Part I. Model Reconstruction

The commons refers to a non-exclusive, public good or space accessible to every individual that is governed by self-restraint through cooperative behavior. This self-restraint refers to acting with consideration of the whole community as opposed to acting upon self-interest alone alongside a minor level of coordination among community members. In order to preserve this space, collective action must be taken by the community. For example, in *Governing the Commons*, Elinor Ostrom emphasizes the success of this action through means of defining boundaries, high levels of monitoring, and establishing tiered punishments for violations of the space. Without these coping mechanisms, there is a risk of individual community members overutilizing and abusing the space, leading to what is known as "the tragedy of the commons."

"The tragedy of the commons" refers to the outcome in which the commons have been destroyed to a point of being unable to regenerate. To illustrate this, consider how the combination of machine-produced gas emissions and high levels of deforestation have led to irreversible damage to the earth's atmosphere. The atmosphere is a specific type of commons and as a result, one individual's action—in this case manufacturing producing gas emissions—directly affects every other individual.

A model frequently used to consider the tragedy of the commons depicts a village with a common pasture used for grazing. There are 50 villagers who each have one cow grazing on the commons, which is a suitable amount for the space to be able to regenerate itself. The villagers each have a choice to keep only one cow in the commons, or they can add another cow to the pasture. By adding another cow to the pasture, a villager would assume a net increased profit by doubling their capacity without paying extra costs due to the pasture being a free space. The immediate or subsequent damage to the pasture would be inconsequential. However if every villager sequentially undergoes this course of action, there would be a 100% increase in grazing, far exceeding the capacity of the pasture and ultimately destroying it. While comparing outcomes, self-interest will be highly prevalent because each individual fears missing an opportunity regardless of whether or not it is a rational decision. This is akin to a Prisoner's Dilemma where each prisoner is afraid of not attempting to take the opportunity of potential freedom even if it isn't the best outcome. The expected utility of this scenario is seen in Table 1. The value of having one cow on the pasture is set as the base value or zero, so the benefit for one villager adding another cow is one. However, if either all villagers add cows or everyone but Villager 1 adds a cow, there is a negative externality with an expected utility of negative one to equate to the ultimate loss of their cow(s).

		All other players	(Villagers 2-50)
		Add another cow	Don't add another cow
Player 1 (Villager 1)	Add another cow	(-1, -1)	(1, 0)
	Don't add another cow	(-1, -1)	(0, 0)

Table 1. The expected utility resulting from one individual (Player 1) adding a cow or not versus all other players (the remaining 49 villagers).

The best action for preserving space is for none of the villagers to add an additional cow to the pasture. However, the decisions of the players or villagers will be determined by the presence of self-governance in the community. If there is a lack of coordination through means such as punishments for adding additional cows, a significant portion of villagers will add cows at the expense of the community and the commons. This is not an issue of cooperation but a conflict of self-interest and coordination. Incentivizing alternative options to adding a cow to the pasture only works if it exceeds the expected benefit of adding the cow. Thus, a player will almost always choose to add a cow for the potential of self-benefit because they are under the assumption that they are the only one doing it. The tragedy of the commons is an occurrence that will always arise when proper social expectations aren't upheld to sustain a common space because every player takes advantage of "free" space for self-interest. This is why many common spaces either become destroyed over time or undergo strict policy to ensure preservation. Without control or coordination of a collective group, people will act for themselves because this behavior is incentivized with no fear of punishment.

Part II. Model Extension

The "tragedy of the commons" model assumes that there are no effective measures in place to prevent abuse of a common place, and thus will always result in its destruction. However, this outcome would change drastically by implementing a punishment system as mentioned in Part I. Continuing with the example established in Part I, the punishment system implemented will focus on the number of cows each villager will be able to have in the pasture. This system has multiple potential outcomes; if the cost of the punishment, C, is less than the benefit of adding an additional cow, B, it will be ineffective. If C equals B the outcome will most likely be to not add a cow because there is no incentive to put in the extra effort. An extra cow that produces a benefit equivalent to the cost still has external costs to take care of it as well as unnecessary effort to acquire it. However, if C is greater than B, everyone will be incentivized to *not* add a cow because it will only negatively affect them. These outcomes can be seen in *Figure 1*.

Benefits (B) vs Cost (C)	Player (Villager) Response	
B - C = >0	The player will add a cow	
B - C = 0	The player most likely will not add a cow	
B - C = <0	The player will not add a cow	

Figure 1. A comparison of cost or loss to the benefit of adding an additional cow to the pasture. A positive value (greater than zero) is required for a player to be motivated to take an action, in this case adding the cow. Otherwise they will be unlikely to take action due to a lack of or net loss from adding the cow.

Thus, in order to change the outcome of the tragedy of the commons model, the punishment system needs to have a cost that exceeds the total benefit of taking an action (adding an additional cow). One potential example of this could be to bar access to the common pasture for any villager (or player) if they violate a limit of 1 cow on the space. This would result in the villagers not only losing out on their additional cow, but they would also be falling below the base value of having one cow already on the pasture. Thus the expected utilities would change as well; if Player 1 does not add a cow they would maintain their present benefits from their single cow without risking any loss. The collective punishment system established by the community plays an important role in these adjustments; if the punishment is too small or there is a lack of coordination to uphold it, Player 1 will continue to benefit from adding another cow (Benefit of +1) at the cost of the community and commons. If the system is too harsh and punishes Player 1 regardless of them not adding a cow, Player 1 would be negatively impacted. However, in the ideal scenario of proper levels of punishment being upheld by the community, they would remove access to the commons only at the expense of the violator (Player 1). These values can be seen in Table 2, noting that the highlighted sections are the ideal outcomes with this addition of punishment. The non-highlighted sections are still possible with a punishment system in place, but these are extremely unlikely without other changes being made to this system and model (i.e. corruption).

		Collective Punishment	
		Do Nothing	Remove Access to the Commons
Player 1 (Villager 1)	Add another cow	(1, -1) [B > C]	(-1, 0) [C > B]
	Don't add another cow	(0, 0) [B = C]	(-1, 0) [C > 0]

Table 2. The expected utilities of Player 1 taking action (adding a cow or not) with different systems of punishment. In this ideal scenario, the punishment should not be taken only if Player 1 does not do anything and it should only be applied if they do violate the agreement of placing only one cow on the pasture.