# Anisotropic Poroelasticity in Cracked and Granular Materials

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## Anisotropic Poroelasticity in Cracked and Granular Materials

- Undrained deformation, the Skempton coefficient and its use in the analysis of earthquakes
- Laboratory measurement of anisotropic poroelastic coefficients in crustal rocks
- Effects of anisotropic cracking on the Skempton tensor

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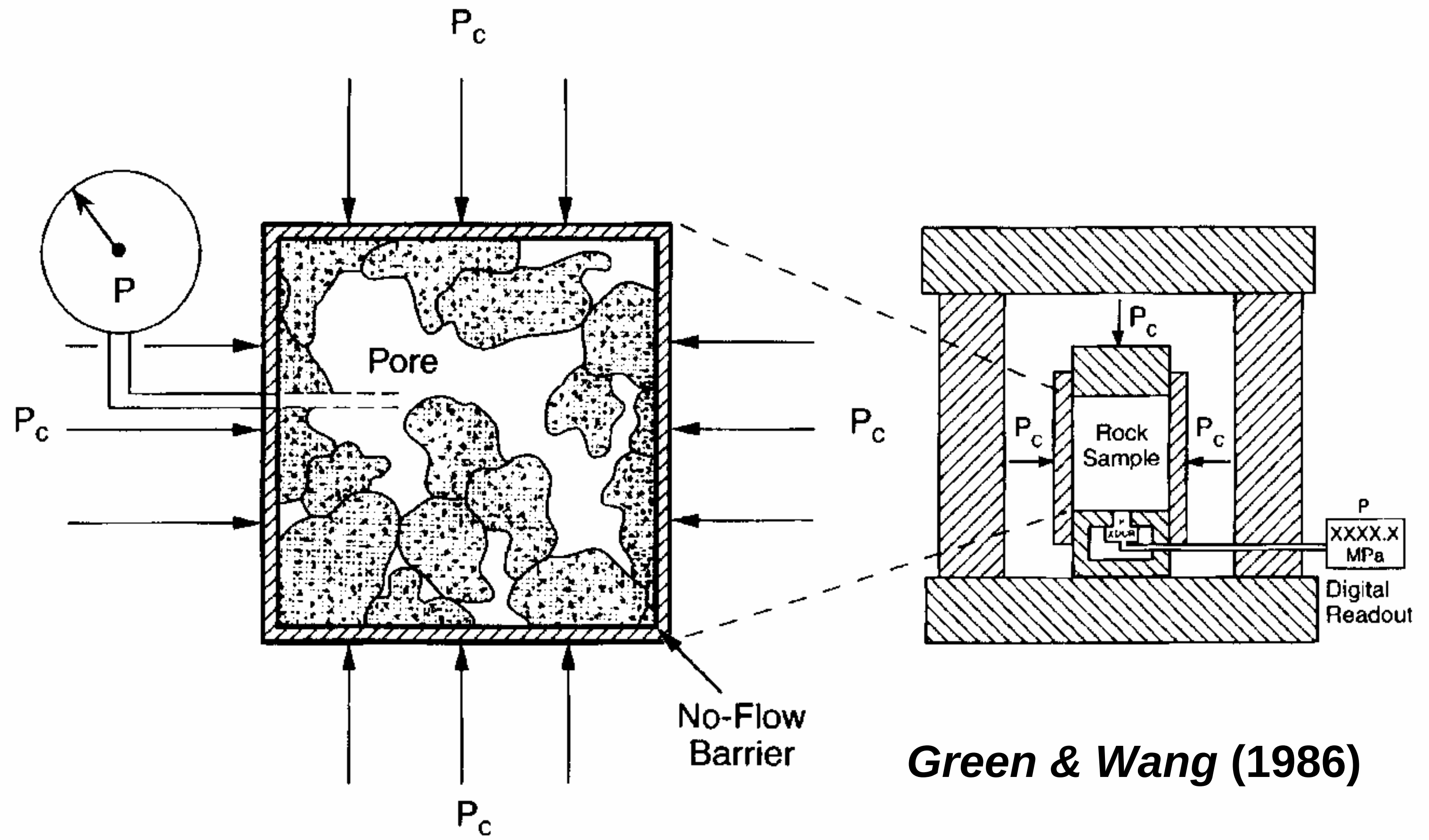



***Green & Wang* (1986)**

Undrained Condition → $\Delta m = 0$

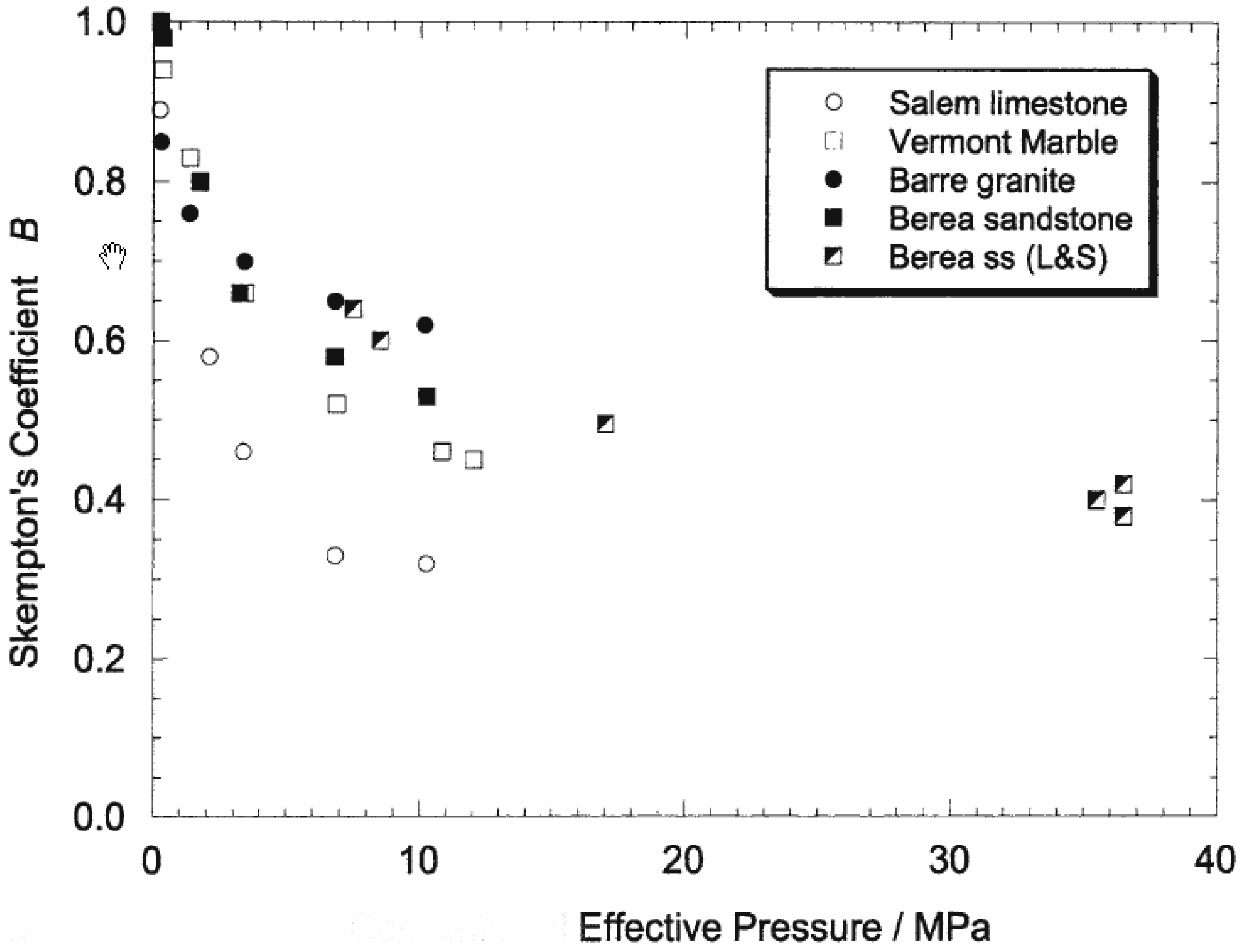
Skempton's Coefficient: $B = \left. \dfrac{dp}{dP_c} \right|_{\Delta m = 0}$

**TABLE C.1.**
Example Poroelastic Moduli for Different Rock Types*

| Rock | $G$(GPa) | $\nu$ | $K$(GPa) | $K_s'$(GPa) | $\alpha$ | $\nu_u$ | $K_u$(GPa) | $B$ | $c$(m$^2$/s) | $\phi$ | $k$(mD) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Berea sandstone | 6 | 0.20 | 8.0 | 36 | 0.79 | 0.33 | 16.0 | 0.62 | $1.6 \times 10^0$ | 0.19 | $1.9 \times 10^2$ |
| Boise sandstone | 4.2 | 0.15 | 4.6 | 42 | 0.85 | 0.31 | 8.3 | 0.50 | $4.0 \times 10^{-1}$ | 0.26 | $8.0 \times 10^2$ |
| Ohio sandstone | 6.8 | 0.18 | 8.4 | 31 | 0.74 | 0.28 | 13.0 | 0.50 | $3.9 \times 10^{-2}$ | 0.19 | $5.6 \times 10^0$ |
| Pecos sandstone | 5.9 | 0.16 | 6.7 | 39 | 0.83 | 0.31 | 14.0 | 0.61 | $5.4 \times 10^{-3}$ | 0.20 | $8.0 \times 10^{-1}$ |
| Ruhr sandstone | 13 | 0.12 | 13 | 36 | 0.65 | 0.31 | 30.0 | 0.88 | $5.3 \times 10^{-3}$ | 0.02 | $2.0 \times 10^{-1}$ |
| Weber sandstone | 12 | 0.15 | 13 | 36 | 0.64 | 0.29 | 25.0 | 0.73 | $2.1 \times 10^{-2}$ | 0.06 | $1.0 \times 10^0$ |
| Tennessee marble | 24 | 0.25 | 40 | 50 | 0.19 | 0.27 | 44.0 | 0.51 | $1.3 \times 10^{-5}$ | 0.02 | $1.0 \times 10^{-4}$ |
| Charcoal granite | 19 | 0.27 | 35 | 45 | 0.27 | 0.30 | 41.0 | 0.55 | $7.0 \times 10^{-6}$ | 0.02 | $1.0 \times 10^{-4}$ |
| Westerly granite | 15 | 0.25 | 25 | 45 | 0.47 | 0.34 | 42.0 | 0.85 | $2.2 \times 10^{-5}$ | 0.01 | $4.0 \times 10^{-4}$ |
| Clay | — | — | 0.062 | $\infty$ | 1 | — | 6.2 | 0.99 | — | — | — |
| Mudstone | — | — | 2.13 | 42 | 0.95 | — | 10.1 | 0.83 | — | — | — |
| Kayenta sandstone | — | — | 9.1 | 37.9 | 0.76 | — | 18.5 | 0.67 | — | — | — |
| Limestone | — | — | 33.3 | 107 | 0.69 | — | 40.2 | 0.25 | — | — | — |
| Hanford basalt | — | — | 46.7 | 59 | 0.23 | — | 45.4 | 0.12 | — | — | — |
| Berea sandstone | 5.6 | 0.17 | 6.6 | 28.9 | 0.77 | 0.34 | 15.8 | 0.75 | $1.5 \times 10^0$ | 0.19 | $1.9 \times 10^2$ |
| Indiana limestone | 12.1 | 0.26 | 21.2 | 72.6 | 0.71 | 0.33 | 31.2 | 0.46 | — | 0.13 | — |

* The top section of the table is a combination of measured and calculated values compiled by Detournay and Cheng (1993), who incorporated values previously presented by Rice and Cleary (1976). Typically, drained moduli are measured values, and the solid-grain modulus is taken from handbooks of mineral properties. The undrained moduli are computed assuming that the solid-grain modulus $K_s$ is equal to both the unjacketed bulk modulus $K_s'$ and the unjacketed pore incompressibility $K_\phi$. The fluid-bulk modulus was assumed to be $K_f = 3.3$ GPa. The second section of the table compiled by Palciauskas and Domenico (1989) and reprinted in Domenico and Schwartz (1998, p. 171) also combined measurement and calculation. The bottom set of values for Berea sandstone and Indiana limestone is a best-fit set of constants based on eight different measurements by Hart and Wang (1995) in which water ($K_f = 2.3$ GPa) was the pore fluid.

***Wang* (2000)**




*Paterson & Wong* (2005)

Constitutive Relations for an Isotropic Poroelastic Material
(*Biot*, 1941; *Rice & Cleary*, 1976)

$$2G\varepsilon_{ij} = \sigma_{ij} - \frac{v}{1+v}\sigma_{kk}\delta_{ij} + \frac{3(v_u - v)}{B(1+v)(1+v_u)}p\delta_{ij}$$

$$m = m_0 + \frac{3\rho_0(v_u - v)}{2GB(1+v)(1+v_u)}\left(\sigma_{kk} + \frac{3}{B}\,p\right)$$

Undrained Condition ($\Delta m$ = 0 or $m$=$m_o$) →

$$\Delta p \;=\; -B\Delta\sigma_{kk}/3$$

$$2G\Delta\varepsilon_{ij} = \Delta\sigma_{ij} - \frac{v_u}{1+v_u}\Delta\sigma_{kk}\delta_{ij}$$

(Tensile stress is considered to be positive here)

- In an isotropic poroelastic material, the pore pressure induced by undrained deformation is proportional to the mean stress and solely characterized by the scalar Skempton coefficient:

$$\Delta p = B \frac{\Delta \sigma_{kk}}{3} = B \Delta \sigma_{m}$$

- The induced pore pressure is independent of the deviatoric stresses in an isotropic poroelastic material.

(Compressive stress is considered to be positive from this point on)

# **Coulomb Stress Analysis of Earthquake Interaction:** Poroelastic effects due to undrained pore pressure excess (*Beeler et al*., 2000)

$$CFF = \tau - \mu(\sigma_n - p)$$

$$\Delta CFF = \Delta\tau - \mu(\Delta\sigma_n - \Delta p)$$

The deformation is undrained, so the induced pore pressure is characterized by the Skempton coefficient

$$\Delta CFF = \Delta\tau - \mu(\Delta\sigma_n - B\Delta\sigma_m)$$

$$= \Delta\tau - \mu\ (1 - B\frac{\Delta\sigma_m}{\Delta\sigma_n})\,\Delta\sigma_n = \Delta\tau - \mu'\,\Delta\sigma_n$$

with $\mu' = \mu\ (1 - B\frac{\Delta\sigma_m}{\Delta\sigma_n})$

The mean stress distribution in the vicinity of a dislocation source is highly heterogeneous, and yet in simulations it is often assumed that μ’ is constant: $\mu' \approx \mu\ (1 - B)$ ?

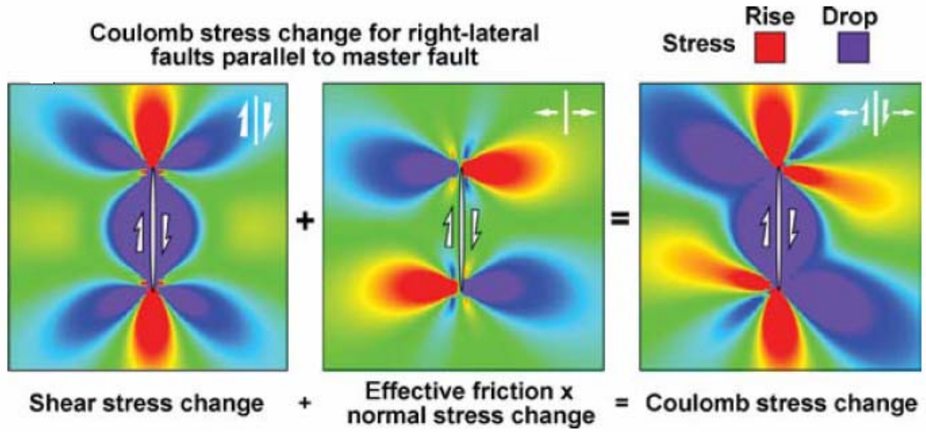
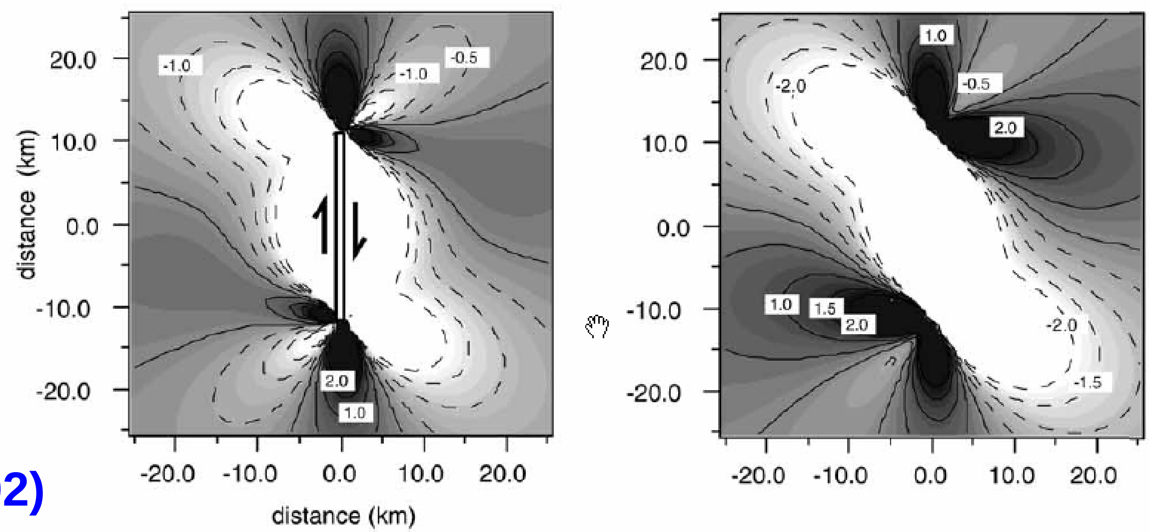



**Figure 2** Illustration of a Coulomb stress change calculation (Equation 4). The panels show a map view of a vertical strike-slip fault embedded in an elastic half-space, with imposed slip that tapers toward the fault ends. Stress changes are depicted by graded colors; green represents no change in stress. (From King et al. 1994.)

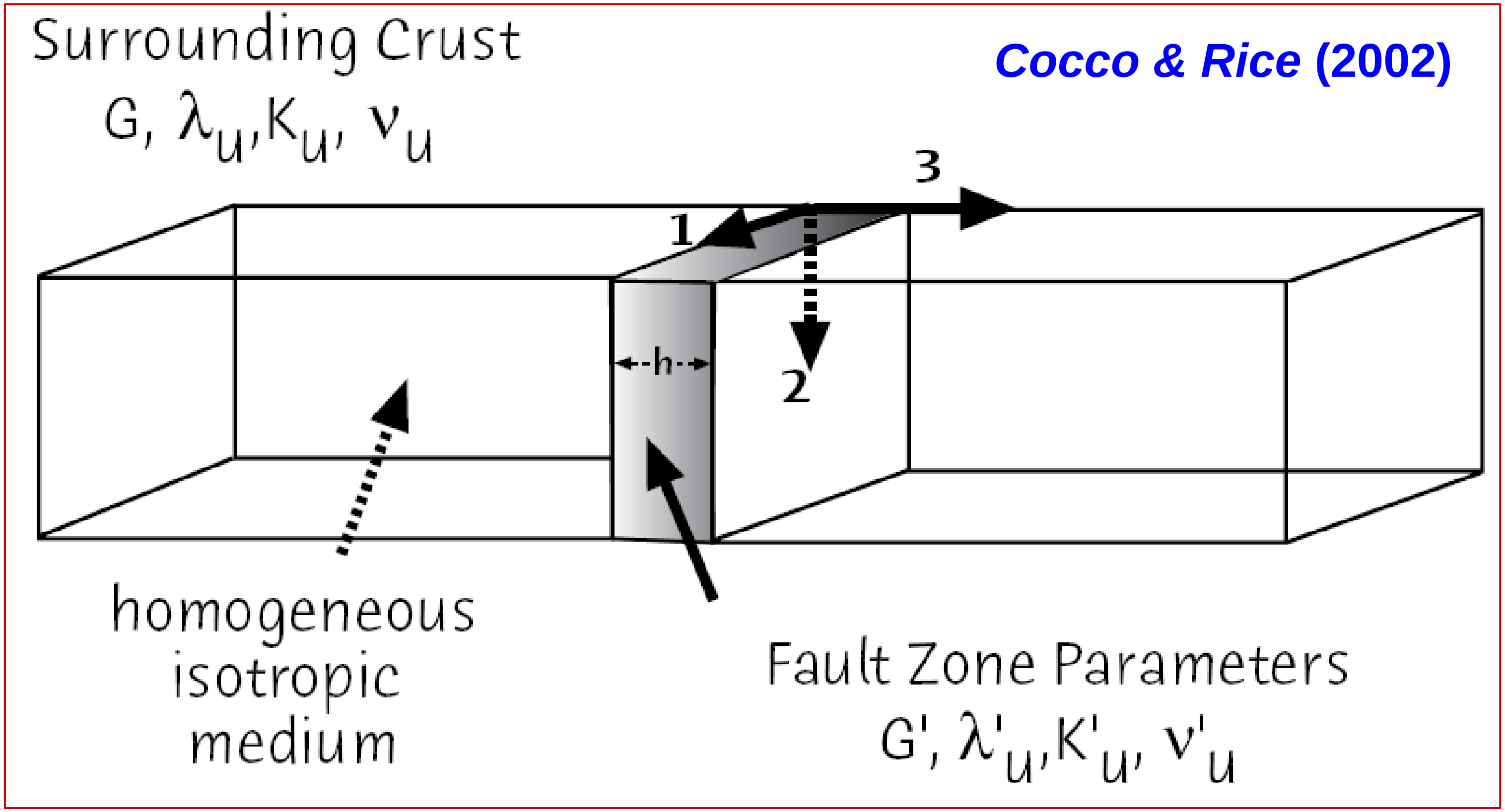
constant apparent friction
$\Delta CFF = \Delta\tau + \mu' \Delta\sigma_n$

$$\Delta CFF = \Delta\tau - \mu(\Delta\sigma_n - B\Delta\sigma_m)$$

$\mu' = 0.4$; $\mu = 0.75$; $B = 0.47$




***Cocco & Rice* (2002)**




The fault zone and country rock have different poroelastic properties. Mechanical equilibrium and strain continuity → pore pressure excess induced under undrained condition is given by

$$\Delta p = B' \frac{(1+\nu'_u)}{(1-\nu'_u)} \left[ \frac{G'}{G} \frac{(1-\nu_u)}{(1+\nu_u)} \frac{\Delta\sigma_{kk}}{3} + \frac{(G-G')}{G} \Delta\sigma_{33} \right]$$

Pore pressure excess that develops under undrained condition in a fault zone with poroelastic properties different from the surrounding.

$$\Delta p = B' \frac{(1+\nu_u')}{(1-\nu_u')} \left[ \frac{G'}{G} \frac{(1-\nu_u)}{(1+\nu_u)} \frac{\Delta\sigma_{kk}}{3} + \frac{(G-G')}{G} \Delta\sigma_{33} \right]$$

Three limiting cases:

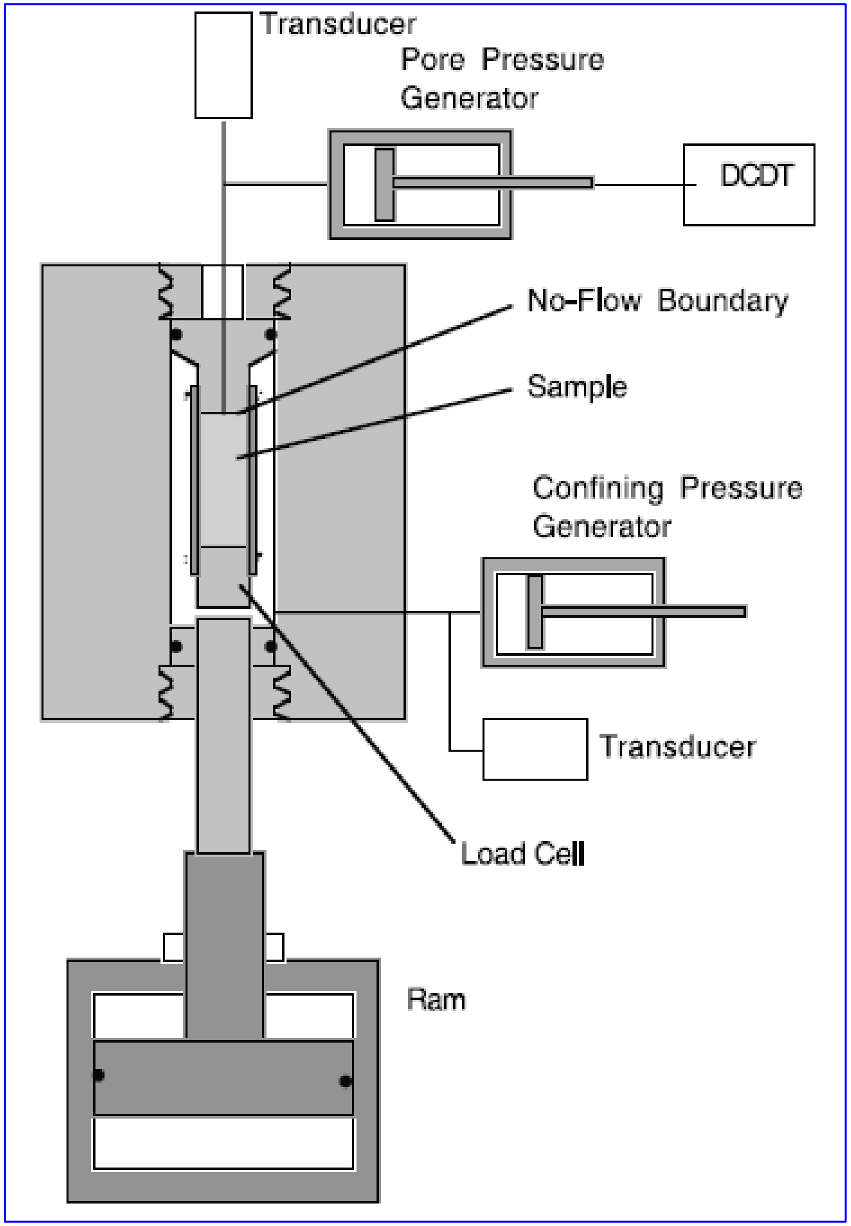
(1) $G' = G$

$$\Delta p = B' \frac{(1+\nu_u')}{(1-\nu_u')} \frac{(1-\nu_u)}{(1+\nu_u)} \frac{\Delta\sigma_{kk}}{3}$$

(2) $G \gg G'$

$$\Delta p = B' \frac{(1+\nu_u')}{(1-\nu_u')} \Delta\sigma_{33}$$

(3) The undrained response is anisotropic, characterized by a Skempton tensor $B_{ij}$. If the tensor component $B_{33}$ is significantly larger than the others, then the pore pressure change is dominated by $(B_{33}/3)\,\Delta\sigma_{33}$ .




The Skempton tensor is defined by:

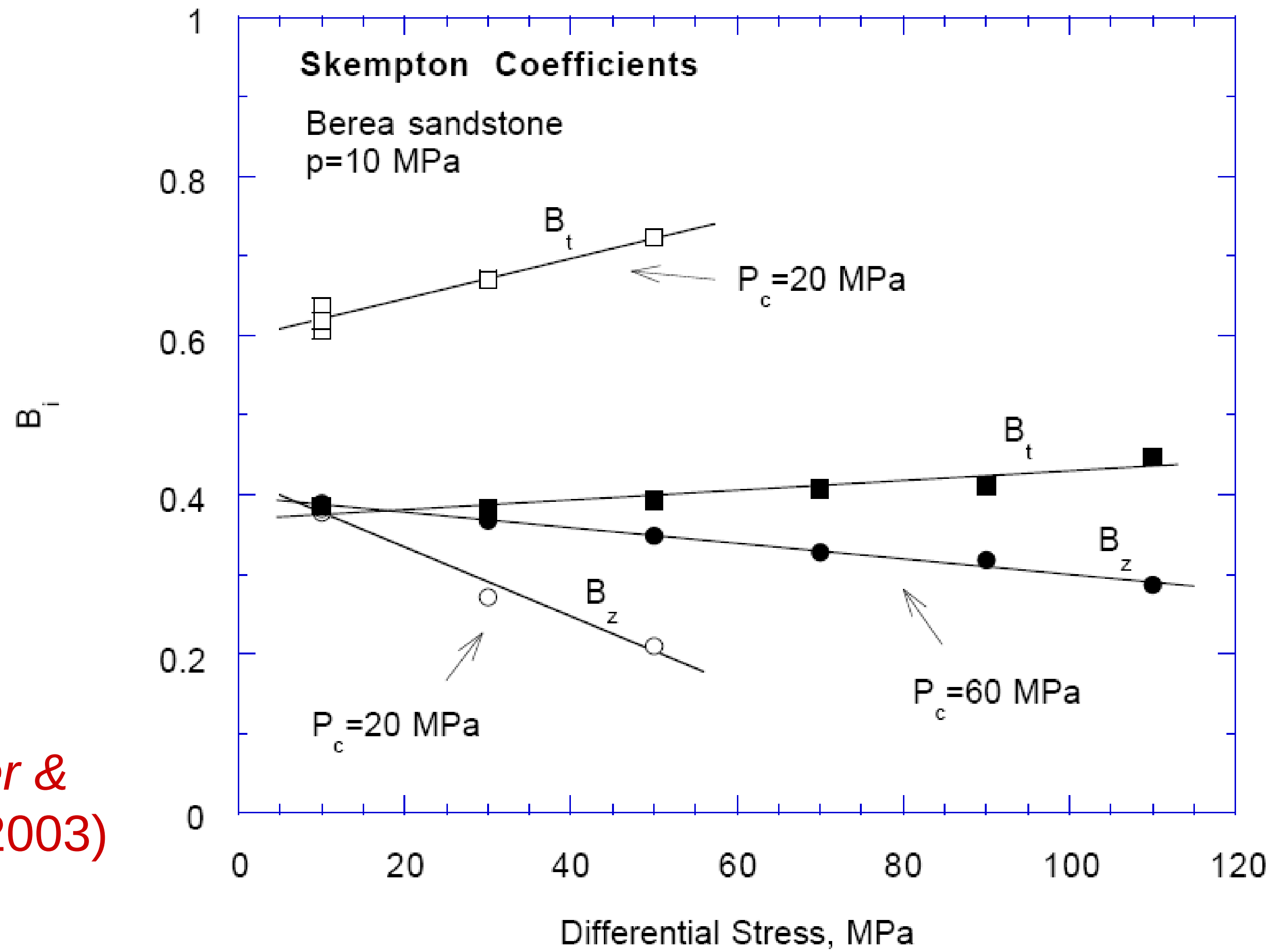
$$\Delta p = B_{ij} \frac{\Delta\sigma_{ij}}{3}\bigg|_{\Delta m=0}$$

❖ Measurement of pore pressure change induced by perturbations in the principal stresses.

❖ Storage in the pore pressure system changes in response to pore pressure, which has to be accounted for by a servo-controlled pore pressure generator.

❖ Systematic measurement of the Skempton tensor in triaxially compressed samples of Berea sandstone, Navajo sandstone and Ottawa sand.

*Lockner and Stanchits* (2002)




*Lockner & Beeler* (2003)

❖ The poroelastic anisotropy is such that the undrained response to the transverse stress is more pronounced.

❖ The anisotropy increases with increasing differential stress and decreases with increasing confinement.

# Anisotropic Poroelasticity

(*Biot*, 1955; *Carroll*, 1979; *Thompson and Willis*, 1991; *Cheng*, 1997)

$$B_1 = B_2 = \frac{\left[3(S_{11} + S_{12} + S_{13}) - \beta_s\right]}{C}$$

$$B_3 = \frac{\left[2S_{13} + S_{33}) - \beta_s\right]}{C}$$

❖ Transversely isotropic material with symmetry axis along the $x_3$ direction.

❖The “drained” effective compliance of the anisotropic poroelastic material is given by $S_{ij}$ (in the Voigt matrix notation)

❖ Compressibility of the solid grains is denoted by $\beta_s$ (Solid assumed to be isotropic).

❖ The storage coefficient $C$ is defined to be the volume of fluid released from unit rock volume per unit decrease in pore pressure under the condition of constant confining stresses.

# Compliance of an Anisotropically Cracked Rock

(*Kachanov*, 1993; *Sayers and Kachanov*, 1995)

$$S_{11} = S_{22} = \frac{1}{E_o} + \alpha_{11} + \beta_{1111}$$

$$S_{33} = \frac{1}{E_o} + \alpha_{33} + \beta_{3333}$$

$$S_{12} = -\frac{\nu_o}{E_o} + \frac{\beta_{1111}}{3}$$

$$S_{23} = S_{13} = -\frac{\nu_o}{E_o} + \beta_{1133}$$

❖ Transversely isotropic material with an axisymmetric distribution of microcracks.

❖ Solid matrix is assumed to be isotropic, with elastic moduli $\nu_o$ and $E_o$.

❖ Crack fabric is characterized by two tensors: $\alpha_{ij}$ and $\beta_{ijkl}$.

# Skempton Tensor of an Anisotropically Cracked Rock

$$B_1 = B_2 = \frac{32}{E_o C} \frac{(1-\nu_o^2)}{(2-\nu_o)} \left( \alpha_{11}^* + \cancel{\frac{4}{3}\beta_{1111}^*} + \cancel{\beta_{1133}^*} \right)$$

$$\Rightarrow \qquad B_t \approx \frac{32(1-\nu_o^2)}{E_o C(2-\nu_o)} \alpha_{11}^*$$

$$B_3 = \frac{32(1-\nu_o^2)}{E_o C(2-\nu_o)} \left( \alpha_{33}^* + \cancel{\beta_{3333}^*} + \cancel{2\beta_{1133}^*} \right)$$

$$\Rightarrow \qquad B_z \approx \frac{32(1-\nu_o^2)}{E_o C(2-\nu_o)} \alpha_{33}^*$$

❖ The fourth-rank crack fabric tensor can be neglected in a saturated rock under drained conditions for $\nu_o << 2$.

❖ The Skempton tensor is directly proportional to the second-rank crack fabric tensor.

# Crack Fabric Tensor and Crack Density

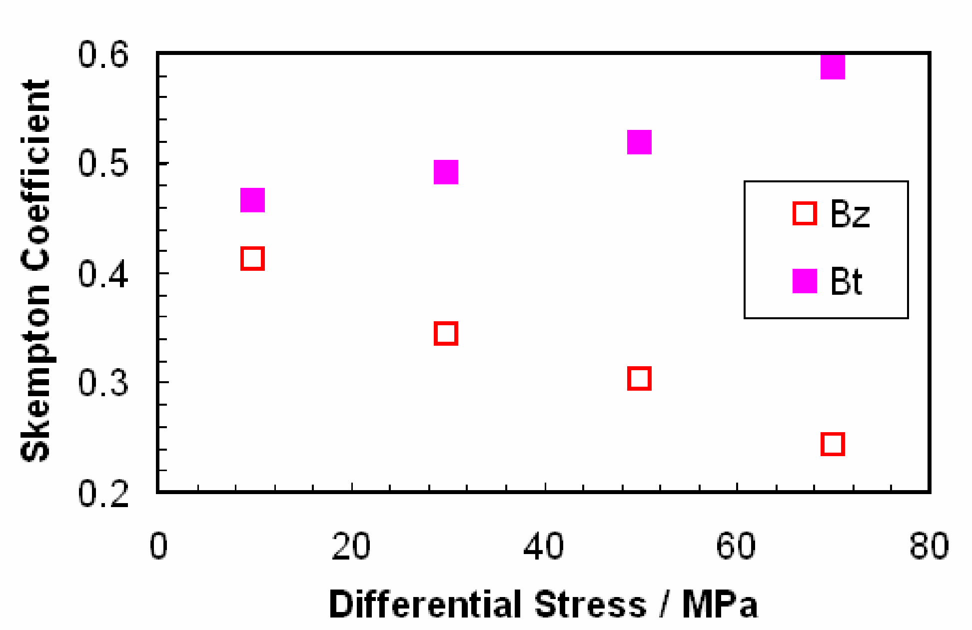
$$\alpha_{ij} = \frac{1}{V}\sum_{r=1}^{N} B_T^r n_i^r n_j^r S_r = \frac{32}{3E_o}\frac{(1-\nu_o^2)}{(2-\nu_o)}\alpha_{ij}^*$$

$$\alpha_{ij}^* = \frac{c^3}{V}\sum_{r=1}^{N} n_i^r n_j^r = \alpha_{mm}^* < n_i n_j >$$

$$\alpha_{mm}^* = \frac{N c^3}{V}$$

❖ The trace of the second-rank crack fabric tensor is identical to the scalar crack density introduced by *Walsh* (1965) and *Budiansky and O'Connell* (1976).

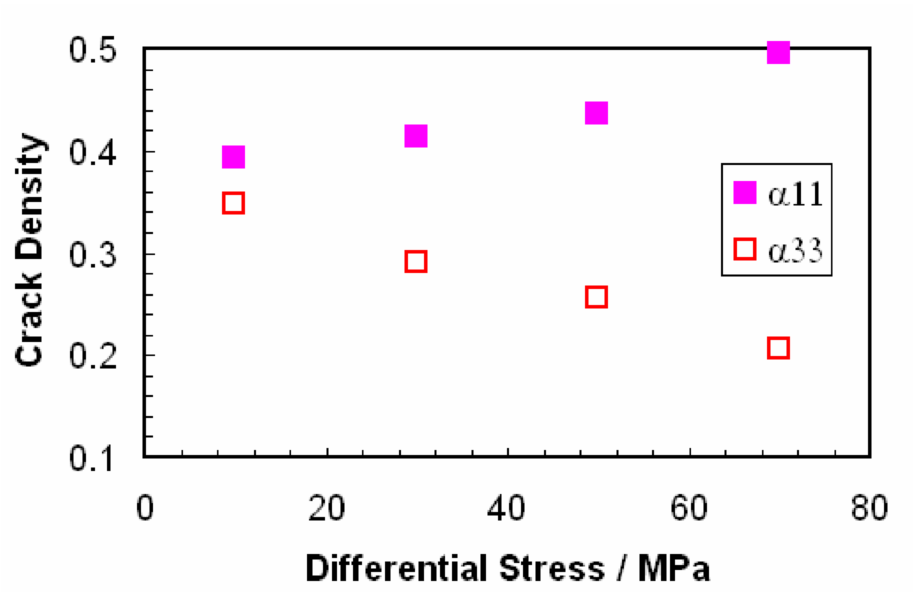
❖ Indices of the tensor are associated with directions of the crack normal and displacement discontinuity.




Berea sandstone data of *Lockner & Beeler* (2003)

Confining Pressure: 30 MPa
Pore Pressure: 7.5 MPa

$E_o$=75 GPa, $\nu_o$ =0.15
$C$ = 0.19 GPa$^{-1}$ (from direct storage measurement)




$$B_t = \frac{32\,(1-\nu_o^2)}{E_o C\,(2-\nu_o)}\alpha_{11}^*$$

$$B_z = \frac{32\,(1-\nu_o^2)}{E_o C\,(2-\nu_o)}\alpha_{33}^*$$

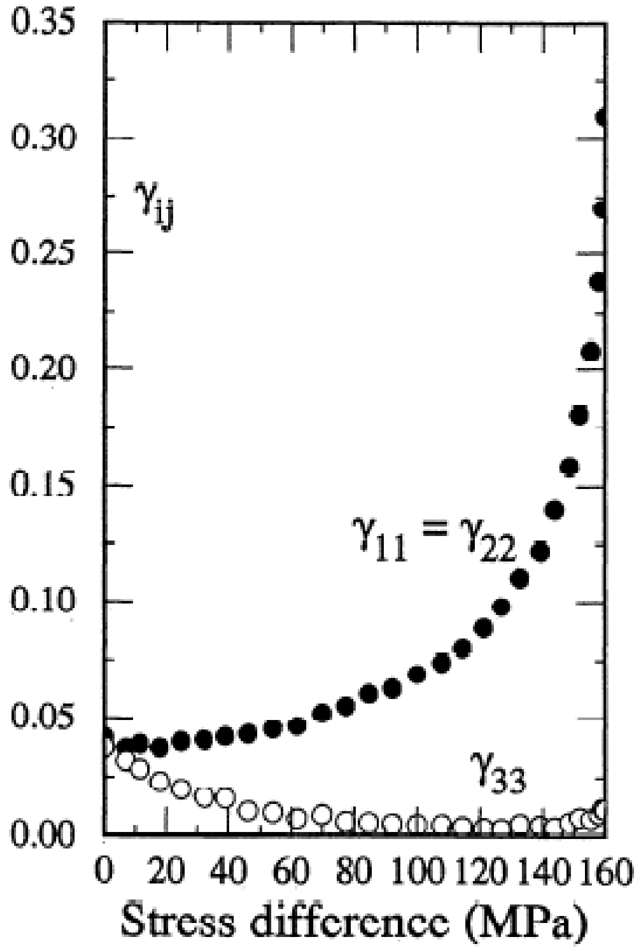



**Figure 8.** Components $\gamma_{11} = \gamma_{22}$ (solid dots) and $\gamma_{33}$ (open dots) of the crack density tensor obtained by inverting the ultrasonic measurements of *Scott et al.* [1993] on a cylinder of Berea sandstone at 20 MPa confining pressure. $Ox_3$ is parallel to the maximum compressive stress.

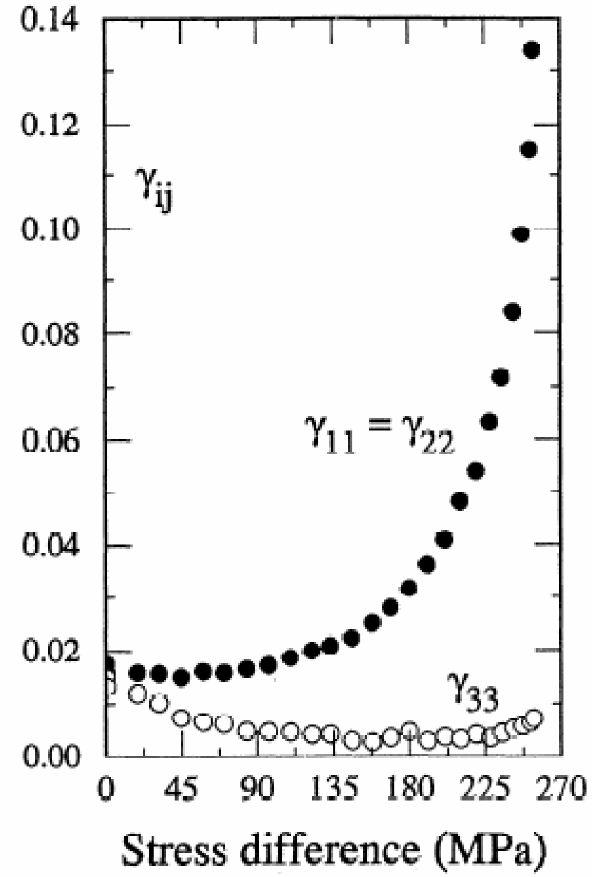
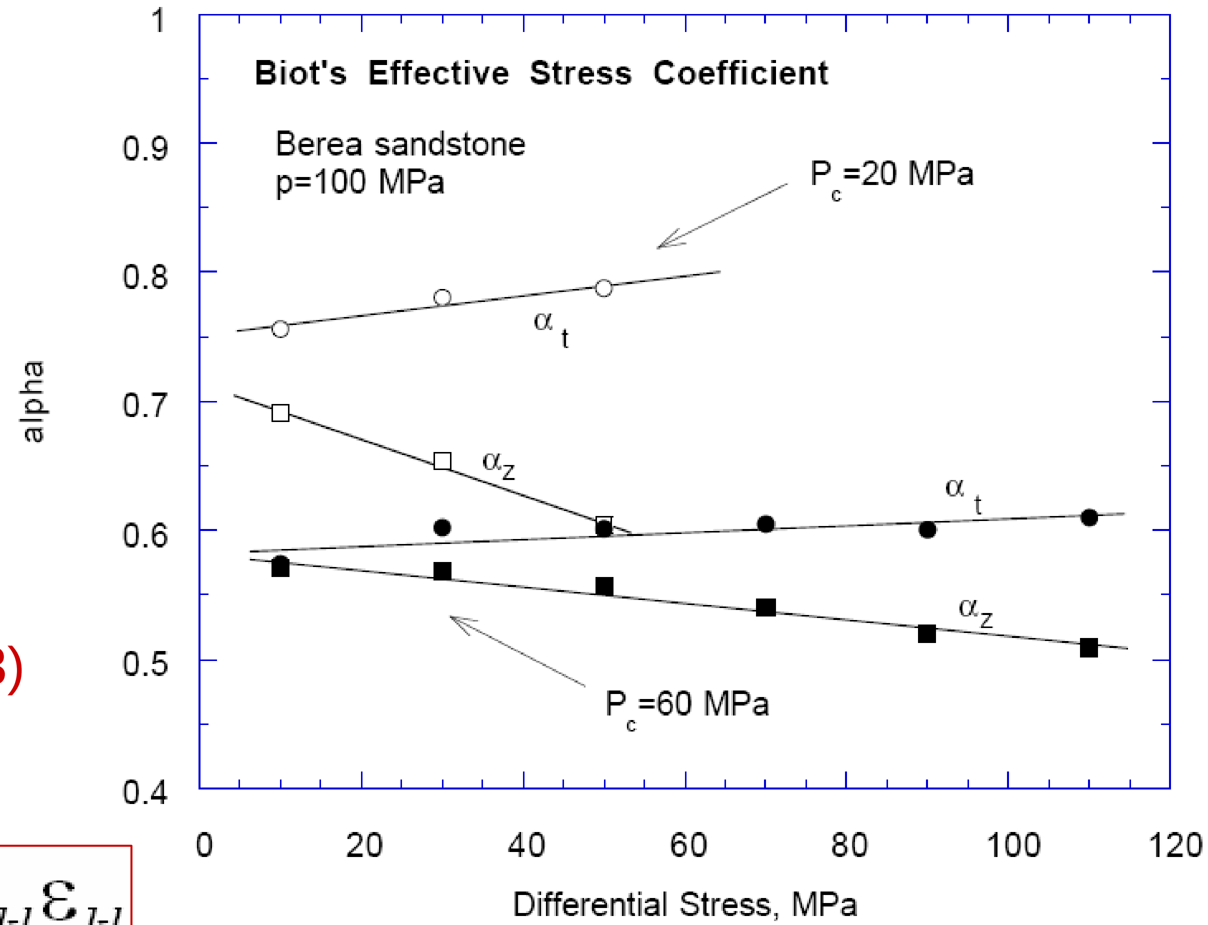



**Figure 9.** Components $\gamma_{11} = \gamma_{22}$ (solid dots) and $\gamma_{33}$ (open dots) of the crack density tensor obtained by inverting the ultrasonic measurements of *Scott et al.* [1993] on a cylinder of Berea sandstone at 60 MPa confining pressure as a function of the difference between the maximum compressive stress and the confining pressure. $Ox_3$ is parallel to the maximum compressive stress.

*Sayers & Kachanov* (1995)




*Lockner &*
*Beeler* (2003)

$$\boxed{\sigma_{ij} - \alpha_{ij} p = C_{ijkl} \varepsilon_{kl}}$$

❖ The poroelastic anisotropy is such that the Biot-Willis coefficient is larger in response to the transverse stress.

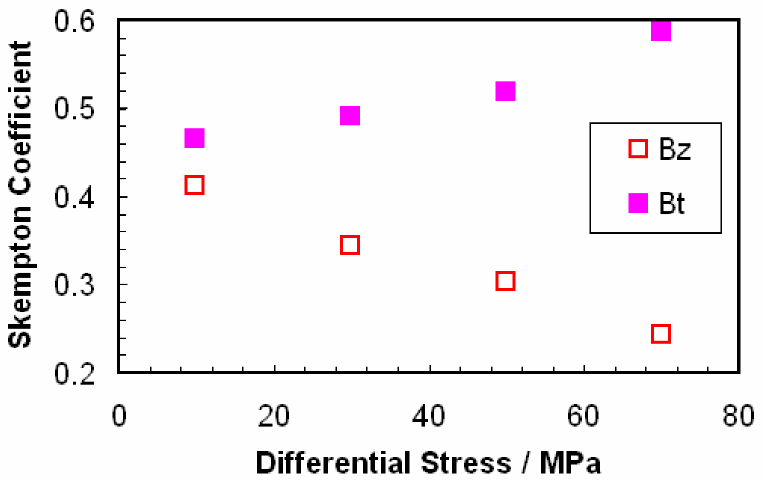
❖ The anisotropy increases with increasing differential stress and decreases with increasing confinement.

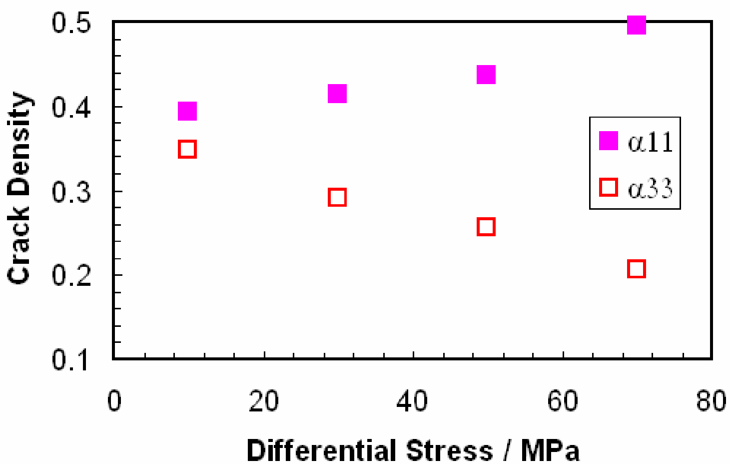
# Biot-Willis Effective Stress Coefficients of an Anisotropic Cracked Rock

$$\alpha_t = \alpha_1 = \alpha_2 = 1 - \frac{(1-2\nu_o)(1+\nu_o+g\alpha_{33}^*)}{(1+g\alpha_{33}^*)(1-\nu_o+g\alpha_{11}^*)-2\nu_o^2}$$

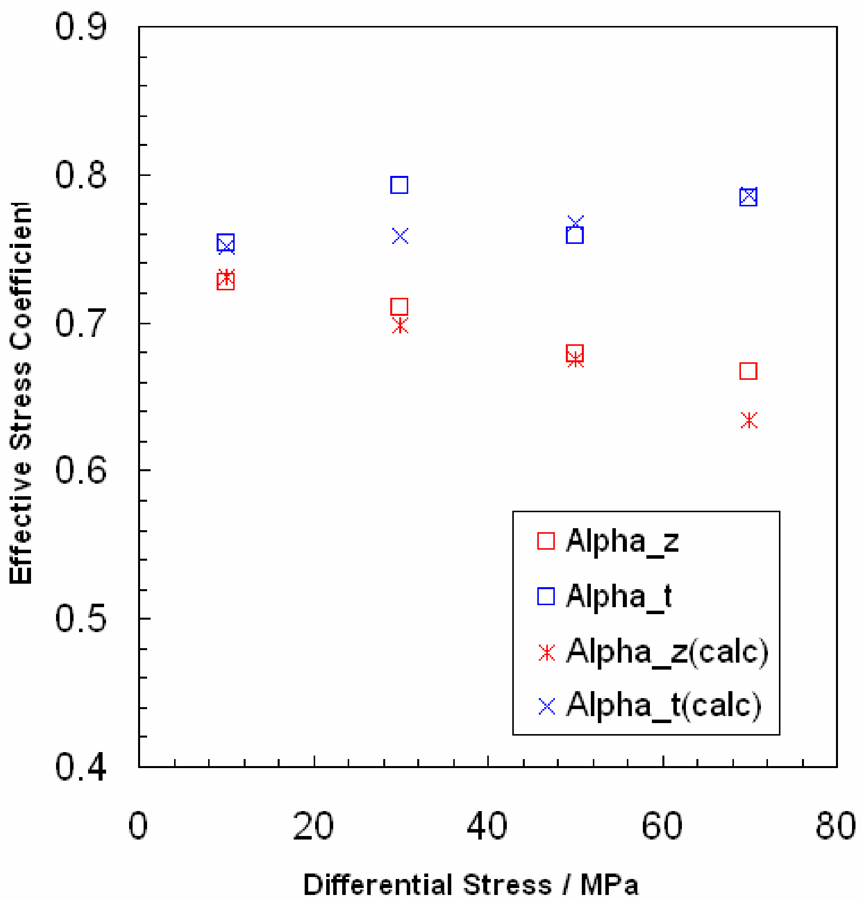
$$\alpha_z = \alpha_3 = 1 - \frac{(1-2\nu_o)(1+\nu_o+g\alpha_{11}^*)}{(1+g\alpha_{33}^*)(1-\nu_o+g\alpha_{11}^*)-2\nu_o^2}$$

with $g = \dfrac{32}{3}\dfrac{(1-\nu_o^2)}{(2-\nu_o)}$




Berea sandstone data of
*Lockner & Beeler* (2003)




Confining Pressure: 30 MPa
Pore Pressure: 7.5 MPa
$E_o$=75 GPa, $\nu_o$ =0.15
$C$ = 0.19 GPa$^{-1}$ (from direct storage measurement)

## **Skempton Tensor of an Anisotropically Cracked Rock:**
## *Dependence on porosity and fracture anisotropy*

❖ Using results of *Cheng* (1997) and *Sayers & Kachanov* (1995) the storage coefficient can be related to the crack density:

$$C = \frac{32(1-\nu_o^2)}{3E_o(2-\nu_o)}\alpha^*_{kk} + E_o\phi(\beta_f - \beta_s)$$

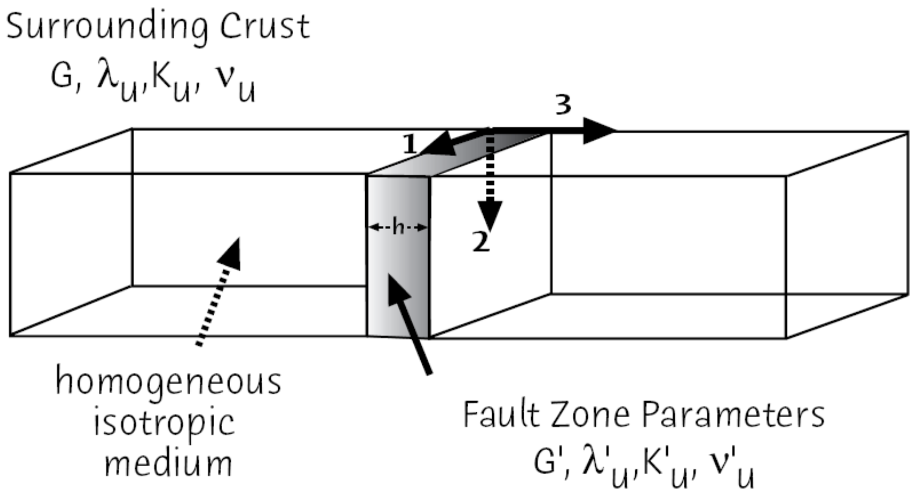
❖ Since typically $\beta_f >> \beta_s$, we can neglect $\beta_s$ and the principal components of the Skempton tensor are simply given by:

$$B_t = B_1 = B_2 = \frac{\alpha^*_{11}}{(\alpha^*_{kk}/3) + \xi\phi}$$

$$B_z = B_3 = \frac{\alpha^*_{33}}{(\alpha^*_{kk}/3) + \xi\phi}$$

with $\xi = \frac{E_o\beta_f\,(2-\nu_o)}{32\,(1-\nu_o^2)}$ (~2 for $\beta_f$=0.45 GPa$^{-1}$, $E_o$=75 GPa, $\nu_o$ =0.15)

## **Skempton Tensor of an Anisotropically Cracked Rock:**
## *Dependence on porosity and fracture anisotropy*




$$B_t = B_1 = B_2 = \frac{\alpha^*_{11}}{(\alpha^*_{kk}/3) + \xi\phi}$$

$$B_z = B_3 = \frac{\alpha^*_{33}}{(\alpha^*_{kk}/3) + \xi\phi}$$

If $\alpha^*_{33} >> \alpha^*_{11}$, then

$$B_3 = \frac{3}{1 + 3\xi\phi/\alpha^*_{33}} >> B_1$$

For an array of cracks with aspect ratio $c/a$

$$B_3 = \frac{3}{1+4\pi\xi(c/a)} \text{ and } \Delta p = \frac{\Delta\sigma_n}{1+4\pi\xi(c/a)}$$

If $c << a$, then $B_3 \rightarrow 3$ and $\Delta p \sim \Delta\sigma_n$

# Anisotropic Poroelasticity in Cracked and Granular Materials

- Under triaxial compression, appreciable stress-induced anisotropy develops in a rock, and as a result, its undrained mechanical response is anisotropic.
- Synthesizing the theoretical results of Cheng (1997) and Kachanov (1993), we derived explicit expressions for the Skempton pore pressure and Biot-Willis effective stress tensors in terms of the crack fabric tensors in a transversely isotropic material.
- Laboratory data on Berea sandstone show that the anisotropies associated with the Skempton and Biot-Willis coefficients increases with increasing differential stress and decreasing effective pressure, and accordingly the crack density is inferred to have similar anisotropic attributes.
- An analogous approach was also used to analyze laboratory data for Skempton tensor of Ottawa sand.