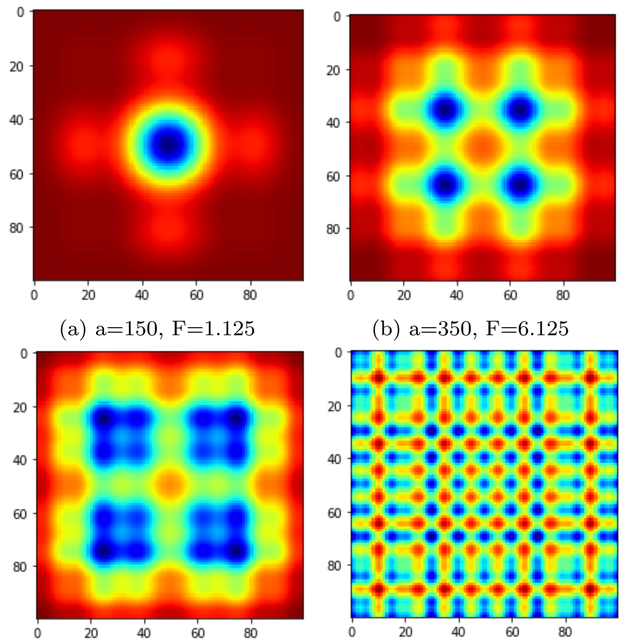
(a) 100 points

(b) 10000 points

FIG. 4: Two diagrams showing the diffraction pattern where F=50 but taking with different numbers of intervals

IV. ANALYSIS: 2-DIMENSIONAL

Th effects of changing z and a will be the same for 2 dimensions but the pattern will be symmetrical on the y and x axis.



(a) a=150, F=1.125

(b) a=350, F=6.125

(c) a=400, F=8

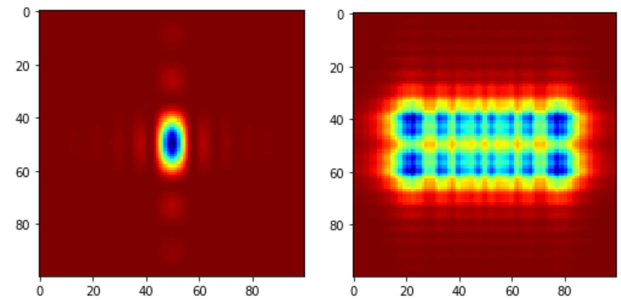
(d) a=1000, F=50

FIG. 5: Diagrams showing the 2d diffraction patterns as the aperture width increases. The blue region show high intensity and the red areas show low intensity

The 2d diagrams correspond to where the peaks are on the 1d diagrams.

Rectangular aperture

The rectangular aperture was created by increasing the integration size of the aperture for the y component.



(a) a=20

(b) a=400

Even though it the integration limits of y were changed, when the diffraction pattern was observed more closely at larger aperture widths the exact opposite occurred, with the pattern being stretched parallel to the x axis.

V. DISCUSSION

Overall the experiment was a success as fresnel patterns were observed where they were expected. The code could have been improved with a better user interface to allow the user to 'zoom' onto a diffraction pattern and allow variables to be changed without the need to close the program.

The composite simpsons rule used was better suited for this task than the traditional simpson's rule as the diffraction pattern at higher fresnel numbers becomes less smooth so it will be better analysed with the composite rule. However, the best approximation for the least amount of calculations would be the adaptive simpsons rule(2), which would make sub intervals in the areas where the function was less smooth.