CSCI 567: Homework 6

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Problem 1. 1 (a)

$$J = \frac{1}{N}\sum_i (x_i - p_{i1}e_1 - p_{i2}e_2)^T(x_i - p_{i1}e_1 - p_{i2}e_2) \tag{1}$$

$$J = \frac{1}{N}\sum_i (x_i^T - (p_{i1}e_1)^T - (p_{i2}e_2)^T)(x_i - p_{i1}e_1 - p_{i2}e_2) \tag{2}$$

$$J = \frac{1}{N}\sum_i (x_i^T - p_{i1}^T e_1^T - p_{i2}^T e_2^T)(x_i - p_{i1}e_1 - p_{i2}e_2) \tag{3}$$

$$J = \frac{1}{N}\sum_i (x_i^T x_i - x_i^T p_{i1}e_1 - x_i^T p_{i2}e_2 - x_i p_{i1}^T e_1^T + p_{i1}^T p_{i1} e_1^T e_1 + p_{i1}^T p_{i2} e_1^T e_2 - x_i p_{i2}^T e_2^T + p_{i2}^T p_{i1} e_1 e_2^T + p_{i2}^T e_2^T p_{i2} e_2) \tag{4}$$

Differentiate wrt e_2

$$\frac{\partial J}{\partial e_2} = \frac{1}{N}\sum_i -x_i^T e_2 - x_i e_2^T + 2||p_{i2}||||e_2||^2 \tag{5}$$

Since, $||e_2||^2 = 1$

$$\frac{\partial J}{\partial e_2} = \frac{1}{N}\sum_i -2e_2^T x_i + 2||p_{i2}|| \tag{6}$$

Equating the equation equal to 0
For all i

$$p_{i2} = e_2^T x_i \tag{7}$$

Problem 1. 1 (b)

$$J = -e_2^T S e_2 + \lambda_2(e_2^T e_2 - 1) + \lambda_{12}(e_2^T S e_1 - 0) \tag{8}$$

$$J = -e_2^T S e_2 + \lambda_2(e_2^T e_2 - \lambda + \lambda_{12}(e_2^T S e_1) \tag{9}$$

$$\frac{\partial J}{\partial e_2} = -S(2 * e_2) + 2 * \lambda_2 * e_2 + \lambda_{12} e_1 \tag{10}$$

Equating equation to 0

$$S(2 * e_2) = 2 * \lambda_2 * e_2 + \lambda_{12} e_1 \tag{11}$$

Multiplying both sides with e_2^T

$$2Se_2^Te_2 = 2\lambda_2 e_2^Te_2 + \lambda_{12}e_2^Te_1 \tag{12}$$

Using $e_2^Te_1 = 0$

$$S||e_2||^2 = \lambda_2||e_2||^2 \tag{13}$$

Differentiate wrt e_2

$$2Se_2 = 2\lambda_2 e_2 \tag{14}$$

Hence,

$$Se_2 = \lambda_2 e_2 \tag{15}$$

Problem 1. 2 (a)

```
from numpy import linalg as LA
import numpy as np

w, v = LA.eigh(np.matrix([
                        [91.43, 171.92, 297.99],
                        [171.92, 373.92, 545.21],
                        [171.92, 297.99, 1297.26],
                       ]), UPLO='U')
print w, v
```

$\lambda_1 = 1626.52$
$\lambda_2 = 128.99$
$\lambda_3 = 7.10$
$\mathrm{u}_1 = [\ 0.21\ 0.41\ 0.88]^T(16)$

$\mathrm{u}_2 = [\ 0.25\ 0.85\ -0.46]^T(17)$

$\mathrm{u}_3 = [\ 0.94\ -0.31\ -0.08]^T(18)$

Problem 1. 2 (b)

If we compute percentages using formula

$$\frac{\lambda_i}{\lambda_1+\lambda_2+\lambda_3} for all i = 1, 2, 3 \tag{19}$$

Then we find $\lambda_1 = 92.28\%$
$\lambda_2 = 7.32\%$
$\lambda_3 = 0.40\%$
$since \mathrm{u}_3$ just explains 0.40% of variation of data hence it can be removed or is redundant.

Problem 1. 3 (c)

To interpret the information, use the absolute value of each vector. Now since, each value of u_1 is positive so we can see increasing any of the parameters will result in increase of size of bird and since 0.88 which corresponds to weight is the maximum value then we can see if we vary the weight than other parameters or size of the bird will vary more.
Now consider the last to values of u_2 which correspond to wingspan and weight which means more the wingspan lesser will be the weight.

Problem 2. (a)

Using forward algorithm
$\alpha_1(1) = \pi_1 * p_{1a} = 0.24$
$\alpha_1(2) = \pi_2 * p_{2a} = 0.08$
$\alpha_2(1) = b_{1c}[a_{11}\alpha_1(1) + a_{21}\alpha_1(2)]$=0.04
$\alpha_2(2) = b_{2c}[a_{12}\alpha_1(1) + a_{22}\alpha_1(2)]$=0.024
$\alpha_3(1) = b_{1c}[a_{11}\alpha_2(1) + a_{21}\alpha_2(2)]$=0.00752
$\alpha_3(2) = b_{2c}[a_{12}\alpha_2(1) + a_{22}\alpha_2(2)]$=0.01056
$\alpha_4(1) = b_{1g}[a_{11}\alpha_3(1) + a_{21}\alpha_3(2)]$=0.0028464
$\alpha_4(2) = b_{2g}[a_{12}\alpha_3(1) + a_{22}\alpha_3(2)]$=0.0008592
$\alpha_5(1) = b_{1t}[a_{11}\alpha_4(1) + a_{21}\alpha_4(2)]$=0.000233616
$\alpha_5(2) = b_{2t}[a_{12}\alpha_4(1) + a_{22}\alpha_4(2)]$=0.000410832
$\alpha_6(1) = b_{1a}[a_{11}\alpha_5(1) + a_{21}\alpha_5(2)]$=0.000131146
$\alpha_6(2) = b_{2a}[a_{12}\alpha_5(1) + a_{22}\alpha_5(2)]$=$6.33168E - 05$

P(O|θ) = $\alpha_6(1)$ + $\alpha_6(2)$ = 0.000194462

Problem 2. (b)

Using backward algorithm
$\beta_6(1) = 1$
$\beta_6(2) = 1$
$\beta_5(1) = \beta_6(1)a_{11}b_{1a} + \beta_6(2)a_{12}b_{2a}$=0.34
$\beta_5(2) = \beta_6(1)a_{21}b_{1a} + \beta_6(2)a_{22}b_{2a}$=0.28
$\beta_4(1) = \beta_5(1)a_{11}b_{1t} + \beta_5(2)a_{12}b_{2t}$=0.049
$\beta_4(2) = \beta_5(1)a_{21}b_{1t} + \beta_5(2)a_{22}b_{2t}$=0.064

$$P(X_6 = S_1|O,\theta) = \frac{\alpha_6(S_1)\beta_6(S_1)}{\alpha_6(S_1)\beta_6(S_1) + \alpha_6(S_2)\beta_6(S_2)} = 0.67440081 \tag{20}$$

Problem 2. (c)

$$P(X_4 = S_1|O,\theta) = \frac{\alpha_4(S_1)\beta_4(S_1)}{\alpha_4(S_1)\beta_4(S_1) + \alpha_4(S_2)\beta_4(S_2)} = 0.717226569 \tag{21}$$

Problem 2. (e)

$$p(O_7 = k|\theta) = (P(X_6 = S_1|\theta) \times a_{11} + P(X_6 = S_2|\theta) \times a_{21}) * b_{1k} + (P(X_6 = S_1|\theta) \times a_{12} + P(X_6 = S_2|\theta) \times a_{22}) * b_{2k} \tag{22}$$

$$p(O_7 = k|\theta) = 0.570330232 * b_{1k} + 0.34969474 b_{2k} \tag{23}$$

$$p(O_7 = A|\theta) = 0.570 * 0.4 + 0.349 * 0.2 = 0.298071041 \tag{24}$$

$$p(O_7 = C|\theta) = 0.570 * 0.2 + 0.349 * 0.4 = 0.2536 \tag{25}$$

$$p(O_7 = G|\theta) = 0.570 * 0.3 + 0.349 * 0.1 = 0.2059 \tag{26}$$

$$p(O_7 = T|\theta) = 0.570 * 0.1 + 0.349 * 0.3 = 0.1617 \tag{27}$$