Theory Assignment 2

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Part 1

(Q1) Consider function $f(x) = (x - a)(x + b)$, for any constant $a, b \in R$ with the natural domain R (set of all real numbers). For each task $(T1), \dots, (T5)$ provide a short answer including justification.

> $(T1)$ Identify the unique **point** $x \in R$ for which the **slope** of the tangent line at $(x, f(x))$ **is** b.
>
> $(T2)$ Identify the unique **point** $x \in R$ for which the **slope** of the tangent line at $(x, f(x))$ **is** $-a$.
>
> $(T3)$ Identify the unique **point** $x \in R$ for which $f(x)$ reaches the **smallest** value.
>
> $(T4)$ Identify the **segment** of R in which all values of $f(x)$ are **negative**.
>
> $(T5)$ Identify a **part** of the domain R in which $f(x)$ is **convex**.

A correct solution to each task is worth **5 points** which gives the total of **25 points**.

First of all, we can simplify f(x) by $f(x) = x^2 + (b - a)x - ab$

The slope of f(x) is $f'(x) = 2x + b - a$

T1:

the slope of f(x) is b.

So, $f'(x) = 2x + b - a = b$ 5

$2x = a$. So, $x = \frac{a}{2}$.

Then the value of f(x) is $f(\frac{a}{2}) = (\frac{a}{2})^2 + (b - a)(\frac{a}{2}) - ab$

$f(\frac{a}{2}) = (\frac{a^2}{4}) + \frac{ab}{2} - \frac{2a^2}{4} - \frac{2ab}{2} = -\frac{a^2}{4} - \frac{ab}{2}$

So, the point is $(\frac{a}{2}, -\frac{a^2}{4} - \frac{ab}{2})$

T2:

the slope of f(x) is -a.

So, $f'(x) = 2x + b - a = -a$

$2x = -b$. So, $x = \frac{-b}{2}$

5

Then the value of f(x) is $f(\frac{-b}{2}) = (\frac{-b}{2})^2 + (b-a)(\frac{-b}{2}) - ab$

$f(\frac{-b}{2}) = (\frac{b^2}{4}) + \frac{ab}{2} - \frac{2b^2}{4} - \frac{2ab}{2} = -\frac{b^2}{4} - \frac{ab}{2}$

So, the point is ($\frac{-b}{2}$, $-\frac{b^2}{4} - \frac{ab}{2}$)

T3:

Since the coefficient of x^2 is positive, the parabola is U-shape, and f(x) reaches the smallest value when the slope is equal to 0.

So, $f'(x) = 2x + b - a = 0$

$2x = a - b$, $x = \frac{a-b}{2}$.

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Then the value of f(x) is $f(\frac{a-b}{2}) = (\frac{a-b}{2})^2 + (b-a)(\frac{a-b}{2}) - ab$

$f(\frac{a-b}{2}) = (\frac{a^2-2ab+b^2}{4}) + \frac{ab}{2} - \frac{b^2}{2} - \frac{a^2}{2} + \frac{ab}{2} - \frac{2ab}{2} = \frac{a^2}{4} + \frac{b^2}{4} - \frac{ab}{2} - \frac{2a^2}{4} - \frac{2b^2}{4}$

$f(\frac{a-b}{2}) = -\frac{a^2}{4} - \frac{b^2}{4} - \frac{ab}{2}$

So, the point is ($\frac{a-b}{2}$, $-\frac{a^2}{4} - \frac{b^2}{4} - \frac{ab}{2}$)

T4:

Case 1: the lowest point of f(x) is above the x-axis (the value of f(x) is always positive). In this case, we can not find any negative values of f(x). So, there is no segment of x that meets these criteria.

Case 2: the lowest point of f(x) is below the x-axis (the value of f(x) is not always positive). In this case, we can find the segment of x that makes the values of f(x) negative by finding the y-intercept of the graph.

We can do that by $f(x) = x^2 + (b-a)x - ab = 0$

And the solution of x can be solved by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, a = 1, b = (b-a), and c = -ab

So, the segment of x that makes the values of f(x) negative are from

$$\frac{-(b-a) - \sqrt{(b-a)^2 - 4(1)(-ab)}}{2(1)} \ to \ \frac{-(b-a) + \sqrt{(b-a)^2 - 4(1)(-ab)}}{2(1)}$$

$$= \frac{-b + a - \sqrt{b^2 - 2ab + a^2 + 4ab}}{2} \ to \ \frac{-b + a + \sqrt{b^2 - 2ab + a^2 + 4ab}}{2}$$

$$= \frac{-b + a - \sqrt{b^2 + 2ab + a^2}}{2} \ to \ \frac{-b + a + \sqrt{b^2 + 2ab + a^2}}{2}$$

$$= \frac{-b + a - \sqrt{(a+b)^2}}{2} \ to \ \frac{-b + a + \sqrt{(a+b)^2}}{2}$$

$$= \frac{-b + a - (a+b)}{2} \ to \ \frac{-b + a + (a+b)}{2}$$

$$= \frac{-2b}{2} \ to \ \frac{2a}{2}$$

= -b to a

4

OR

a to -b

T5:

Since the shape of the function f(x) is a U-shape parabola, all the domain R will make the graph convex (At the left side of the minimum point, the slope will increase from the negative value to 0. At the right side of the minimum point, the slope will increase from 0 to the positive value).

Also, we can calculate the second derivative of f(x) to find the rate of change of the slope.

4

$f'(x) = 2x + b - a$

So, $f''(x) = 2$ (positive value)

2

Then, we can conclude that the slope of all domain R will increase (convex).

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-1

(Q2) A **permutation** of sequence $< a_1, \dots, a_n >$ is an arbitrary arrangement of all elements of this sequence. For example, sequence $< 1,2,3 >$ has 6 different permutations $< 1,2,3 >, < 1,3,2 >, < 2,1,3 >$, $< 2,3,1 >, < 3,1,2 >$, and $< 3,2,1 >$. Coincidentally 6=3! (**factorial function**), and in general any sequence of n different elements has $n!$ permutations.

Given matrix $A[n \times n]$ is called a **permutation matrix** if the product $A \times \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$, where $\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$ is interpreted as sequence $< a_1, \dots, a_n >$, generates another vector which can be interpreted as one of the permutations of this sequence. For example, for $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, which is interpreted as sequence $< 1,2,3 >$, $A \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ which is interpreted as sequence $< 2,1,3 >$.

Note that a permutation matrix is formed of only 1s and 0s, such that each row and column contains exactly single 1.

Your first task is to create 5 permutation matrices A, B, C, D and E of size $[5 \times 5]$ which produce the following 5 permutations:

Matrix $\boldsymbol{A}$: $\boldsymbol{A} \times \begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix} = \begin{bmatrix} s \\ t \\ a \\ l \\ e \end{bmatrix}$ *(slate to stale)*

Matrix **B:** $\boldsymbol{B} \times \begin{bmatrix} s \\ t \\ a \\ l \\ e \end{bmatrix} = \begin{bmatrix} t \\ a \\ l \\ e \\ s \end{bmatrix}$ *(stale to tales)*

Matrix **C:** $\boldsymbol{C} \times \begin{bmatrix} t \\ a \\ l \\ e \\ s \end{bmatrix} = \begin{bmatrix} s \\ t \\ e \\ a \\ l \end{bmatrix}$ *(tales to steal)*

Matrix **D:** $\boldsymbol{D} \times \begin{bmatrix} s \\ t \\ e \\ a \\ l \end{bmatrix} = \begin{bmatrix} t \\ e \\ a \\ l \\ s \end{bmatrix}$ *(steal to teals)*

Matrix **E:** $\boldsymbol{E} \times \begin{bmatrix} t \\ e \\ a \\ l \\ s \end{bmatrix} = \begin{bmatrix} l \\ e \\ a \\ s \\ t \end{bmatrix}$ *(teals to least)*

A correct content of each matrix is worth **4 points** which gives the total of **20 points**.

We can easily find those metrics by using the dot product method.

Matrix A

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix} = \begin{bmatrix} s \\ t \\ a \\ l \\ e \end{bmatrix}$$

4

proof by
1(s) + 0(l) + 0(a) + 0(t) + 0(e) = s
0(s) + 0(l) + 0(a) + 1(t) + 0(e) = t
0(s) + 0(l) + 1(a) + 0(t) + 0(e) = a

0(s) + 1(l) + 0(a) + 0(t) + 0(e) = l
0(s) + 0(l) + 0(a) + 0(t) + 1(e) = e

Matrix B

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} s \\ t \\ a \\ l \\ e \end{bmatrix} = \begin{bmatrix} t \\ a \\ l \\ e \\ s \end{bmatrix}$$

4

proof by
0(s) + 1(t) + 0(a) + 0(l) + 0(e) = t
0(s) + 0(t) + 1(a) + 0(l) + 0(e) = a
0(s) + 0(t) + 0(a) + 1(l) + 0(e) = l
0(s) + 0(t) + 0(a) + 0(l) + 1(e) = e
1(s) + 0(t) + 0(a) + 0(l) + 0(e) = s

Matric C

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} t \\ a \\ l \\ e \\ s \end{bmatrix} = \begin{bmatrix} s \\ t \\ e \\ a \\ l \end{bmatrix}$$

4

proof by
0(t) + 0(a) + 0(l) + 0(e) + 1(s) = s
1(t) + 0(a) + 0(l) + 0(e) + 0(s) = t
0(t) + 0(a) + 0(l) + 1(e) + 0(s) = e
0(t) + 1(a) + 0(l) + 0(e) + 0(s) = a
0(t) + 0(a) + 1(l) + 0(e) + 0(s) = l

Matrix D

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} s \\ t \\ e \\ a \\ l \end{bmatrix} = \begin{bmatrix} t \\ e \\ a \\ l \\ s \end{bmatrix}$$

4

proof by

0(s) + 1(t) + 0(e) + 0(a) + 0(l) = t

0(s) + 0(t) + 1(e) + 0(a) + 0(l) = e

0(s) + 0(t) + 0(e) + 1(a) + 0(l) = a

0(s) + 0(t) + 0(e) + 0(a) + 1(l) = l

1(s) + 0(t) + 0(e) + 0(a) + 0(l) = s

Matrix E

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} t \\ e \\ a \\ l \\ s \end{bmatrix} = \begin{bmatrix} l \\ e \\ a \\ s \\ t \end{bmatrix}$$

4

proof by

0(t) + 0(e) + 0(a) + 1(l) + 0(s) = l

0(t) + 1(e) + 0(a) + 0(l) + 0(s) = e

0(t) + 0(e) + 1(a) + 0(l) + 0(s) = a

0(t) + 0(e) + 0(a) + 0(l) + 1(s) = s

1(t) + 0(e) + 0(a) + 0(l) + 0(s) = t

Let $\boldsymbol{F}$ be a matrix, such that $\boldsymbol{F} \times \begin{bmatrix} l \\ e \\ a \\ s \\ t \end{bmatrix} = \begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix}$.

How can we express $\boldsymbol{F}$ in terms of $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ and $\boldsymbol{E}$?

This is an extra question for **5 points.**

Matrix F

First, $\boldsymbol{E} \times \begin{bmatrix} t \\ e \\ a \\ l \\ s \end{bmatrix} = \begin{bmatrix} l \\ e \\ a \\ s \\ t \end{bmatrix}$ So, $F \text{ x } E \text{ x } \begin{bmatrix} t \\ e \\ a \\ l \\ s \end{bmatrix} = \begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix}$

Then, $\boldsymbol{D} \times \begin{bmatrix} s \\ t \\ e \\ a \\ l \end{bmatrix} = \begin{bmatrix} t \\ e \\ a \\ l \\ s \end{bmatrix}$ So, $F \text{ x } E \text{ x } D \text{ x } \begin{bmatrix} s \\ t \\ e \\ a \\ l \end{bmatrix} = \begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix}$

Then, $\boldsymbol{C} \times \begin{bmatrix} t \\ a \\ l \\ e \\ s \end{bmatrix} = \begin{bmatrix} s \\ t \\ e \\ a \\ l \end{bmatrix}$ So, $F \text{ x } E \text{ x } D \text{ x } C \text{ x } \begin{bmatrix} t \\ a \\ l \\ e \\ s \end{bmatrix} = \begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix}$

Then, $\boldsymbol{B} \times \begin{bmatrix} s \\ t \\ a \\ l \\ e \end{bmatrix} = \begin{bmatrix} t \\ a \\ l \\ e \\ s \end{bmatrix}$ So, $F \text{ x } E \text{ x } D \text{ x } C \text{ x } B \text{ x } \begin{bmatrix} s \\ t \\ a \\ l \\ e \end{bmatrix} = \begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix}$

Then, $A \times \begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix} = \begin{bmatrix} s \\ t \\ a \\ l \\ e \end{bmatrix}$ So, $F \ x \ E \ x \ D \ x \ C \ x \ B \ x \ A \ x \begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix} = \begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix}$

Then, multiply both sides of the equation with the inverse of

$\begin{bmatrix} s \\ l \\ a \\ t \\ e \end{bmatrix}$, A, B, C, D, E to get matrix F

So, we can write F in terms of A, B, C, D, and E like this:

$F = A^{-1} \ x \ B^{-1} \ x \ C^{-1} \ x \ D^{-1} \ x \ E^{-1}$

25

5

45

Part 2

Q1 (2) Compute the formulae of the **first derivatives** of the following functions and compute values of the relevant derivatives in point $x = 1$.

(a) $f(x) = x^2$-3x+2,
(b) $g(x) = \frac{1}{2-x}$,
(c) $f(x) + g(x)$,
(d) $f(x) \cdot g(x)$,
(e) $f(x)/g(x)$.

(a)

$f'(x) = 2x - 3$

$f'(1) = 2(1) - 3 =$ - 1

2

(b)

$g(x) = (2-x)^{-1},\ g'(x) = \ -(2-x)^{-2} \bullet \frac{d(2-x)}{dx} = \ -(2-x)^{-2} \bullet (-1) = (2-x)^{-2}$

$g'(1) = (2-1)^{-2} = 1^{-2} =$ 1

2

(c)

f(x) + g(x) = $x^2 - 3x + 2 + (2-x)^{-1}$

$f'(x) + g'(x) = 2x - 3 + (2-x)^{-2}$

$f'(1) + g'(1) = 2(1) - 3 + (2-1)^{-2} = \ -1 + 1 =$ 0

2

(d)

$f(x) \bullet g(x) = (x^2 - 3x + 2) \bullet (2-x)^{-1}$

$\frac{d(f(x) \bullet g(x))}{dx} = f(x)g'(x) + g(x)f'(x) = (x^2 - 3x + 2)((2-x)^{-2}) + (2-x)^{-1}(2x-3)$

$f(1)g'(1) + g(1)f'(1) = (1^2 - 3(1) + 2)((2-1)^{-2}) + (2-1)^{-1}(2(1) - 3)$

= -1

2

(e)

$f(x)/g(x) = (x^2 - 3x + 2) \cdot (2 - x)$

Use the same fomula with (d)

The derivative of $f(x)/g(x) = (x^2 - 3x + 2)\frac{d(2-x)}{dx} + (2 - x)\frac{d(x^2 - 3x + 2)}{dx}$

$= (x^2 - 3x + 2)(-1) + (2 - x)(2x - 3)$

When x = 1, the result $= (1^2 - 3(1) + 2)(-1) + (2 - 1)(2(1) - 3) =$

-1

2 10

Q2 (1) Compute the formulae of the **second derivatives** of the following functions.

(a) $f(x) = 3x^3 + 2x^2$,

(b) $g(x) = 2x^2 + 3x$,

(c) $f(x) \cdot g(x)$,

(d) $f(x)/g(x)$,

(e) What is the $(n-1)th$ derivative of x^n computed in point $x = 1$?

(a)

$f'(x) = 9x^2 + 4x$, $f''(x) = 18x + 4$

2

(b)

$g'(x) = 4x + 3$, $g''(x) = 4$

2

(c)

$f(x)g(x) = (3x^3 + 2x^2)(2x^2 + 3x) = 6x^5 + 13x^4 + 6x^3$

First derivative:

$30x^4 + 52x^3 + 18x^2$

Second derivative:

$120x^3 + 156x^2 + 36x$

2

(d)

$$f(x)/g(x) = \frac{3x^3 + 2x^2}{2x^2 + 3x} = \frac{3x^2 + 2x}{2x + 3}$$

First derivative:

Use divide rule

$$\frac{(2x+3)\frac{d(3x^2+2x)}{dx} - (3x^2+2x)\frac{d(2x+3)}{dx}}{(2x+3)^2} = \frac{(2x+3)(6x+2) - (3x^2+2x)(2)}{4x^2 + 12x + 9}$$

$$= \frac{12x^2 + 18x + 4x + 6 - 6x^2 - 4x}{4x^2 + 12x + 9} = \frac{6x^2 + 18x + 6}{4x^2 + 12x + 9}$$

Second derivative:

Use divide rule

$$\frac{(4x^2 + 12x + 9)\frac{d(6x^2+18x+6)}{dx} - (6x^2 + 18x + 6)\frac{d(4x^2+12x+9)}{dx}}{(4x^2 + 12x + 9)^2}$$

$$= \frac{(4x^2 + 12x + 9)(12x + 18) - (6x^2 + 18x + 6)(8x + 12)}{(4x^2 + 12x + 9)^2}$$

$$= \frac{48x^3+144x^2+108x+72x^2+216x+162-48x^3-144x^2-48x-72x^2-216x-72}{(4x^2 + 12x + 9)^2}$$

$$= \frac{60x + 90}{(4x^2 + 12x + 9)^2} = \frac{30(2x+3)}{(2x+3)^4} = \frac{30}{(2x+3)^3}$$

2

(e)

The first derivative of x^n is nx^{n-1}

The second derivative is $n(n-1)x^{n-2}$

So, the n-1 derivative is $n(n-1)(n-2)\cdots(2)x^1$

When x = 1, the answer is n!

2

10

Q3 (2) Determine whether the following systems of linear equations have **one**, **none** or **many solutions**.

(a) $x + y = -1;\ x + y = 0;$
(b) $x + y = 0;\ x - y = -1;$
(c) $x + y = 1;\ -x - y = -1;$
(d) $y = 1;\ x = 1;$
(e) $\sqrt{25}x + 5y = \frac{25}{\sqrt{25}};\ x + y = 1;$

(a)
The given equations contradict each other (x + y = -1, and x + y = 0). So, the equations have no solution. 2

(b)
Add two equations together, 2x = -1, So, x = -0.5
Then we can find y by y = -x = 0.5
So, the equations have one solution which is x = -0.5 and y = 0.5

2

(c)
In the second equation, when we multiply it by -1, it is the same as the first equation (the same graph). So, the equations have many solutions. 2

(d)
According to the given equations, the equations have one solution which is x = 1 and y = 1 2

(e)
The first equation can be simplified as $5x + 5y = 5$
or $x + y = 1$ which is the same as the second equation.
So, the equations have many solutions. 2

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Q4 (2) Rewrite the following products as the relevant **systems** of **linear equations**.

(a) $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \times \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} b \\ -a \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}^T \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(a)

$1(x) + 4(y) = 5$

$2(x) + 3(y) = 6$ 2

$x + 4y = 5$

$2x + 3y = 6$

(b)

$-1(y) + 0(x) = 1$

$0(y) + 1(x) = -1$

$-y = 1$ 2

$x = -1$

(c)
a(y) + b(x) = b
-b(y) + a(x) = -a

ay + bx = b
-by + ax = -a 2

(d)
0(x) + 2(y) +1(z) = 3
2(x) + 0(y) + 2(z) = 3
1(x) + 2(y) + 0(z) = 3

2y + z = 3
2x + 2z = 3 2
x + 2y = 3

(e)
The transpose matrix can be simplified as follow.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

So, the equations are
0(x) + 1(y) + 0(z) = 1
1(x) + 1(y) + 1(z) = 0
0(x) + 1(y) + 0(z) = 1

y = 1
x + y + z = 0 2
y = 1

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Q5 (2) Correct one entry in each 2x2 matrix **to satisfy** the following **equations.**

(a) $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} \times \begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}a\\-b\end{bmatrix}$

(b) $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} \times \begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$

(c) $\begin{bmatrix}\sqrt{2} & \sqrt{2}\\\sqrt{2} & \sqrt{2}\end{bmatrix} \times \begin{bmatrix}\sqrt{2}\\\sqrt{2}\end{bmatrix} = \begin{bmatrix}4\\6\end{bmatrix}$

(d) $\begin{bmatrix}0\\-1\end{bmatrix}^T \times \begin{bmatrix}2 & -1\\-1 & -2\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}^T$

(e) $\begin{bmatrix}-1 & 1 & 0\\0 & 1 & -1\\-1 & 0 & -1\end{bmatrix} \times \begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}-a+b\\b-c\\-c+a\end{bmatrix}$

(a)

According to the given matrix equations, the second row is incorrect since 0(a) + 0(b) is not equal to -b.

So, we can correct it by the following matrix equations.

$$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} \times \begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}a\\-b\end{bmatrix}$$

2

(b)

According to the given matrix equations, the second row is incorrect since 1(1) + 1(1) is not equal to 0.

So, we can correct it by the following matrix equations.

$$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} \times \begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

0

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(c)

According to the given matrix equations, the second row is incorrect since $\sqrt{2}(\sqrt{2}) + \sqrt{2}(\sqrt{2})$ is not equal to 6.

So, we can correct it by the following matrix equations.

$$\left[\begin{array}{cc} \sqrt{2} & \sqrt{2} \\ 3 & 3 \\ \hline \sqrt{2} & \sqrt{2} \end{array}\right] \times \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

D

(d)

First, we can simplify the matrix equations by calculation the transpose matrix as follow.

$$\begin{bmatrix} 0 & -1 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

D

The second column is incorrect since 0(-1) + (-1(-2)) is not equal to 0.

So, we can correct it by the following matrix equations.

$$\begin{bmatrix} 0 & -1 \end{bmatrix} x \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

(e)

According to the given matrix equations, the third row is incorrect since -1(a) + 0(b) + -1(c) is not equal to -c + a.

So, we can correct it by the following matrix equations.

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -a + b \\ b - c \\ -c + a \end{bmatrix}$$

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