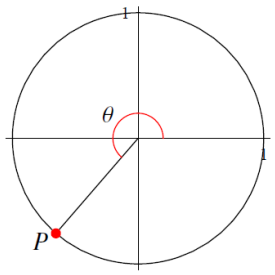
University of Liverpool
Department of Computer Science
Autumn 2021

Maths and Stats for AI and Data Science
COMP533

Theory Assignment 1 (study problems) 50/100



(Q1) Let θ be an angle defined on a unit circle, and P is the point on this circle defined by θ, see the picture to the right. Recall also that the angle determining the full circle is 360^o (360 degrees).

What are the cardinalities (sizes) of the followings 3 sets of points?

A= { P : P is defined on all $\theta = 5i \cdot 1^o$, for all integer i } **10 points**

B= { P : P is defined on all $\theta = \frac{1}{5}i \cdot 1^o$, for all integer i } **10 points**

C= { P : P is defined on all $\theta = (5 + \frac{1}{5})\, i \cdot 1^o$, for all integer i } **5* points**

Provide short justification to your answers.

(Q2) Identify the first 4 (four) values in each of the following sequences, decide whether they have limits when $n \to \infty$, and determine the relevant limits if they exist.

Sequence $A(n)$ such that

$A(n) = \frac{2n}{1^n} \cdot \frac{(-1)^n}{n}$, for any integer $n \geq 0$. **10 points**

Sequence $B(n)$ such that

$B(n) = \frac{(-1)^n}{n} \cdot \frac{(-1)^{3n}}{n}$, for any integer $n > 0$. **10 points**

Sequence $C(n)$ such that

$C(0) = \frac{1}{2}$ and $C(n) = C(n-1) + \frac{1}{2^{n+1}}$, for any integer $n > 0$. **5* points**

Hint: Use the fact that for any integer $k \geq 1$, and $x < 1$, we get

$$(1 - x) \cdot \left(x + x^2 + \cdots + x^k\right) = x - x^{k+1}.$$

Provide short justification to your answers.

Please note that questions with * require an extra effort and understanding of the problem.

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In each group of questions **Q1**, **Q2**, **Q3**, **Q4** and **Q5** provide short answers to **only one** block **(1)** or **(2)** of 5 questions. Each individual question is worth **2 points**, which gives the total of **50 points**.

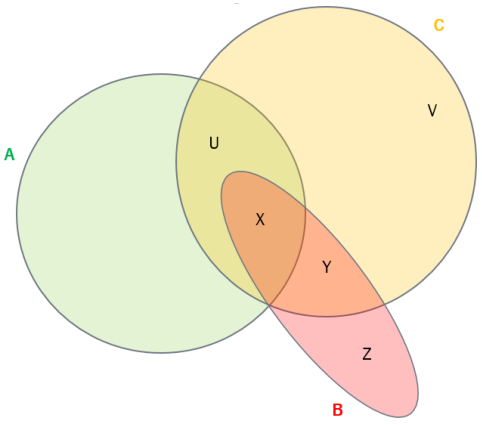
Q1 (1) Given two sets: $A = \{a, c, e\}$, and $B = \{b, d, e\}$. Compute:

(a) $A \cup B =$
(b) $A \cap B =$
(c) $A - B =$
(d) $B - A =$
(e) Do pairs (e, c) and (c, e) belong to $A \times B$?

Q1 (2) Given two finite **non-empty** sets A and B. Compare the cardinalities of the two sets when

(a) $A \subseteq B$
(b) $B \subset A$
(c) $|A \cup B| = 5$
(d) $A \cap B \neq \emptyset$
(e) $|A \times B| = 2$

Q2 (1) Given three sets A (green circular region), B (red oval region), and C (orange circular region) by the Venn diagram shown on the right. Using set operators express in terms of A, B, and C the content of the 5 (five) sets represented by regions U, V, X, Y and Z.



(a) $U =$
(b) $V =$
(c) $X =$
(d) $Y =$
(e) $Z =$

Q2 (2) Answer the following questions about function $f: A \rightarrow B$,

(a) What are the names/roles of sets A and B?
(b) If $\forall$(for every) $b \in B$ $\exists$(there is) $a \in A$ such that $f(a) = b$, function f is ……..
(c) If $\forall a, a' \in A$ such that $a \neq a'$ then also $f(a) \neq f(a')$, function f is ……..
(d) Let $f(x) = \frac{x+3}{3}$, and $A = \{6,9,12\}$. Compute the content of B.

What is the inverse of the function f from (d) defined as $f^{-1}: B \rightarrow A$?

Q3 (1) What are the natural domains of the following functions defined on real numbers?

(a) $f(x) = x + 5$
(b) $g(x) = \frac{1}{x-5}$
(c) $g(f(x))$
(d) $f(x) \cdot g(x)$
(e) $f(x)/g(x)$

Q3 (2) Answer the following questions.

(a) Which angle is greater, $\alpha = 3$ radians or $\beta = 180^o$ (degrees)?
(b) For α and β defined in (a) which value is greater: $sin\ \alpha$ or $sin\ \beta$?
(c) We know that $sin(x \pm 360^o) = sin(x)$. How do we call this phenomenon?
(d) What does letter S stand for in the CAST rule?
(e) If $sin^2(x) = \frac{1}{4}$, what is the value of $cos^2(x)$?

Q4 (1) Compute the limits of the following sequences when n $\to \infty$.

(a) $A(n) = 2 - \frac{1}{n}$
(b) $B(n) = 3 + \frac{1}{n}$
(c) $C(n) = A(n) + B(n)$
(d) $D(n) = A(n) \cdot B(n)$
(e) $E(n) = n \cdot (A(n) + B(n))$

Q4 (2) We say that function $f: R \to R$ has the limit L in infinity (when argument $x \to \infty$), if for any (as small as you want) proximity $\varepsilon > 0$ there is $x_\varepsilon \in R$, such that, for any $x > x_\varepsilon$ we have $|L - f(x)| < \varepsilon$.

(a) What is the limit of function $f(x) = \frac{1-x}{3x}$, when $x \to \infty$?
(b) What is the limit of function $g(x) = \frac{x+1}{3x}$, when $x \to \infty$?
(c) Propose x_ε for function $f(x)$, for $\varepsilon = \frac{1}{5}$.
(d) Propose x_ε for function $g(x)$, for $\varepsilon = \frac{1}{3}$.
(e) What is the limit of $g(x) - f(x)$, when $x \to \infty$?

Q5 (1) Answer the following questions about limits in points.

(a) Compute $\lim_{x \to 1+} \frac{x-1}{2x}$.
(b) Compute $\lim_{x \to 1-} \frac{2x+1}{2x}$.
(c) Compute $\lim_{x \to 0+} \frac{x-1}{2x}$.
(d) Compute $\lim_{x \to 0-} \frac{2x+1}{2x}$.
(e) Does the (two-sided) limit $\lim_{x \to 0}(\frac{x+1}{2x} - \frac{4x-1}{4x})$ exist?

Q5 (2) Answer the following questions about continuity of the given functions.

(a) Is function $f(x) = \frac{x-1}{x}$ continuous in point $x = 1$?
(b) Is function $g(x) = \frac{2x+1}{-x}$ continuous in point $x = 0$?
(c) Is function $f(x)/g(x)$ continuous in point $x = 0$?
(d) Is function $f(x) - g(x)$ continuous in the interval $(-1,2) \in R$?
(e) Is function $h(x) = \frac{1-x^2}{(x-1)}$ continuous (in its whole natural domain R)?

Make sure the solutions to the study problems and the coursework are submitted in Canvas by

Friday October 22nd 2021.