Recurrence for merge sort

$$T(n) = \begin{cases} \Theta(1) \text{ if } n = 1; \\ 2T(n/2) + \Theta(n) \text{ if } n > 1. \end{cases}$$

- We shall usually omit stating the base case when $T(n) = \Theta(1)$ for sufficiently small n, but only when it has no effect on the asymptotic solution to the recurrence.

Recursion tree

Solve $T(n) = 2T(n/2) + cn$, where $c > 0$ is constant.

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$T(n)$

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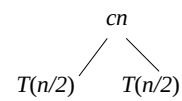

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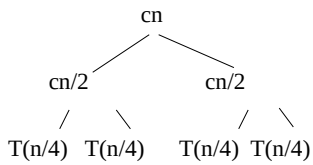

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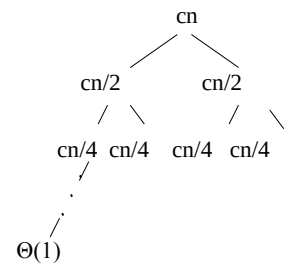

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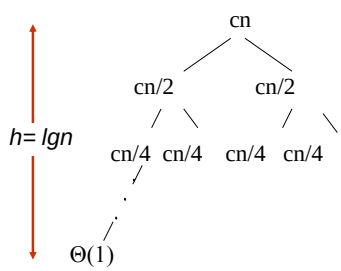

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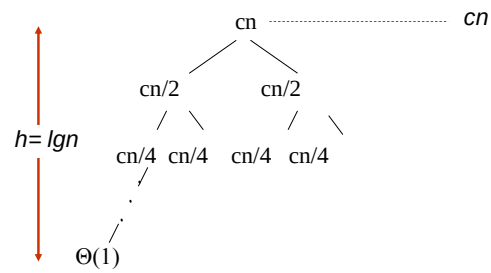

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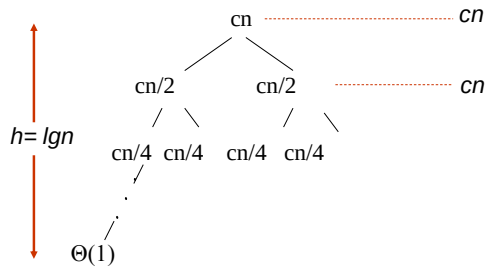

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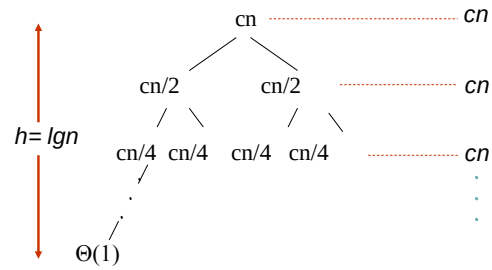

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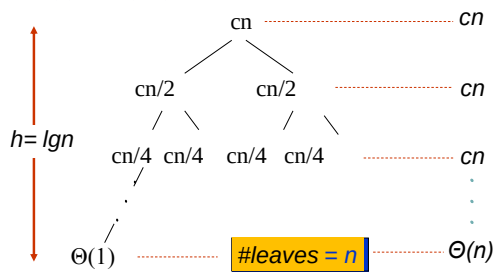

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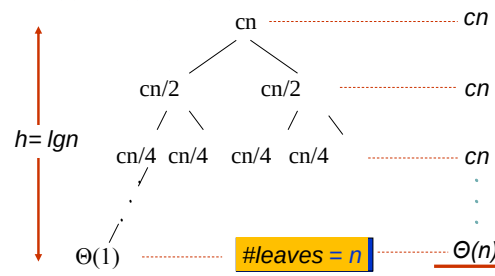

Total= Θ(n lg n)

Conclusions

- $\Theta(n \lg n)$ grows more slowly than $\Theta(n^2)$.
- Therefore, merge sort asymptotically beats insertion sort in the worst case.
- In practice, merge sort beats insertion sort for $n > 30$ or so.
- Go test it out for yourself! Try it out with random, ascending, and descending inputs
- https://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html
- ***https://www.toptal.com/developers/sorting-algorithms***

L2.13