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Chapitre 3. Effective Stress¹ and Pore Water Pressure²

3.1 Stresses in a Saturated Soil

To have a good understanding of what are the stresses in a soil, imagine you are standing in your favourite swimming pool which, at first, is empty (Fig. 1a). Try to memorize the level of the stresses (pressure) you feel under your feet. Now, stay there in the bottom of the swimming pool and ask someone to start filling it. As the water level rises, you will feel the pressure under your feet progressively decreasing, until you are completely immersed (Fig. 1b).

Now that you are completely immersed, the level of the water may rise higher and higher, without affecting the pressure under your feet.

Archimede had the same feeling as yours more than 2000 years ago!



Fig.1a

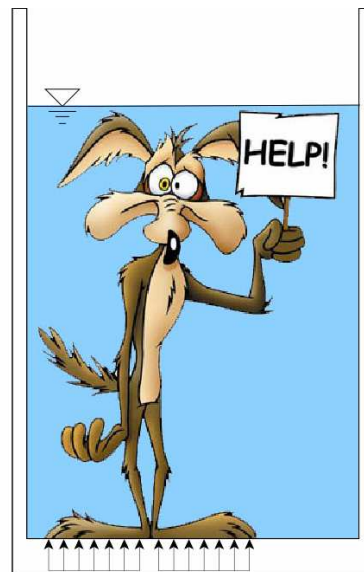


Fig. 1b

The same phenomenon occurs with soil particles. The contact stresses between the soil particles decreases in presence of water, because of the buoyancy³.

¹ Contrainte effective

² Pression interstitielle

³ Poussée d'Archimède

Moreover, it has been found, by experiments, that settlement in soil is only depending upon the stress between the particles and not upon the fluid pressure around them.

In examining the reasons for this observed behaviour it is helpful to use the following quantities:

$$\sigma_v = \text{Total Vertical Stress} = \frac{\text{Total Vertical Force}}{\text{Cross Sectional Area}} \quad (1)$$

and to define an additional quantity : the vertical **effective stress**, by the relation

$$\sigma'_v = \sigma_v - u_w \quad (2)$$

Where u_w is called the **pore water pressure**.

Another situation in which effective stresses are important is the case of two rough blocks sliding over one another, with water pressure in between them as shown in Fig 2.

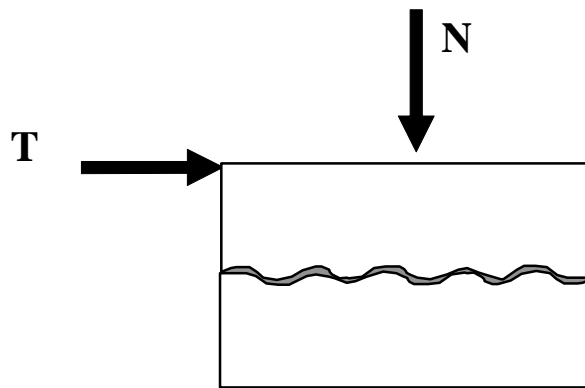


Fig 2 Two pieces of Rock in contact

The effective normal thrust⁴ transmitted through the points of contact will be

$$N' = N - U \quad (3a)$$

where U is the force provided by the water pressure

The frictional force will then be given by $T = \mu N'$ where μ is the coefficient of friction. For soils and rocks the actual contact area is very small compared to the cross-sectional area so that U/A is approximately equal to u_w the pore water pressure. Hence dividing through by the cross sectional area A this becomes:

$$\tau = \mu \sigma'_v \quad (3b)$$

where τ is the average shear stress and σ'_v is the vertical effective stress.

Of course it is not possible to draw a general conclusion from a few simple experiments, but there is now a large body of experimental evidence to suggest that **both deformation and strength of**

⁴ poussée

soils depend upon the effective stress. This was originally suggested by Terzaghi in the 1920's, and equation 2 and similar relations are referred to as the Principle of Effective Stress.

3.2 Calculation of Effective Stress

It is clear from the definition of effective stress that in order to calculate its value it is necessary to know both the total stress and the pore water pressure.

3.2.1 Calculation of Vertical (Total) Stress

Consider the horizontally "layered" soil deposit shown schematically in Fig.3,

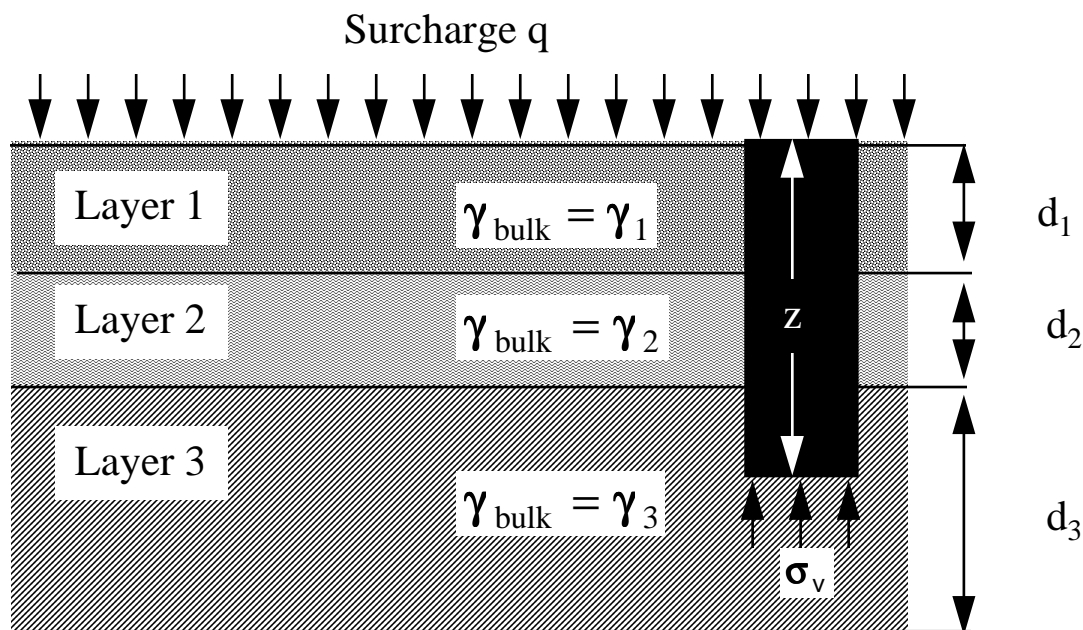


Fig 3 Soil Profile

If we consider the equilibrium of a column of soil of cross sectional area A it is found that

$$\begin{aligned}
 \text{Force on base} &= \text{Force on Top} + \text{Weight of Soil} \\
 A\sigma_v &= Aq + A\gamma_1 d_1 + A\gamma_2 d_2 + A\gamma_3 (z - d_1 - d_2) \\
 \sigma_v &= q + \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3 (z - d_1 - d_2)
 \end{aligned} \tag{4}$$

3.2.2 Calculation of Pore Water Pressure

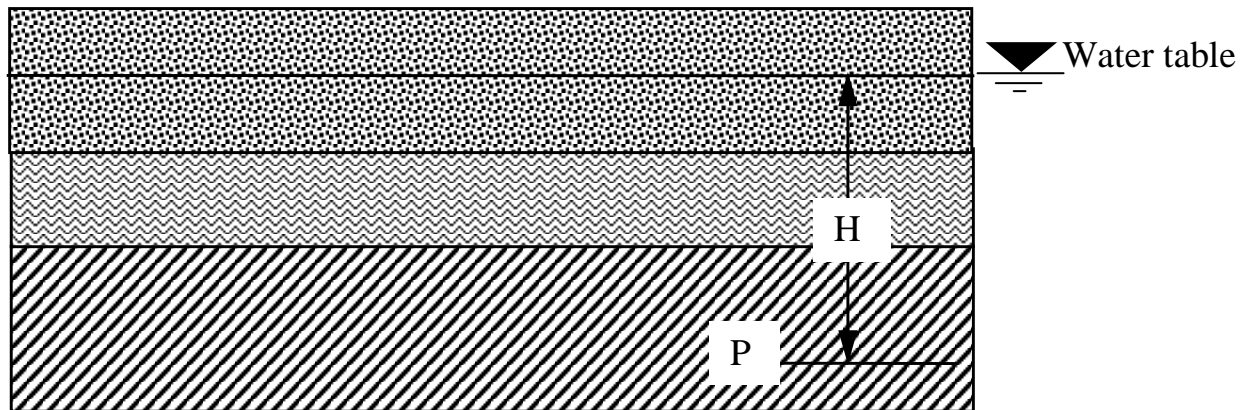


Fig 4 Soil with a static water table

Suppose the soil deposit shown in Fig. 4 has a static **water table**⁵ as indicated. The water table is the water level in a borehole⁶, and at the water table $u_w = 0$. The water pressure at a point P is given by

$$u_w(P) = \gamma_w H \quad (5)$$

3.2.2.a) Example

A uniform layer of sand 10 m deep overlays bedrock. The water table is located 2 m below the surface of the sand which is found to have a voids ratio $e = 0.7$. Assuming that the soil particles have a specific gravity $G_s = 2.7$ calculate the effective stress at a depth 5 m below the surface.

Step one: Draw ground profile showing soil stratigraphy and water table

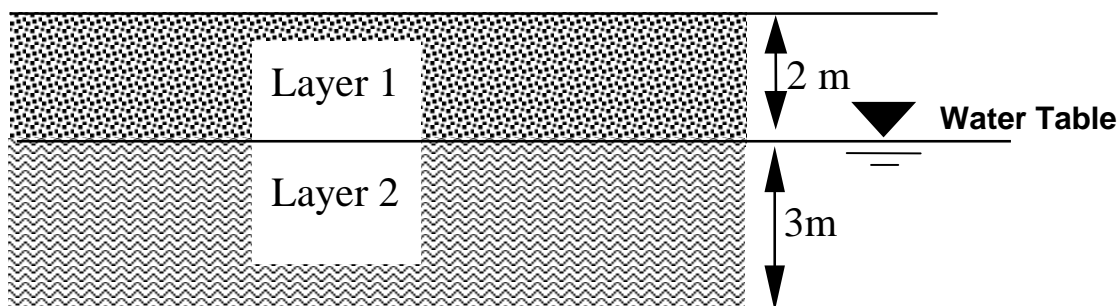


Fig 5 Soil Stratigraphy

⁵ Nappe phréatique

⁶ Trou de forage

Step two: Calculation of Dry and Saturated Unit Weights. (Assume a volume of solid of 1m³)

	Distribution by Volume (m ³)	Distribution by Weight for the dry soil (kN)	Distribution by Weight for the saturated soil (kN)
Voids	$V_v = e \cdot V_s = e = 0.7$	0	$W_v = V_v \cdot \gamma_w = 7$
Solid	1	$W_s = G_s \cdot V_s \cdot \gamma_w = 27$	27
TOTAL	1.7	27	34

Fig 6 Calculation of dry and saturated unit weight

$$\gamma_{dry} = \frac{27 \text{ kN}}{1.70 \text{ m}^3} = 15.88 \text{ kN} / \text{m}^3 \quad (6)$$

$$\gamma_{sat} = \frac{34 \text{ kN}}{1.70 \text{ m}^3} = 20 \text{ kN} / \text{m}^3$$

Step three: Calculation of (Total) Vertical Stress

$$\sigma_v = 15.88 \times 2 + 20 \times 3 = 91.76 \text{ kN} / \text{m}^2 \quad (7)$$

Step four: Calculation of Pore Water Pressure

$$u_w = 3 \times 10 = 30 \text{ kN} / \text{m}^2 \quad (8)$$

Step five: Calculation of Effective Vertical Stress

$$\sigma'_v = \sigma_v - u_w = 91.76 - 30 = 61.76 \text{ kN} / \text{m}^2 \quad (9)$$

3.2.2.b) Example – Effects of groundwater⁷ level changes

Initially a 50 m thick deposit of a clayey soil has a groundwater level 1 m below the surface. Due to groundwater extraction the regional groundwater level is lowered by 2 m. By considering the changes in effective stress at a depth, z , in the clay investigate what will happen to the ground surface.

Due to decreasing demands for water the groundwater rises (possible reasons include de-industrialisation and climate changes due to the **greenhouse effect**⁸) back to the initial level. What will happen now to the ground surface?

⁷ nappe souterraine

⁸ effet de serre

Assume

γ_{bulk} is constant with depth

γ_{bulk} is the same above and below the water table (clays may remain saturated for many metres above the groundwater table due to capillary suctions)

The vertical total and effective stresses at depth z are given in the Table below.

	Initial GWL	Lowered GWL
σ_v	$\gamma_{bulk} \times z$	$\gamma_{bulk} \times z$
u	$\gamma_w \times (z - 1)$	$\gamma_w \times (z - 3)$
σ_v'	$z \times (\gamma_{bulk} - \gamma_w) + \gamma_w$	$z \times (\gamma_{bulk} - \gamma_w) + 3 \times \gamma_w$

At all depths the effective stress increases and as a result the soil compresses. The cumulative effect throughout the clay layer can produce a significant settlement of the soil surface.

When the groundwater rises the effective stress will return to its initial value, and the soil will swell⁹ and the ground surface heave¹⁰ (up). However, due to the inelastic nature of soil, the ground surface will not, in general, return to its initial position.

3.3 Exercises

1. Calculate the total vertical stress, the pore water pressure and the effective vertical stress at a depth of 5 m in a soil where the water table is located :

- (1) at a height h above the ground level;
- (2) at the ground level ;
- (3) at a depth of 2.5m.

Assume a constant $\gamma_{bulk} = 18 \text{ kN/m}^3$ in the soil whatever the water table level could be

2. A stratum of sand 2.5 m thick overlies a stratum of saturated clay 3 m thick. The water table is 1 m below the surface. For the sand, $G_s = 2.65$, $e = 0.50$ and for the clay $G_s = 2.72$, $e = 1.1$. Calculate the total and effective vertical stresses at depths of 1 m, 2.5 m and 5.5 m below the surface assuming that the sand above the water table is completely dry.

3. The same soil stratum¹¹ as in the previous question lies 2 m below the surface of a lake. Taking the properties to be the same as before, calculate the total and effective vertical stresses at depths of 1 m, 2.5 m and 5.5 m below the soil surface.

4. In a deep deposit of clay the water table lies 3 m below the soil surface. Calculate the effective stresses at depths of 1 m, 3 m and 5 m below the surface. Assume that the soil remains fully saturated above the water table, and that $\gamma_{sat} = 16.5 \text{ kN/m}^3$.

⁹ gonfler

¹⁰ se soulever

¹¹ couche