Depth-First Search

- *Depth-first search* is another strategy for exploring a graph
  - Explore "deeper" in the graph whenever possible
  - Edges are explored out of the most recently discovered vertex v that still has unexplored edges
  - When all of v's edges have been explored, backtrack to the vertex from which v was discovered

Depth-first Search (DFS)

- Explore edges out of the most recently discovered vertex v.
- When all edges of v have been explored, backtrack to explore other edges leaving the vertex from which v was discovered (its *predecessor*).
- "Search as deep as possible first."
- Continue until all vertices reachable from the original source are discovered.
- If any undiscovered vertices remain, then one of them is chosen as a new source and search is repeated from that source.

Depth-first Search

- **Input:** $G = (V, E)$, directed or undirected. No source vertex given!
- **Output:**
  - 2 **timestamps** on each vertex. Integers between 1 and 2|V|.
    - $d[v]$ = ***discovery time*** (v turns from white to gray)
    - $f[v]$ = ***finishing time*** (v turns from gray to black)
  - $\pi[v]$: predecessor of $v = u$, such that v was discovered during the scan of u's adjacency list.
- Uses the same coloring scheme for vertices as BFS.

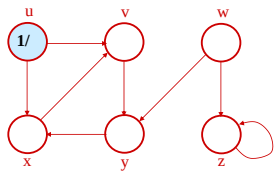
Pseudo-code

```
DFS(G)
1. for each vertex u ∈ V[G]
2.     do color[u] ← white
3.        π[u] ← NIL
4. time ← 0
5. for each vertex u ∈ V[G]
6.     do if color[u] = white
7.           then DFS-Visit(u)
```

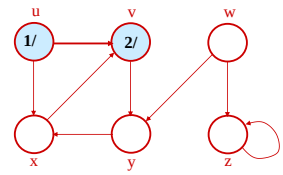
Uses a global timestamp **time**.

```
DFS-Visit(u)
1.  color[u] ← GRAY  ∇ White vertex u has been discovered
2.  time ← time + 1
3.  d[u] ← time
4.  for each v ∈ Adj[u]
5.      do if color[v] = WHITE
6.            then π[v] ← u
7.                 DFS-Visit(v)
8.  color[u] ← BLACK   ∇ Blacken u; it is finished.
9.  f[u] ← time ← time + 1
```

Example (DFS)



Example (DFS)



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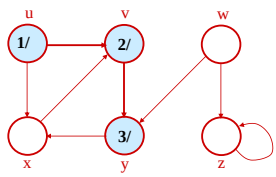

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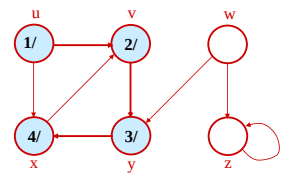

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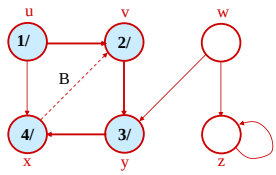

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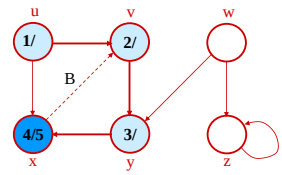

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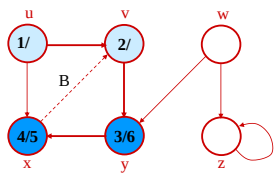
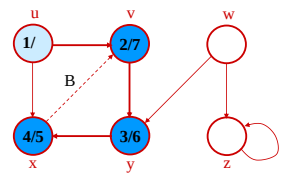
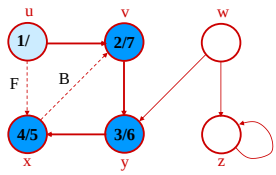


Example (DFS)



Example (DFS)



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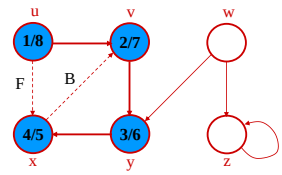

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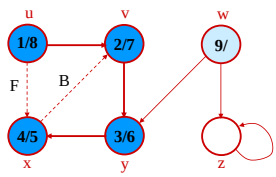

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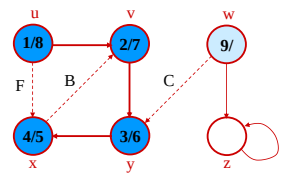

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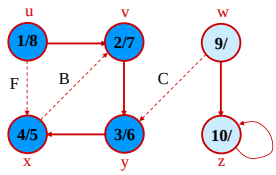
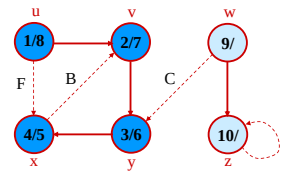
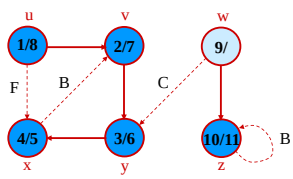


Example (DFS)



Example (DFS)



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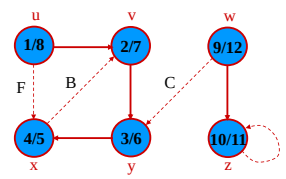
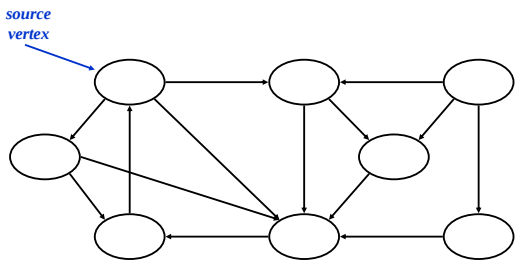
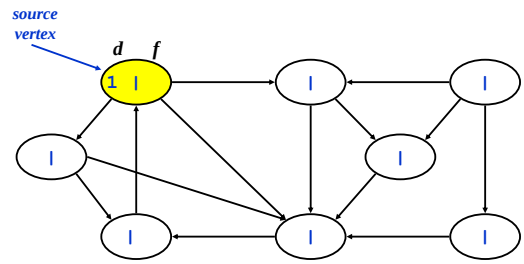


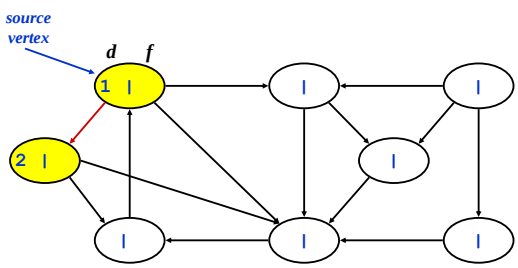
Another DFS Example



DFS Example



DFS Example



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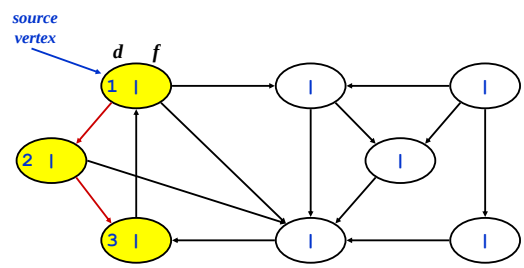

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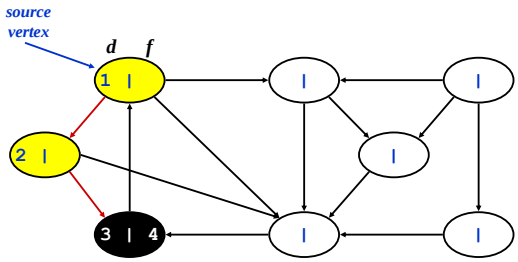

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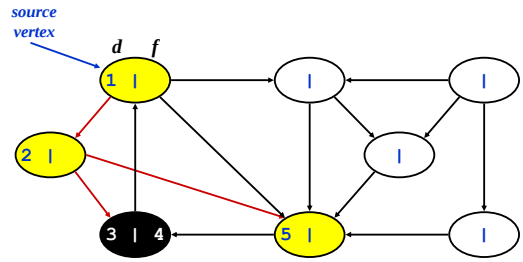

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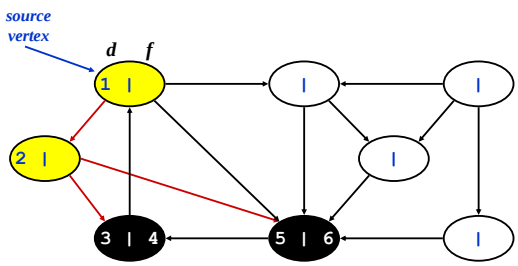

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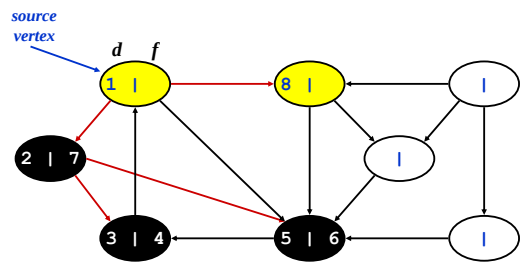

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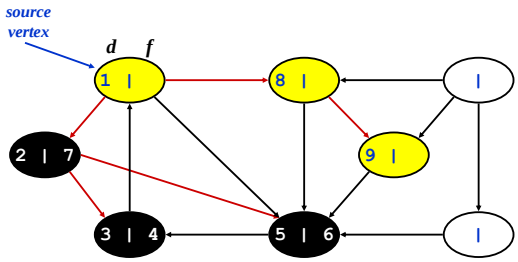

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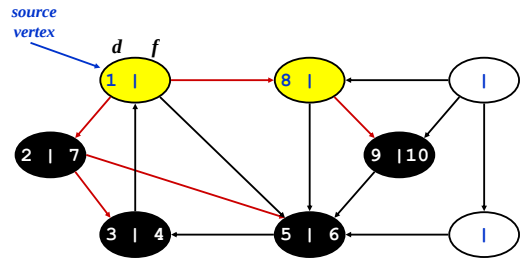

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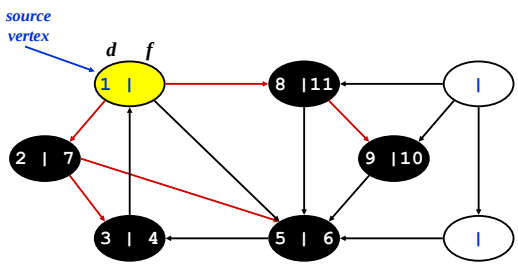

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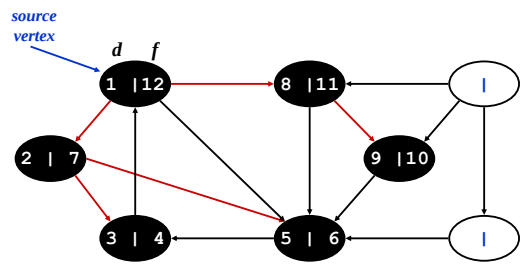

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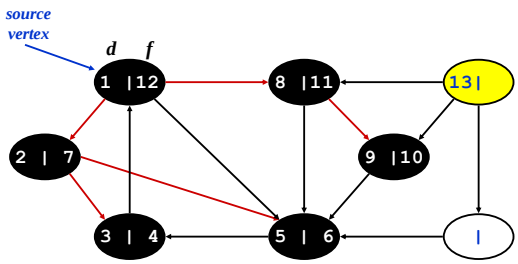

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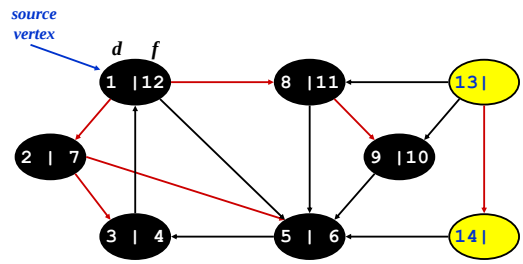

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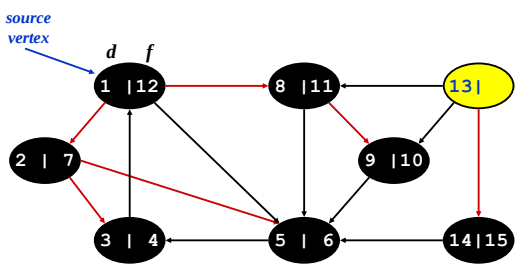
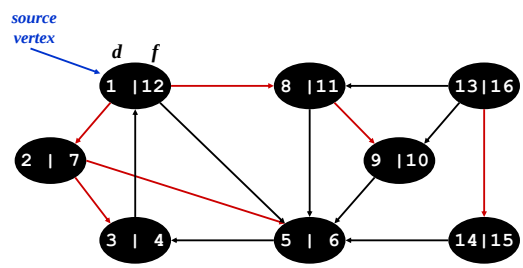


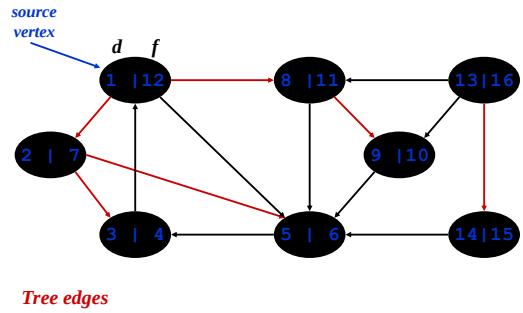
DFS Example



DFS: Kinds of edges

- DFS introduces an important distinction among edges in the original graph:
  - *Tree edge*: encounter new (white) vertex
    - The tree edges form a spanning forest
    - *Can tree edges form cycles? Why or why not?*

DFS Example

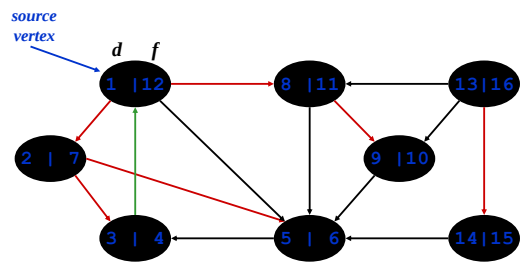


Tree edges

DFS: Kinds of edges

- DFS introduces an important distinction among edges in the original graph:
  - *Tree edge*: encounter new (white) vertex
  - *Back edge*: from descendent to ancestor
    - Encounter a yellow vertex (yellow to yellow)

DFS Example

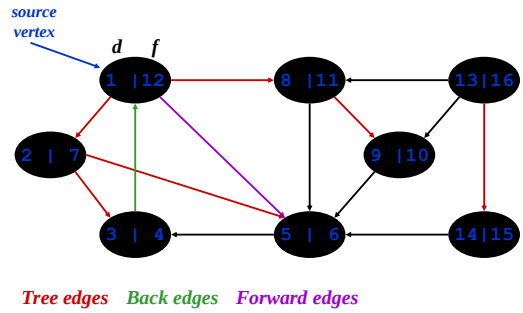


Tree edges *Back edges*

DFS: Kinds of edges

- DFS introduces an important distinction among edges in the original graph:
  - *Tree edge*: encounter new (white) vertex
  - *Back edge*: from descendent to ancestor
  - *Forward edge*: from ancestor to descendent
    - Not a tree edge, though
    - From yellow node to black node

DFS Example

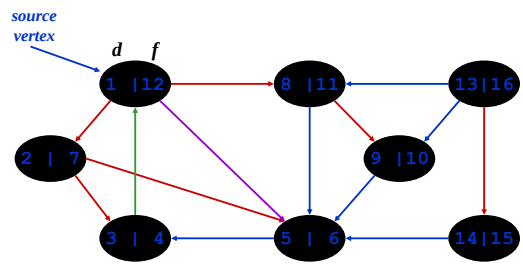


Tree edges *Back edges* *Forward edges*

DFS: Kinds of edges

- DFS introduces an important distinction among edges in the original graph:
  - *Tree edge*: encounter new (white) vertex
  - *Back edge*: from descendent to ancestor
  - *Forward edge*: from ancestor to descendent
  - *Cross edge*: between a tree or subtrees
    - From a yellow node to a black node

DFS Example



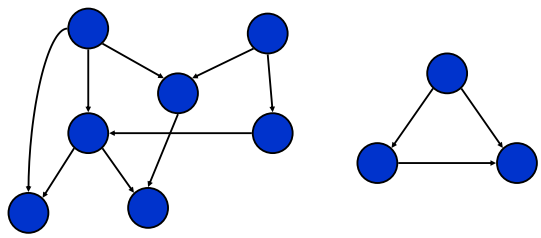
Tree edges *Back edges* *Forward edges* *Cross edges*

DFS: Kinds of edges

- DFS introduces an important distinction among edges in the original graph:
  - *Tree edge*: encounter new (white) vertex
  - *Back edge*: from descendent to ancestor
  - *Forward edge*: from ancestor to descendent
  - *Cross edge*: between a tree or subtrees
- Note: tree & back edges are important; most algorithms don't distinguish forward & cross

Directed Acyclic Graphs

- A *directed acyclic graph* or *DAG* is a directed graph with no directed cycles:



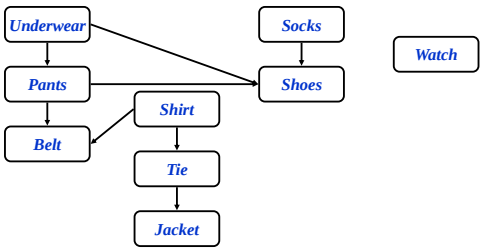
DFS and DAGs

- Argue that a directed graph G is acyclic iff a DFS of G yields no back edges:
  - Forward: if G is acyclic, will be no back edges
    - Trivial: a back edge implies a cycle
  - Backward: if no back edges, G is acyclic
    - Argue contrapositive: G has a cycle $\Rightarrow \exists$ a back edge
      - Let v be the vertex on the cycle first discovered, and u be the predecessor of v on the cycle
      - When v discovered, whole cycle is white
      - Must visit everything reachable from v before returning from DFS-Visit()
      - So path from u→v is yellow→yellow, thus (u, v) is a back edge

Topological Sort

- *Topological sort* of a DAG:
  - Linear ordering of all vertices in graph G such that vertex u comes before vertex v if edge $(u, v) \in$ G
- Real-world example: getting dressed

Getting Dressed



Getting Dressed