Derivation of the Proactive Consistency Function (PCF)

Epistemological Modeling

October 27, 2025

Abstract

This document outlines the step-by-step analytical solution for the Proactive Consistency Function (PCF) differential equation, assuming a constant maintenance proactivity. The resulting function describes the exponential decay of system consistency over time, moderated by ongoing preventative effort.

1 Initial Differential Equation

The Proactive Consistency Function (PCF) models how system consistency, $C(t)$, changes over time (t). The rate of change is defined by the following first-order differential equation:

$$\frac{dC}{dt} = -k \cdot C(t) \cdot (1 - P(t))$$

Where:

- $C(t)$: The system Consistency at time t.
- $P(t)$: The Proactivity level of the maintenance effort.
- k: The inherent Decay Rate constant ($k > 0$).

2 Simplifying Assumption: Constant Proactivity

To find an explicit function $C(t)$, we assume a constant, time-invariant proactivity level, P_0, where $0 \leq P_0 \leq 1$.

$$P(t) = P_0$$

We define the **Residual Risk Factor,** R, as the proportion of risk remaining after proactivity is applied:

$$R = (1 - P_0)$$

The simplified differential equation becomes:

$$\frac{dC}{dt} = -k \cdot C(t) \cdot R$$

Let $A = kR$. Since $k > 0$ and $0 \leq R \leq 1$ (and typically $R > 0$), A is a positive constant representing the net effective decay rate.

$$\frac{dC}{dt} = -A \cdot C$$

3 Analytical Solution

This is a standard first-order linear homogeneous ordinary differential equation. We solve it using separation of variables:

3.1 Separation of Variables

$$\frac{1}{C} dC = -A \, dt$$

3.2 Integration

Integrating both sides yields:

$$\int \frac{1}{C} dC = \int -A \, dt$$

$$\ln |C| = -At + \text{const.}$$

3.3 Solving for $C(t)$

Exponentiating both sides:

$$C(t) = e^{-At+\text{const.}} = e^{\text{const.}} \cdot e^{-At}$$

Let $C_0 = e^{\text{const.}}$ be the initial consistency (at $t = 0$).

$$C(t) = C_0 e^{-At}$$

3.4 Final PCF Expression

Substituting the original terms back, $A = k(1 - P_0)$:

$$C(t) = C_0 \cdot e^{-k(1-P_0)t}$$

4 Conclusion

The derived function shows that system consistency follows an **exponential decay model**, where the decay is mitigated by the proactivity level P_0.